TITLE: Effect of a Frequency Perturbation in a Chain of Syntonized Transparent Clocks

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Abstract

This contribution analyzes the propagation of error in syntonized frequency in a chain of syntonized transparent clocks due to a sinusoidal frequency perturbation at the first clock downstream from the master clock. It is shown that if residence time is much less than frequency update interval a very large number of hops is required before the effect of a frequency offset perturbation on downstream accumulates to the level of the perturbation (e.g., for residence time on the order of 0.001 of the frequency update interval, the frequency offset amplitude due to the applied perturbation after 100 hops is 0.2% of the applied perturbation amplitude). For AVB networks, where residence time could be as large as one-tenth the frequency update interval, the frequency offset accumulates to the level of the perturbation after 11 hops. After 7 hops, the frequency offset is approximately half the level of the applied perturbation.

1 Introduction

Recent discussion on the P1588 reflector [1], [2] has raised concerns that a chain of syntonized transparent clocks may exhibit excessive jitter amplification and accumulation due to gain peaking. Specifically, it is stated in [1] that the jitter accumulation may be excessive when the period of a frequency perturbation is on the order of twice the frequency measurement interval. It is added in [2] that the accumulation occurs due to effective gain peaking in the syntonization mechanism.

It is well-known that large jitter and wander accumulation in a chain of phase-locked loops may occur due to excessive gain peaking. This effect was analyzed in [3] for the case of a chain of digital regenerators. The transfer function and frequency response for such a chain was computed, and the frequency response was integrated analytically to obtain the mean-square jitter as a function of bandwidth and gain peaking (the integration was necessary because the jitter generation was a random process). The regenerator model of [3] was linear, second-order with 40 dB/decade roll-off; this model reflected the regenerators commonly used in PDH telecommuncations networks at the time. The effect was subsequently analyzed for SDH, SONET, and OTN networks for phase-locked-loop (PLL) based regenerators, using linear, second-order models with 20 dB/decade roll-off. The analysis for OTN is documented in [4]. This analysis uses the same techniques as that in [3], i.e., the frequency response for the chain of second-order, linear filters is computed and integrated over frequency to obtain mean-square jitter. The analysis of [4] shows that, with 0.1 dB of gain peaking, the jitter network limits for PDH, SDH/SONET, and OTN are met for a chain of 50 regenerators (the OTN regenerator hypothetical reference model consists of a chain of 50 regenerators; see Appendix III of [4]). The analysis assumes that the regenerators also meet the respective jitter transfer and generation requirements given in [4]. As a result of the analyses of [3] and [4] and other similar analyses, the gain peaking for regenerators and clocks used in telecommuncations networks is limited to 0.1 dB.

The analyses of [3] and [4] assume a chain of 2nd order PLLs, with either 40 dB/decade roll-off [3] or 20 dB/decade roll-off [4]. Such PLLs have inherent gain peaking. However, the syntonization process for a transparent clock (TC) described in section 12.1.2 of [5] is not a second-order PLL. In particular, there is no apparent gain peaking present in the syntonization process. In addition, the quantity of interest in a chain of TCs is the frequency offset relative to the master clock at the beginning of the chain, and how this frequency offset varies with the number of hops. The primarey uses of this frequency are for measurement of residence time and link propagation time. The purpose of the present contribution is to explicitly evaluate the accumulation of frequency offset error, if any, that occurs in a chain of syntonized TCs.

The contribution is organized as follows. A difference equation for the frequency offset relative to the master clock is derived in section 2. This is actually a partial difference equation, i.e., its independent variables are discrete time index and hop number index. The stability of this equation as a function of hop number is considered in section 3. Conclusions are given in section 4. References are contained in section 5.

2 Transfer function for the syntonization of a TC

Consider a chain of TCs timed by a master at the beginning of the chain. The master is labeled node 0, and the TCs are labeled nodes 1 through N, respectively. We assume each TC performs a frequency update every M^{th} synch interval on receipt of the Sync and Follow_Up message for that synch interval, and that the frequency update is done before the measurement of residence time (i.e., residence time is measured using the new frequency rather than the old). We assume the master frequency is perfect, and neglect phase measurement granularity. Define the following notation:

i = index of Sync message sent for which a frequency update is performed (if a frequency update is performed every M Synch messages, then this is actually the Mi^{th} Sync message; Sync messages for which no frequency update is performed need not be considered in this analysis). The first Sync message is labeled $i = 0$.

k = index of node number. The master clock at the beginning of the chain is labeled $k = 0$; the TCs are labeled $1, 2, ..., N$.

$\nu_{k,i}$ = actual frequency offset of TC k relative to the master clock, during the frequency update interval between the i^{th} and $(i+1)^{st}$ Sync messages

$\mu_{k,i}$ = measured frequency offset of TC k relative to the master clock, when the i^{th} Sync and Follow_Up messages arrive. Note that if the measurement process were perfect, then the measured value would be equal to the actual frequency offset during the previous frequency update interval, i.e., $\mu_{k,i} = \nu_{k,i}$. However, in general the measurement process is imperfect and this is not true.

$r_{k,i}$ = measured residence time of Sync message i at TC k. Note that this will differ from ideal residence time, relative to the master clock, due to the frequency offset of TC k.

T_r = ideal residence time, relative to the perfect master.

T_I = frequency update interval, relative to the perfect master.

$m_{k,i}$ = corrected master time at TC k when i^{th} Sync message is received (i.e., corrected for the residence times of TCs $1, 2, ..., k$-1). Note that $m_{0,i}$ is the time the i^{th} Sync message is actually sent, relative to the master clock.

The corrected master time is related to the time the Sync message is sent by the master clock and the residence times in the upstream TCs by

$$m_{k,i} = m_{0,i} + \sum_{j=1}^{k-1} r_{k,i} \; . \tag{2-1}$$

The residence time at TC k is related to the frequency offset of TC k by

$$r_{k,i} = (1+\nu_{k,i})T_r \; . \tag{2-2}$$

Note that $\nu_{k,i}$ appears in Eq. (2-2) rather than $\nu_{k,i\text{-}1}$ because it is assumed the residence time is computed using the updated frequency. Combining Eqs. (2-1) and (2-2) produces

$$m_{k,i} = m_{0,i} + kT_r + T_r \sum_{j=1}^{k-1} \nu_{k,i} \; . \tag{2-3}$$

The measured frequency offset of TC k is equal to the ratio of the elapsed local time during a frequency update interval to the elapsed corrected master time during the same frequency update interval, minus one. For the frequency offset measured at the arrival of the i^{th} Sync message, this may be written

$$\begin{aligned}
\mu_{k,i} &= \frac{T_I(1+\nu_{k,i-1})}{m_{0,i} + kT_r + T_r \sum_{j=1}^{k-1} \nu_{j,i} - \left[m_{0,i-1} + kT_r + T_r \sum_{j=1}^{k-1} \nu_{j,i-1} \right]} - 1 \\
&= \frac{T_I(1+\nu_{k,i-1})}{T_I + T_r \sum_{j=1}^{k-1} \left(\nu_{j,i} - \nu_{j,i-1} \right)} - 1 \\
&= \frac{T_I \nu_{k,i-1} - T_r \sum_{j=1}^{k-1} \left(\nu_{j,i} - \nu_{j,i-1} \right)}{T_I + T_r \sum_{j=1}^{k-1} \left(\nu_{j,i} - \nu_{j,i-1} \right)} \\
&= \frac{\nu_{k,i-1} - \frac{T_r}{T_I} \sum_{j=1}^{k-1} \left(\nu_{j,i} - \nu_{j,i-1} \right)}{1 + \frac{T_r}{T_I} \sum_{j=1}^{k-1} \left(\nu_{j,i} - \nu_{j,i-1} \right)}
\end{aligned} \; . \tag{2-4}$$

The frequency is adjusted by reducing the current frequency by an amount equal to the measured frequency offset of the TC relative to the master. Then, the new actual frequency offset is related to the old actual frequency offset by

$$\nu_{k,i} = \nu_{k,i-1} - \mu_{k,i} \,. \tag{2-5}$$

Inserting Eq. (2-4) into Eq. (2-5) produces

$$\begin{aligned}
\nu_{k,i} &= \nu_{k,i-1} - \frac{\nu_{k,i-1} - \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)}{1 + \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)} \\
&= \frac{\nu_{k,i-1} - \nu_{k,i-1} + \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)}{1 + \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)} + O(\nu_{k,i}^2) \\
&= \left[\frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)\right]\left[1 - \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)\right] + O(\nu_{k,i}^2) \\
&= \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right) + O(\nu_{k,i}^2)
\end{aligned} \,. \tag{2-6}$$

Now, the frequency offsets are small compared to 1, i.e., $|\nu_{k,i}| << 1$. Then the terms of $O(\nu_{k,i}{}^2)$ are small compared to the terms of $O(\nu_{k,i})$, and Eq. (2-6) may be approximated to first order in $\nu_{k,i}$ as

$$\nu_{k,i} = \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right). \tag{2-7}$$

Eq. (2-7) holds for $k \ge 2$. For $k > 2$, it may be converted to a recursive form by rewriting it for index k-1 and subtracting the result from Eq. (2-7). The result of this is

$$\begin{aligned}
\nu_{k,i} - \nu_{k-1,i} &= \frac{T_r}{T_I}\sum_{j=1}^{k-1}(\nu_{j,i} - \nu_{j,i-1}) - \frac{T_r}{T_I}\sum_{j=1}^{k-2}(\nu_{j,i} - \nu_{j,i-1}) \\
&= \frac{T_r}{T_I}\left(\nu_{k-1,i} - \nu_{k-1,i-1}\right) \qquad k > 2
\end{aligned} \,. \tag{2-8}$$

An equation for $k = 2$ is obtained by simply substituting $k = 2$ in Eq. (2-7). Then, Eq. (2-7) is equivalent to

$$\begin{aligned}
\nu_{2,i} &= \frac{T_r}{T_I}\left(\nu_{1,i} - \nu_{1,i-1}\right) \\
\nu_{k,i} &= \nu_{k-1,i}\left(1 + \frac{T_r}{T_I}\right) - \frac{T_r}{T_I}\nu_{k-1,i-1} \qquad k > 2
\end{aligned} \,. \tag{2-9}$$

3 Stability Analysis

We are interested in how a small frequency perturbation applied at the first TC downstream from the master ($k = 1$) propagates through the successive TC. The master frequency is assumed perfect. Then, we set

$$\nu_{0,i} = 0$$
$$\nu_{1,i} = a^i \quad \text{with } a = Ae^{j\omega} \tag{3-1}$$

where ω is the discrete frequency, A is the amplitude of the perturbation, and $j = \sqrt{-1}$ (i.e., in this section we do not use j as an index). The first of Eq. (2-9) may be rewritten

$$\nu_{2,i} = A\frac{T_r}{T_I}e^{j\omega i}(1 - e^{-j\omega}) . \tag{3-2}$$

As expected for linear systems, $\nu_{2,i}$ is equal to $\nu_{1,i}$ multiplied by a complex frequency response $H_1(e^{j\omega})$. This means that the resulting frequency offset at any TC is sinusoidal with a respective phase (as expected), and we may use the second of Eq. (2-9) to compute the frequency response between TCs k and $k+1$. To do this, take the z-transform of the second of Eq. (2-9) and then set $z = e^{j\omega}$. Taking the z-transform produces

$$N_k(z) = N_{k-1}(z)\left(1 + \frac{T_r}{T_I} - \frac{T_r}{T_I}z^{-1}\right), \tag{3-3}$$

where $N_k(z)$ is the z-transform of $\nu_{k,i}$, and the result for the z-transform of a shifted sequence has been used (i.e., the the z-transform of a sequence obtained by replacing the index by the index minus one is equal to z^{-1} multiplied by the z-transform of the unshifted sequence). The complex freqeuncy response between TCs k-1 and k, with $k \geq 3$, is

$$H(e^{j\omega}) = 1 + \frac{T_r}{T_I} - \frac{T_r}{T_I}e^{-j\omega} . \tag{3-4}$$

The frequency response at TC k is obtained by multiplying Eq. (3-2) by Eq. (3-4) raised to the k-2 power (the power is k-2 because, in getting from TC 2 to TC k, k-2 hops are traversed). The result is

$$\nu_{k,i} = A\frac{T_r}{T_I}e^{j\omega i}(1 - e^{-j\omega})\left(1 + \frac{T_r}{T_I} - \frac{T_r}{T_I}e^{-j\omega}\right)^{k-2} . \tag{3-5}$$

The amplitude of the response may be obtained, as a function of frequency, by computing the complex amplitude of Eq. (3-5). This is done omitting the constant A as that is the amplitude of the input perturbation (and what is of interest here is how the perturbation grows or decays with the number of hops). The result is

$$\left|N_k(e^{j\omega})\right|^2 = \left(\frac{T_r}{T_I}\right)^2 [2(1 - \cos\omega)]\left[1 + \frac{2T_r}{T_I}\left(1 + \frac{T_r}{T_I}\right)(1 - \cos\omega)\right]^{k-2} \qquad k \geq 2 , \tag{3-6}$$

where $N_k(e^{j\omega})$ is the frequency response between the input perturbation and TC k.

The minimum value of the frequency response is zero, and occurs at zero frequency. The maximum value occurrs at odd multiples of π (recall that ω is the discrete frequency). The maximum value is

$$\begin{aligned} \left| N_k(e^{j\omega}) \right|^2_{\max} &= 4\left(\frac{T_r}{T_I}\right)^2 \left[1 + \frac{4T_r}{T_I}\left(1 + \frac{T_r}{T_I}\right)\right]^{k-2} \qquad k \ge 2 \\ &= 4\left(\frac{T_r}{T_I}\right)^2 \left[\left(1 + \frac{2T_r}{T_I}\right)^2\right]^{k-2} \end{aligned} \qquad (3\text{-}7)$$

Then

$$\left| N_k(e^{j\omega}) \right|_{\max} = 2\left(\frac{T_r}{T_I}\right)\left[1 + \frac{2T_r}{T_I}\right]^{k-2} \qquad k \ge 2 \qquad (3\text{-}8)$$

It is seen from Eq. (3-8) that the magnitude of the worst-case frequency offset accumulation depends on how long the frequency update interval is compared to the residence time. In general IEEE 1588 networks, $T_r << T_I$, and a large number of hops are required for the worst case amplitude to exceed 1. For example, if $T_r / T_I = 0.001$, then the worst case frequency offset amplitude after 100 hops accumulates be a factor of

$$\left| N_k(e^{j\omega}) \right|_{\max} = 2(0.001)[1.002]^8 = 0.00203 \qquad k = 100$$

In AVB networks, it is planned to hold Sync messages until Follow_Up messages arrive and, in worst-case, the residence time could be as long as the synch interval. The frequency update interval is likely to be 10 times the synch interval, i.e., $T_r / T_I = 0.1$. For a seven hop network, the worst-case frequency offset accumulates by a factor of

$$\left| N_k(e^{j\omega}) \right|_{\max} = 2(0.1)[1.2]^5 = 0.498 \qquad k = 7$$

For 10 hops, the result is

$$\left| N_k(e^{j\omega}) \right|_{\max} = 2(0.1)[1.2]^8 = 0.86 \qquad k = 10$$

For 11 hops, the result is

$$\left| N_k(e^{j\omega}) \right|_{\max} = 2(0.1)[1.2]^9 = 1.03 \qquad k = 11$$

It is seen that the frequency offset at sucessive nodes does not accumulate to the magnitude of the initial perturbation until the 11^{th} hop.

The factor by which the frequency offset amplitude accumulates, as a function of TC number, is plotted in Figure 1 for several ratios of residence time to frequency update interval. Note that the horizontal axis starts at TC 2 because the perturbation is applied at TC 1. It is seen that, for ratios of residence time to frequency update interval of 0.01 and 0.001, the perturbation has negligible effect. In all cases, the effect of the perturbation is diminished at the succeeding 9 nodes after TC 1 where the perturbation is introduced.

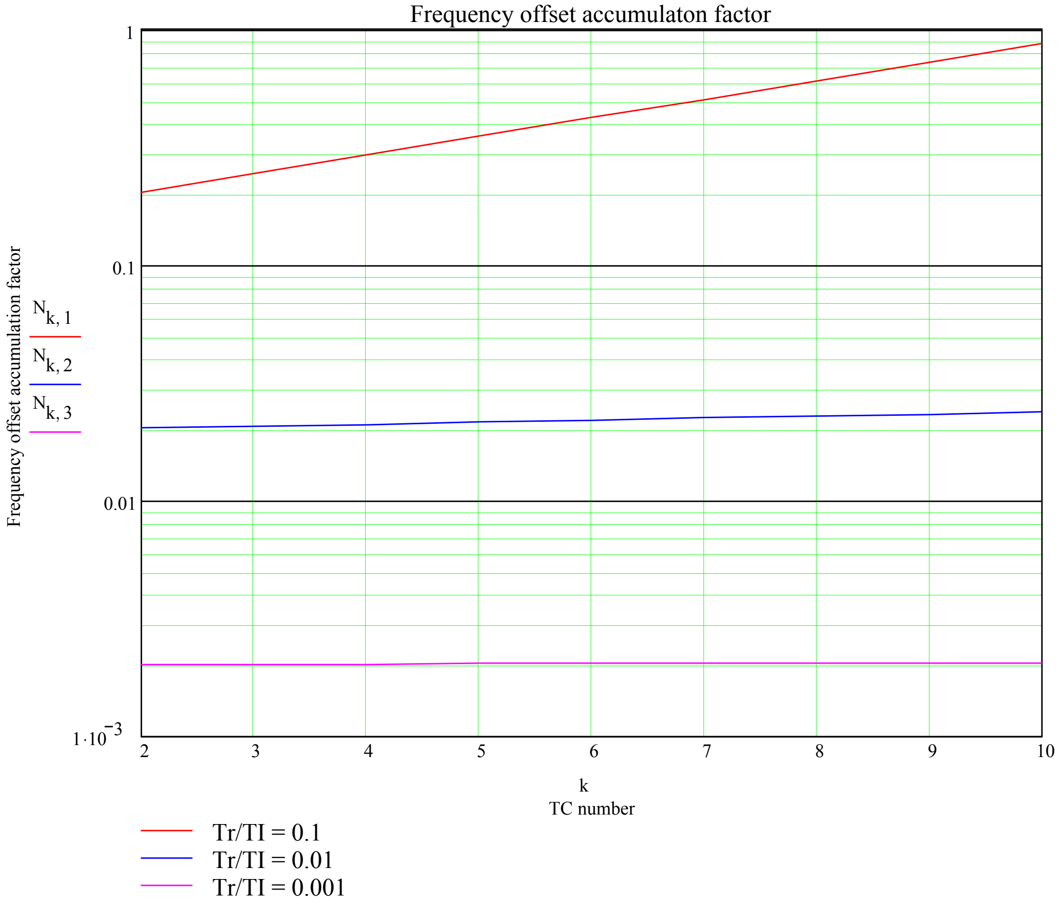

Figure 1. Factor by which frequency offset amplitude, due to applied sinusoidal perturbation, accumulates, as a function of TC number, for several ratios T_r / T_I.

4 Conclusions

The analysis shows that if $T_r << T_I$, a very large number of hops is required before the effect of a frequency offset perturbation on downstream accumulates to the level of the perturbation. For example, for residence time on the order of 0.001 of the frequency update interval, the frequency offset amplitude due to the applied perturbation after 100 hops is 0.2% of the applied perturbation amplitude. For AVB networks, where it is likely that $T_r / T_I = 0.1$ in very worst case, the frequency offset accumulates to the level of the perturbation after 11 hops. After 7 hops, the frequency offset is approximately half the level of the applied perturbation.

It may be asked why the frequency offset downstream of a perturbation should not be at least at the level of the perturbation itself. The reason is that any given TC is still syntonizing to the master. The frequency offset at an intermediate TC affects only the residence time measurement. Furthermore, the frequency offset measured at a particular TC is impacted only to the extent that successive residence times at upstream TCs *are different* (by successive, we mean at successive frequency update times). If intermediate residence times are in error but did not vary, the syntonization of a downstream TC would not be affected (though the time synchronization of the system would most definitely be affected).

5 References

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