PREDICTION OF COVERAGE FOR DS MODULATION IN A FREQUENCY SELECTIVE CHANNEL

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(Rev. 03/01/93

PERFORMANCE PREDICTION

The performance of a wireless modulation technique can be evaluated by its ability to transmit lata at a specified rate and quality of service (QOS) over a given percentage of a coverage area. Quality of service in this context refers to bit error rate (BER) or probability of a bit error (P_e). For example, wireless data can be transmitted at 2 Mbps with a maximum 1x10-6 P_e over 60 % of a room. A summary of this performance prediction for a specified data rate can be displayed by a cumulative distribution function (CDF) of P_e versus percent of locations in the coverage area with that P_e or less. A method for performing this prediction with measured impulse responses will now be described.

COVERAGE PREDICTION METHOD

If the data symbol period is less than the delay spread of the channel, the receiver sampling window will have more than one data symbol within its limits. The received signal strength and P_e are dependent on the values of the data symbols within the sampling window. Thus the P_e must be averaged over all possible data sequences within the sampling window. All data sequences are assumed to be equally likely since each symbol's value occurs with equal probability.

The maximum and minimum indexes of the data symbols within the sampling window are first determined. The maximum and minimum data indexes are computed by

d(min) = floor((ts-tds)/T)

d(max) = ceil(ts/T)

where ts is the sampling time in seconds

tds is the delay spread in seconds

T is the symbol period in seconds

ceil function rounds to higher integer

floor function rounds to lower integer

An array of all possible data sequences within these limits is then built. Data symbols can be either +1 or -1.

The P_e for a BPSK modulator in Additive White Gaussian Noise (AWGN) without Intersymbol Interference (ISI) is

$$P_e = Q\sqrt{(2Eb/No)}$$

where Eb is the signal energy per bit

No is the noise energy per Hertz

This expression can also be used for BPSK DS provided no wideband or narrowband jammers are present.

To determine the P_e with ISI and AWGN given the data sequence and impulse response measurement, h(t), we must determine the demodulator's response to the "direct path" and the remaining "multiple paths". The direct path component (DPC) is assumed to have zero delay and the same phase as the sampling clock.

$$\tau_o = 0$$
$$\theta_o = \theta_s$$

The remaining multiple path component's (MPC) delays and phases are referenced to the DPC's delay and phase. Using the demodulators response, ro, from the appendix

$$r_o = r\,(\tau_s + mT\,) =$$

$$\sqrt{E}\left(d_o\beta_o + \sum_{l=-\infty}^{\infty}\sum_{K=1}^{\infty} d_l\ \ \beta_k \cos\left(\omega_o(\tau_K - \tau_s\,)\right)\ldots\right.$$

$$\left.(\theta_s - \theta_K\,)\ R'_{cc'}\ (t_s, n)\right) + n'(+)$$

Further simplifying

$$r_o = \sqrt{E}\,(DPC + MPC) + n'\ (t)$$

where

$$n'(t) = \int_{t-T}^{t} n(t)\sqrt{\frac{2}{T}}\cos\ (\omega' t + \theta')\,c'(t)\,dt$$

For BPSK demodulation, an error occurs if a 1 is sent but a voltage less than 0 is detected or a -1 is sent and a voltage greater than or equal to 0 is detected. Since the data sequence and h(t) are known in r(t), n(t) is the only random variable. Thus

$$Pe = \frac{1}{2}\,Pr\,(\sqrt{E}\ (DPC + MPC)\ + N < 0 \mid 1\ sent)$$

$$+\ \frac{1}{2}\,Pr\,(-\sqrt{E}\ (DPC + MPC)\ + N \geq 0 \mid -1\ sent)$$

where N is a noise voltage chosen from the noise probability density function (PDF).

If we assume n(t) to be zero mean AWGN we need only compute the variance of the demodulated noise to determine its PDF.

$$\sigma_{nn}^2 = \mathcal{E}\left(\int_{s-T}^{s} n(s)\sqrt{\frac{2}{T}}\cos(\omega' s + \phi')\ c(s)\ ds\ \ldots \right)$$

$$\left(\int_{t-T}^{t} n(t)\sqrt{\frac{2}{T}}\cos(\omega' t + \phi')\ c(t)\ dt \right)$$

$$\sigma_{nn}^2 = \frac{2}{T}\int_{s-T}^{s}\int_{t-T}^{t} \mathcal{E}(n(s)\ n(t))\ c(s)c(t) \ldots$$

$$\cos\ (\omega' s + \phi')\ \cos(\omega'\ t + \phi')\ dt\ ds$$

but

$$\mathscr{E}(\,n(s)\;n\;(t)\;) = \begin{array}{cc} \frac{No/2}{0} & \frac{t=s}{otherwise} \end{array}$$

and at $t = s$

$$\cos(\omega'_s + \Phi')\;\cos\;(\omega'\,t + \Phi') = \frac{1}{2}\;(\cos\;(2\omega' + 2\Phi') + 1)$$

$$c(s)\;c(t) = 1$$

therefore

$$\sigma^2_{nn} = \frac{No}{2}$$

Knowing the mean and variance of the AWGN we can write the noise PDF as

$$f_n(\eta) = \frac{e^{-\eta^2/2(No/2)}}{\sqrt{2\pi\,(\frac{No}{2})}}$$

$$= \frac{e^{-\eta^2/No}}{\sqrt{\pi No}}$$

if

$$P_e = Pr\,(-\sqrt{E}\;(DPC + MPC)\;+N \geq 0\;|\;-1\;sent\;)$$

$$P_e = Pr\,(N \geq \sqrt{E}\;(DPC + MPC)\;|\;-1\;sent\;)$$

$$P_e = \int_{\sqrt{E}(DPC + MPC)}^{\infty} e^{-\eta^2/No} / \sqrt{\pi No}\, d\eta$$

$$u = \eta / \sqrt{N_o}$$

$$du \sqrt{N_o} = d\eta$$

then

$$P_e = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E}{N_o}}(DPC + MPC)}^{\infty} e^{-u^2} du$$

$$P_e = \frac{1}{2}\, erfc\left(\sqrt{\frac{E}{No}}\, (DPC + MPC)\right)$$

$$P_e = Q\left(\sqrt{\frac{2E}{N_o}}\, (DPC + MPC)\right)$$

In the event that excessive ISI causes a symbol error, an N which causes the voltage to cross zero again corrects the ISI error. In this case the P_e is

$$P_e = 1.0 - Q\left(\sqrt{\frac{2E}{N_o}}\, (DPC + MPC)\right)$$

When the P_e is computed for all possible data sequences for a given impulse response measurement the ensemble of P_e are averaged and stored.

OTHER METHODS OF PERFORMANCE PREDICTION

Chen [Chen, 1992] computed the average P_e for a DS transceiver in a frequency selective channel. The P_e was averaged over all likely multipath component amplitudes, phases, and delays. Chen made the assumption that delay spread did not exceed 2 symbol periods which reduced the number of possible data sequences to 8 since only the current and two previous data symbols could appear in the sampling window. Probability densities for the multipath component amplitudes, phases and delays were taken from Saleh's [Saleh,1987] indoor model.

Chuang [Chuang,1987] generated P_e as a function of normalized rms delay spread using Devasirvathem's [Devasirvathem, 1987] indoor power delay profile measurements. The study was inspired by Bello's GWSSUS channel research [Bello,1963] which predicted P_e as a function of the power delay profile's rms delay spread to symbol period ratio, d. A large number of impulse responses were stochastically generated from one of Devasirvatham's averaged power delay profiles. The generated impulse responses are reasonable estimates of impulse responses that may exist in the neighborhood of Devasirvatham's 4 foot measurement square. For each simulated impulse response, P_e was calculated. This P_e was then averaged over other impulse responses having the same d. A graph of average P_e versus d was constructed and compared with Bello's prediction methods. The results compared favorably for $d < .2$.

Winter [Winter,1985] predicted the outage of a receiver with maximal ratio combining antenna diversity as a function of rms delay spread to symbol period, d. Computation of the BER and probability of an outage was derived in terms of the squared sum of the antenna weighting function. Outage was defined as the probability that a communication link cannot meet a specified BER requirement.

Thoma [Thoma,1992] predicted the performance of pi/4 DQPSK modulation while moving. Rappaport's [Rappaport,1990] channel model was used to simulate 1,125 complex impulse responses over 1 meter. The error distribution was then calculated for a given data rate and velocity. At a highway speed of 60 mph and a data rate of 1 Mbps the detection of 37,313 symbols can be simulated. At a walking speed of 3.75 mph and a data rate of 1 Mbps the detection of 597,014 symbols can be simulated. Performance was measured by outage probability where outage probability is defined as the probability that the number of errors in a code block exceeds a threshold. The number of errors in a code block below this threshold are assumed to be correctable with forward error correction and therefore unimportant.

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APPENDIX

This expression for the response of the demodulator assumes that delay spread is larger than symbol period. No limit on delay spread to symbol period ratio is imposed. The sampling clock can have any value between zero and the maximum delay spread.

WHERE

ℓ	Transmitted Symbol Index
m	Received Symbol Index
d_ℓ	ℓTH transmitted symbol
$\beta_k, \theta_k, \tau_K$	Amplitude, phase, and delay of kth. multipath component
τ_S, θ_s	Delay and phase of sample clock
ω_o	Carrier frequency
$R'_{cc'}(t)$	Partial cross correlation function
t_ϵ	Sub chip offset
nT_c	Integral chip offset
$\theta'(n)$	Code correlation
T_c	Chip period
N_c	Number of chips in PW word
c (t)	PN code waveform
T	PN word period
N_A, N_D	Number of chips that agree/disagree

APPENDIX

$$r(\tau + mT) = r_o =$$

$$\sqrt{E} \sum_{\ell=-\infty}^{\infty} d_\ell \sum_{K} (\beta_K \cos(\omega_o(\tau_k - \tau_s) + \theta_s - \theta_k)\ R'_{cc'}(t_\epsilon, n)$$

$$+ \int_{t-T}^{t} n(t) \sqrt{\frac{2}{T}} \cos(\omega'_o t + \theta')\ c'(t)\ dt$$

where

$$R'_{cc'}(t_\epsilon, \eta) = \left\{ (1 - \frac{t_\epsilon}{Tc})\ \theta'(n) + \frac{t_\epsilon}{Tc}\ \theta'(n+1)\ \ if - T \le \tau_s - (\tau_K - (m-\ell)T) < T \right\}$$

$$t_\epsilon + nT_c = \tau_s - (\tau_K - (m-\ell)\ T)\ ,\quad n = \{0, 1, \ldots Nc-1\},\ 0 \le t_\epsilon < T_c$$

$$\theta'(n) = (N_a - N_D) / N_c$$