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Abstract	Enhanced LDPC coding scheme
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8.4.9.2.4 Low Density Parity Check Code (optional)

8.4.9.2.4.1 Code Description

The fundamental LDPC code is a systematic linear block code with (N_c , N_k), rate = N_k / N_c where N_c is length of code and N_k is information bit size. There is some code definition for the system and variety code rates may be constructed for good code performance for each code rate. Changing the consistent matrix size accommodates varying data field lengths. Explanation about consistent matrix is in Packet Encoding section.

8.4.9.2.4.2 LDPC encoding

In a general analysis, an (N_c, N_k) LDPC code has N_k information bits and N_c coded bits with code rate r = N_k /N_c. The parity-check matrix $\boldsymbol{H}$ is of dimension (N_c - N_k)_N_c, and it defines a set of equations. For simplicity, let us put N_c - N_k = N_p , where N_p denotes the number of parity bits.

We can get relation parity check matrix $\boldsymbol{H}$ between codewords $\boldsymbol{v}$ as belows

$$\boldsymbol{H} \cdot \boldsymbol{v}^t = \boldsymbol{0} \qquad (1)$$

for all codewords $\boldsymbol{v}$.

An example parity-check matrix is shown below for an LDPC code (8, 4) as well as the expanded parity-check equations

For efficient encoding of LDPC, $\boldsymbol{H}$ are divided into the form

(2)

where is , is , is , is , is , and finally, is . Further, all these matrices are sparse and is lower triangular with ones along the diagonal.

Let $\boldsymbol{v}$=($\boldsymbol{u}$, $\boldsymbol{p}_1$, $\boldsymbol{p}_2$) that $\boldsymbol{u}$ denotes the systematic part, $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ combined denote the parity part, $\boldsymbol{p}_1$ has length g, and $\boldsymbol{p}_2$ has length (N_p-g). The definition equation $\boldsymbol{H} \cdot \boldsymbol{v}^t = 0$ splits into two equations, as in equation 3 and 4 namely

(3)

and

(4)

Define and when we use the parity check matrix as indicated appendix we can get . Then from (4) we conclude that

(5)

and

. (6)

As a result, the encoding procedures and the corresponding operations can be summarized below and illustrated in Fig. 1.

Encoding procedure

Step 1) Compute and .

Step 2) Compute .

Step 3) Compute by .

Step 4) Compute by .

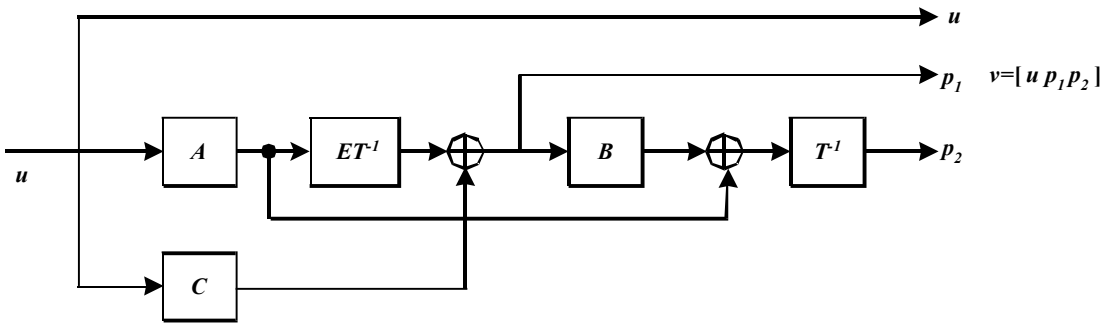

Fig. 1 Block diagram of the encoder architecture for the block LDPC code.

A detailed description of the ***A, B, T, C, D*** and ***E*** matrices of the code is contained in the Appendix LDPC Code Definition. The code is fully described by the definition of the H matrix as indicated in the appendix.

8.4.9.2.4.3 Rate Adjustment

As the code is rate flexible we can easily construct ***H*** matrix for various code rates (e.g. 1/2, 2/3, 3/4 and 5/6). For each code rate, we design fundamental LDPC codes that achieve good performance whose ***H*** matrix described in appendix.

8.4.9.2.4.4 Packet Encoding

In this section, we describe construction of parity check matrix of block LDPC codes to obtain packet data length flexibility. A block LDPC code is an almost structured LDPC codes whose parity-check matrix consists of small square blocks which are the zero matrix or a circulant permutation matrix. Let ***P*** be the permutation matrix given by

Note that $\boldsymbol{P}^i$ is just the circulant matrix of the identity matrix $\boldsymbol{I}$ to the right by (mod) times for any integer . For simple notation, denotes the zero matrix.

Let $\boldsymbol{H}$ be the matrix defined by

where . When $\boldsymbol{H}$ has full rank, then its codeword size N_c is and information bit size N_k is . Therefore, its code rate is given by

regardless of its block length .

Therefore, we can obtain larger size block LDPC codes by increasing the size of circulant permutation matrices $\boldsymbol{P}$ which is an element matrix of $\boldsymbol{H}$ matrix. Also, we can straightforwardly get small size block LDPC codes by decreasing the size of $\boldsymbol{P}$.

For example, when we use with code rate 1/2 case in appendix, we can construct (24, 12) block LDPC codes and for the same case. If we set , then we can obtain (240, 120) block LDPC codes.

Appendix (normative); LDPC Code Definition

A full definition of an LDPC code can be accomplished through identification of the locations of the "edges" between the variable nodes (codeword bits) and check nodes (parity relationships). Figure 2 shows a Tanner graph of an example LDPC code, depicting the arrangement of the check nodes, variable nodes, and the "edges" connecting them.

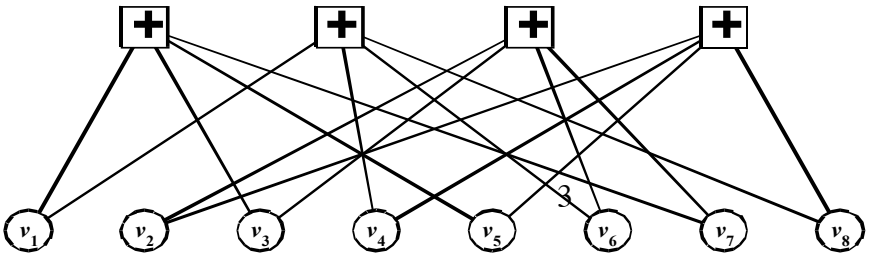

Fig. 2 This is an example Tanner graph of an LDPC code, showing the check nodes, variable nodes, and edges. The codeword is made up of the bits represented by the variable nodes. In this case the codeword has eight bits.

Each check node represents a parity relationship between the codeword bits represented by the variable nodes connected to it by the edges. The number of edges connected to a check node is the "degree" of the check node, and the number of edges connected to a variable node is the "degree" of the variable node. For the specified code all check nodes are of degree eighteen, all variable nodes related to the systematic information bits are of degree four, and all variable nodes corresponding to parity bits are of degree two except for the last, which is of degree one.

This LDPC code list file contains six parts to describe the parity check matrix ***H***. When matrix H split into the form

we describe the matrices , , , , and .

An example for a code with n=8, m=4, we design the parity check matrix

where is zero matrix.

Then we describe matrices A, B, T, C, D and E as below

, ,

, ,

Code rate = 1/2

$$A = \begin{bmatrix} 0 & 0 & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 & \infty & \infty \\ \infty & \infty & 0 & 0 & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 \\ \infty & \infty & \infty & \infty & 0 & 0 & \infty & \infty & \infty & 26 & \infty & 9 \\ \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 & \infty & 13 & 39 & \infty \\ \infty & \infty & 5 & \infty & \infty & 18 & \infty & \infty & 35 & \infty & 16 & \infty \\ 30 & \infty & \infty & \infty & \infty & \infty & \infty & 29 & 28 & 17 & \infty & \infty \\ \infty & \infty & \infty & 33 & \infty & \infty & 19 & \infty & 34 & \infty & \infty & \infty \\ \infty & 25 & 26 & \infty & \infty & \infty & \infty & \infty & \infty & 15 & \infty & 4 \\ \infty & \infty & \infty & \infty & 36 & \infty & \infty & 1 & 38 & \infty & 9 & 7 \\ \infty & 23 & \infty & \infty & \infty & \infty & 2 & \infty & \infty & \infty & 30 & 21 \\ \infty & \infty & \infty & 29 & \infty & 27 & \infty & \infty & \infty & 12 & 20 & \infty \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \infty \\ \infty \\ \infty \\ \infty \\ \infty \\ Ns-1 \\ \infty \\ \infty \\ \infty \\ \infty \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 0 & 0 & \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 & 0 & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & 0 & 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 39 & \infty & \infty & \infty & 18 & \infty & \infty & \infty & 22 & \infty & \infty & 16 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 \end{bmatrix}$$

$$E = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 1 \end{bmatrix}$$

Code Rate = 2/3

$$A = \begin{bmatrix}
0 & 0 & 0 & 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 & 0 & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & 0 & 0 & 0 & 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 & 0 & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 & 0 & 0 & \infty & \infty & \infty & \infty & 7 & \infty & \infty & \infty & \infty & \infty & 0 & 0 \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 & 0 & 0 & \infty & 3 & 5 & \infty & 1 & \infty & \infty & 15 \\
\infty & \infty & \infty & 1 & 5 & 11 & \infty & \infty & 4 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 4 & 13 & \infty & 10 & \infty & \infty & 7 & \infty \\
\infty & 6 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 6 & \infty & \infty & 2 & \infty & 15 & \infty & \infty & \infty & \infty & 14 & 8 & \infty & 3 \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & 6 & \infty & 7 & \infty & 5 & \infty & \infty & 9 & \infty & 11 & \infty & \infty & 3 & \infty & 15 & \infty & \infty \\
4 & \infty & 6 & \infty & 1 & \infty & 2 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 8 & \infty & \infty & 12 & \infty & 9 & \infty \\
\infty & \infty & 3 & \infty & \infty & \infty & \infty & \infty & 12 & \infty & \infty & \infty & \infty & 3 & \infty & 14 & \infty & \infty & 2 & 14 & \infty & 11 & \infty & 1 \\
13 & \infty & \infty & \infty & \infty & 6 & \infty & \infty & \infty & \infty & 8 & \infty & 15 & \infty & \infty & \infty & \infty & \infty & 9 & \infty & \infty & 4 & 13 & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & 9 & \infty & \infty & 7 & \infty & 10 & \infty & \infty & 5 & \infty & \infty & 6 & \infty & \infty & 13 & \infty & \infty & 14
\end{bmatrix}$$

$$B = \begin{bmatrix}
0 \\ \infty \\ \infty \\ \infty \\ \infty \\ \infty \\ Ns-1 \\ \infty \\ \infty \\ \infty \\ \infty
\end{bmatrix}$$

$$T = \begin{bmatrix}
0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
0 & 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & 0 & 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & 0 & 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & 0 & 0 & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & 0 & 0 & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & 0 & 0 & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0
\end{bmatrix}$$

$$C = \begin{bmatrix} \infty & 4 & \infty & 2 & \infty & \infty & \infty & 5 & \infty & \infty & \infty & \infty & 7 & \infty & \infty & \infty & 3 & \infty & 13 & 15 & \infty & \infty & 10 & \infty \end{bmatrix}$$

$$D = \begin{bmatrix} 1 \end{bmatrix}$$

$$E = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 1 \end{bmatrix}$$

Code Rate = 3/4

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 & 0 & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 & 0 & 0 & 0 & 0 & \infty & \infty & \infty & \infty & \infty & 2 & \infty & \infty & \infty & 0 & 0 \\ \infty & 5 & \infty & 9 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 & 0 & 0 & 0 & \infty & 1 & \infty & \infty & 8 & \infty \\ 2 & \infty & \infty & \infty & 3 & \infty & \infty & 13 & \infty & \infty & 15 & \infty & 2 & \infty & \infty & 10 & \infty & 0 & \infty & 7 & 5 & \infty & 4 \\ \infty & \infty & \infty & \infty & \infty & 4 & \infty & \infty & 12 & 11 & \infty & 10 & 7 & \infty & 17 & \infty & \infty & \infty & 13 & \infty & 15 & 14 & \infty \\ \infty & \infty & 1 & 6 & \infty & \infty & 3 & 4 & \infty & \infty & 2 & \infty & \infty & \infty & 13 & \infty & 11 & 2 & \infty & 5 & \infty & 7 & \infty \\ \infty & 8 & \infty & \infty & 10 & 3 & \infty & \infty & \infty & 12 & \infty & 7 & \infty & 5 & \infty & \infty & \infty & \infty & \infty & \infty & 13 & 15 & 14 \end{bmatrix}$$

$$B = \begin{bmatrix} \infty \\ 0 \\ 10 \\ \infty \\ \infty \\ \infty \\ 11 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & 0 & \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 0 & 0 & \infty & \infty & \infty \\ Ns\text{-}1 & \infty & \infty & \infty & 0 & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & 0 & 0 & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 11 & \infty & 6 & \infty & \infty & \infty & 9 & \infty & 16 & \infty & \infty & \infty & \infty & 1 & \infty & 5 & 6 & \infty & 14 & 7 & \infty & \infty & \infty \end{bmatrix}$$

$$D = \begin{bmatrix} 2 \end{bmatrix}$$

$$E = \begin{bmatrix} 0 & \infty & \infty & \infty & \infty & \infty & \infty & 0 \end{bmatrix}$$

Code rate = 5/6

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 & 0 & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \infty & 1 & \infty & 0 \\ 1 & 6 & \infty & 4 & \infty & 10 & \infty & 5 & \infty & 11 & \infty & 8 & 7 & \infty & \infty & 15 & \infty & \infty & \infty & \infty & 3 & 12 & \infty & 9 & 2 & 14 & 3 & \infty & \infty & 13 \\ \infty & \infty & 6 & 3 & 2 & 12 & \infty & \infty & \infty & \infty & 9 & \infty & 0 & 9 & \infty & \infty & \infty & 8 & \infty & 12 & \infty & 2 & 3 & \infty & 5 & \infty & \infty & 11 & 5 & \infty \\ \infty & \infty & \infty & \infty & 11 & \infty & 15 & 1 & 10 & 6 & 4 & \infty & \infty & 5 & 4 & \infty & 1 & 7 & 2 & 6 & \infty & \infty & 8 & \infty & \infty & \infty & 13 & \infty & 12 & 10 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \infty \\ \infty \\ Ns-1 \\ \infty \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & \infty & \infty & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \\ \infty & \infty & 0 & 0 & \infty \\ \infty & \infty & \infty & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 6 & 13 & 15 & \infty & \infty & \infty & 2 & \infty & \infty & \infty & \infty & 12 & \infty & \infty & 4 & 13 & 11 & \infty & 15 & \infty & 14 & \infty & \infty & 10 & \infty & 8 & 3 & 8 & 0 & 5 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 \end{bmatrix}$$

$$E = \begin{bmatrix} \infty & \infty & \infty & \infty & 1 \end{bmatrix}$$

Supplementary information on the proposed text: Simulation results of Intel LDPC coding vs. Samsung B-LDPC coding.
The following two graphs in Figure 1 and 2 highlights the performance comparison of the two proposed LDPC encoders, i.e., Intel LDPC (illustrated in IEEE 802.11-03/0865r1) and Samsung B-LDPC (proposed in this contribution).

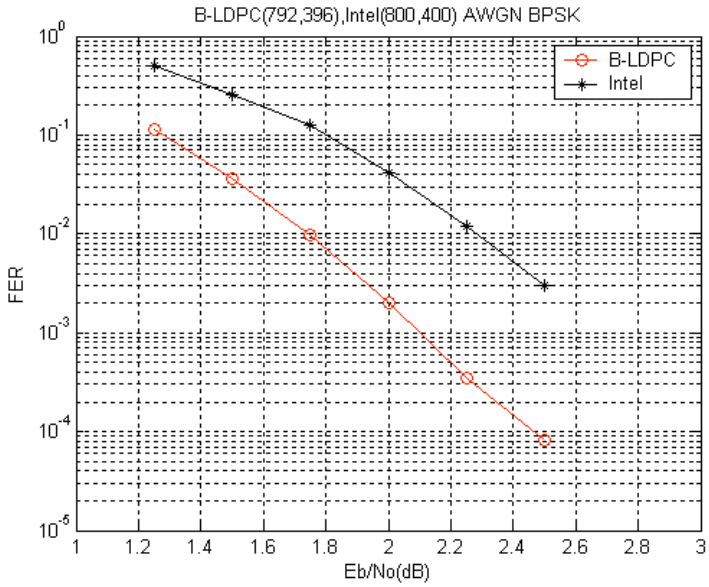

Fig. 1. Performance comparison between a (792,396) B-LDPC code and a (800,400) Intel LDPC code

The performance of a B-LDPC code with column is compared with that of a Intel LDPC code frame length 800 in Fig. 1, in terms of FER (frame error rate). The parity check matrix of B-LDPC code has the length of the B-LDPC code is 792 and the code rate 1/2.

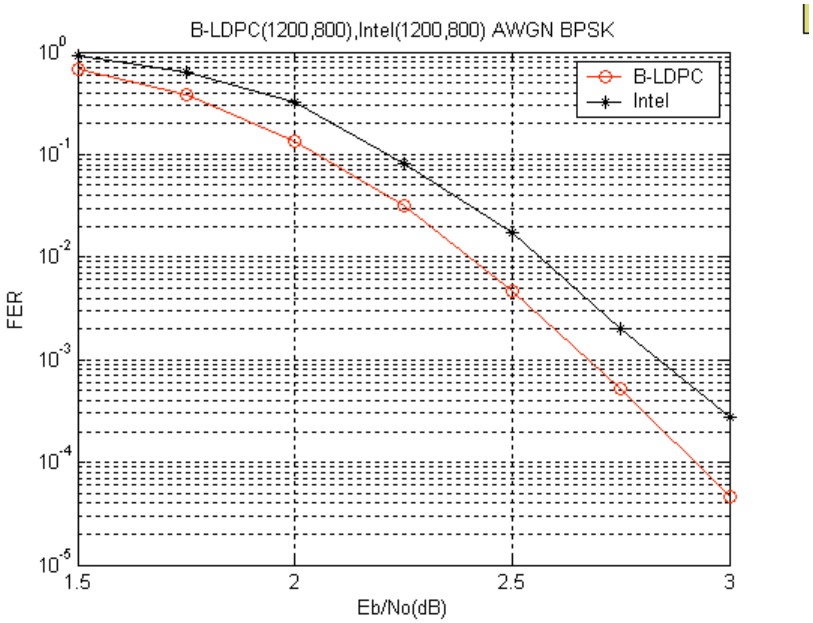

Fig. 2 Performance comparison between a (1200,800) B-LDPC code and a (1200,800) Intel LDPC code

The performance of a B-LDPC code with column is compared with that of a Intel LDPC code frame length 1200 in Fig. 2, in terms of FER (frame error rate). The parity check matrix of B-LDPC code has the length of the B-LDPC code is 1200 and the code rate 2/3.