Effect of a Frequency Perturbation in a Chain of Syntonized Transparent Clocks

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Outline

- Introduction
- Transfer function for a chain of syntonized TCs
- Stability analysis for variation of frequency offset as a function of number of hops
- Unfiltered phase error accumulation over multiple hops
- Wander tolerance requirements
- Filtered phase accumulation at endpoint
- Split path syntonization scenario
- Conclusions
- References

Introduction - 1

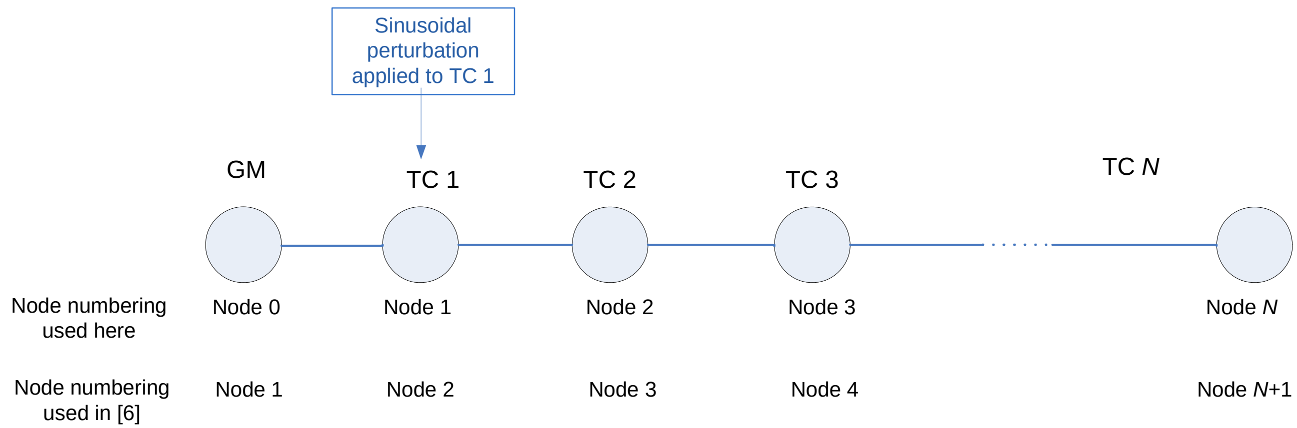
- Recent discussions on the 1588 (see [1] and [2]) and 802.1 reflectors, and in AVB Timing calls and general AVB calls, have raised concerns that a chain of syntonized clocks may exhibit excessive jitter amplification and accumulation due to gain peaking
  - It is stated in [1] that the jitter accumulation may be excessive when the period of the perturbation is of the order of twice the frequency measurement interval
- It is well-known that large jitter and wander accumulation in a chain of PLLs may occur due to excessive gain peaking
  - Early analysis done in [3] analytically using chain of 2nd order PLLs with 40 dB/decade roll-off
    - Model reflected PDH regenerators commonly used in telecommunications networks at the time (1982)
  - Later analyses done for SDH/SONET and OTN (see [4] for details of the OTN analyses)
    - These models were similar to those in [3]; main difference was the use of PLL models with 20 dB/decade roll-off
  - All the analyses showed that, if gain peaking does not exceed 0.1 dB, the respective jitter requirements are met for chains of 50 regenerators (assuming regenerators also meet respective jitter transfer and generation requirements)
    - 50 regenerators is a common Hypothetical Reference Model (see Appendix III of [4] for OTN)

Introduction - 2

- **As a result of these and similar analyses, the gain peaking of regenerators and clocks in telecommunications networks is limited to 0.1 dB**
- The above analyses are for 2nd order PLLs, which have inherent gain peaking
- However, the syntonization process for a transparent clock (TC) is not a 2nd order PLL
  - Previously, there has not been a careful evaluation of the transfer function to determine if gain peaking is present
    - Since the TC output is a syntonized frequency, the immediate transfer function of interest is the relation between input and output frequency offsets
    - However, effect on phase accumulation at a collocated ordinary clock (OC) is also of interest
- In the present contribution, we evaluate the transfer function for a TC that syntonizes, and evaluate accumulation of
  - Frequency offset
  - Phase error
- We also discuss wander tolerance and show initial results for filtered phase error at an endpoint
- Finally, we investigate phase error accumulation using the alternative, *split path syntonization* described in [8]

Transfer Function for a TC - 1

Node Numbering conventions (initial node is GM, followed by TCs)



- In the following notation and analysis, we use the node numbering convention above (the node numbering of [6] is shown for use when comparing results here with those of [6])
- Each TC performs a frequency update every M^{th} synch interval (on receipt of Sync and Follow_Up for that interval), and that frequency update is done before the measurement of residence time
- Assume perfect master (i.e., grandmaster)
- Neglect phase measurement granularity

Transfer Function for a TC - 2

❑Notation

i = index of Sync message sent for which a frequency update is performed (if a frequency update is performed every M Synch messages, then this is actually the Mi^{th} Sync message). The first Sync message is labeled $i = 0$.

k = index of node number. The master clock at the beginning of the chain is labeled $k = 0$; the TCs are labeled $1, 2,..., N$.

$\nu_{k,i}$ = actual frequency offset of TC k relative to the master clock, during the frequency update interval between the i^{th} and $(i+1)^{st}$ Sync messages

$\mu_{k,i}$ = measured frequency offset of TC k relative to the master clock, when the i^{th} Sync and Follow_Up messages arrive. Note that if the measurement process were perfect, then the measured value would be equal to the actual frequency offset during the previous frequency update interval, i.e., $\mu_{k,i} = \nu_{k,i}$. However, in general the measurement process is imperfect and this is not true.

$r_{k,i}$ = measured residence time of Sync message i at TC k. Note that this will differ from ideal residence time, relative to the master clock, due to the frequency offset of TC k.

T_r = ideal residence time, relative to the perfect master.

T_I = frequency update interval, relative to the perfect master.

$m_{k,i}$ = corrected master time at TC k when i^{th} Sync message is received (i.e., corrected for the residence times of TCs $1, 2, ..., k$-1). Note that $m_{0,i}$ is the time the i^{th} Sync message is actually sent, relative to the master clock.

Transfer Function for a TC - 3

- Corrected master time at node *k* is given by

$$m_{k,i} = m_{0,i} + \sum_{j=1}^{k-1} r_{j,i}$$

- Residence time at TC *k* is related to frequency offset by

$$r_{k,i} = (1 + \nu_{k,i})T_r$$

- Then

$$m_{k,i} = m_{0,i} + kT_r + T_r \sum_{j=1}^{k-1} \nu_{j,i}$$

- The measured frequency offset at TC *k* is equal to the ratio of the elapsed local time during a frequency update interval to the elapsed corrected master time during the same interval, minus 1
  - This may be written as (see next slide)

Transfer Function for a TC - 4

$$\mu_{k,i} = \frac{T_I(1+\nu_{k,i-1})}{m_{0,i} + kT_r + T_r\sum_{j=1}^{k-1}\nu_{j,i} - \left[m_{0,i-1} + kT_r + T_r\sum_{j=1}^{k-1}\nu_{j,i-1}\right]} - 1$$

$$= \frac{T_I(1+\nu_{k,i-1})}{T_I + T_r\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)} - 1$$

$$= \frac{T_I\nu_{k,i-1} - T_r\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)}{T_I + T_r\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)}$$

$$= \frac{\nu_{k,i-1} - \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)}{1 + \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)}$$

Transfer Function for a TC - 5

- Frequency is adjusted by reducing the current frequency by an amount equal to the measured frequency offset of the TC relative to its master
  - The new frequency offset is related to the old by

$$\nu_{k,i} = \nu_{k,i-1} - \mu_{k,i}$$

  - Inserting the previous equation for $\mu_{k,i}$ into the above produces

$$\nu_{k,i} = \nu_{k,i-1} - \frac{\nu_{k,i-1} - \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)}{1 + \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)}$$

$$= \frac{\nu_{k,i-1} - \nu_{k,i-1} + \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)}{1 + \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)} + O(\nu_{k,i}^2)$$

$$= \left[\frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)\right]\left[1 - \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)\right] + O(\nu_{k,i}^2)$$

$$= \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right) + O(\nu_{k,i}^2)$$

Transfer Function for a TC - 6

- The result, to first order in the frequency offsets, is

$$\nu_{k,i} = \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)$$

- May obtain a recursive form by rewriting the above for index *k*-1 and subtracting; the boundary condition for *k* = 2 is obtained by substituting *k* = 2 in the above. The result is

$$\nu_{2,i} = \frac{T_r}{T_I}\left(\nu_{1,i} - \nu_{1,i-1}\right)$$

$$\nu_{k,i} = \nu_{k-1,i}\left(1 + \frac{T_r}{T_I}\right) - \frac{T_r}{T_I}\nu_{k-1,i-1} \qquad k > 2$$

- The above is a partial difference equation with a boundary condition

Stability Analysis - 1

- Assume the master is perfect, and impose a sinusoidal frequency perturbation at TC 1 (k = 1)

$$\nu_{0,i} = 0$$

$$\nu_{1,i} = Ae^{j\omega i}$$

  - A = amplitude of the frequency offset perturbation (in convenient units, e.g., ppm, ns/s, dimensionless, etc.)
  - ω = frequency of perturbation in rad/s
  - $j = \sqrt{-1}$ (note that j will not be used as an index here)

- The boundary condition (equation above that relates the frequency offsets at TCs 1 and 2) becomes

$$\nu_{2,i} = A\frac{T_r}{T_I}e^{j\omega i}(1 - e^{-j\omega})$$

Stability Analysis - 2

- To obtain the complex frequency response to a sinusoidal perturbation for the frequency offset at TC *k*, first take the z-transform of the partial difference equation, and then set $z = e^{j\omega}$

$$N_k(z) = N_{k-1}(z)\left(1 + \frac{T_r}{T_I} - \frac{T_r}{T_I} z^{-1}\right)$$

$$H(e^{j\omega}) = 1 + \frac{T_r}{T_I} - \frac{T_r}{T_I} e^{-j\omega}$$

  - Where $N_k(z)$ = z-transform of $\nu_{k,i}$
  - $\mathrm{H}(e^{j\omega})$ = sinusoidal transfer function

- Frequency response at TC *k* obtained by multiplying the frequency response at TC 2 by the sinusoidal transfer function raised to the *k*-2 power

Stability Analysis - 3

$$v_{k,i} = A\frac{T_r}{T_I}e^{j\omega i}(1-e^{-j\omega})\left(1+\frac{T_r}{T_I}-\frac{T_r}{T_I}e^{-j\omega}\right)^{k-2}$$

- The amplitude of the frequency response is given by (see the accompanying contribution for intermediate steps)

$$\left|N_k(e^{j\omega})\right|^2 = \left(\frac{T_r}{T_I}\right)^2 [2(1-\cos\omega)]\left[1+\frac{2T_r}{T_I}\left(1+\frac{T_r}{T_I}\right)(1-\cos\omega)\right]^{k-2} \qquad k \ge 2$$

- Maximum value of the above expression occurs for $\omega = \pi$
  - Perturbation frequency is the Nyquist rate, i.e., one-half the frequency update rate
  - Perturbation period is twice the frequency update period

$$\left|N_k(e^{j\omega})\right|_{\max} = 2\left(\frac{T_r}{T_I}\right)\left[1+\frac{2T_r}{T_I}\right]^{k-2} \qquad k \ge 2$$

Stability Analysis - 4

- Magnitude of worst-case frequency offset accumulation depends on how long the frequency update interval is compared to the residence time, i.e., T_r/T_I, and on the number of hops
- For example, for T_r/T_I = 0.001, the worst-case frequency offset amplitude after 100 hops is

$$\left|N_k(e^{j\omega})\right|_{\max} = 2(0.001)[1.002]^8 = 0.00203 \qquad k = 100$$

- In AVB networks, it is planned to hold Sync until Follow_Up arrives
  - In worst case, residence time could be equal to the synch interval
  - A value considered for the frequency update interval is 10 synch intervals
  - Then could have, in worst case, T_r/T_I = 0.1
  - For a 7-hop network, the worst case frequency offset accumulation is

$$\left|N_k(e^{j\omega})\right|_{\max} = 2(0.1)[1.2]^5 = 0.498 \qquad k = 7$$

Stability Analysis - 5

- For 10 hops

$$\left|N_k(e^{j\omega})\right|_{\max} = 2(0.1)[1.2]^8 = 0.86 \qquad k = 10$$

- For 11 hops

$$\left|N_k(e^{j\omega})\right|_{\max} = 2(0.1)[1.2]^9 = 1.03 \qquad k = 11$$

- Frequency offset does not accumulate to the magnitude of the perturbation until the 11th hop
- May be asked why the frequency offset downstream of the perturbation is not at least as large as the perturbation itself
  - TC is still syntonizing to the master
  - Frequency at an intermediate TC only affects the residence time, and only to the extent that successive residence times (i.e., at successive frequency update times) are different

Stability Analysis - 6

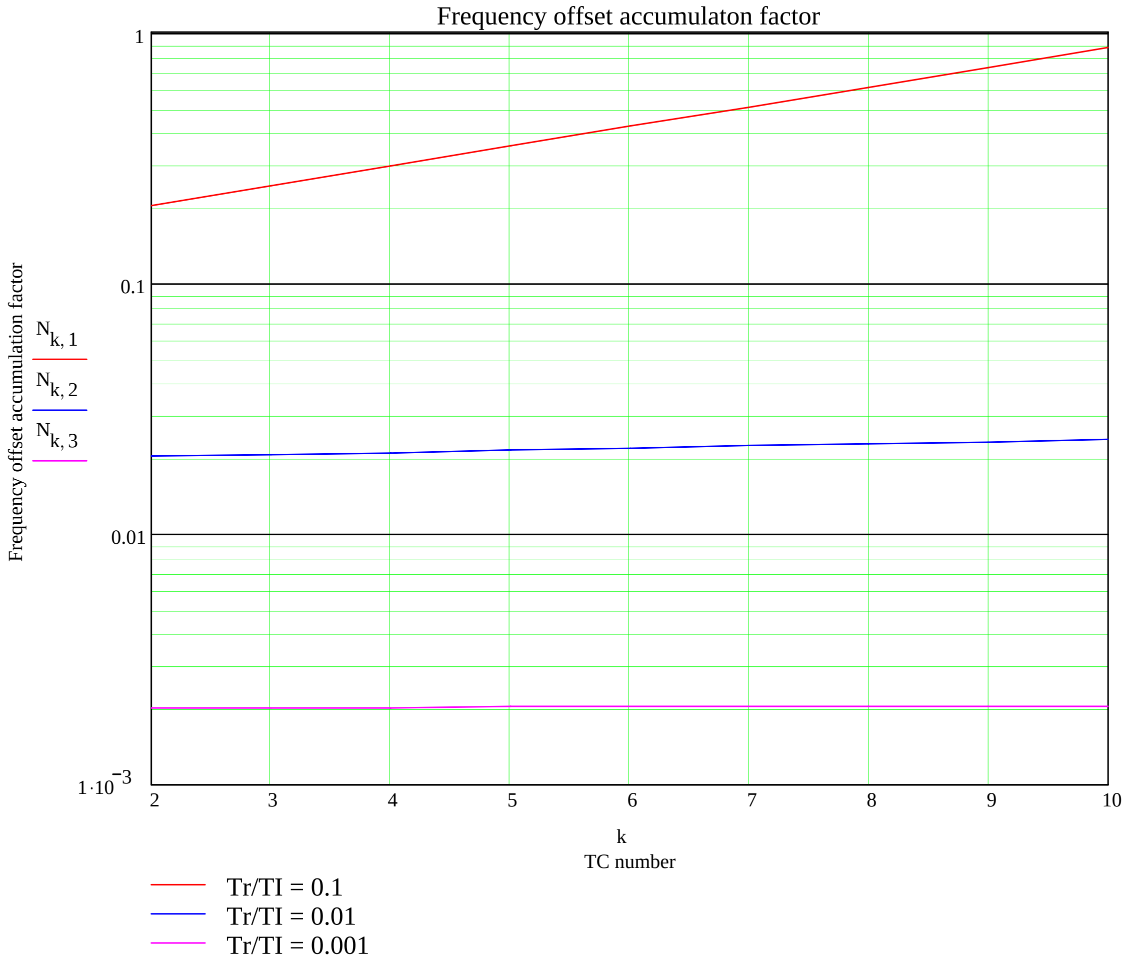
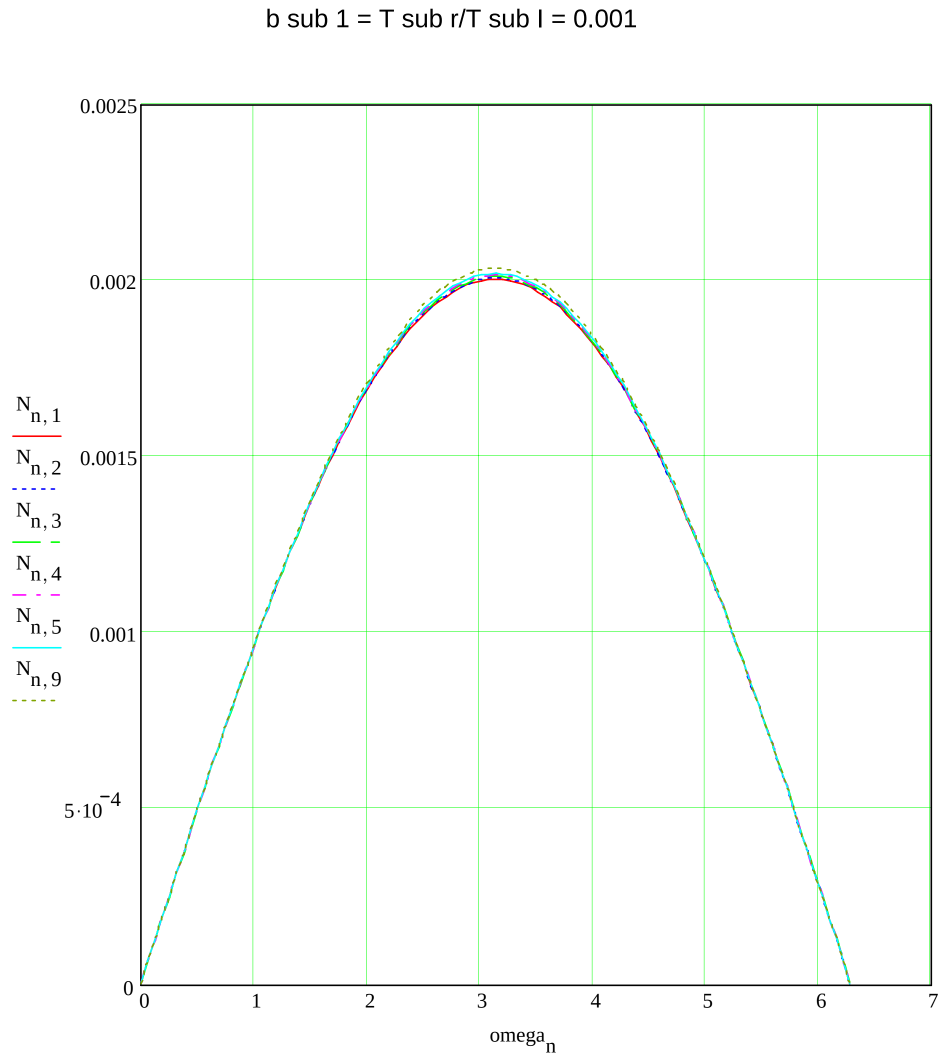


Figure 2. Factor by which frequency offset amplitude, due to applied sinusoidal perturbation, accumulates, as a function of TC number, for several ratios T_r / T_I.

Stability Analysis - 7

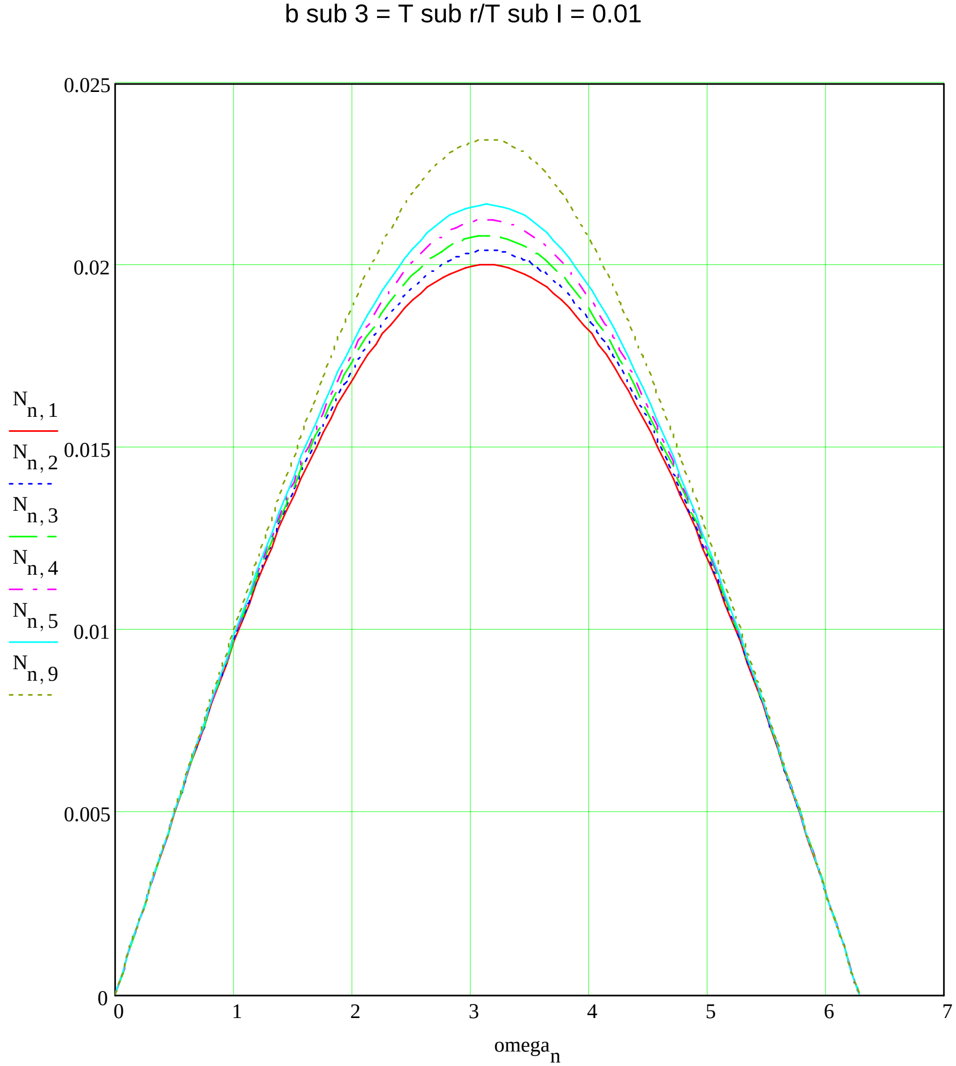
- Frequency response at successive TCs, for T_r/T_I = 0.001(obtained using Mathcad)

  - $N_{n,k}$
  - k+1= TC number
  - Perturbation applied at TC 1



Stability Analysis - 8

- Frequency response at successive TCs, for T_r/T_I = 0.01(obtained using Mathcad)



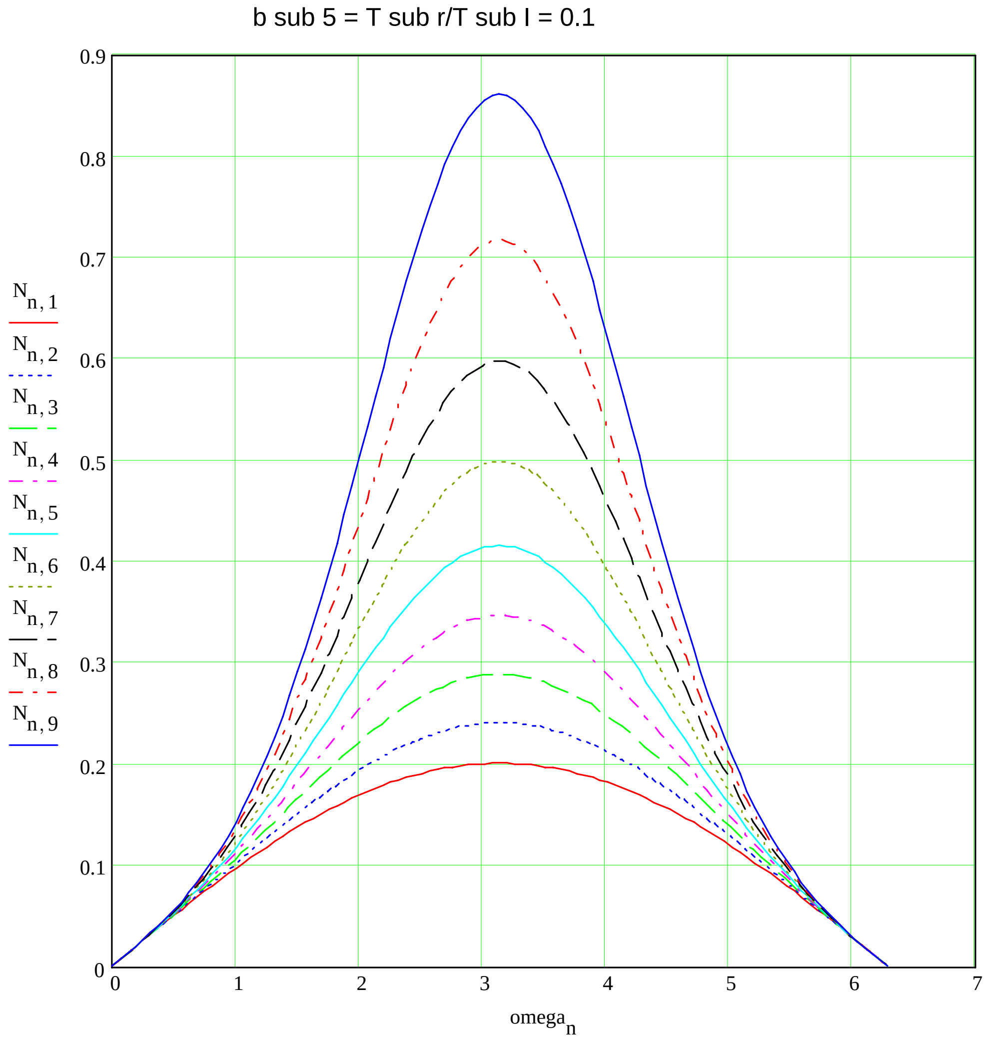
- $N_{n,k}$
- k+1= TC number
- Perturbation applied at TC 1

Note: The curve for *k* = 10 in the contribution is in error

Stability Analysis - 9

- Frequency response at successive TCs, for T_r/T_I = 0.1(obtained using Mathcad)

  - $N_{n,k}$
  - k+1= TC number
  - Perturbation applied at TC 1



Unfiltered Phase Error Accumulation - 1

- Reference [6] cited a general form for phase error accumulation

$$G = G_0 \left(1 + a \frac{T_r}{T_I} \right)^n$$

  - Specific values of G_0 and *a* obtained in [6] were different for those obtained in above for accumulation of frequency offset
  - In this section this discrepancy is further investigated and resolved
- Use above results for frequency offset accumulation to compute total phase error accumulation under the assumption that an OC is collocated with each TC
  - Interested in component of phase error accumulation due to sinusoidal perturbation applied at TC 1
- The phase error at a TC is the difference between the timestamp of a Sync message when it arrives at a TC and the timestamp of the Sync message when it left the master, corrected for
  - Propagation times on upstream links
  - Residence times at upstream TCs
- If residence and propagation time measurements were perfect, we would correct for these errors exactly and could reduce phase error accumulation to near zero

Unfiltered Phase Error Accumulation - 2

- However, residence time measurements are not perfect
  - We are interest here in the component of the residence time measurement error due to errors in the TC frequency offset measurement
  - For TC *k*, Sync message *i*, this error is equal to the frequency offset measurement error multiplied by the residence time, i.e., $\nu_{k,i}T_r$
  - For TCs 2 through *m*, the total phase error is at the time of the i^{th} Sync message is

$$\phi_{m,i} = \sum_{k=2}^{m-1} \nu_{k,i} T_r$$

    - Note: must add to this the perturbation itself (i.e., contribution from TC 1). However, we are mainly interested here in the additional phase error due to the effect of the perturbation at downstream TCs
  - Substitute into the above the result for $\nu_{k,i}$ (slide 11 (TC 2) and slide 13 (TCs *k* > 2)); the result is

$$\phi_{m,i} = A\frac{T_r^2}{T_I}(1-e^{-j\omega})e^{j\omega i}\sum_{k=2}^{m-1}\left(1+\frac{T_r}{T_I}-\frac{T_r}{T_I}e^{-j\omega}\right)^{k-2}$$

Unfiltered Phase Error Accumulation - 2

- Performing the summation produces

$$\phi_{m,i} = A\frac{T_r^2}{T_I}(1-e^{-j\omega})e^{j\omega i}\frac{1-\left(1+\frac{T_r}{T_I}-\frac{T_r}{T_I}e^{-j\omega}\right)^{m-2}}{1-\left(1+\frac{T_r}{T_I}-\frac{T_r}{T_I}e^{-j\omega}\right)}$$

$$= A\frac{T_r^2}{T_I}(1-e^{-j\omega})e^{j\omega i}\frac{\left(1+\frac{T_r}{T_I}-\frac{T_r}{T_I}e^{-j\omega}\right)^{m-2}-1}{\frac{T_r}{T_I}\left(1-e^{-j\omega}\right)}$$

$$= AT_r e^{j\omega i}\left[\left(1+\frac{T_r}{T_I}-\frac{T_r}{T_I}e^{-j\omega}\right)^{m-2}-1\right]$$

- Amplitude of phase error frequency response is obtained by computing absolute value of the above

$$\left|\phi_{m,n}\right|_{\max} = AT_r\cdot\left\|\left[\left(1+\frac{T_r}{T_I}-\frac{T_r}{T_I}e^{-j\omega}\right)^{m-2}-1\right]\right\|$$

Unfiltered Phase Error Accumulation - 3

- The above result is expressed in terms of the amplitude of the sinusoidal frequency offset perturbation, *A*, applied at TC 1
  - Reference [6] uses as input a sinusoidal *phase* perturbation
  - To convert from a frequency perturbation to phase perturbation, we note a frequency offset at TC 1 of magnitude *A* results in a phase offset at TC 1 of AT_r. Substituting in the above produces for cumulative phase offset (due to the effect of the phase perturbation on downstream TCs)

$$\left|\phi_{m,n}\right|_{\max} = B \cdot \left\| \left[\left(1 + \frac{T_r}{T_I} - \frac{T_r}{T_I} e^{-j\omega} \right)^{m-2} - 1 \right] \right\|$$

- The maximum phase perturbation occurs for $\omega = \pi$; the result is

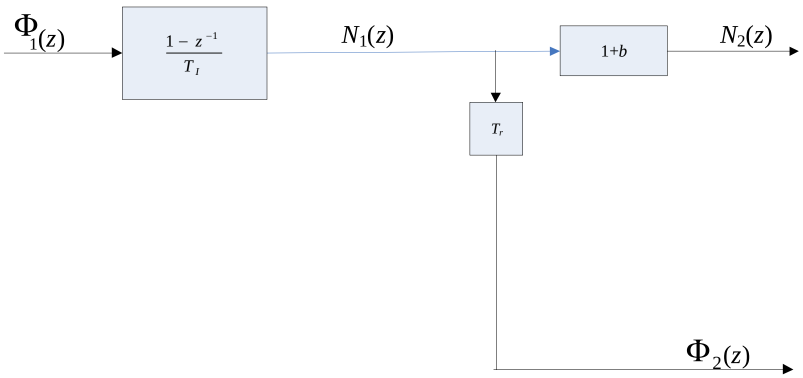
$$\left|\phi_{m,n}(\pi)\right|_{\max} = B \cdot \left\| \left[\left(1 + 2\frac{T_r}{T_I} \right)^{m-2} - 1 \right] \right\|$$

- This corresponds to the general form given earlier, with $G_0 = B$ and $a = 2$

Unfiltered Phase Error Accumulation - 4

❑Block diagrams for frequency offset and phase error accumulation

Stage 1 to 2
(i.e., GM is stage 0 and perturbation is applied at stage 1, to either frequency $N_1(z)$ or phase $_1(z)$)



$N_k(z)$ = z-transform of frequency at node *k*

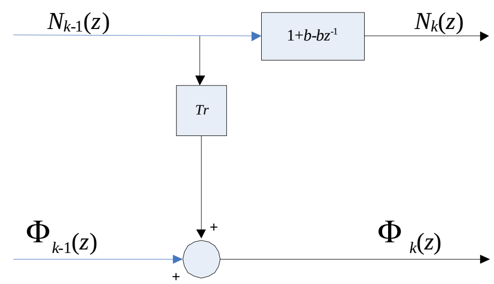
$_k(z)$ = z-transform of phase at node *k*

$$b = T_r / T_I$$

$$N_2(z) = (1 + b)N_1(z)$$

$$N_1(z) = \frac{1 - z^{-1}}{T_I}\Phi_1(z)$$

Stages k-1 > 1, to k
(i.e., GM is stage 0 and perturbation is applied at stage 1



$N_k(z)$ = z-transform of frequency at node *k*

$_k(z)$ = z-transform of phase at node *k*

$$b = T_r / T_I$$

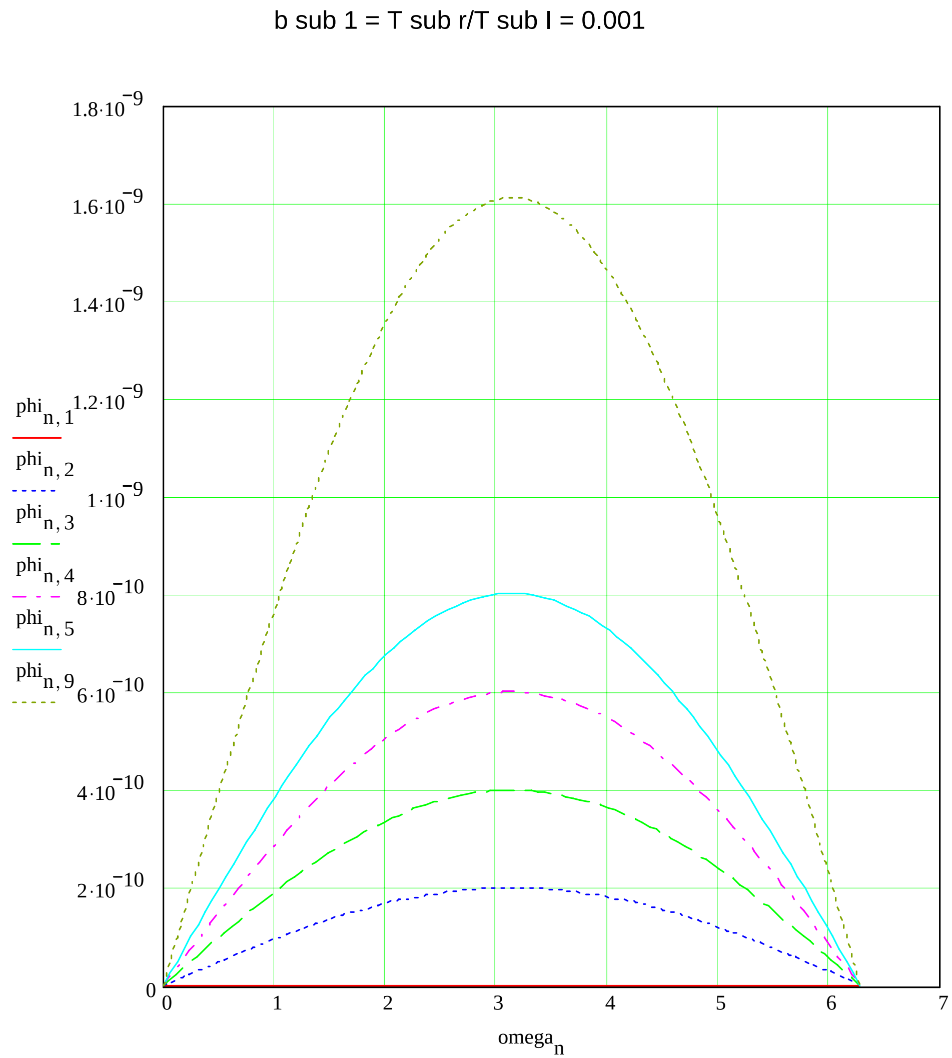
$$N_k(z) = (1 + b - bz^{-1})N_{k-1}(z)$$

$$\Phi_k(z) = \Phi_{k-1}(z) + T_r N_k(z)$$

Unfiltered Phase Error Accumulation - 4

- Notation
  - $N_{n,k}$
  - k+1= TC number
  - Perturbation applied at TC 1

Perturbation Amplitude B = 100 ns (taken from [6])

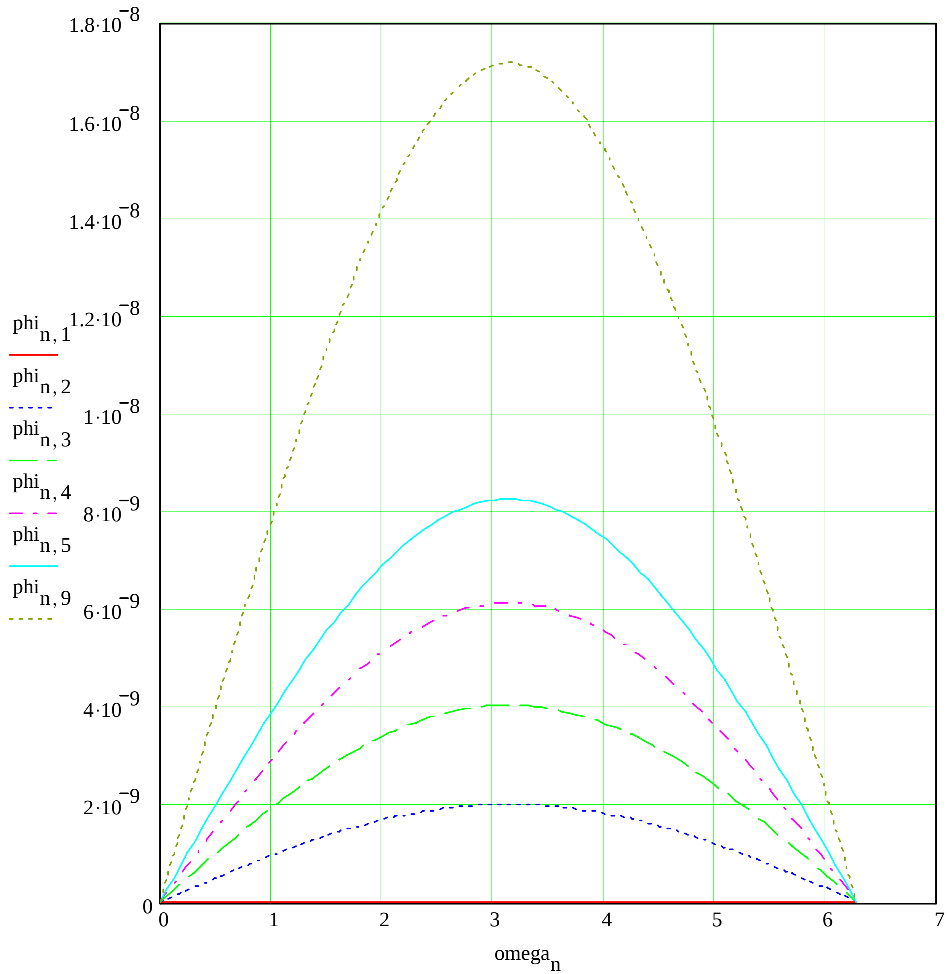


Unfiltered Phase Error Accumulation - 5

- Notation
  - $phi_{n,k}$
  - k+1= TC number
  - Perturbation applied at TC 1

Perturbation Amplitude B = 100 ns (taken from [6])

b sub 1 = T sub r/T sub I = 0.01



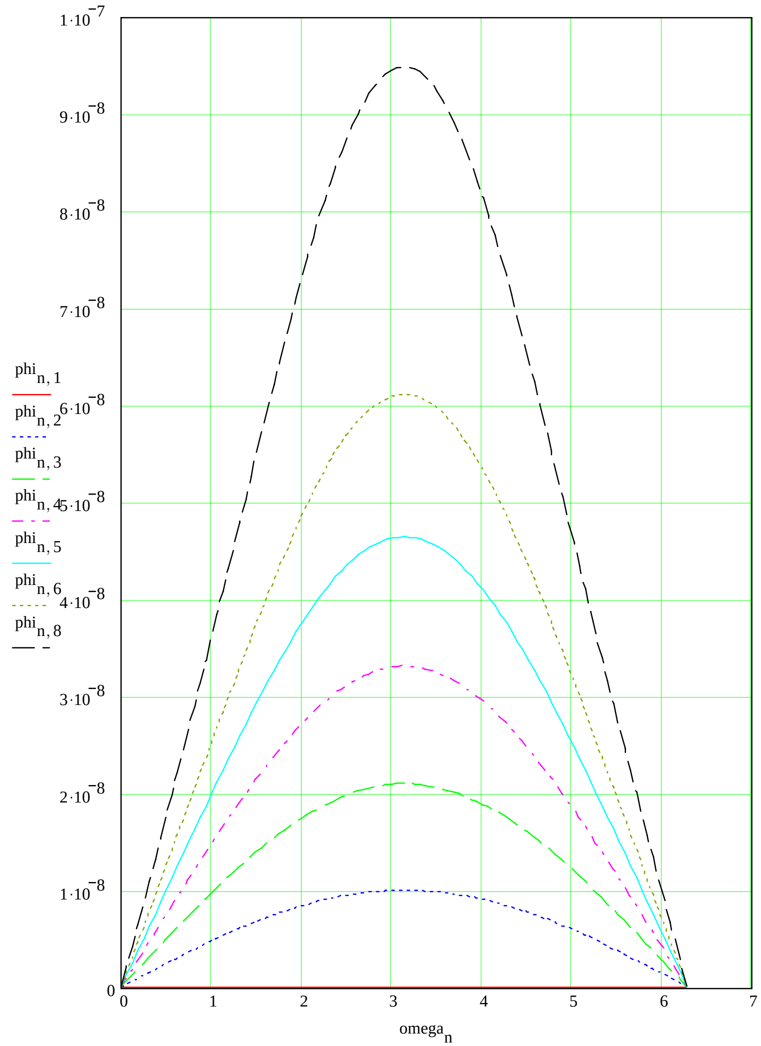
Unfiltered Phase Error Accumulation - 6

❑Notation

- $phi_{n,k}$
- k+1= TC number
- Perturbation applied at TC 1

Perturbation Amplitude B = 100 ns (taken from [6])

b sub 1 = T sub r/T sub I = 0.05

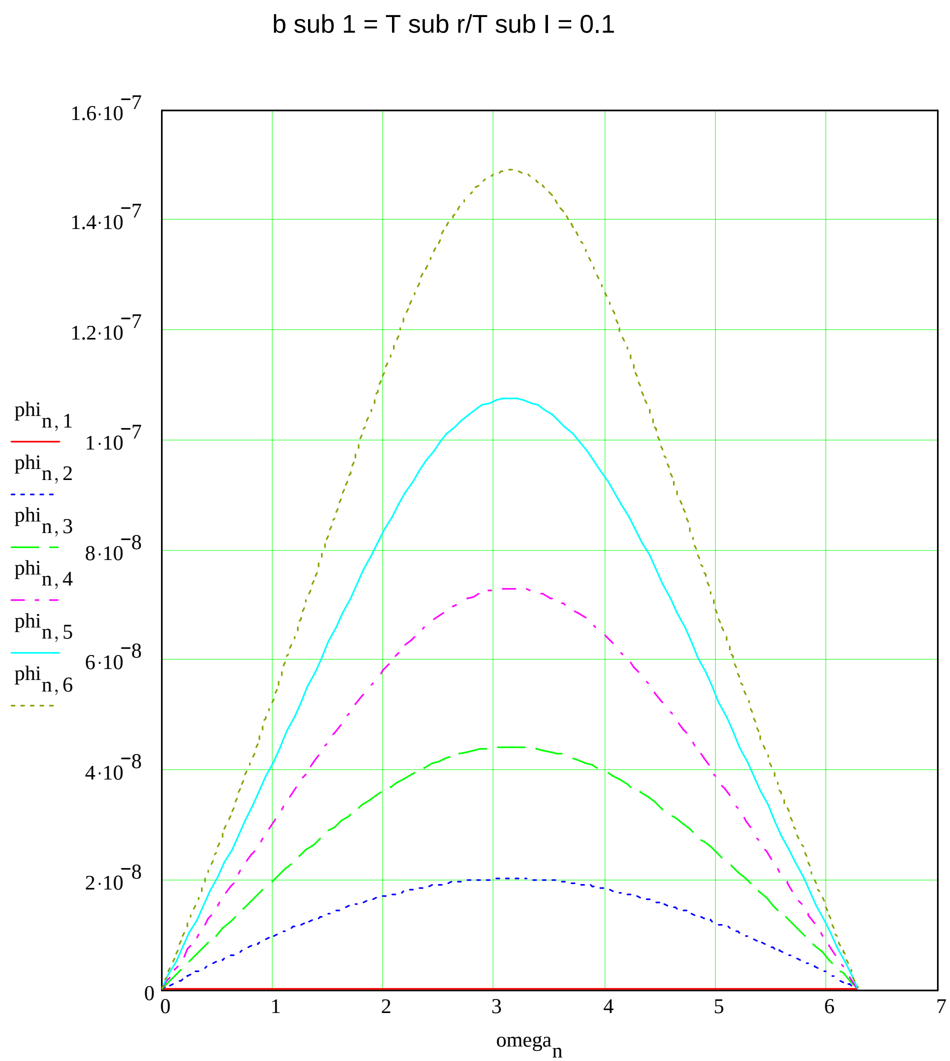


Unfiltered Phase Error Accumulation - 7

Notation

- $phi_{n,k}$
- k+1= TC number
- Perturbation applied at TC 1

Perturbation Amplitude B = 100 ns (taken from [6])



Unfiltered Phase Error Accumulation - 8

- The above result for phase error accumulation amplitude corresponds to the quantity (outputdeviation – testdeviation) of [6]
  - It is also equal to residencetime * (syntonizedrate –1) accumulated over the nodes
  - Essentially, it is the amount the accumulated phase error exceeds the initial phase error due to the perturbation
- Relation between discrete and continuous frequency of perturbation
  - Discrete frequency ω and continuous frequency Ω are related by equivalent sampling time, i.e., frequency update interval T_I. If the period of the perturbation is T, then

$$\omega = \Omega T_I = 2\pi T_I / T$$

$$T = 2\pi T_I / \omega$$

  - $\omega = \pi$ corresponds to perturbation period equal to twice the frequency update period, or $T = 2T_I$
  - $\omega = 2$ corresponds to perturbation period equal (approximately) to 3.1 times the frequency update period, or $T = 3.1T_I$

Unfiltered Phase Error Accumulation - 9

- Results from [6] for T_r/T_l = 0.05, B = 100 ns, 2nd TC after TC where perturbation is applied (i.e., node 3 in notation here; node 4 in notation of [6]). $T = 3.1T_l$ (ω = 2)

  - Peak amplitude is approximately 8 ns
  - Agrees with result on slide 27 for phi sub n,2 (TC 3) for ω = 2)

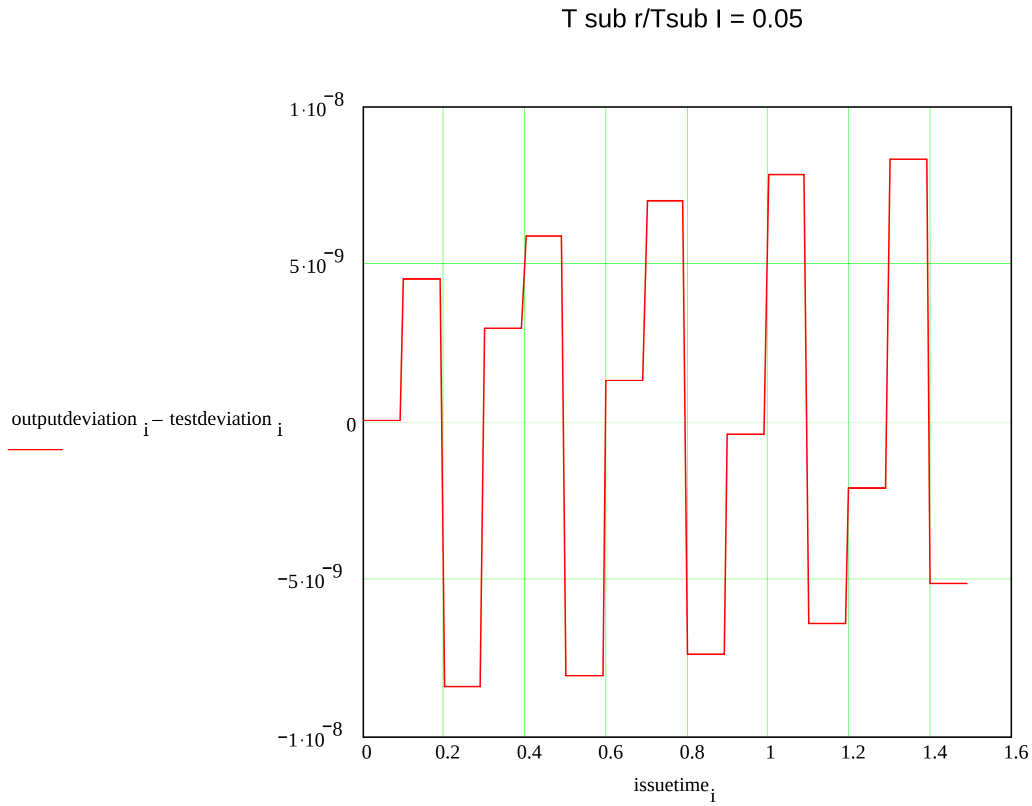


Figure 6. Results for (outputdeviation-testdeviation) using Mathcad file provided in [6]. This quantity is the frequency offset at the first node after the node where the perturbation is applied, multiplied by the ideal residence time. The period of the applied perturbation is $3.1T_l$.

Unfiltered Phase Error Accumulation - 10

- Results from [6] for T_r/T_I = 0.05, B = 100 ns, 2nd TC after TC where perturbation is applied (i.e., node 3 in notation here; node 4 in notation of [6]). $T = 2T_I$ ($\omega = \pi$)

– Peak amplitude is a
pproximately 10 ns
– Agrees with result o
n slide 27 for phi sub
n,2 (TC 3) for $\omega = \pi$)

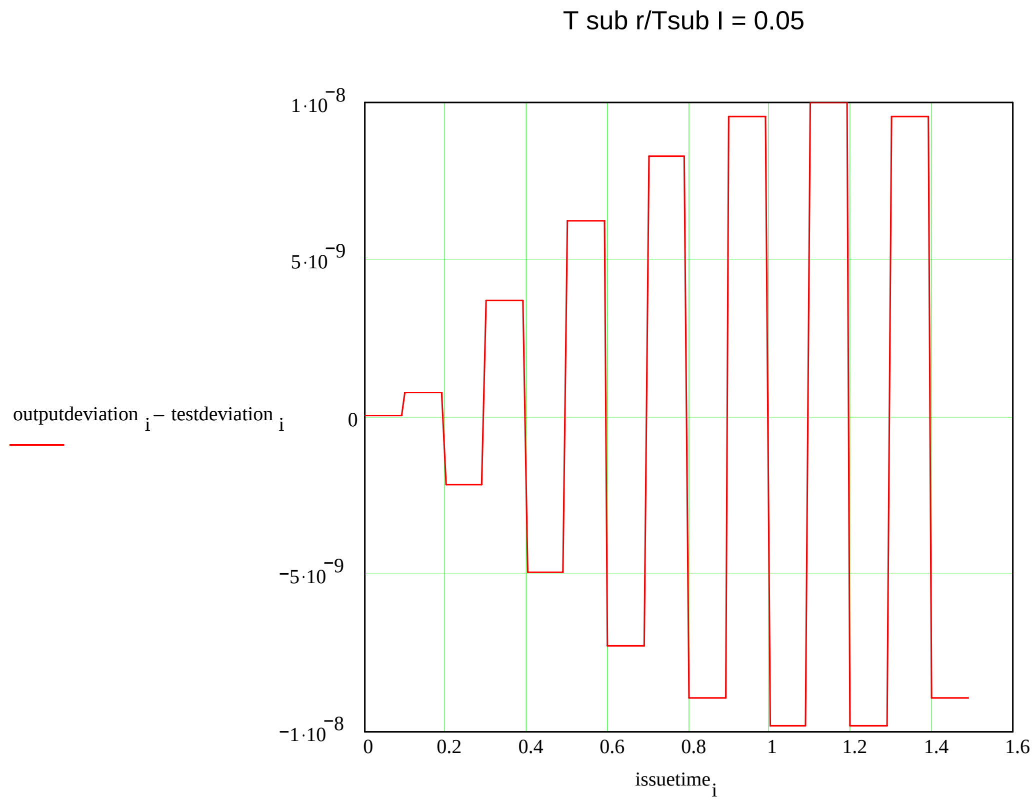


Figure 7. Results for (outputdeviation-testdeviation) using Mathcad file provided in [6]. This quantity is the frequency offset at the first node after the node where the perturbation is applied, multiplied by the ideal residence time. The period of the applied perturbation is $2.1T_I$.

Unfiltered Phase Error Accumulation - 10

- Table 1 of [6] listed some differences between analysis there and here
  - When Table 1 was prepared, comparison was with frequency offset accumulation study here (phase offset accumulation study had not yet been done)
    - Also, comparison used different perturbation frequencies

Unfiltered Phase Error Accumulation - 10

- Differences resolved by comparing with phase accumulation study and using same perturbation frequency
  - Table 1 of [6] indicates G_0 = 1 there and $2T_r/T_l$ here
    - the latter is for the frequency offset accumulation; it is 1 for phase accumulation, in agreement with [6]
  - Table 1 of [6] indicates worst case gain peaking was obtained for perturbation period of 3.1 times frequency update interval, and not 2 times frequency update interval
    - However using Mathcad listing supplied in [6], a larger outputdeviation – testdeviation is found for perturbation period of 2 times frequency update interval
      - This is maximum, and is in agreement with results obtained here
    - Stated in [9] that discrepancy is due to comparing different quantities in [6] versus here
      - Here, we compared |outputdeviation – testdeviation|
      - [6] compared |outputdeviation| / |testdeviation| and noted that difference is due to different phases of difference signal (outputdeviation – testdeviation) and test signal (testdeviation); in [6] these signals are 90 degrees out of phase

Wander Tolerance

- The phase accumulation results on the previous slides assume a 100 ns sinusoidal phase perturbation amplitude
- This can be considered in the context of wander tolerance
  - Apply a sinusoidal phase wander at a TC, and require that the syntonization and synchronization transport meet the respective requirements (e.g., MTIE, time synchronization)
    - This assumes that the system parameters (e.g., sync interval, frequency update interval, are set appropriately
    - We would choose the syntonization algorithm and the endpoint filter specifications such that the application jitter, wander, and synchronization requirements could be met when the sinusoidal perturbation is applied
- It is stated in [9] that the 100 ns amplitude for the sinusoidal perturbation is arbitrary
  - Would more likely want to tolerate 40 ns peak-to-peak (resulting from phase measurement granularity combined with beating of 2 oscillators whose frequencies are close

Filtered Phase Error Accumulation - 1

- Note that the phase error accumulation results obtained above are unfiltered
- The levels would likely be reduced by endpoint filtering
- For example, consider a first-order, linear digital filter with an equivalent 1 s time constant

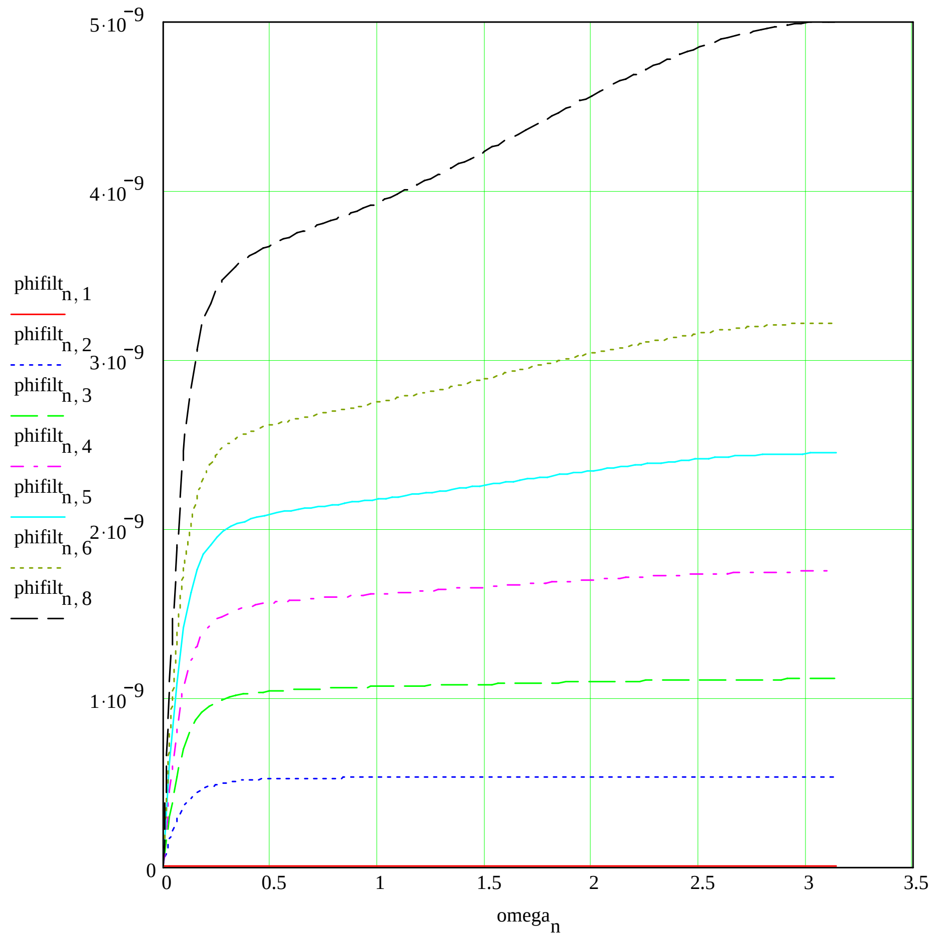
$$H(z) = \frac{0.1}{1-0.9z^{-1}}$$

  - The equivalent time constant is seen to be 1 s by noting that this filter reduces a step input to $1/e \cong 0.37$ of its value after 10 steps, i.e., $(0.9)^{10} = 0.35$, and that 10 steps here corresponds to 1s because $T_l = 0.1$ s
- To obtain the filtered frequency response
  - Substitute $z = e^{j\omega}$ in the above z-transform for the filter
  - Multiply the unfiltered phase error frequency response on slide 23 (first equation on that slide) by this filter frequency response
  - The result is on the next slide

Filtered Phase Error Accumulation - 2

- Comparing this with the figure on slide 27, the reduction is seen to be approximately a factor of 20

b sub 1 = T sub r/T sub I = 0.05



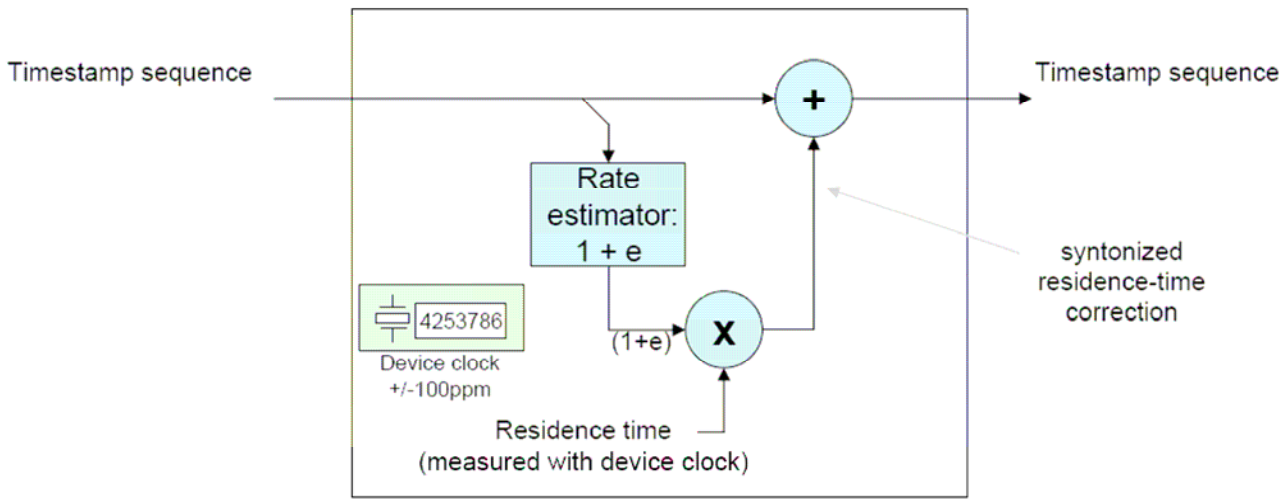
Split Path Syntonization Scenario - 1

- Reference [8] proposes a scheme that attempts to avoid phase error accumulation in nodes subsequent to the TC where the sinusoidal phase perturbation is applied
  - Use the free-running node clock frequencies to measure residence times
  - Use these residence times to compute the corrected master event timestamps (timestamp when Sync message leaves master plus accumulated residence time)
  - Use these corrected master event timestamps to estimate frequency offset of each TC relative to master
  - Use the estimated frequency offset to compute residence time error, and accumulate in a separate field of the Follow_Up message
  - Use this error accumulation to correct the synchronized time at the respective node
- The next 3 slides, taken (i.e., directly copied) from [8], illustrate the method (the first slide shows the current syntonization method; the second and third slides show the split path scenario)

Split Path Syntonization Scenario - 2

Taken from [8]

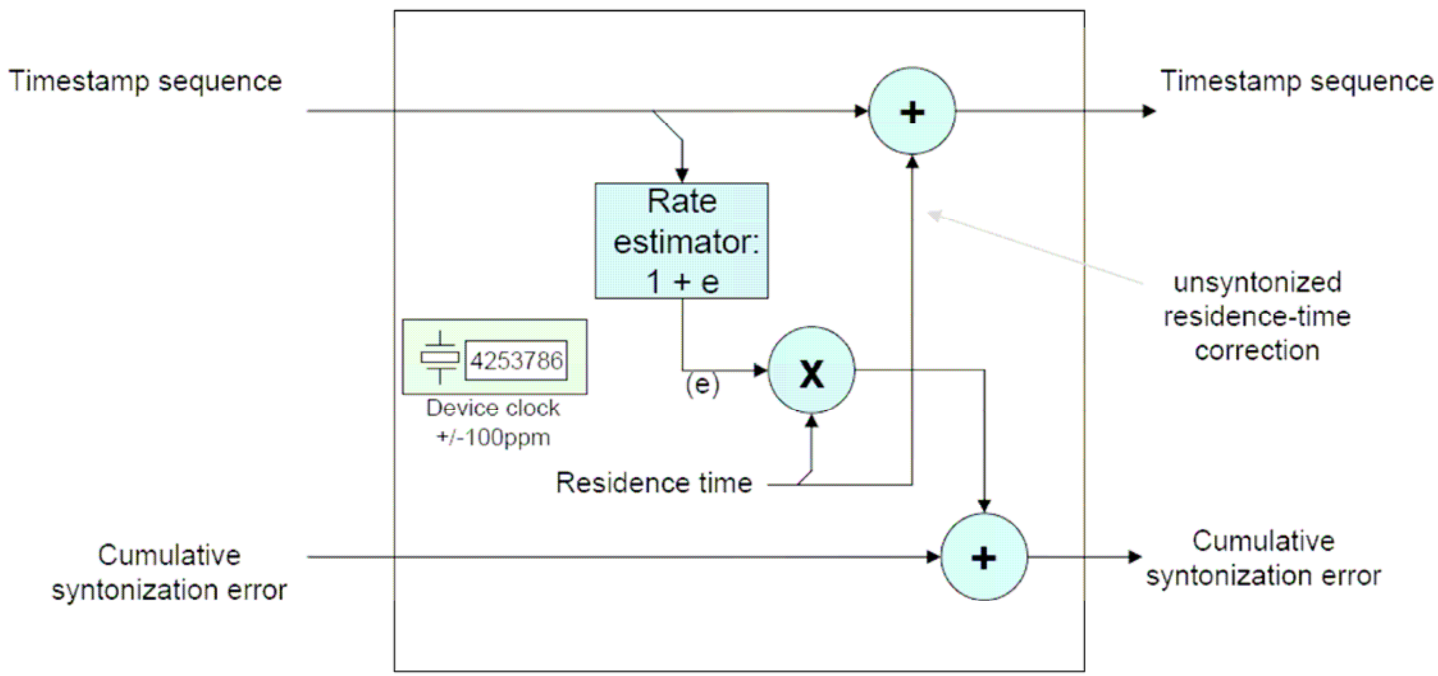
Syntonized TC model



Split Path Syntonization Scenario - 3

Taken from [8]

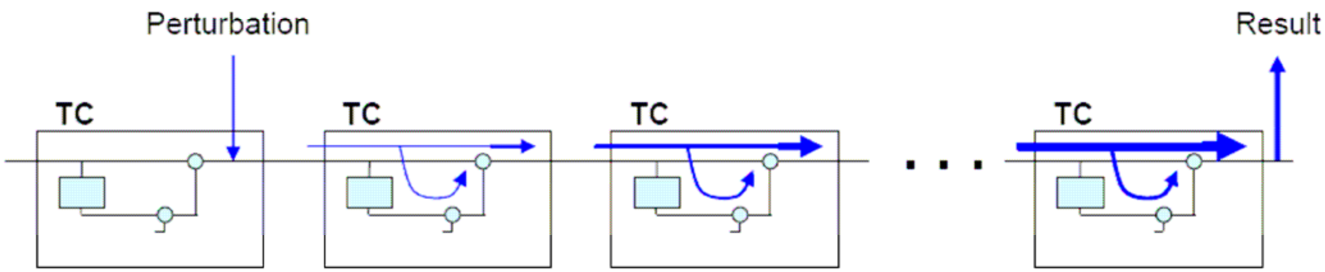
Split-path Syntonized TC model



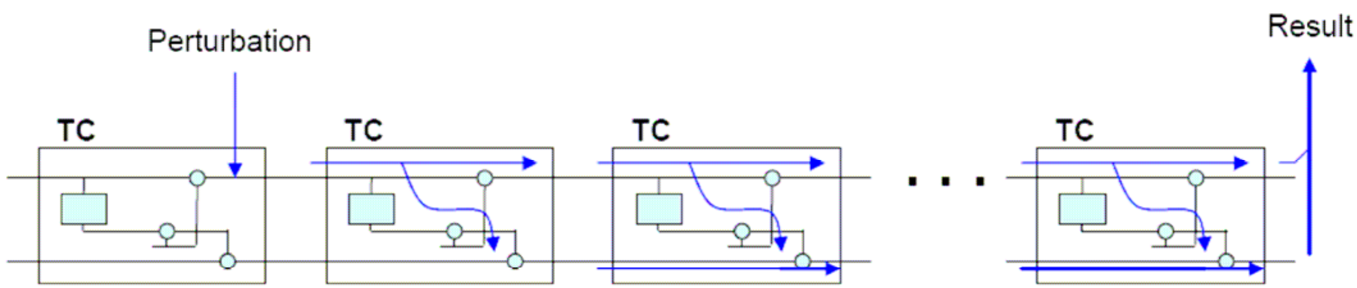
Split Path Syntonization Scenario - 4

Taken from [8]

Breaking the syntonization cascade



Gain cascade effect



Error summation effect

Split Path Syntonization Scenario - 4

- Can analyze the split path syntonization scenario in a manner analogous to the conventional analysis
  - Use the equation for $\mu_{k,j}$ on slide 8 to compute the measured frequency offset, but now the $\nu_{k,j}$ are fixed (free-running) frequency offsets (except at TC 1, where $\nu_{1,j}$ is a sinusoidal perturbation
  - Express this equation to first order in the $\nu_{k,j}$

$$\mu_{k,i} = \frac{\nu_{k,i-1} - \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)}{1 + \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)}$$

$$\cong \left[\nu_{k,i-1} - \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)\right]\left[1 - \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)\right]$$

$$\cong \nu_{k,i-1} - \frac{T_r}{T_I}\sum_{j=1}^{k-1}\left(\nu_{j,i} - \nu_{j,i-1}\right)$$

Split Path Syntonization Scenario - 5

- Next, note that the frequency offsets at nodes other than TC 1 are fixed, and therefore the terms in the above summation for *j* > 1 vanish
- Then

$$\mu_{k,i} = \nu_{k,i-1} - \frac{T_r}{T_I}\left(\nu_{j,i} - \nu_{j,i-1}\right)$$

- The total phase error due to components at nodes 2 through *k* is

$$T_r \sum_{j=2}^{k-1} \mu_{j,i} = T_r \sum_{j=2}^{k-1} \nu_{j,i-1} - \frac{T_r^2}{T_I}(k-2)\left(\nu_{1,i} - \nu_{1,i-1}\right)$$

  - We have omitted the perturbation itself at node 1 because we are interested in the accumulation over and above this

- The actual time Sync message *i* arrives at TC *k* is

$$m_{actual,k,i} = m_{0,i} + (k-1)T_r$$

- The corrected master time at TC *k*, based on the free-running clocks, is

$$m_{k,i} = m_{0,i} + (k-1)T_r + T_r \sum_{j=1}^{k-1} \nu_{j,i}$$

Split Path Syntonization Scenario - 6

- Using the measured frequency offset to correct the actual frequency offset in the above sum produces

$$m_{k,i} = m_{0,i} + (k-1)T_r + T_r \nu_{1,i} + T_r \sum_{j=2}^{k-1} (\nu_{j,i} - \nu_{j,i-1}) + \frac{T_r^2}{T_I}(k-2)\left(\nu_{1,i} - \nu_{1,i-1}\right)$$

- Note that the first summation in the above equation is zero, because the frequency offsets at nodes 2 through *k* do not vary in time
  - Omitting that term, and comparing the above with the actual time the Sync message arrives at TC *k* produces for the phase error accumulation

$$\phi_{k,i} = T_r \nu_{1,i} + + \frac{T_r^2}{T_I}(k-2)\left(\nu_{1,i} - \nu_{1,i-1}\right)$$

- Inserting a sinusoidal frequency variation amplitude *A*, and relating this to the sinusoidal phase perturbation amplitude *B* through $B = At_r$ produces

$$\phi_{k,i} = B \frac{T_r}{T_I}(k-2)(1 - e^{-j\omega})e^{j\omega i}$$

Split Path Syntonization Scenario - 7

- Then, the magnitude of the phase error accumulation is

$$\left|\phi_{k,i}\right| = B\frac{T_r}{T_I}(k-2)\sqrt{2(1-\cos\omega)}$$

- The maximum amplitude occurs at $\omega = \pi$, i.e., when the perturbation period is twice the frequency adjustment time. The result is

$$\left.\left|\phi_{k,i}\right|\right|_{\max} = 2B\frac{T_r}{T_I}(k-2)$$

- The corresponding result for a TC syntonized using the conventional method is

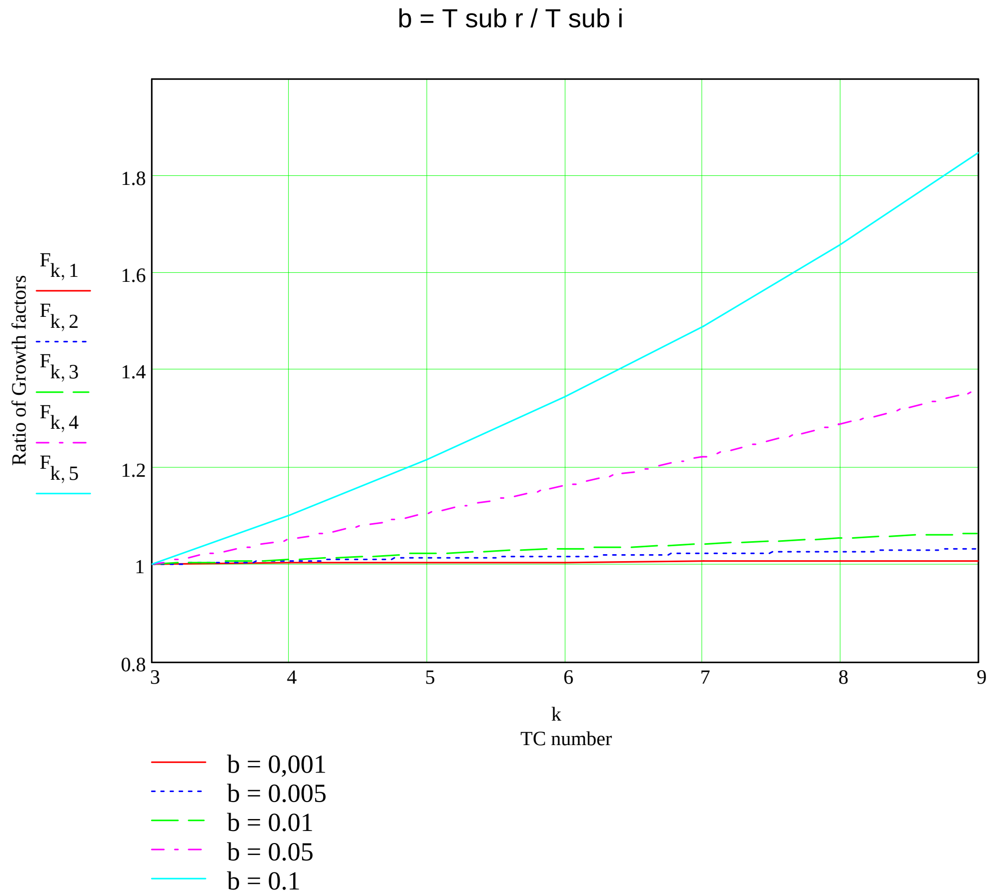
$$\left.\left|\phi_{k,i}\right|\right|_{\max} = B\cdot\left[\left(1+2\frac{T_r}{T_I}\right)^{k-2} - 1\right]$$

Split Path Syntonization Scenario - 8

- The ratio of the accumulation factor for the conventional approach to that of the split path approach is

$$F = \frac{\left(1 + 2\frac{T_r}{T_I}\right)^{k-2} - 1}{2(k-2)\frac{T_r}{T_I}}$$

We see that for $T_r << T_I$, the ratio is close to 1. When T_r is appreciable compared to T_I, the ratio can be large after a sufficient number of hops



Conclusions - 1

- If $T_r \ll T_l$, a very large number of hops is required for the effect of a frequency offset to result in appreciable frequency or phase offset accumulation
  - For ratio of 0.001, the frequency offset after 100 hops is 0.2% of the applied frequency perturbation amplitude; the unfiltered phase offset is 0.21 of the applied phase offset amplitude
- If T_r is appreciable compared to T_l, the accumulation is faster
  - For ratio of 0.1, the frequency offset accumulates to the level of the perturbation after 11 hops; after 7 hops it is approximately half the level of the perturbation
  - For ratio of 0.1, the unfiltered phase offset accumulation is about twice the applied perturbation amplitude after 7 hops
- For the split path syntonization scheme, the benefit is mainly for networks with T_r is appreciable compared to T_l and a sufficiently large number of hops

Conclusions - 2

- If it is decided that 802.1AS networks must tolerate sinusoidal phase wander applied at a clock, consideration should be given to the wander amplitude that must be tolerated
  - One amplitude to consider is 40 ns peak-to-peak, due to phase measurement granularity and beating of adjacent oscillators that are close in frequency
- Phase error accumulation will be reduced by endpoint filtering; this should be accounted for in considering ability of systems to tolerate sinusoidal wander

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7. Geoffrey M. Garner, *Analysis of Clock Synchronization Approaches for Residential Ethernet*, presentation to IEEE 802.3 Residential Ethernet SG, San Jose, September, 2005.
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9. Chuck Harrison, *Email to P1588 and IEEE 802.1 Reflectors*, March 12, 2007.