Considerations in FSK Tx pulse shaping and Rx filtering  
for low ISI at high bit rates

Naftali Chayat, LANNAIR inc. 
Tel-Aviv, ISRAEL

Introduction

This document proposes a family of M-FSK modulations. It is shown that with proper Tx and Rx signal shaping 2.1 Mbit/sec (8-FSK) performance can be achieved (with implementation margin). The proposition has the following benefits:

- Compatibility with many of the CPFSK flavours
- Convenient fallback rates: 1.4 Mbit/sec (4-FSK) and 0.7 Mbit/sec (2-FSK) with common RF hardware.
- Possible simple discriminator implementation (without sophisticated DSP)
- Improved performance (3 dB) through coherent demodulation and trellis codes (needs implementation using DSP techniques)

The document will discuss the issues of filtering (shaping) at the transmit and the receive sides. The techniques discussed will be applied as an example to improve the reception of Gaussian FSK.

LANNAIRs MODEM PROPOSAL

A modulation format with the following parameters is proposed as a standard:

- Bit rate: 2.1 Mb/s (8FSK) ; Foldback bit rates: 1.4 Mb/s (4FSK) and 0.7 Mb/s (2FSK)
- Tx filter: Sqrt-rolloff (alpha=0.5) + 1dB preemphasis @ 1/2Fs
- IF filter: Sqrt-rolloff (alpha=1.0)
- Rx filter: Sqrt-rolloff (alpha=0.5) + 1 dB deemphasis @ 1/2Fs + IF filter compensation
- Modulation factor: hM=2/3, M=2,4,8
- Baud rate: 700 ksymbols/sec
ISI removal at the receive side

In many documents presented to IEEE 802.11 graphs are presented of the "degraded eye diagram" of Gaussian FSK. We are going to show how with proper filter after the discriminator the "eye" gets wide open again.

Let us look at the Gaussian FSK with BT=0.5, wide IF filter and Integrate&Dump at the receive side. With a wide IF filter the { FM mod + IF filtering + FM demod } can be viewed as a transparent operation, so only baseband processing will be discussed. At the frequency of 0.5 the baud rate (the Nyquist frequency) there is an attenuation relatively to f=0 due to three factors: the rectangular shape of baseband pulses (-4 dB), the Gaussian filter (-3 dB) and the Integrate&Dump (-4 dB), which amounts to -11 dB overall. A pulse shape with low ISI has typically -6 dB at Nyquist frequency (e.g. rolloff pulses). A preemphasis of 4 dB prior to I&D around the Nyquist frequency reduces the ISI (opens the eye) drastically. There is a penalty in noise for this preemphasis, but the ISI removal is absolutely essential in multilevel modulations. Another method, to be discussed later is to preemphasize at the transmit side.

![Figure 1: Received eye diagram of GFSK, BT=0.5 with I&D filter only](image)
Figure 2: Received eye diagram of GFSK, with preemphasis and I&D filter

For small modulation factors, the IF filter can (approximately) be interchanged with the FM demodulator, causing the IF filter to become part of the baseband filtering chain. This brings us to the following rule of thumb for low ISI:

$\text{(Tx pulse shape @ } 1/2 \text{ Fs}) + \text{(IF filter @} +/\text{-}1/2 \text{ Fs}) + \text{(Rx filter @ } 1/2 \text{ Fs}) = -6 \text{ dB}$

For binary FSK this rule of thumb is sufficient in most cases. For multilevel modulations, more accurate treatment of the filter is required.

In practical case the IF filter will not drop sharply immediately after $+/\text{-} 1/2 \text{ Fs}$. Typically, the IF filter will achieve high rejection only after $+/\text{-} 0.7-1 \text{ Fs}$. For a given bit rate 4-FSK has a symbol rate two times lower than 2-FSK, enabling thus a narrower IF filter and consequently a lower ACI (adjacent channel interference).

FSK as an incremental PSK - transmit pulse shaping
The frequency shift created during FSK induces a phase increment over the symbols duration. A vivid example of this is the MSK modulation, which can be viewed on one hand as FSK modulation with rectangular pulse shape and \( h=0.5 \), and on the other hand as an incremental PSK modulation with phase increments of -90 or +90 degrees. Another example is 4-level FSK with rectangular pulses and \( h=0.25 \), which forms, at the sampling instants, an \( \pi/4 \)-QPSK modulation. Figure 3 shows a sort of "phase eye diagram" of the 4-level FSK:

![Figure 3: Phase eye diagram of 4-FSK, rectangular pulses, \( h=0.25 \)](image)

The "phase-ISI-free" property achieved in previous example can be achieved by various pulse shapes. When a spectrally limited waveform is wanted, an attenuation of -2 dB at Nyquist frequency with respect to \( f=0 \) is required. In the case of our proposition, shown in figure 4, the Tx pulse shape is an \( \alpha=0.5 \) square-root-rolloff filter (-3 dB @ Nyquist frequency) with additional 1 dB preemphasys, in this case with 4-level modulation and modulation factor \( h=1/6 \). This can be viewed as "\( \pi/6 \)-6PSK" modulation. Of the 6 next phases only 4 transitions are legal ones, while 2 phases are inaccessible.
Figure 4: Phase eye diagram of 4-FSK, h=1/6, preemphasized sqrt-rolloff filter

Please note that in contrast to GFSK in which the Nyquist frequency is attenuated by 3 dB in addition to the -4 dB of the rectangular pulse, here a 2 dB preemphasys is required. This change enables smaller h for the same BER.

The strength of this approach is the ability to describe the FSK modulation as an incremental (or differential) PSK modulation, which can be demodulated by both an analog means (discriminator + postdetection filter), and by digital PSK-oriented means. In the case of digital implementation coherent demodulation can be introduced (3 dB gain). Additional growth path in this framework can be an introduction of trellis coding. All this future improvements preserve a common RF hardware.

Transmit spectrum shape

The sensitivity of FSK and its spectral shape is dominated by the modulation factor h, the number of levels M and the transmit pulse shaping. The spectral shape depends mostly on the product hM and the pulse shape. This enables to define a family of modulations with a common baseband filter driven by a D/A converter with varying number of most significant bits toggled: 1 bit will correspond to 2-FSK, 2 bits to 4-FSK, 3 bits to 8-FSK et cetera. In the case of
$h_M=2/3$ and the proposed pulse shape, as described in the previous section, this formats produce power spectra shown on fig. 5. The spectra are plotted at 250 KHz/div, for a baud rate of 700 ksymbols/sec.

![Figure 5: PSD of 2,4,8-FSK with $h_M=2/3$ and preemphasized sqrt-rolloff filter](image)

**Rx eye diagrams**

With the proposed Tx and Rx filters and the modulation factors, extremely low ISI is obtained, as shown in figs. 6 and 7.

**BER and ACI performance**

The following table summarizes the $E_s/N_0$, $E_b/N_0$ and $C/N@1MHz$ at $BER=1e-5$, as well as first and second ACI of the proposed modulation method (with respect to the proposed IF filter):

<table>
<thead>
<tr>
<th>Modulation</th>
<th>2-FSK</th>
<th>4-FSK</th>
<th>8-FSK</th>
</tr>
</thead>
<tbody>
<tr>
<td>99% pwr BW</td>
<td>735 KHz</td>
<td>830 KHz</td>
<td>840 KHz</td>
</tr>
<tr>
<td></td>
<td>815 KHz</td>
<td>910 KHz</td>
<td>920 KHz</td>
</tr>
<tr>
<td>----------------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>-20 dB BW</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>bits/symbol</td>
<td>1 bit/s</td>
<td>2 bit/s</td>
<td>3 bit/s</td>
</tr>
<tr>
<td>Bit rate</td>
<td>0.7 Mb/s</td>
<td>1.4 Mb/s</td>
<td>2.1 Mb/s</td>
</tr>
<tr>
<td>Adj ch intrfr</td>
<td>-32 dB</td>
<td>-29 dB</td>
<td>-28 dB</td>
</tr>
<tr>
<td>Alt ch intrfr</td>
<td>&lt;-90 dB</td>
<td>-87 dB</td>
<td>-86 dB</td>
</tr>
<tr>
<td>Es/No @ 1e-5</td>
<td>14.5 dB</td>
<td>20 dB</td>
<td>26 dB</td>
</tr>
<tr>
<td>Eb/No @ 1e-5</td>
<td>14.5 dB</td>
<td>17 dB</td>
<td>21 dB</td>
</tr>
<tr>
<td>C/N @ 1 MHz</td>
<td>13 dB</td>
<td>18.5 dB</td>
<td>24.5 dB</td>
</tr>
<tr>
<td>(with coherent demod)</td>
<td>(10.5 dB)</td>
<td>(15.5 dB)</td>
<td>(21.5 dB)</td>
</tr>
</tbody>
</table>

The baud rate proposed produces a 920 kHz bandwidth. A reserve is left to allow implementation inaccuracies, as well as practical, suboptimal, filters. The methods for practical signal shaping will be finalized and proposed in the following submissions.

Figure 6: received eye diagram of the proposed 4-FSK
SUMMARY

- Anti ISI prescription was given - emphasis of Nyquist frequency region
- Improved transmit pulse shaping was discussed

A high speed modulation method was proposed:

- 2.1 Mb/s / 1.4 Mb/s
- 8-CPFSK / 4-CPFSK with anti-ISI shaping
- On Tx: preemphasized sqrt-rolloff baseband filter
- On Rx: deemphasized sqrt-rolloff filter
- Works with GFSK at 0.7 Mb/s
- Best with DSP - possible with analog implementation