PERFORMANCE PREDICTION

The performance of a wireless modulation technique can be evaluated by its ability to transmit data at a specified rate and quality of service (QOS) over a given percentage of a coverage area. Quality of service in this context refers to bit error rate (BER) or probability of a bit error (Pc). For example, wireless data can be transmitted at 2 Mbps with a maximum 1x10^{-6} Pc over 60% of a room. A summary of this performance prediction for a specified data rate can be displayed by a cumulative distribution function (CDF) of Pc versus percent of locations in the coverage area with that Pc or less. A method for performing this prediction with measured impulse responses will now be described.

COVERAGE PREDICTION METHOD

If the data symbol period is less than the delay spread of the channel, the receiver sampling window will have more than one data symbol within its limits. The received signal strength and Pc are dependent on the values of the data symbols within the sampling window. Thus the Pc must be averaged over all possible data sequences within the sampling window. All data sequences are assumed to be equally likely since each symbol's value occurs with equal probability.
The maximum and minimum indexes of the data symbols within the sampling window are first determined. The maximum and minimum data indexes are computed by

\[ d(\text{min}) = \text{floor}\left(\frac{(ts - tds)}{T}\right) \]
\[ d(\text{max}) = \text{ceil}\left(\frac{ts}{T}\right) \]

where \(ts\) is the sampling time in seconds
\(tds\) is the delay spread in seconds
\(T\) is the symbol period in seconds

ceil function rounds to higher integer
floor function rounds to lower integer

An array of all possible data sequences within these limits is then built. Data symbols can be either +1 or -1.

The \(P_c\) for a BPSK modulator in Additive White Gaussian Noise (AWGN) without Intersymbol Interference (ISI) is

\[ P_c = \sqrt{\frac{2Eb}{No}} \]

where \(Eb\) is the signal energy per bit
\(No\) is the noise energy per Hertz

This expression can also be used for BPSK DS provided no wideband or narrowband jammers are present.
To determine the $P_e$ with ISI and AWGN given the data sequence and impulse response measurement, $h(t)$, we must determine the demodulator's response to the "direct path" and the remaining "multiple paths". The direct path component (DPC) is assumed to have zero delay and the same phase as the sampling clock.

$$\tau_\phi = 0$$
$$\theta_\phi = \theta_s$$

The remaining multiple path component's (MPC) delays and phases are referenced to the DPC's delay and phase. Using the demodulators response, $r_0$, from the appendix

$$r_0 = r_\phi (\tau_s + mT) = \sqrt{E} \left( d_0 \beta_\phi + \sum_{t=-\infty}^{\infty} \sum_{k=1}^{\delta} d_k \beta_k \cos (\omega_\phi (\tau_k - \tau_s)) \right) \ldots$$

$$= (\theta_s - \theta_k) R'_{cc'} (t_k, n) + n'(+)$$

Further simplifying

$$r_0 = \sqrt{E} (DPC + MPC) + n'(t)$$

where

$$n'(t) = \int_{t-T}^{t} n(t) \sqrt{ \frac{2}{T} \cos (\omega't + \theta') c'(t) } \, dt$$
For BPSK demodulation, an error occurs if a 1 is sent but a voltage less than 0 is detected or a -1 is sent and a voltage greater than or equal to 0 is detected. Since the data sequence and h(t) are known in r(t), n(t) is the only random variable. Thus

\[ P_e = \frac{1}{2} \Pr \left( \sqrt{E} \left( DPC + MPC \right) + N < 0 \mid 1 \text{ sent} \right) \]

\[ + \frac{1}{2} \Pr \left( -\sqrt{E} \left( DPC + MPC \right) + N \geq 0 \mid -1 \text{ sent} \right) \]

where N is a noise voltage chosen from the noise probability density function (PDF).

If we assume n(t) to be zero mean AWGN we need only compute the variance of the demodulated noise to determine its PDF.

\[ \sigma_{nn}^2 = \mathbb{E} \left( \int_{s-T}^{s} n(s) \sqrt{\frac{2}{T}} \cos (\omega's + \phi') c(s) \, ds \ldots \right) \]

\[ \left( \int_{t-T}^{t} n(t) \sqrt{\frac{2}{T}} \cos (\omega't + \phi') c(t) \, dt \right) \]

\[ \sigma_{nn}^2 = \frac{2}{T} \int_{s-T}^{s} \int_{t-T}^{t} \mathbb{E} \left( n(s) \, n(t) \right) c(s) c(t) \ldots \]

\[ \cos (\omega's + \phi') \cos (\omega't + \phi') \, dt \, ds \]
but

\[ \mathbb{E}(n(s) n(t)) = \begin{cases} \frac{N_o}{2} & t = s \\ 0 & \text{otherwise} \end{cases} \]

and at \( t = s \)

\[
\cos(\omega_s' + \Phi') \cos(\omega t + \Phi') = \frac{1}{2} \left( \cos(2\omega' + 2\Phi') + 1 \right)
\]

\[ c(s) c(t) = 1 \]

therefore

\[ \sigma_{nn}^2 = \frac{N_o}{2} \]

Knowing the mean and variance of the AWGN we can write the noise PDF as

\[
f_n(\eta) = \frac{e^{-\eta^2/2(N_o/2)}}{\sqrt{2\pi \left( \frac{N_o}{2} \right)}}
\]

\[
= \frac{e^{-\eta^2/N_o}}{\sqrt{\pi N_o}}
\]

if

\[ P_e = \Pr \left( -\sqrt{E(DPC + MPC)} + N \gtrless 0 \mid -1 \text{ sent} \right) \]

\[ P_e = \Pr \left( N \gtrless \sqrt{E(DPC + MPC)} \mid -1 \text{ sent} \right) \]
\[
\begin{align*}
    P_e &= \int_{\sqrt{B(DPC + MPC)}} e^{-\eta^2/\sqrt{\pi N_o}} d\eta \\
    u &= \eta / \sqrt{N_o} \\
    du \sqrt{N_o} &= d\eta \\
	hen \end{align*}
\]

\[
\begin{align*}
    P_e &= \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{B}{N_o} (DPC + MPC)}} e^{-u^2} du \\
    P_e &= \frac{1}{2} \text{erfc} \left( \sqrt{\frac{2E}{N_o} (DPC + MPC)} \right) \\
    P_e &= Q \left( \sqrt{\frac{2E}{N_o} (DPC + MPC)} \right)
\end{align*}
\]

In the event that excessive ISI causes a symbol error, an N which causes the voltage to cross zero again corrects the ISI error. In this case the \( P_e \) is

\[
\begin{align*}
    P_e &= 1.0 - Q \left( \sqrt{\frac{2E}{N_o} (DPC + MPC)} \right)
\end{align*}
\]
When the $P_e$ is computed for all possible data sequences for a given impulse response measurement the ensemble of $P_e$ are averaged and stored.

**OTHER METHODS OF PERFORMANCE PREDICTION**

Chen [Chen, 1992] computed the average $P_e$ for a DS transceiver in a frequency selective channel. The $P_e$ was averaged over all likely multipath component amplitudes, phases, and delays. Chen made the assumption that delay spread did not exceed 2 symbol periods which reduced the number of possible data sequences to 8 since only the current and two previous data symbols could appear in the sampling window. Probability densities for the multipath component amplitudes, phases and delays were taken from Saleh's [Saleh, 1987] indoor model.

Chuang [Chuang, 1987] generated $P_e$ as a function of normalized rms delay spread using Devasirvatham's [Devasirvatham, 1987] indoor power delay profile measurements. The study was inspired by Bello's GWSSUS channel research [Bello, 1963] which predicted $P_e$ as a function of the power delay profile's rms delay spread to symbol period ratio, $d$. A large number of impulse responses were stochastically generated from one of Devasirvatham's averaged power delay profiles. The generated impulse responses are reasonable estimates of impulse responses that may exist in the neighborhood of Devasirvatham's 4 foot measurement square. For each simulated impulse response, $P_e$ was calculated. This $P_e$ was then averaged over other impulse responses having the same $d$. A graph of average $P_e$ versus $d$ was constructed and compared with Bello's prediction methods. The results compared favorably for $d < .2$.

Winter [Winter, 1985] predicted the outage of a receiver with maximal ratio combining antenna diversity as a function of rms delay spread to symbol period, $d$. Computation of the BER and
probability of an outage was derived in terms of the squared sum of the antenna weighting function. Outage was defined as the probability that a communication link cannot meet a specified BER requirement.

Thoma [Thoma, 1992] predicted the performance of pi/4 DQPSK modulation while moving. Rappaport's [Rappaport, 1990] channel model was used to simulate 1,125 complex impulse responses over 1 meter. The error distribution was then calculated for a given data rate and velocity. At a highway speed of 60 mph and a data rate of 1 Mbps the detection of 37,313 symbols can be simulated. At a walking speed of 3.75 mph and a data rate of 1 Mbps the detection of 597,014 symbols can be simulated. Performance was measured by outage probability where outage probability is defined as the probability that the number of errors in a code block exceeds a threshold. The number of errors in a code block below this threshold are assumed to be correctable with forward error correction and therefore unimportant.

REFERENCES


APPENDIX

This expression for the response of the demodulator assumes that delay spread is larger than symbol period. No limit on delay spread to symbol period ratio is imposed. The sampling clock can have any value between zero and the maximum delay spread.

WHERE

\( \ell \) Transmitted Symbol Index
\( m \) Received Symbol Index
\( d_{\ell} \) \( \ell \)TH transmitted symbol
\( \beta_{k}, \theta_{k}, \tau_{k} \) Amplitude, phase, and delay of kth. multipath component
\( \tau_{s}, \theta_{s} \) Delay and phase of sample clock
\( \omega_{0} \) Carrier frequency
\( R'_{cc}(t) \) Partial cross correlation function
\( t_{s} \) Sub chip offset
\( nT_{c} \) Integral chip offset
\( \theta'(n) \) Code correlation
\( T_{c} \) Chip period
\( N_{c} \) Number of chips in PW word
\( c(t) \) PN code waveform
\( T \) PN word period
\( N_{A}, N_{D} \) Number of chips that agree/disagree
APPENDIX

\[ r(\tau + mT) = r_0 = \]

\[ \sqrt{E} \sum_{t=0}^{\infty} d_t \sum_{K} (\beta_K \cos(\omega_0(\tau_K - \tau_s) + \theta_s - \theta_K)) \ R_{CC}'(t_s, n) \]

\[ + \int_{T-T}^{T} n(t) \sqrt{2 \over T} \cos(\omega'_0 t + \theta') c'(t) \ dt \]

where

\[ R_{CC}'(t_s, \eta) = \left\{ \begin{array}{ll}
(1 - \frac{t_s}{T_c}) \theta'(n) + \frac{t_s}{T_c} \theta'(n+1) & \text{if } T \leq t_s - (\tau_x - (m-\ell)T) < T
\end{array} \right\} \]

\[ t_s + nT_c = \tau_x - (m-\ell)T, \ n = \{0, 1, \ldots, N_c-1\}, \ 0 \leq t_s < T_c \]

\[ \theta'(n) = (N_a - N_D) / N_c \]