Modulation Schemes for Frequency Hopped PHYs

by
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Abstract
Three possible waveforms that appear to be close to the emission mask requirements imposed by FCC Part 15.247 for frequency hopped PHY's operating at 1 MBps are presented. One of these is .39 GMSK as proposed by Motorola at Agoura Hills, one is a variant of quaternary CPFSK as proposed by California Microwave and the final one is a duobinary scheme being introduced in this paper by KII. The intent of this document is to provide a first cut at a very explicit definition of the waveform for a common air interface that might be standardized.

Transmitted Waveform Generation
The following cells define the transmit waveform for an arbitrary CPFSK waveform. In general, the first set of cells build an information table containing random data. The next set of cells defines the necessary parameters of the modulation scheme including the modulation index h and the iffrequency to be used. Also defined are the alphabet size M the symbol time T and the amplitude. The third cell calculates the phase history of the signal for later use in the generation of the transmit signal. The fourth cell defines the modulation waveshaping function q. A cell farther down in the code illustrates a function that generates the q function for techniques that use gaussian premodulation filters.

Finally, the cell that actually defines the transmit signal is listed. This signal generating function uses the parameter, duration, that tells how long it takes for the impact of a particular information bit to be fully reflected in the phase of the transmitted signal and uses this information along with the phase history vector calculated earlier to generate the waveform. The cells listed below were used to generate streams of data 10000 bits long that were stored to disk. The illustrations of power spectrum read this stored data from disk and perform the required calculations.

(*Execute this cell if you are using a binary alphabet (including Duobinary)*)
Clear[info];SeedRandom[1];
info=Table[2 Random[Integer]-1,{10000}];

(*Execute this cell if you are using a quaternary alphabet*)
Clear[info];SeedRandom[1];
info=Table[2 Random[Integer,\{1,4\}]-5,{10000}];
(*This cell defines and executes the differential encoding function used for Duobinary signalling*)

differentialinfo[\(a, b\)] := 1/; (\(a \oplus b\) == 1)

differentialinfo[\(a, b\)] := 1/; (\(a \oplus b\) == 1)

diffinfo[information_] := FoldList[differentialinfo[1, information]]

(*This cell defines the parameters for .39 GMSK*)
amplitude = 1.; ifrequency = 1.0; T = 1; M = 2; h = .5;
makegaussianq[.39, 1];

(*This cell defines the parameters for Duobinary*)
amplitude = 1.; ifrequency = 1.0; T = 1; M = 2; h = .5; duration = 2;
Clear[q]
q[t_] := t/(2 duration T)/; t == 0 & t <= duration T;
q[t_] := 0/; t < 0;
q[t_] := 1/2; t > duration T;

(*This cell defines the parameters for quaternary CPFSK*)
amplitude = 1.; ifrequency = 1.0; T = 2; M = 4; h = .25; duration = 1;
Clear[q]
q[t_] := t/(2 duration T)/; t == 0 & t <= duration T;
q[t_] := 0/; t < 0;
q[t_] := 1/2; t > duration T;

(*This cell calculates the phase history vector for any of the schemes*)
Clear[theta]; x = 0;
theta = Table[x += Pi h info[[k]] // N, {k, 1, Length[info]}];

(*This cell defines the actual transmit waveform function for all cases*)
Clear[s];
s[t_] := 0./; t <= (duration T);
s[t_] := N[amplitude Cos[2 Pi ifrequency t + theta[[n = Floor[t/T] - duration + 1]] +
     Apply[Plus, Table[2 Pi h info[[k + 1]] q[t - k T],
     {k, n - duration + 1, n}]]]//t > (duration T);

Plot[s[t], {t, 0, 7}, PlotRange -> All, PlotStyle -> Thickness[.001]]

The following cell defines a function that generates the q function required if a
 gaussian premodulation filter is used. This function accepts two parameters, the first
 being the bandwidth symbol time product and the second being the symbol time.

makegaussianq[bt_, bitperiod_] :=
  (kg = N[Sqrt[2 Log[2]] Pi];
   filterbt = bt;
   width = 1 (Ceiling[1/(2 bt)]);
   duration = (-2 width) + 1;
p[t_] := (Erf[kg filterbt (t - bitperiod/2)] +
     Erf[kg filterbt (t + bitperiod/2)])/(bitperiod 4);
   qfunc = Table[NIntegrate[p[t], {t, width, k}],
     {k, width, 1 width, 1}];
   gauss = Interpolation[Table[{n/10, qfunc[[n + 1]]}//N,
     {n, 0, Length[qfunc] - 1}]];
   Clear[q];
   q[t_] := 0/; t < 0;
   q[t_] := gauss[[t]]; 0 < t <= (-2 width);
   q[t_] := 5/; t > (-2 width)
Power Spectra of Proposed Waveforms

The cells that follow display simulated power spectrum of the above defined waveforms based on a sample of 200 bits of random data. Also displayed is the power spectrum of MSK data for comparison purposes.

**MSK Transmit Signal**

The next set of cells calculates and plots the theoretical spectrum of an MSK signal transmitting random data.

```math
\text{In[12]} := \text{Table}\left[\frac{16}{(2\pi f)} \left(\cos(2\pi f)/(1-16 f f)\right)^2, \{f, 0, 4, 0.005\}\right];
\text{In[13]} := 10 \log[10, \text{N[\%]}] + 10;
\text{In[14]} := \text{ListPlot}\left[\text{Flatten}\left[\text{Join}\left[\text{Reverse}\left[\text{Take}[\%],\{1,200\}\right],\text{Take}[\%,\{1,600\}]\right]\right], \text{PlotRange} -> \{20, -60\}, \text{PlotJoined} -> \text{True}, \text{PlotStyle} -> \{\text{Thickness}[.001]\}\right]
```

![Power Spectrum of MSK](image)

**Duobinary Transmit Signal**

The next set of cells calculates and plots the spectrum of a duobinary data stream. This spectrum is plotted in conjunction with the MSK plot generated above and the 20 dB bandwidth is calculated.

```math
\text{insig=OpenRead["Hard Disk:DUOBINSIG.MAC"]};
\text{txsignal=Table[N[Read[insig]],\{8000\}]};
\text{Fourier[txsignal]};
\text{test2Q=Abs[\%] Abs[\%]};
\text{ListPlot[10 \log[10, Take[test2Q, 800]],}
\text{PlotJoined} -> \text{True}, \text{PlotRange} -> \text{All},
\text{PlotStyle} -> \{\text{Thickness}[.001], \text{RGBColor}[0,1,0]\}\]
\text{Close[insig]};
\text{tt=Max[test2Q]};
\text{ttt=Flatten[Position[Take[test2Q, 400],x_/;x>(tt/100)]]};
\text{Print["Twenty dB width in KHz is ",N[5 \{Max[ttt]-Min[ttt]\}]]}
```

Twenty dB width in KHz is 860.
.39 GMSK Transmit Signal

The next set of cells calculates and plots the spectrum of a .39 GMSK data stream. This spectrum is plotted in conjunction with the MSK plot generated above and the 20 dB bandwidth is calculated.

```
In[1]:=
insig=OpenRead["Hard Disk:GMSKSIG.MAC"];
txsignal=Table[N[Read[insig]],{8000}];
Fourier[txsignal];
test2Q=Abs[txsignal] Abs[txsignal];
ListPlot[10 Log[10,Take[test2Q,800]],PlotJoined->True,PlotRange->All,
PlotStyle->{Thickness[.001],RGBColor[0,0,1]}]
Close[insig];
```

```
In[8]:=

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In[16]:=
  insig=OpenRead["QUADCPSK.SIG"];
  txsignal=Table[N[Read[insig]],{8000}];
  Fourier[txsignal];
  test2Q=Abs[%] Abs[%];
  ListPlot[10 Log[10,Take[test2Q,800]],
    PlotJoined->True,PlotRange->All,
    PlotStyle->{Thickness[.001],RGBColor[0,0,1]}]
  Close[insig];

In[22]:=
  tt=Max[test2Q];
  ttt=Flatten[Position[Take[test2Q,400],x /;x>(tt/100)]];
  Print["Twenty dB width in KHz is ",N[5 (Max[ttt]-Min[ttt])]]

Twenty dB width in KHz is 750.

\section*{Conclusion}

The cells in this notebook provide the ability to generate an CPFSK waveform and to exhibit a sample of the transmit power spectrum of that waveform. The waveforms generated above can be used in conjunction with channel, interference, noise and filter simulations to determine the expected BER performance of a specific modulation/demodulation combination.