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TITLE: **Performance Comparision Between Direct Sequence and Slow Frequency Hopped Spread Spectrum Transmission in Indoor Multipath Fading Channels**

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AUTHOR:

Kwang-Cheng Chen
Department of Electrical Engineering
National Tsing Hua University
Hsinchu, Taiwan 30043,R.O.C.
TEL: +886-35-715131 ext. 4054
FAX: +886-35-715971
E-Mail: chenkc@ee.nthu.edu.tw

Computer and Communication Laboratories
Industrial Technology Research Institute
Bldg. 11, M000, 195 Chung Hsing Rd.,
Section 4, Chutung, Hsinchu,
Taiwan 31015, R.O.C.
TEL: +886-35-917282
FAX: +886-35-941447

Tsung-Cheng Wu
Chi-chao Chao
Department of Electrical Engineering
National Tsing Hua University
Hsinchu, Taiwan 30043,R.O.C.

Abstract

Applying spread spectrum transmission in indoor channels has attracted very much attention recently due to personal communication and wireless data communication. As the indoor channels are so dynamically changing and suffering from severe multipath fading, a good channel model from measurement results in always hard to achieve. Based on the latest channel model, we analyze the point-to-point spread spectrum system performance in indoor environments for direct-sequence and slow frequency-hopped systems that are two most likely spread spectrum systems for commercial wireless personal communication and wireless data communication. While both approaches are considered in real applications with processing gain from teens up to hundred or higher, our results precisely demonstrate the performance and the trade off between direct-sequence and slow frequency hopped spread spectrum communication systems in indoor multipath fading channels.

I Introduction

The spread spectrum communication systems attract more and more attention in the areas of indoor wireless data communication and personal communication services. Since the indoor environments are different from the urban fading channels, adequate statistical models for indoor multipath fading have been proposed in [4, 5, 10]. Some statistical measurements and resulting indoor multipath fading channels were introduced in [8, 9, 12]. Based on these latest measurement results , we target to analyze the point-to-point transmission performance of direct-sequence and slow frequency-hopped spread spectrum communication.

The performance of direct-sequence communications have already developed of the flat fading channel in [15, 18]. The performance of coherent direct-sequence for multipath fading channel [2] and frequency-selective, nonselective Rician fading channels [3], both employing BPSK, were analyzed by the characteristic function method. The error probability of Slow FH/binary FSK is approximated in [1] and fast FH/BFSK in [6]. In indoor wireless environment, the bit error probability based on multipath fading channel in fast FH/BFSK was studied in [6]. In addition, DS/DPSK with diversity consideration was studied in [7]. Performance simulations from measured channel data with two-ray of equal variance were considered in [11]. However, detailed analysis based on more precise indoor channel model(s) has not been available. Even more important questions in wireless personal communication system design such as how to choose between direct-sequence and frequency-hopped, and thus the appropriate system parameters, have not been answered.

In this paper, we will derive the average bit error probability on direct-sequence with BPSK modulation and slow frequency hopped system with BFSK modulation under indoor multipath fading channel based on a more precise channel model for indoor environments. We assume that the two systems do not use equalization, error correction techniques, and data bit rate from 1 Mbps to 10 Mbps [13]. From the numerical results of average bit error probability of two systems, the performance comparison of two systems for specific channel parameters has been made. This points out the critical direction to design appropriate spread-spectrum communication systems for wireless data communication or personal communication services.

This paper is organized as follows. In Section II, a channel model from the latest measurement results is introduced. Bit error rate analysis of DS system is treated in Section III. The analysis of slow frequency hopped system is in Section IV. Finally the numerical results are summaried in Section V.

II Channel Model

The multipath fading channel used for the performance analyses of spread spectrum communications in [15, 18] treat mostly as frequency-nonselective and slowly fading channel. A two-ray model with uniformly distributed delay-time was usually employed to describe multipath fading for the analyses of performance. Those channel models are not adequate for indoor channels and thus not suitable to analyze to reach any conclusion. To understand more about the indoor channel characteristics, many measurements were performed and they were reported in [4, 5, 8, 9, 10, 12, 14]. Some indoor multipath fading channel models were proposed in [4, 5, 19].

According to these measurements suggest, we modify the indoor channel model in [4, 5, 19] to approach the channel dynamics by introducing Doppler frequency shift. We describe the equivalent impulse response as follows

$$h(t) = \sum_{k=0}^{\infty} \beta_k \delta(t - \tau_k) \exp(j\theta_k + j\omega_{dk} t)$$

where β_k : amplitude of the k-th arrival which follows a Rayleigh distribution.

θ_k : phase difference between k-th arrival and local oscillator of receiver that is assumed to be well synchronized to the first arrival.

τ_k : time delay of the k-th arrival relative to the first arrival.
The arrivals are modeled as a Poisson process.

ω_{dk} : frequency shift for the k-th arrived path relative to the first arrival, which is independent of other later arrivals.

We assume that local oscillator of receiver precisely track the frequency and phase deviation of the first arrival. Thus, the second, the third,$\cdots$ arrivals will have frequency drifts and phase deviations relative to the receiver oscillator, denoted as ω_{dk}, θ_k. We assume that each arrival has its own frequency drift different to others comes from Doppler effect due to the channel dynamic characteristics. The k-th arrival has the frequency difference ω_{dk} comparing to the first arrival. Furthermore, based on the channel model, arrival times of paths follow a Poisson process with parameter λ. From the measured power profile, it is assumed that the average power of each arrival follows exponential decay relative to the first arrival and the decay depends on the time delay τ_k. That is, the k-th arrival have average power $\overline{\beta_0^2}\exp(-\tau_k/\Gamma)$, where $\overline{\beta_0^2}$ denotes the first arrival average power, and Γ denotes a time constant, the measurement result. We consider the behavior of system within $[0,T_b]$, symbol time-interval. Thus, in average, there are N paths arriving within T_b time-interval, and N=min($\lambda T_m, \lambda T_b$), where λ is Poisson process parameter and T_m is the multipath spread. We further assume that the arrivals after 2 symbol periods are small and can be neglected. This coincides with previous measurements such as [8]. In the subsequent sections about direct-sequence and slow frequency-hopped systems, we assume $N = \lambda T_b$ for general derivation. If $\lambda T_m \leq \lambda T_b$, then $N = \lambda T_m$ and there will be no intersymbol-interference.

III Direct Sequence System

A. Receiver System and Received Signal

Let the direct-sequence spread spectrum system employ binary phase shift keying (BPSK) modulation. Under ideal synchronization, the correlation receiver functions as Figure 1. Let the transmitted signal be $x(t;\theta) = c(t)d(t)\sqrt{2S}\cos[\omega_0 t + \theta]$, where $c(t)$ is the spreading

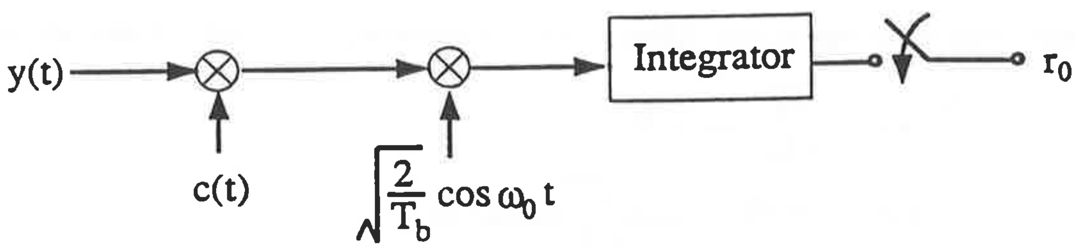

Figure 1: Receiver for Direct Sequence System

sequence, $d(t)$ represent data signal, and S is the power of signal. Then the received signal at the receiver input is represented by (after 2 symbol periods truncation),

$$
\begin{aligned}
y(t) &= \sum_{k=0}^{2N} \beta_k x(t-\tau_k, \theta_k, \omega_{dk}) \\
&= \beta_0 c(t)d(t)\sqrt{2S}\cos\omega_0 t \\
&\quad + \sum_{k=1}^{N} \beta_k c(t-\tau_k)d(t-\tau_k)\sqrt{2S}\cos[(\omega_0-\omega_{dk})(t-\tau_k)-\theta_k] \\
&\quad + \sum_{l=N+1}^{2N} \beta_l c(t+T_b-\tau_l)d(t+T_b-\tau_l)\sqrt{2S}\cos[(\omega_0-\omega_{dl})(t+T_b-\tau_l)-\theta_l] \\
&\quad + n(t) \qquad (1)
\end{aligned}
$$

where $n(t)$ is additive white Gaussian noise with zero mean and spectral density $N_0/2$. and

$$p(\beta_k) = \frac{\beta_k}{\rho_k^2}\exp(\frac{-\beta_k^2}{2\rho_k^2})$$

The first arrival is assumed to be well synchronized so that there does not exit frequency shift, time delay and phase ambiguity in the first term of (1). The second term of (1) is the multipath arrivals within current symbol period and τ_k, $k = 1, \cdots N$, are in $[0\ , T_b]$. τ_1 denotes the second arrival in time interval $[0,\ T_b]$, and so on.

In the third term of (1), we use the notation $t+T_b$ to represent the timing from previous symbol. Similarly, the amplitudes $\{\beta_l\}$ come from previous symbol that are delayed into current symbol time interval, $\{\beta_l\}$, $l = N+1, \cdots, 2N$. These β_l within $[T_b,\ 2T_b]$ are all random variables in our model. After coherent correlator, the output of sampler is

$$r_0 = \int_0^{T_b} c(t)y(t)\sqrt{\frac{2}{T_b}}\cos\omega_0 t\ dt \tag{2}$$

$$\begin{aligned} &= \int_0^{T_b} \beta_0 c^2(t)d(t)\sqrt{2S}\cos\omega_0 t\sqrt{\frac{2}{T_b}}\cos\omega_0 t\ dt \\ &+ \sum_{k=1}^{N}\beta_k\int_0^{T_b} c(t)c(t-\tau_k)d(t-\tau_k)\sqrt{2S}\cos[(\omega_0-\omega_{dk})(t-\tau_k)-\theta_k]\sqrt{\frac{2}{T_b}}\cos\omega_0 t\ dt \\ &+ \sum_{l=N+1}^{2N}\beta_l c(t+T_b-\tau_l)d(t+T_b-\tau_l)\sqrt{2S}\cos[(\omega_0-\omega_{dl})(t+T_b-\tau_l)-\theta_l] \\ &\quad \cdot\sqrt{\frac{2}{T_b}}\cos\omega_0 t\ dt \\ &+ \int_0^{T_b} n(t)c(t)\sqrt{\frac{2}{T_b}}\cos\omega_0 t\ dt \end{aligned} \tag{3}$$

After some algebraic manipulations, we have

$$\begin{aligned} r_0 &= \beta_0 d_0\sqrt{ST_b} \\ &+ \sum_{k=1}^{N}\beta_k d_0 2\sqrt{\frac{S}{T_b}}\int_0^{T_b} c(t)c(t-\tau_k)\{\frac{1}{2}[\cos\omega_{dk}t \\ &\qquad +(\omega_0-\omega_{dk})\tau_k+\theta_k]+\frac{1}{2}\cos[(2\omega_0-\omega_{dk})t-(\omega_0-\omega_{dk})\tau_k-\theta_k]\} \\ &+ \sum_{l=N+1}^{2N}\beta_l d_0' 2\sqrt{\frac{S}{T_b}}\int_0^{T_b} c(t)c(t+T_b-\tau_l)\{\frac{1}{2}\cos[\omega_{dl}t+(\omega_0-\omega_{dl})\tau_l-T_b(\omega_0-\omega_{dl})+\theta_l] \\ &\qquad +\frac{1}{2}\cos[(2\omega_0-\omega_{dl})t-(\omega_0-\omega_{dl}\tau_l-\theta_l]\} \\ &+ \int_0^{T_b} n(t)c(t)\sqrt{\frac{2}{T_b}}\cos\omega_0 t\ dt \end{aligned} \tag{4}$$

where $d_0, d_0' \in \{-1, 1\}$, d_0 represent the current data bit in $[0,\ T_b]$, d_0' is the previous data bit just before d_0.

In our interesting cases, we assume $\mid \omega_{dk}T_b \mid \ll 1$. Without loss of generality, (4) can be further simplified as

$$
\begin{aligned}
r_0 &= \beta_0 d_0 \sqrt{ST_b} \\
&+ \sum_{k=1}^{N} \beta_k \sqrt{\frac{S}{T_b}} \cos[(\omega_0 - \omega_{dk})\tau_k + \theta_k] \int_0^{T_b} d(t)c(t)c(t-\tau_k)\,dt \\
&- \sum_{k=1}^{N} \beta_k \frac{S}{T_b} \omega_{dk} T_b \sin[(\omega_0 - \omega_{dk})\tau_k + \theta_k] \int_0^{T_b} d(t)c(t)c(t-\tau_k)\,dt \\
&+ \sum_{l=N+1}^{2N} \beta_l \sqrt{\frac{S}{T_b}} \cos[(\omega_0 - \omega_{dl})(\tau_l - T_b) + \theta_l] \int_0^{T_b} d'(t)c(t)c(t+T_b-\tau_l)\,dt \\
&- \sum_{l=N+1}^{2N} \beta_l \sqrt{\frac{S}{T_b}} \omega_{dl} T_b) \sin[(\omega_0 - \omega_{dl})(\tau_l - T_b) + \theta_l] \int_0^{T_b} d'(t)c(t)c(t+T_b-\tau_l)\,dt \\
&+ \int_0^{T_b} n(t)c(t)\sqrt{\frac{2}{T_b}} \cos\omega_0 t\,dt
\end{aligned} \tag{5}
$$

Because the delayed spreading sequence $c(t-\tau_k)$ multiplying with spreading sequence $c(t)$ will introduce extra terms, we reformulate the integration term in the follow,

$$
\begin{aligned}
&\int_0^{T_b} d(t-\tau_k)c(t)c(t-\tau_k)\,dt \\
&= d_0' \int_0^{\tau_k} c(t)c(t-\tau_k)\,dt + d_0 \int_{\tau_k}^{T_b} c(t)c(t-\tau_k)\,dt \\
&= d_0' \int_0^{\tau_k} c(t)c(t-\tau_k)\,dt + d_0 \int_0^{T_b} c(t)c(t-\tau_k)\,dt - d_0 \int_0^{\tau_k} c(t)c(t-\tau_k)\,dt \\
&= d_0 T_b R_c(\tau_k) + (d_0' - d_0) T_b \theta_c(\tau_k)
\end{aligned} \tag{6}
$$

where

$$R_c(\tau_k) = \frac{1}{T_b}\int_0^{T_b} c(t)c(t-\tau_k)dt \quad \text{autocorrelation of } c(t)$$

$$
R_c(\tau) = \begin{cases} 1 - \frac{\tau}{T_c}(1+\frac{1}{L}) & 0 \le \tau \le T_c \\ -\frac{1}{L} & T_c < \tau < (L-1)T_c \\ \frac{\tau}{T_c}(1+\frac{1}{L}) - \frac{1}{L} & (L-1) \le \tau < LT_c \end{cases}
$$

and L denotes the period of spreading code.

$$\theta_c(\tau_k) := \frac{1}{T_b}\int_0^{\tau_k} c(t)c(t-\tau_k)dt$$

Similarly,

$$\int_0^{T_b} d(t+T_b-\tau_l)c(t)c(t+T_b-\tau_l)\ dt = d_0'T_bR_c(\tau_l-T_b)+(d_0''-d_0')T_b\theta_c(\tau_l-T_b) \tag{7}$$

where d_0'' is the data bit before d_0'. Finally, we can get

$$\begin{aligned} r_0 &= \beta_0 d_0\sqrt{ST_b} \\ &+ \sqrt{ST_b}\sum_{k=1}^{N}\beta_k\cos[(\omega_0-\omega_{dk})\tau_k+\theta_k]\cdot[d_0R_c(\tau_k)+(d_0'-d_0)\theta_c(\tau_k)] \\ &- \sqrt{ST_b}\sum_{k=1}^{N}\beta_k\sin[(\omega_0-\omega_{dk})\tau_k+\theta_k]\omega_{dk}T_b\cdot[d_0R_c(\tau_k)+(d_0'-d_0)\theta_c(\tau_k)] \\ &+ \sqrt{ST_b}\sum_{l=N+1}^{2N}\beta_l\cos[(\omega_0-\omega_{dl})(\tau_l-T_b)+\theta_l]\cdot[d_0'R_c(\tau_l-T_b)+(d_0''-d_0')\theta_c(\tau_l-T_b)] \\ &- \sqrt{ST_b}\sum_{l=N+1}^{2N}\beta_l\sin[(\omega_0-\omega_{dl})(\tau_l-T_b)+\theta_l]\omega_{dk}T_b\cdot[d_0'R_c(\tau_l-T_b)+(d_0''-d_0')\theta_c(\tau_l-T_b)] \\ &+\eta \end{aligned} \tag{8}$$

where $\eta = \int_0^{T_b} n(t)c(t)\sqrt{\frac{2}{T_b}}\cos\omega_0 t\ dt$.

B. Average Bit Error Probability for DS System

We know that $\beta\cos\theta, \beta\sin\theta$ are Gaussian distributed random variables with zero mean, variance σ^2 and both are statistically independent when β follows a Rayleigh distribution and θ follows a uniformly distribution. The p.d.f.'s of β and θ are,

$$f(\beta) = \frac{\beta}{\sigma^2}\exp(\beta^2/2\sigma^2) \qquad f(\theta) = \frac{1}{2\pi} \quad 0 \le \theta \le 2\pi$$

$\beta\cos(C+\theta), \beta\sin(C+\theta)$ also have the same property when C is a constant.

Using the property, we derive that r_0 is Gaussian distribution with mean $E(r_0)$, variance $Var(r_0)$ given β_0, ω_{dk} and τ_k.

Let $B_k = \sqrt{ST_b}\beta_k \cos[(\omega - \omega_{dk})\tau_k + \theta_k]$, $A = \beta_0\sqrt{ST_b}$. We have

$$E(r_0) = A \tag{9}$$

$$\begin{aligned} Var(r_0) &= E(r_0^2) - E(r_0)^2 \\ &= \sum_{k=1}^{N}[d_0 R_c(\tau_k) + (d_0' - d_0)\theta_c(\tau_k)]^2 Var(B_k) \\ &+ \sum_{k=1}^{N}[d_0 R_c(\tau_k) + (d_0' - d_0)\theta_c(\tau_k)]^2 \omega_{dk}^2 T_b^2 Var(B_k) \\ &+ \sum_{l=N+1}^{2N}[d_0' R_c(\tau_l - T_b) + (d_0'' - d_0')\theta_c(\tau_l - T_b)]^2 Var(B_k) \\ &+ \sum_{l=N+1}^{2N}[d_0' R_c(\tau_l - T_b) + (d_0'' - d_0')\theta_c(\tau_l - T_b)]^2 \omega_{dl}^2 T_b^2 Var(B_k) \\ &+ Var(\eta) \end{aligned} \tag{10}$$

where $Var(B_k) = ST_b\rho_0^2 = \frac{1}{2}ST_b\overline{\beta_0^2}\exp(-\tau_k/\Gamma)$, which is a result of measurement in indoor wireless channel from [4] , that is $\overline{\beta_k^2} = \overline{\beta_0^2}\exp(-\tau_k/\Gamma)$, $\overline{\beta_k^2} = 2\rho_k^2$, and ST_b is the signal energy. Since $| \omega_{dk}^2 T_b^2 | \ll 1$, The second and forth terms are small enough to be neglected compared with the first and third terms in (10).

$$\begin{aligned} Var(\eta) &= E(\eta^2) = E[\int_0^{T_b} n(t)c(t)\sqrt{\frac{2}{T_b}}\cos\omega_0 t \, dt \int_0^{T} n(s)c(s)\sqrt{\frac{2}{T_b}}\cos\omega_0 s \, ds] \\ &= \int_0^T \int_0^T E[n(t)n(s)]\frac{2}{T_b}c(t)c(s)[\frac{1}{2}\cos\omega_0(t-s) + \frac{1}{2}\cos(\omega_0 t + \omega_o s)]ds \, dt \\ &= N_0/2 \end{aligned} \tag{11}$$

Then the p.d.f of r_0 is

$$f(r_0) = \frac{1}{\sqrt{2\pi Var(r_0)}}\exp[-(r_0 - E(r_0))^2/2Var(r_0)]dr_0 \tag{12}$$

The conditional error probability given β_0, τ_k is

$$P_b|_{\beta_0,\underline{\tau}_k} = P\{r_0 \le 0 \mid d_0 = 1, d_0' = 1, d_0'' = 1\}P\{d_0' = 1, d_0'' = 1\}$$

$$
\begin{aligned}
&+ \quad P\{r_0 \le 0 \mid d_0 = 1, d_0^{'} = 1, d_0^{''} = -1\}P\{d_0^{'} = 1, d_0^{''} = -1\} \\
&+ \quad P\{r_0 \le 0 \mid d_0 = 1, d_0^{'} = -1, d_0^{''} = 1\}P\{d_0^{'} = -1, d_0^{''} = 1\} \\
&+ \quad P\{r_0 \le 0 \mid d_0 = 1, d_0^{'} = -1, d_0^{''} = -1\}P\{d_0^{'} = -1, d_0^{''} = -1\} \qquad (13)
\end{aligned}
$$

where $P\{d_0^{'} = 1, d_0^{''} = 1\} = P\{d_0^{'} = 1, d_0^{''} = -1\} = P\{d_0^{'} = -1, d_0^{''} = 1\} = P\{d_0^{'} = -1, d_0^{''} = -1\}$=1/4, under the assumption that the source emits symbol independently and with equal probability.

Denote $\underline{\tau_k} = \tau_1, \tau_2, \cdots, \tau_{2N}$, $d\underline{\tau_k} = d\tau_1 d\tau_2 \cdots d\tau_{2N}$, and $p\{\underline{\tau_k}\} = p(\tau_1, \tau_2, \cdots, \tau_{2N})$. Let $p\{\underline{\tau_k}\}$ be the joint probability of $\tau_1, \tau_2, \cdots, \tau_{2N}$, we get the conditional error probability.

$$
\begin{aligned}
P & \quad \{r_0 \le 0 \mid d_0 = 1, d_0^{'} = 1, d_0^{''} = 1\} \\
&= \int_{\underline{\tau_k}} P\{r_0 \le 0 \mid d_0 = 1, d_0^{'} = -1, d_0^{''} = 1, \underline{\tau_k}\} p\{\underline{\tau_k}\} d\underline{\tau_k} \\
&= \int_{\underline{\tau_k}} \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi Var(r_0)}} \exp[\frac{-(r_0 - E(r_0))^2}{2Var^{(1)}(r_0)}] dr_0 \; p\{\underline{\tau_k}\} d\underline{\tau_k}
\end{aligned}
$$

According to four kinds of combinations of $d_0, d_0^{'}$ and $d_0^{''}$, we describe those variances with $Var^{(i)}(r_0)$, for i=1 to 4, as follows.

$$
\begin{aligned}
Var^{(1)}(r_0) &= \sum_{k=1}^{N} R_c^2(\tau_k) Var(B_k) + \sum_{l=N+1}^{2N} R_c^2(\tau_l - T_b) Var(B_l) + N_0/2 \\
&= E_b \rho_0^2 \{\sum_{k=1}^{N} R_c^2(\tau_k) \exp(\frac{-\tau_k}{\Gamma}) + \sum_{l=N+1}^{2N} R_c^2(\tau_l - T_b) \exp(\frac{-\tau_l}{\Gamma})\} + N_0/2 \\
&= E_b \rho_0^2 K_1(\underline{\tau_k}) + N_0/2 \qquad (14) \\
Var^{(2)}(r_0) &= \sum_{k=1}^{N} R_c^2(\tau_k) Var(B_k) + \sum_{l=N+1}^{2N} [R_c(\tau_l - T_b) - 2\theta_c(\tau_l - T_b)]^2 Var(B_l) + N_0/2 \\
&= E_b \rho_0^2 \{\sum_{k=1}^{N} R_c^2(\tau_k) \exp(\frac{-\tau_k}{\Gamma}) + \sum_{l=N+1}^{2N} [R_c(\tau_l - T_b) - 2\theta_c(\tau_l - T_b)]^2 \exp(\frac{-\tau_l}{\Gamma})\} + N_0/2 \\
&= E_b \rho_0^2 K_2(\underline{\tau_k}) + N_0/2 \qquad (15) \\
Var^{(3)}(r_0) &= \sum_{k=1}^{N} [R_c(\tau_k) - 2\theta_c(\tau_k)]^2 Var(B_k) + \sum_{l=N+1}^{2N} [R_c(\tau_l - T_b) + 2\theta_c(\tau_l - T_b)]^2 Var(B_l) + N_0/2 \\
&= E_b \rho_0^2 \{\sum_{k=1}^{N} [R_c(\tau_k) - 2\theta_c(\tau_k - T_b)]^2 \exp(\frac{-\tau_k}{\Gamma}) + \sum_{l=N+1}^{2N} [R_c(\tau_l - T_b) + 2\theta_c(\tau_l - T_b)]^2
\end{aligned}
$$

$$\cdot \exp(\frac{-\tau_l}{\Gamma})\} + N_0/2$$

$$= E_b\rho_0^2 K_3(\underline{\tau}_k) + N_0/2 \quad (16)$$

$$Var^{(4)}(r_0) = \sum_{k=1}^{N}[R_c(\tau_k) - 2\theta_c(\tau_k)]^2 Var(B_k) + \sum_{l=N+1}^{2N} R_c^2(\tau_l - T_b) + N_0/2$$

$$= E_b\rho_0^2\{\sum_{k=1}^{N}[R_c(\tau_k) - 2\theta_c(\tau_k)]^2 \exp(\frac{-\tau_k}{\Gamma}) + \sum_{l=N+1}^{2N} R_c^2(\tau_l - T_b)\exp(\frac{-\tau_l}{\Gamma})\} + N_0/2$$

$$= E_b\rho_0^2 K_4(\underline{\tau}_k) + N_0/2 \quad (17)$$

Define

$$Q(\frac{x-\eta}{\sigma}) = \frac{1}{\sqrt{2\pi}\sigma}\int_x^\infty \exp[-(s-\eta)^2/2\sigma^2]ds$$

The conditional average bit error probability over $\beta_0, \underline{\tau}_k$ is,

$$P_b\,|_{\beta_0,\underline{\tau}_k} = \frac{1}{4}\sum_{i=1}^{4} Q(\sqrt{\frac{A^2}{Var^{(i)}(r_0)}}) \quad (18)$$

Let $E_b = ST_b$, $A^2 = \beta_0^2 E_b$. To obtain average error probability, we must average $P_b\,|_{\beta_0^2,\underline{\tau}_k}$ over β_0^2 and $\underline{\tau}_k$. That is,

$$P_b = \frac{1}{4}\int_{\underline{\tau}_k}\int_{\beta_0^2} P_b\,|_{\beta_0^2,\underline{\tau}_k}\, p(\beta_0^2)p\{\underline{\tau}_k\}d\beta_0^2 d\underline{\tau}_k$$

$$= \frac{1}{4}\int_{\underline{\tau}_k}\sum_{i=1}^{4}\int_{\gamma_i} P_b\,|_{\gamma_i,\underline{\tau}_k}\, p(\gamma_i)p\{\underline{\tau}_k\}d\gamma_i\, d\underline{\tau}_k \quad (19)$$

where

$$\gamma_i = \frac{\beta_0^2 E_b}{2Var^{(i)}(r_0)}, \text{ and } \quad P_b\,|_{\gamma_i,\underline{\tau}_k} = \frac{1}{2}erfc(\sqrt{\gamma_i}),\ i = 1,2,3,4 \quad (20)$$

and the p.d.f of γ_i is

$$p(\gamma_i) = \frac{1}{\overline{\gamma_i}}exp(\frac{-\gamma_i}{\overline{\gamma_i}}),\quad \gamma_i \geq 0 \quad (21)$$

where $\overline{\gamma_i}$ is the average value of γ_i. For example i=1,

$$\gamma_1 = \frac{\beta_0^2 E_b}{2E_b\rho_0^2\{\sum_{k=1}^{N} R_c^2(\tau_k)\exp(\frac{-\tau_k}{\Gamma}) + \sum_{l=N+1}^{2N} R_c^2(\tau_l - T_b)\exp(\frac{-\tau_l}{\Gamma})\} + N_0} \quad (22)$$

$$\overline{\gamma}_1 = \frac{\overline{\beta_0^2} E_b}{2E_b\rho_0^2[\sum_{k=1}^{N} R_c^2(\tau_k)\exp(\frac{-\tau_k}{\Gamma}) + \sum_{l=N+1}^{2N} R_c^2(\tau_l - T_b)\exp(\frac{-\tau_l}{\gamma})] + N_0} \quad (23)$$

With the help of integration result in [18], the average probability of error is

$$P_b = \frac{1}{8}\int_{\tau_k}\sum_{i=1}^{4}\{1 - \sqrt{\frac{2\frac{E_b}{N_0}\rho_0^2}{2\frac{E_b}{N_0}\rho_0^2[1 + K_i(\tau_k)] + 1}}\}p\{\tau_k\}d\tau_k \tag{24}$$

C. Rician channel

If the first arrival is the primary signal which may be line of sight, we assume the first arrival is a constant value instead of a Rayleigh distributed random variable. That is, $A = \sqrt{E_b}$. Such a channel model is known as the Rician channel in [3, 6]. The consequent average bit error probability for Rician channel will be

$$\begin{aligned} P_b &= \frac{1}{4}\int_{\tau_k}\sum_{i=1}^{4}Q(\frac{A}{\sqrt{Var^{(i)}(r_0)}})p\{\tau_k\}\, d\tau_k \\ &= \frac{1}{4}\int_{\tau_k}\sum_{i=1}^{4}Q(\sqrt{\frac{\frac{E_b}{N_0}}{\frac{E_b}{N}K_i(\tau_k) + 1}})p\{\tau_k\}\, d\tau_k \end{aligned} \tag{25}$$

due to $\overline{\beta_k^2} = E_b \exp(-\tau_k/\Gamma)$ and $\rho_k^2 = \frac{1}{2}E_b \exp(-\tau_k/\Gamma)$.

After $x(t)$ passing through bandpass filter, with some algebraic manipulations, $y(t)$ is

$$\begin{aligned} y(t) &= \sum_{k=0}^{N}\sum_{m=-\infty}^{\infty} \beta_k\, p(t-mT-\tau_k)\sqrt{2S}\cos[(\omega_{IF}+\omega_i+\omega_{dk})t-(\omega_0+\omega_j+\omega_i+\omega_{dk})\tau_k+\theta_k] \\ &+ \sum_{l=N+1}^{2N}\sum_{m=-\infty}^{\infty} \beta_l\, p(t-mT-\tau_l)\sqrt{2S}\cos[(\omega_{IF}+\omega_i+\omega_{dl})t+(\omega_0+\omega_j+\omega_i+\omega_{dl})(T-\tau_l)+\theta_l] \\ &+ n(t)2\cos(\omega_0-\omega_{IF}+\omega_j)t \end{aligned} \quad (28)$$

Then, we shall consider the outputs of integrators in I-Q channels which operate from $t=0$ to $t=T$ respectively, i.e., y_{c1}, y_{s1} for ω_1 detector, y_{c2}, y_{s2} for ω_2 detector. In the hopped narrowband ω_j, there are four kinds of outputs of I-Q integrators due to d_0, d_0', d_0'', defined in section III. We assume data "1" is modulated by ω_1, data "0" by ω_2. Now assume system transmit signal with ω_1. Due to different d_0, d_0', d_0'', there are four cases about $y_{c1}, y_{s1}, y_{c2}, y_{s2}$ listed in the following.

Case I: When $d_0 = d_0' = d_0''=1$, y_{c1} and y_{s1} become

$$\begin{aligned} y_{c1} &= \int_0^T y(t)2\cos(\omega_{IF}+\omega_1)t\, dt \\ &= \sqrt{2S}\int_0^T \sum_{k=0}^{N}\beta_k\cos[\omega_{dk}t-(\omega_0+\omega_j+\omega_1+\omega_{dk})\tau_k+\theta_k]\, dt \\ &+ \sqrt{2S}\int_0^T \sum_{l=N+1}^{2N}\beta_l\cos[\omega_{dl}t+(\omega_0+\omega_j+\omega_1+\omega_{dl})(T-\tau_l)+\theta_l]\, dt \\ &+ \eta_1 \end{aligned} \quad (29)$$

and,

$$\begin{aligned} y_{s1} &= -\sqrt{2S}\int_0^T \sum_{k=1}^{N}\beta_k\sin[\omega_{dk}t-(\omega_0+\omega_j+\omega_1)\tau_k+\theta_k]\, dt \\ &+\sqrt{2S}\int_0^T \sum_{l=N+1}^{2N}\beta_l\sin[\omega_{dl}t+(\omega_0+\omega_j+\omega_1)(T-\tau_l)+\theta_l]\, dt \\ &+\nu_1 \end{aligned} \quad (30)$$

where $i=1,2$,

$$\eta_i = \int_0^T n(t)2\cos(\omega_0-\omega_{IF}+\omega_j)t\cdot 2\cos(\omega_{IF}+\omega_i)t\, dt \quad (31)$$

$$\nu_i = \int_0^T n(t)2\cos(\omega_0-\omega_{IF}+\omega_j)t\cdot 2\sin(\omega_{IF}+\omega_i)t\, dt \quad (32)$$

and $y_{c2} = \eta_2$, $y_{s2} = \nu_2$.

Since $|\, \omega_{dk}T \,| \ll 1$ in our interested cases, $\cos\omega_{dk}t \approx 1$, $\sin\omega_{dk}t \approx \omega_{dk}t$ in (29), we get

$$
\begin{aligned}
y_{c1} &= \sqrt{2S}T \sum_{k=0}^{N} \beta_k \cos[(\omega_0 + \omega_j + \omega_1 + \omega_{dk})\tau_k - \theta_k] \\
&+ \sqrt{2S} \sum_{k=0}^{N} \beta_k \omega_{dk} T \sin[(\omega_0 + \omega_j + \omega_1 + \omega_{dk})\tau_k - \theta_k] \\
&+ \sqrt{2S}T \sum_{l=N+1}^{2N} \beta_l \cos[(\omega_0 + \omega_j + \omega_1 + \omega_{dl})(T - \tau_l) - \theta_l] \\
&- \sqrt{2S} \sum_{l=N+1}^{2N} \beta_l \omega_{dl} T \sin[(\omega_0 + \omega_j + \omega_1 + \omega_{dl})(T - \tau_l) - \theta_l] \\
&+ \eta_1 \qquad (33)
\end{aligned}
$$

The second and forth terms in (33) can be neglected since they are small. y_{s1}, y_{c2}, y_{s2} have similar situation to neglect such small terms.

Case II: When $d_0 = 1, d_0' = 1, d_0'' = -1$, y_{c1} and y_{c2} become

$$
\begin{aligned}
y_{c1} &= \sqrt{2S} \int_0^T \sum_{k=0}^{N} \beta_k \cos[\omega_{dk}t - (\omega_0 + \omega_j + \omega_1 + \omega_{dk})\tau_k + \theta_k]\, dt \\
&+ \sqrt{2S} \int_{\tau_l - T}^{T} \sum_{l=N+1}^{2N} \beta_l \cos[\omega_{dl}t + (\omega_0 + \omega_j + \omega_1 + \omega_{dl})(T - \tau_l) + \theta_l]\, dt \\
&+ \eta_1 \qquad (34)
\end{aligned}
$$

$$
\begin{aligned}
y_{c2} &= \sqrt{2S} \int_0^{\tau_l - T} \sum_{l=N+1}^{2N} \beta_l \cos[\omega_{dl}t + (\omega_0 + \omega_j + \omega_2 + \omega_{dl})(T - \tau_l) + \theta_l]\, dt \\
&+ \eta_2 \qquad (35)
\end{aligned}
$$

Since $|\, \omega_{dk}T \,| \ll 1$, $\cos\omega_{dk}t \approx 1$, $\sin\omega_{dk}t \approx \omega_{dk}t$, we obtain

$$
\begin{aligned}
y_{c1} &= \sqrt{2S}T \sum_{k=0}^{N} \beta_k \cos[(\omega_0 + \omega_j + \omega_1 + \omega_{dk})\tau_k - \theta_k] \\
&+ \sqrt{2S} \sum_{k=0}^{N} \beta_k \omega_{dk} T \sin[(\omega_0 + \omega_j + \omega_1 + \omega_{dk})\tau_k - \theta_k] \\
&+ \sqrt{2S}T \sum_{l=N+1}^{2N} (2 - \frac{\tau_l}{T}) \beta_l \cos[(\omega_0 + \omega_j + \omega_1 + \omega_{dl})(T - \tau_l) - \theta_l]
\end{aligned}
$$

IV Slow Frequency Hopped System

A. Receiver System and Received Signal

Another attractive way to implement spread spectrum systems for wireless personal communication is slow frequency hopped (SFH). In this section, we would like to analyze the performance of applying SFH in indoor channels by adopting the channel model in section II. We assume that modulation for SFH is binary frequency shift keying (BFSK), and a non-coherent receiver with square-law envelope detection is used. This is depicted in Figure 2 and Figure 3.

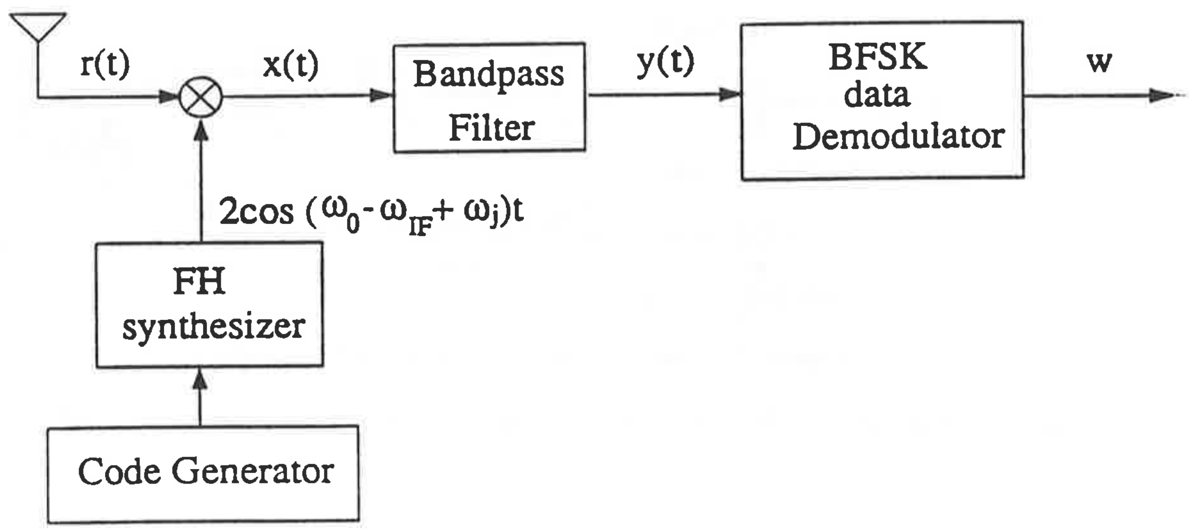

Figure 2: Receiver structure for slow frequency hopped communication

In ideal case with perfect hopped frequency recovery, the SFH synthesizer outputs the same narrowband transmission whose carrier frequency is $\omega_0 + \omega_j$ within the received signal hopped frquency band. First of all, let us consider the transmitter output to be

$$\sqrt{2S} \sum_{m=-\infty}^{\infty} p(t - mT) \ \cos(\omega_0 + \omega_j + \omega_i)t \tag{26}$$

where ω_0 denotes carrier frequency; ω_i denotes the frequency deviation to ω_0 according to input symbols, $i = 1, \cdots, M$, M=2^K; K is the number of bits per symbol, (M=1 in the following analysis); ω_j is the center frequency of hopped narrowband, $j=1,\cdots,2^k$; k denotes the number of shift registers to generate spreading sequences; unitary baseband pulse waveform is denoted by

$$p(t) = \begin{cases} 1 & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

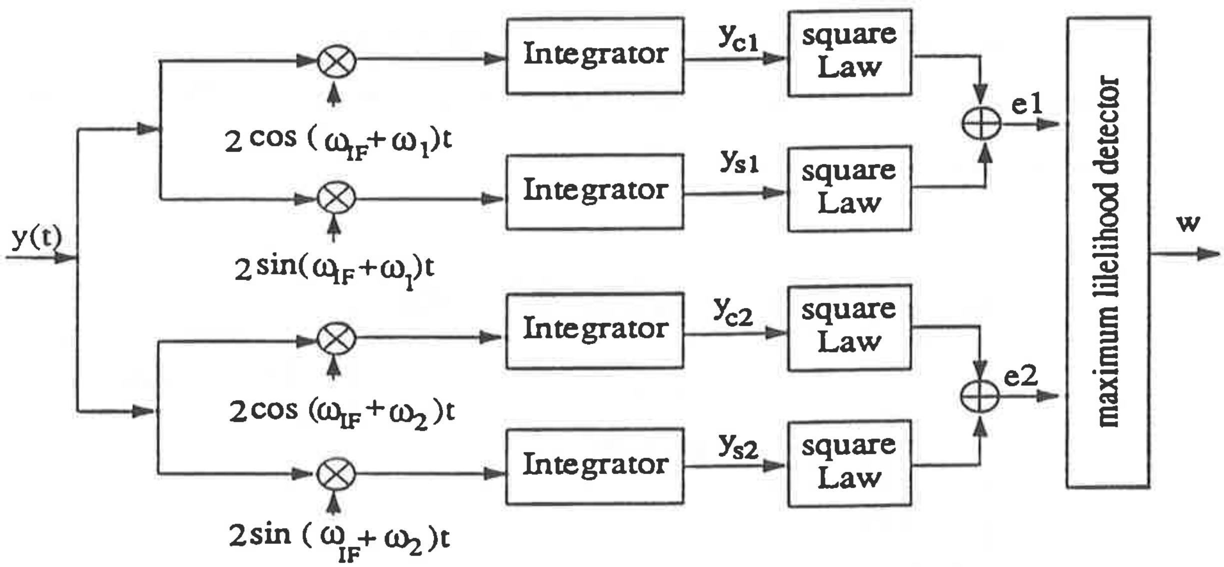

Figure 3: Binary FSK non-coherent demodulator

Applying the same indoor multipath fading channel in section II, we describe the received signal $r(t)$ as

$$\begin{aligned} r(t) &= \sum_{k=0}^{N} \sum_{m=-\infty}^{\infty} \beta_k\, p(t - mT - \tau_k)\sqrt{2S}\cos[(\omega_0 + \omega_j + \omega_i + \omega_{dk})(t - \tau_k) + \theta_k] \\ &+ \sum_{l=N+1}^{2N} \sum_{m=-\infty}^{\infty} \beta_l\, p(t - mT - \tau_l)\sqrt{2S}\cos[(\omega_0 + \omega_i + \omega_j + \omega_{dl})(t + T - \tau_l) + \theta_l] \end{aligned} \quad (27)$$

Because SFH system, the intersymbol interference delayed from previous symbol still exists as the case of the DS system. Let T denotes the symbol period equal to T_b, the bit period of DS system.

Then, $x(t) = r(t)2\cos(\omega_0 - \omega_{IF} + \omega_j)t$.

$$-\sqrt{2S}\sum_{l=N+1}^{2N}(2-\frac{\tau_l}{T})\beta_l\omega_{dl}T\sin[(\omega_0+\omega_j+\omega_1+\omega_{dl})(T-\tau_l)-\theta_l] \quad (36)$$

$$+\eta_1 \quad (37)$$

and y_{s1} has the same form as y_{c1} except sin(·) replace cos(·), and y_{s2} has similar form to y_{c2}. We can neglect the second and forth terms of (37)

$$\begin{aligned} y_{c1} &= \sqrt{2S}T\sum_{k=0}^{N}\beta_k\cos[(\omega_0+\omega_j+\omega_1+\omega_{dk})\tau_k-\theta_k] \\ &+\sqrt{2S}T\sum_{l=N+1}^{2N}(2-\frac{\tau_l}{T})\beta_l\cos[(\omega_0+\omega_j+\omega_1+\omega_{dl})(T-\tau_l)-\theta_l] \\ &+\eta_1 \end{aligned} \quad (38)$$

$$y_{c2}=\sqrt{2S}T\sum_{l=N+1}^{2N}(\frac{\tau_l}{T}-1)\beta_l\cos[(\omega_0+\omega_j+\omega_2+\omega_{dl})(T-\tau_l)-\theta_l]+\eta_2 \quad (39)$$

Case III : When $d_0=1, d_0'=-1, d_0''=1$, y_{c1} and y_{c2} become

$$\begin{aligned} y_{c1} &= \sqrt{2S}\int_{\tau_k}^{T}\sum_{k=0}^{N}\beta_k\cos[\omega_{dk}t-(\omega_0+\omega_j+\omega_1+\omega_{dk})\tau_k+\theta_k]\,dt \\ &+\sqrt{2S}\int_0^{\tau_l-T}\sum_{l=N+1}^{2N}\beta_l\cos[\omega_{dl}t+(\omega_0+\omega_j+\omega_1+\omega_{dl})(T-\tau_l)+\theta_l]\,dt \\ &+\eta_1 \end{aligned} \quad (40)$$

$$\begin{aligned} y_{c2} &= \sqrt{2S}\int_0^{\tau_k}\sum_{k=1}^{N}\beta_k\cos[\omega_{dk}t-(\omega_0+\omega_j+\omega_2+\omega_{dk})\tau_k+\theta_l]\,dt \\ &+\sqrt{2S}\int_{\tau_l-T}^{T}\sum_{l=N+1}^{2N}\beta_l\cos[\omega_{dl}t+(\omega_0+\omega_j+\omega_2+\omega_{dl})(T-\tau_l)+\theta_l]\,dt \\ &+\eta_2 \end{aligned} \quad (41)$$

Based on the same reasons as before, we obtain

$$\begin{aligned} y_{c1} &= \sqrt{2S}T\sum_{k=0}^{N}(1-\frac{\tau_k}{T})\beta_k\cos[(\omega_0+\omega_j+\omega_1+\omega_{dk})\tau_k-\theta_k] \\ &+\sqrt{2S}T\sum_{l=N+1}^{2N}(\frac{\tau_l}{T}-1)\beta_l\cos[(\omega_0+\omega_j+\omega_1+\omega_{dl})(T-\tau_l)-\theta_l] \\ &+\eta_1 \end{aligned} \quad (42)$$

$$
\begin{aligned}
y_{c2} &= \sqrt{2S}T\sum_{k=1}^{N}\frac{\tau_k}{T}\beta_k\cos[(\omega_0+\omega_j+\omega_2+\omega_{dk})\tau_k-\theta_k] \\
&+\sqrt{2S}T\sum_{l=N+1}^{2N}(2-\frac{\tau_l}{T})\beta_l\cos[(\omega_0+\omega_j+\omega_2+\omega_{dl})(T-\tau_l)-\theta_l] \\
&+\eta_2 \qquad (43)
\end{aligned}
$$

Case IV : When $d_0 = 1, d_0' = -1, d_0'' = -1$, y_{c1} and y_{c2} become

$$
\begin{aligned}
y_{c1} &= \sqrt{2S}\int_{\tau_k}^{T}\sum_{k=0}^{N}\beta_k\cos[\omega_{dk}t-(\omega_0+\omega_j+\omega_1+\omega_{dk})\tau_k+\theta_k]\,dt \\
&+\eta_1 \qquad (44)
\end{aligned}
$$

$$
\begin{aligned}
y_{c2} &= \sqrt{2S}\int_{0}^{\tau_k}\sum_{k=1}^{N}\beta_k\cos[\omega_{dk}t-(\omega_0+\omega_j+\omega_2+\omega_{dk})\tau_k+\theta_k]\,dt \\
&+\sqrt{2S}\int_{0}^{T}\sum_{l=N+1}^{2N}\beta_l\cos[\omega_{dl}t+(\omega_0+\omega_j+\omega_2+\omega_{dl})(T-\tau_l)+\theta_l]\,dt \\
&+\eta_2 \qquad (45)
\end{aligned}
$$

As earlier cases,

$$
\begin{aligned}
y_{c1} &= \sqrt{2S}T\sum_{k=0}^{N}(1-\frac{\tau_k}{T})\beta_k\cos[(\omega_0+\omega_j+\omega_1+\omega_{dk})\tau_k-\theta_k] \\
&+\eta_1 \qquad (46)
\end{aligned}
$$

$$
\begin{aligned}
y_{c2} &= \sqrt{2S}T\sum_{k=1}^{N}\frac{\tau_k}{T}\beta_k\cos[(\omega_0+\omega_j+\omega_2+\omega_{dk})\tau_k-\theta_k] \\
&+\sqrt{2S}T\sum_{l=N+1}^{2N}\beta_l\cos[(\omega_0+\omega_j+\omega_2+\omega_{dl})(T-\tau_l)-\theta_l] \qquad (47) \\
&+\eta_2 \qquad (48)
\end{aligned}
$$

In cases II,III,IV, y_{s1}, y_{s2} have similar form as y_{c1}, y_{c2}, respectively. To make this paper concise, we do not derive details here.

B. Average Bit Error Probability for Slow FH System

We use the property that $\beta\cos\theta$, $\beta\sin\theta$ are Gausian distributed random variables to derive the probability distribution. That is, y_{c1}, y_{s1}, y_{c2} and y_{s2} are Gaussian distributed random variables with zero mean and variances $Var(\cdot)$, where

$$\begin{aligned} Var(y_{c1}) &= ST^2\overline{\beta_0^2}\{1+\sum_{k=1}^{2N}\exp(-\tau_k/\Gamma)\}+2NT \\ &= Var(y_{s1}) \end{aligned} \tag{49}$$

where the variances of η_1, ν_1, η_2, and ν_2 can be shown to be all $2NT$, and $Var(\beta_0) = 1/2\overline{\beta_0^2}$.

Because the sum of the squares of two independent Gaussion random variables is a random variable with chi-square distribution of degree of freedom 2, we can easily derive the probability distributions of e_1, e_2 , where $e_1 = y_{c1}^2 + y_{s1}^2$, $e_2 = y_{c2}^2 + y_{s2}^2$. Their p.d.f.'s are,

$$f(e_1 = r_1) = \frac{1}{2\sigma_1^2}\exp(\frac{-r_1}{2\sigma_1^2}) \qquad f(e_2 = r_2) = \frac{1}{2\sigma_2^2}\exp(\frac{-r_2}{2\sigma_2^2}) \tag{50}$$

where σ_1, σ_2 is the variance of y_{c1}, y_{c2} respectively. Under the equally probable assumption, the bit error probability is

$$\begin{aligned} P_e &= P(r_1 < r_2 \mid d_0 = 1)P(d_0 = 1) + P(r_1 > r_2 \mid d_0 = -1)P(d_0 = -1) \\ &= \frac{1}{2}P(r_1 < r_2 \mid d_0 = 1) + \frac{1}{2}P(r_1 > r_2 \mid d_0 = -1) \end{aligned} \tag{51}$$

Because of the symmetry, $P(r_1 < r_2 \mid d_0 = 1) = P(r_1 > r_2 \mid d_0 = -1)$, we only need to derive the $P(r_1 < r_2 \mid d_0 = 1)$. The previous symbol will interfere current symbol due to the intersymbol interference caused by the channel. Consequently,

$$\begin{aligned} P(r_1 < r_2 \mid d_0 = 1) &= P(r_1 < r_2 \mid d_0 = 1, d_0' = 1, d_0'' = 1)P(d_0' = 1, d_0'' = 1) \\ &+ P(r_1 < r_2 \mid d_0 = 1, d_0' = 1, d_0'' = -1)P(d_0' = 1, d_0'' = -1) \\ &+ P(r_1 < r_2 \mid d_0 = 1, d_0' = -1, d_0'' = 1)P(d_0' = -1, d_0'' = 1) \\ &+ P(r_1 < r_2 \mid d_0 = 1, d_0' = -1, d_0'' = -1)P(d_0' = -1, d_0'' = -1) \end{aligned}$$

From our assumption, $P(d_0' = 1, d_0'' = 1) = P(d_0' = -1, d_0'' = 1) = P(d_0' = 1, d_0'' = -1) = P(d_0' = -1, d_0'' = -1) = 1/4$. Because

$$\begin{aligned} P(r_1 < r_2) &= \int_0^\infty \int_{r_1}^\infty f(r_1, r_2) dr_2 dr_1 \\ &= \int_0^\infty \frac{1}{2\sigma_1^2} \exp(\frac{-r_1}{2\sigma_1^2}) \int_{r_1}^\infty \frac{1}{2\sigma_2^2} \exp(\frac{-r_2}{2\sigma_2^2}) dr_2 dr_1 \\ &= \int_0^\infty \frac{1}{2\sigma_1^2} \exp(\frac{-r_1}{2\sigma_1^2}) \exp(\frac{-r_1}{2\sigma_2^2}) dr_1 \\ &= \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} \end{aligned} \tag{52}$$

the conditional bit error probability depends on the variances of two channels of the narrowband detectors, $Var(y_{c1})$ and $Var(y_{c2})$, respectively. Let $E = ST$ be signal energy. According to the four cases as before, we derive the variances as follows.

Case I

$$\begin{aligned} Var^{(1)}(y_{c1}) &= ST^2\overline{\beta_0^2}\{1 + \sum_{k=1}^{2N} \exp(\frac{-\tau_k}{\Gamma})\} + 2N_0T \\ Var^{(1)}(y_{c2}) &= 2N_0T \end{aligned}$$

Case II

$$\begin{aligned} Var^{(2)}(y_{c1}) &= ST^2\overline{\beta_0^2}\{1 + \sum_{k=1}^{N} \exp(\frac{-\tau_k}{\Gamma})\} + ST^2\overline{\beta_0^2} \sum_{l=N+1}^{2N} (2 - \frac{\tau_l}{T})^2 \exp(\frac{-\tau_l}{\Gamma}) + 2N_0T \\ Var^{(2)}(y_{c2}) &= ST^2\overline{\beta_0^2} \sum_{l=N+1}^{2N} (\frac{\tau_l}{T} - 1)^2 \exp(\frac{-\tau_l}{\Gamma}) + 2N_0T \end{aligned}$$

Case III

$$\begin{aligned} Var^{(3)}(y_{c1}) &= ST^2\overline{\beta_0^2}[1 + \sum_{k=1}^{N} (1 - \frac{\tau_k}{T})^2 \exp(\frac{-\tau_k}{\Gamma})] \\ &\quad + ST^2\overline{\beta_0^2} \sum_{l=N+1}^{2N} (\frac{\tau_l}{T} - 1)^2 \exp(\frac{-\tau_l}{\Gamma}) + 2N_0T \\ Var^{(3)}(y_{c2}) &= ST^2\overline{\beta_0^2} \sum_{k=1}^{N} (\frac{\tau_k}{T})^2 \exp(\frac{-\tau_k}{\Gamma}) + ST^2\overline{\beta_0^2} \sum_{l=N+1}^{2N} (2 - \frac{\tau_l}{T})^2 \exp(\frac{-\tau_l}{\Gamma}) \\ &+ 2N_0T \end{aligned}$$

Case IV

$$Var^{(4)}(y_{c1}) = ST^2\overline{\beta_0^2}[1+\sum_{k=1}^{N}(1-\frac{\tau_k}{T})^2\exp(\frac{-\tau_k}{\Gamma})] + 2N_0T$$

$$Var^{(4)}(y_{c2}) = ST^2\overline{\beta_0^2}\sum_{k=1}^{N}(\frac{\tau_k}{T})^2\exp(\frac{-\tau_k}{\Gamma}) + ST^2\overline{\beta_0^2}\sum_{l=N+1}^{2N}\exp(\frac{-\tau_l}{\Gamma}) + 2N_0T$$

Then, we get the conditional error probability over $\underline{\tau}_k$

$$P_b|_{\underline{\tau}_k} = \frac{1}{4}\sum_{i=1}^{4}\frac{Var^{(i)}(y_{c2})}{Var^{(i)}(y_{c1})+Var^{(i)}(y_{c2})} = \frac{1}{4}\sum_{i=1}^{4}\frac{\frac{E}{N_0}\overline{\beta_0^2}D_i(\underline{\tau}_k)+2}{\frac{E}{N_0}\overline{\beta_0^2}G_i(\underline{\tau}_k)+4} \quad (53)$$

where

$$D_1(\underline{\tau}_k) = 0$$

$$G_1(\underline{\tau}_k) = 1+\sum_{k=1}^{2N}\exp(\frac{-\tau_k}{\Gamma}) \quad (54)$$

$$D_2(\underline{\tau}_k) = \sum_{l=N+1}^{2N}(\frac{\tau_l}{T}-1)^2\exp(\frac{-\tau_l}{\Gamma}) \quad (55)$$

$$G_2(\underline{\tau}_k) = 1+\sum_{k=1}^{N}\exp(\frac{-\tau_k}{\Gamma}) + \sum_{l=N+1}^{2N}[(2-\frac{\tau_l}{T})^2+(\frac{\tau_l}{T}-1)^2]\exp(\frac{-\tau_l}{\Gamma}) \quad (56)$$

$$D_3(\underline{\tau}_k) = \sum_{k=1}^{N}(\frac{\tau_k}{T})^2\exp(\frac{-\tau_k}{\Gamma}) + \sum_{l=N+1}^{2N}(2-\frac{\tau_l}{T})^2\exp(\frac{-\tau_l}{\Gamma}) \quad (57)$$

$$G_3(\underline{\tau}_k) = 1+\sum_{k=1}^{N}[(1-\frac{\tau_k}{T})^2+(\frac{\tau_k}{T})^2]\exp(\frac{-\tau_k}{\Gamma}) + \sum_{l=N+1}^{2N}[(2-\frac{\tau_l}{T})^2+(\frac{\tau_l}{T}-1)^2]\exp(\frac{-\tau_l}{\Gamma}) \quad (58)$$

$$D_4(\underline{\tau}_k) = \sum_{k=1}^{N}(\frac{\tau_k}{T})^2\exp(\frac{-\tau_k}{\Gamma}) + \sum_{l=N+1}^{2N}\exp(\frac{-\tau_l}{\Gamma}) \quad (59)$$

$$G_4(\underline{\tau}_k) = 1+\sum_{k=1}^{N}[(1-\frac{\tau_k}{T})^2+(\frac{\tau_k}{T})^2]\exp(\frac{-\tau_k}{\Gamma}) + \sum_{l=N+1}^{2N}\exp(\frac{-\tau_l}{\Gamma}) \quad (60)$$

and $\overline{\beta_0^2} = 2\rho_0^2$, ρ_0^2 may typically be obtained from measurement results of indoor multipath fading environments.

Consequently, the overall average bit error probability is

$$P_b = \int_{\underline{\tau}_k} P_b|_{\underline{\tau}_k}\, p(\underline{\tau}_k)d\underline{\tau}_k \quad (61)$$

C. Rician channel

Similar to the Rician channel model in [3, 6], we assume that the first arrival is a constant and the rest arrivals have Rayleigh distributed amplitudes, then y_{c1} (y_{s1}) is Gaussian distributed random variable with mean $\sqrt{2S}T\cos\phi$ ($\sqrt{2S}T\sin\phi$), ϕ is uniformly distribution, and y_{c2}, y_{s2} are zero mean Gaussian random variables. After the square law device in Figure 3, $e_1 = y_{c1}^2 + y_{s1}^2$ is a non-central chi-square distribution, $e_2 = y_{c2}^2 + y_{s2}^2$ is a central chi-square distribution. According to these distributions which are different from Rayleigh channel derivations, the error probability is modified as follows:

$$
\begin{aligned}
P(r_1 < r_2) &= \int_0^\infty \int_{r1}^\infty f(r_1) f(r_2)\, dr_2\, dr_1 \\
f(r_1) &= \frac{1}{2\sigma_1^2}\exp(\frac{(-s^2+r_1)}{2\sigma_1^2}) I_0(\sqrt{r_1}\frac{s}{\sigma_1^2}), \qquad s^2 = m_I^2 + m_Q^2 \\
f(r_2) &= \frac{1}{2\sigma_2^2}\exp(\frac{-r_2}{2\sigma_2^2})
\end{aligned}
$$

where $r_1 = \sqrt{X_1^2 + Y_1^2}$, X_1 and Y_1 are Gaussian distributed random variables with mean m_I, m_Q respectively, i.e., $m_I = E(X_1)$, $m_Q = E(Y_1)$. Then,

$$
\begin{aligned}
P(r_1 < r_2) &= \int_0^\infty \frac{1}{2\sigma_1^2}\exp(\frac{(-s^2+r_1)}{2\sigma_1^2}) I_0(\sqrt{r_1}\frac{s}{\sigma_1^2}) \exp(-\frac{r_1}{2\sigma_2^2})\, dr_1 \\
&= \int_0^\infty \frac{1}{2\sigma_1^2}\exp(-\frac{s^2}{2\sigma_1^2}) \exp\{\frac{-r_1}{2}(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2})\} I_0(\sqrt{r_1}\frac{s}{\sigma_1^2})\, dr_1 \\
&= \frac{2\sigma_e^2}{2\sigma_1^2}\exp(-\frac{s^2}{2\sigma_1^2})\exp(\frac{s_e^2}{2\sigma_e^2}) \\
&\quad \cdot \int_0^\infty \frac{1}{2\sigma_e^2}\exp[-\frac{s_e^2}{2\sigma_e^2}]\exp(\frac{r_1}{2\sigma_e^2}) I_0(\sqrt{r_1}\frac{s_e}{\sigma_e^2})\, dr_1 \\
&= \frac{\sigma_e^2}{\sigma_1^2}\exp(-\frac{s^2}{2\sigma_1^2})\exp(\frac{s_e^2}{2\sigma_e^2}) \\
&= \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}\exp\{s^2(\frac{\sigma_2^2}{2\sigma_1^2(\sigma_1^2+\sigma_2^2)} - \frac{1}{2\sigma_1^2})\} \\
&= \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}\exp\{-\frac{s^2}{2(\sigma_1^2+\sigma_2^2)}\} \qquad (62)
\end{aligned}
$$

where $\sigma_e = \sigma_1^2\sigma_2^2/(\sigma_1^2+\sigma_2^2)$, $s_e = s\sigma_2^2/(\sigma_1^2+\sigma_2^2)$, $s^2 = 2ST^2$. From the result of (62), we can derive the conditional error probability over τ_k,

$$\begin{aligned} P_b|_{\tau_k} &= \frac{1}{4}\sum_{i=1}^{4}\frac{Var^{(i)}(y_{c2})}{Var^{(i)}(y_{c1})+Var^{(i)}(y_{c2})}\exp\{\frac{-2ST^2}{2Var^{(i)}(y_{c1})+2Var^{(i)}(y_{c2})}\} \\ &= \frac{1}{4}\sum_{i=1}^{4}\frac{\frac{E}{N_0}D_i(\tau_k)+2}{\frac{E}{N_0}(G_i(\tau_k)-1)+4}\cdot\exp\{\frac{-2\frac{E}{N_0}}{\frac{E}{N_0}[G_i(\tau_k)-1]+4}\} \end{aligned} \qquad (63)$$

V Numerical Results and Comparison

The average error probability performance of Direct Sequence and Frequency Hopped systems involve integration over $\{\tau_k\}$ which is difficult to numerically evaluate the multiple integrals. However, we numerically illustrate the analytic performance of both cases by choosing some "typical" channel parameters. Since multipath arrived follows a Poisson process, we assume that every path arrives receiver with delay k/λ relative to the first arrival. $k = 1, 2, \cdots, N$. Within symbol time interval $[0, T_b]$, there are N arrivals and N is a random variable in general. By substituting the expected value of every τ_k, we can compare the bit error rate for these two systems in typical cases.

We choose some channel model parameters based on [4, 5, 8] to get the numerical results,

$$\frac{1}{\lambda} = 10 \text{ ns, } 20 \text{ ns} \qquad \Gamma = 20 \text{ ns} \qquad T_m = 200 \text{ ns}$$

and assume ρ_0^2=0.5 according to [5] such that $\overline{\beta_0^2}$=1.

We assume that data bit rate is from 1 Mbps to 10 Mbps and spreading sequences are maximum-length sequences with periods L = 15, 31, 63, and 127 while these lengths are popular for some wireless personal communication applications. We establish these codes to evaluate $\theta_c(\tau_k)$, defined on spreading sequence delay along with the autocorrelation function. The numerical data of SFH and DS systems in Rayleigh and Rician fading channels are provided. In Figure 4, severe multipath effect but no other arrivals within a chip time delay relative to the first arrival (due to high data rate), SFH system is worse than DS system. In Figure 5, with lighter multipath effect and no arrivals in a chip time, SFH system is also worse than DS system. Consider 1 Mbps data rate and 10 ns mean time of arrival path in Figure 6, we observe that the SFH system has lower average bit error probability than that for DS system with L=63 when E_b/N_0 larger than a specific value. Please keep in mind that higher processing gains for DS systems have better resistance to multipath fading at the price of hardware complexity. Moderate data rate 1 Mbps and lighter multipath results, $1/\lambda$=20 ns, is shown in Figure 7. DS system with L=127, has better performance than SFH system shown in Figure 7, while L=63 DS system has similar performance. From Figure 8 to Figure 11, we illustrate the results over Rician channel. In Figure 8 and Figure 9, bit error rates of SFH systems are higher than those of DS systems because of DS systems' good effects in rejecting Rician multipath fading. In Figure 10, SFH has bit error rate lower than DS system with L=31 when E_b/N_0 is not small. Similarly, under less severe multipath effects, SFH has only better performance for L=15 when E_b/N_0 is not small in Figure 11.

VI Conclusion

We have derived and evaluated average bit error probability for specific channel parameters under an updated indoor multipath channel model coming from measurement results. Comparing the performance of two systems without any equalization and error-correcting codes, we observe that the SFH system is more competitive than the DS system under moderate data rate, especially when we apply short-period spreading sequence for DS systems in Rayleigh and Rician channel. On the other hand, for higher data rate, the DS system even with short PN code lengths can provide better performance than the SFH system in Rayleigh and Rician channel. We can also note that severe multipath fading can improve performance of SFH system due to its employing noncoherent modulation/detection, but reduce that of the DS system due to coherent modulation/detector. Further, as our intuition, increasing processing gain for DS system can reduce error probability, particularly in Rician channel which may possess a direct path. Finally, comparing Rician channels to Rayleigh channels, we observe that the SFH system improves performance ability much less than the DS system does. In other words, SFH system does provide advantages over DS system when data rate is not high and is in severe fading (Rayleigh) environments. For more desirable Rician fading (a direct-path), the advantages of SFH system in above conditions are not that obvious. However, severe fading as our Rayleigh model exits in many indoor channels though Rician model is possible or partially possible (partial direct path). The observation of analysis in this paper indeed provides a highlight for the spread spectrum system design of future wireless data communications or personal communications in indoor environments.

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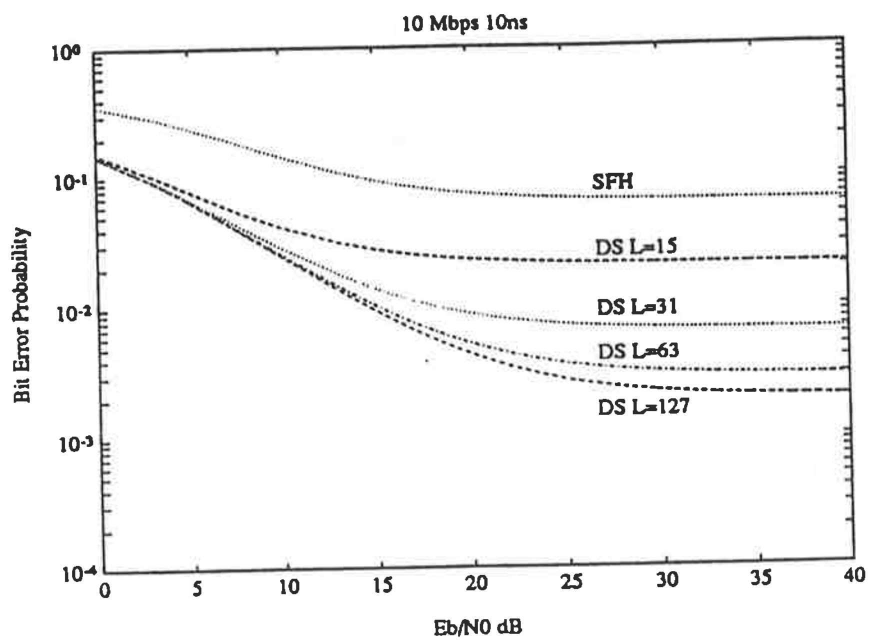

Figure 4: Rayleigh channel; Bit Error Probability of DS and SFH systems for bit rate 10 Mbps and $1/\lambda=10$ ns (parameter of Poisson process). L is the period of spreading sequence of DS system.

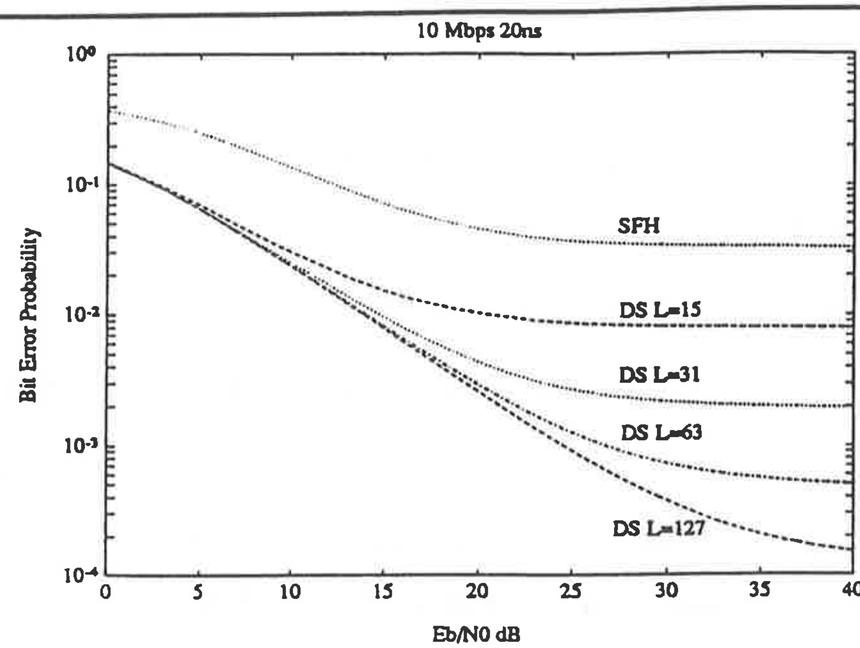

Figure 5: Rayleigh channel; Bit Error Probability of DS and SFH systems for bit rate 10 Mbps and $1/\lambda$=20 ns (parameter of Poisson process). L is the period of spreading sequence of DS system.

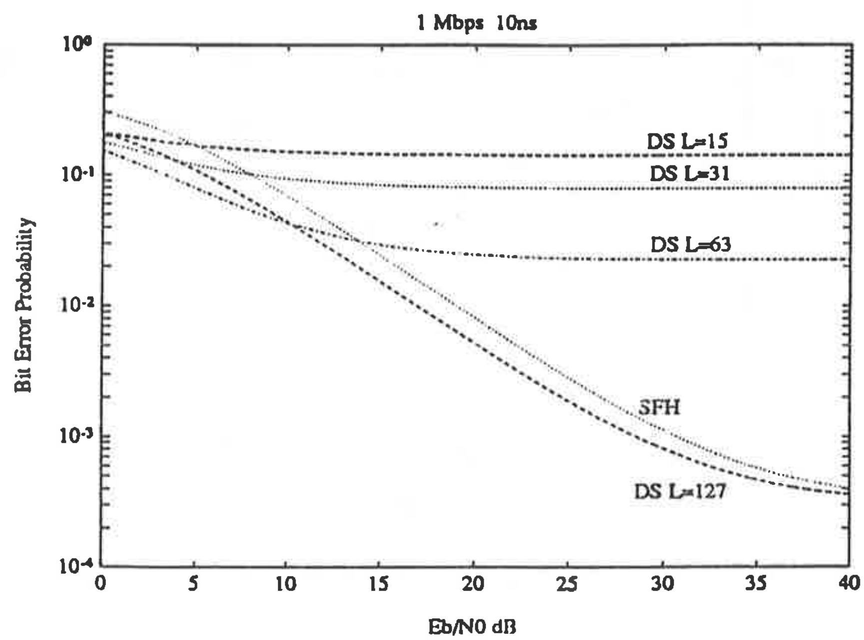

Figure 6: Rayleigh channel; Bit Error Probability of DS and SFH systems for bit rate 1 Mbps and $1/\lambda$=10 ns (parameter of Poisson process). L is the period of spreading sequence of DS system.

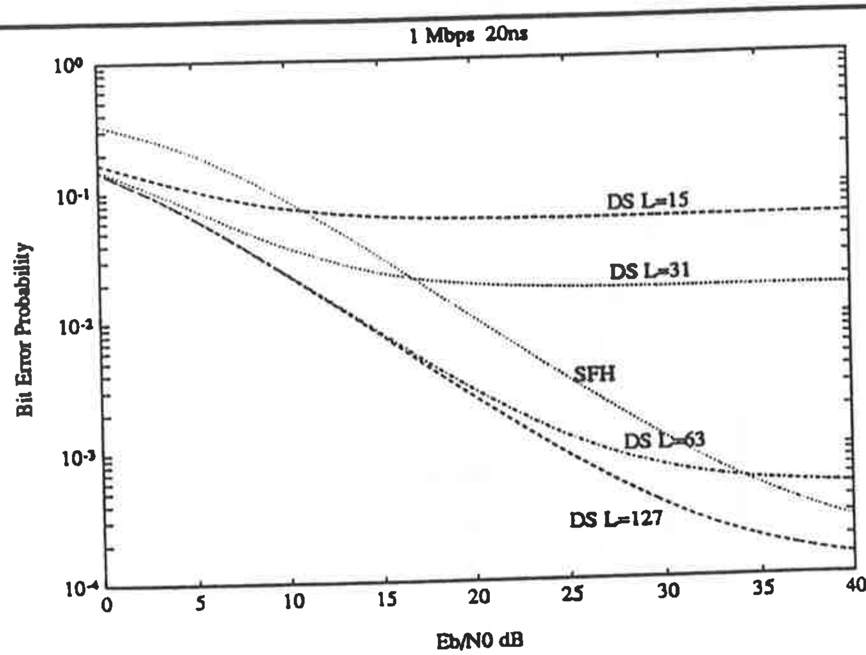

Figure 7: Rayleigh channel; Bit Error Probability of DS and SFH systems for bit rate 1 Mbps and $1/\lambda$=20 ns (parameter of Poisson process). L is the period of spreading sequence of DS system.

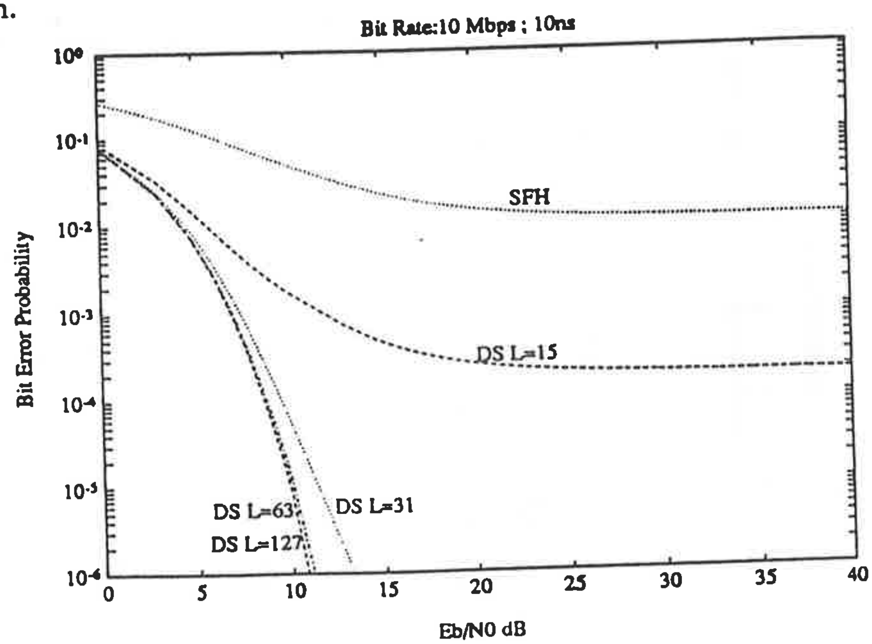

Figure 8: Rician channel; Bit Error Probability of DS and SFH systems for bit rate 10 Mbps and $1/\lambda$=10 ns (parameter of Poisson process). L is the period of spreading sequence of DS system.

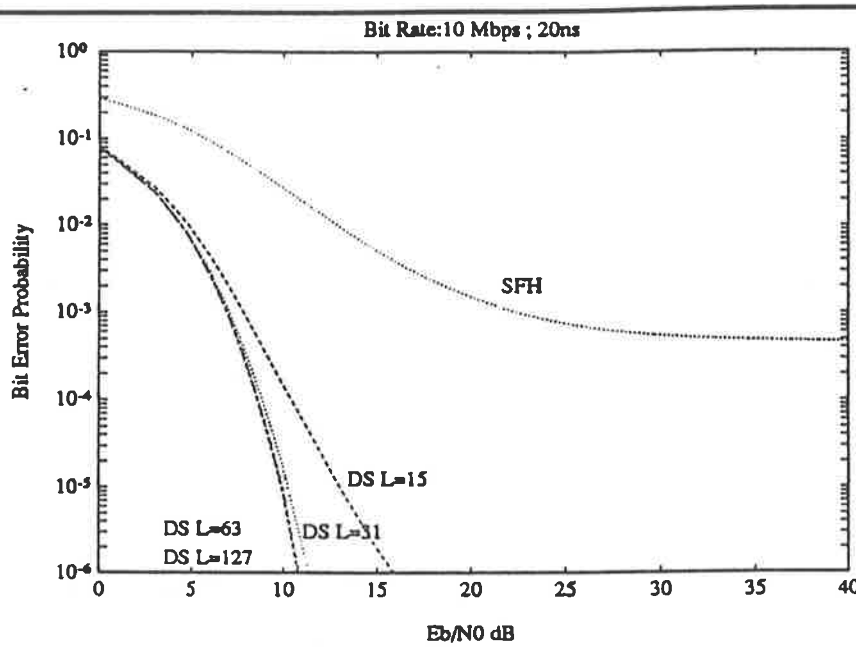

Figure 9: Rician channel; Bit Error Probability of DS and SFH systems for bit rate 10 Mbps and $1/\lambda$=20 ns (parameter of Poisson process). L is the period of spreading sequence of DS system.

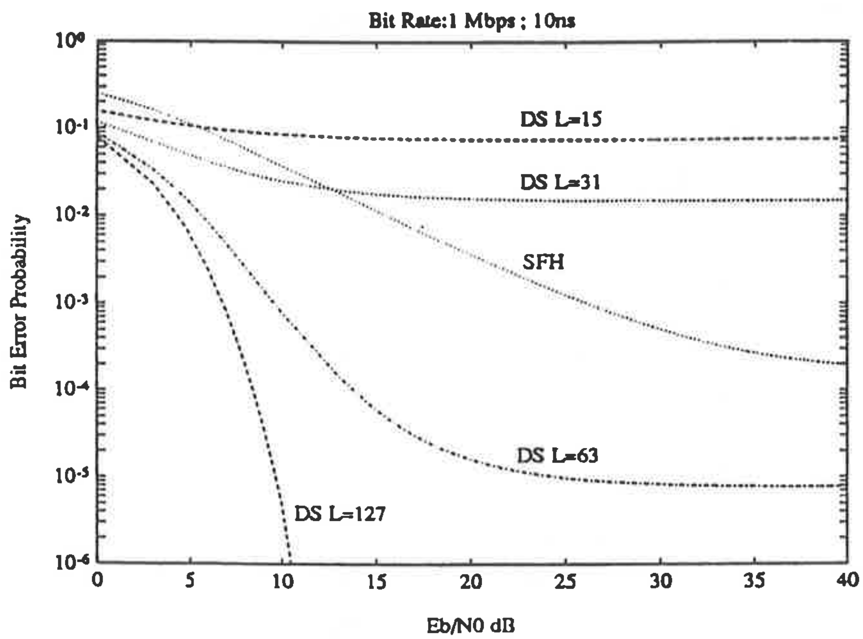

Figure 10: Rician channel; Bit Error Probability of DS and SFH systems for bit rate 1 Mbps and $1/\lambda$=10 ns (parameter of Poisson process). L is the period of spreading sequence of DS system.

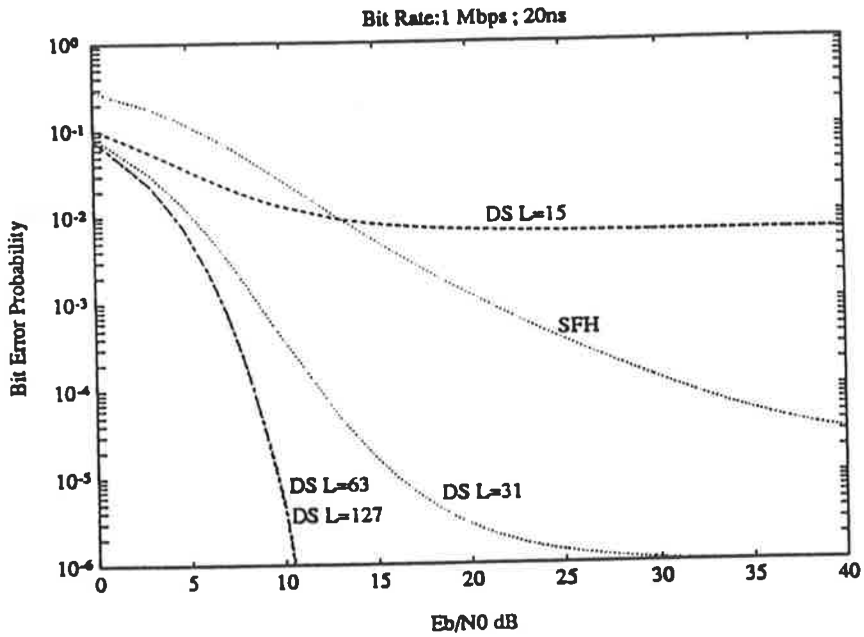

Figure 11: Rician channel; Bit Error Probability of DS and SFH systems for bit rate 1 Mbps and $1/\lambda$=20 ns (parameter of Poisson process). L is the period of spreading sequence of DS system.