This document proposes a simple and novel closed-loop transmit precoding scheme to improve the frame-error rate and bit-error rate performance of OFDM/OFDMAPHY in IEEE 802.16e. For $P \times Q$ MIMO, the codebook consists of unitary rotation matrices and is known both to the transmitter and to the receiver. With a careful design, the size of the codebook is kept small implying minimal feedback from the receiver to the transmitter. Processing for selecting optimum rotation matrix is required only at the receiver, whereas the transmitter simply precodes the transmitted symbols with the chosen rotation matrix. Simulation results are provided to illustrate the performance gain of the proposed closed-loop technique over the existing open-loop operation.
An enhanced closed-loop MIMO design for OFDM/OFDMA-PHY
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1. Introduction

The current IEEE 802.16-2004 standard [1] and its IEEE 802.16e-D4 amendment [2] suggest open-loop MIMO operation. This mode of operation assumes no knowledge, whatsoever, of the communication channel at the transmitter. In this contribution, we propose a simple closed-loop MIMO transmission methodology, where the transmitted symbols are precoded using a finite set of pre-defined unitary rotation matrices. This set of matrices belongs to a carefully crafted codebook and is known both to the receiver and to the transmitter. Given the received data, the receiver determines the optimum rotation matrix for each OFDM sub-carrier that will result in best performance. It will then transmit only the index of the optimum rotation matrix to the transmitter, where it is reconstructed and used to precode the transmitted symbols. With a very few number of rotation matrices in the basis codebook, the amount of feedback involved in such a scheme is much less than if the full set of channel coefficients are sent back from the receiver to the transmitter.

Using numerical simulations, we will show that significant performance gain is achieved from the proposed closed-loop operation over the default open-loop case. We also illustrate the performance/feedback-rate tradeoff and suggest possible options in easing this tradeoff.

2. MIMO Setup

Consider a MIMO OFDM setup with $P$ transmit antennas and $Q$ receive antennas. The $Q$-dimensional baseband received signal vector $\mathbf{r} = [r_1, r_2, \ldots, r_Q]^T$ can be written as

$$
\mathbf{r} = \sum_{p=1}^{P} \mathbf{h}_p^T \mathbf{s}_p + \mathbf{w} = \mathbf{H} \mathbf{s} + \mathbf{w},
$$

where $\mathbf{h}_i = [h_{i1}, h_{i2}, \ldots, h_{iQ}]^T$ is a $Q$-dimensional vector containing channel coefficients from $i$th transmitter to $Q$ receivers, $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_P]$ is the $Q \times P$ channel matrix,

$s = [s_1, s_2, \ldots, s_P]^T$ is the $P$-dimensional transmit signal vector, and $\mathbf{w} = [w_1, w_2, \ldots, w_Q]^T$ is the $Q$-dimensional vector of zero-mean noise with variance $\sigma^2$. The received signal can be processed by using either the optimal maximum-likelihood method or a sub-optimal method like zero-forcing or linear minimum mean squared error processing.

Note that the vector $\mathbf{s}$ is represented by

$$
\mathbf{s} = \mathbf{V} \mathbf{d},
$$

where $\mathbf{d} = [d_1, d_2, \ldots, d_P]^T$ is the $P$-dimensional vector of symbols to be transmitted and $\mathbf{V}$ is the $P \times P$ precoding rotation matrix. The reason of introducing this notation is to bring in the flexibility of treating closed-loop and open-loop options within the same framework. Note that for the open loop case, $\mathbf{V}$ is the $P \times P$ identity matrix. The effective (rotated) channel matrix is, therefore, denoted by
H' = HV.

3. Closed-Loop MIMO

It is known that if perfect channel state information is available at the transmitter, then one can precode the transmitted symbols with the eigenvectors \( \mathbf{V} \) of the matrix \( H^H H \), where \( (\cdot)^H \) denotes conjugate transposition. In this case, we can perfectly separate the transmitted symbols at the receiver, thereby achieving capacity. However, the transmittal of complete channel state information from receiver to the transmitter is prohibitively expensive. Furthermore, the cost of feedback is even higher in an OFDM system, where a different eigenvector is associated with each sub-carrier.

An alternative to sending the complete channel state information is to define a codebook containing a finite set of \( N \) unitary rotation matrices, which is known to both the transmitter and the receiver. Based on a metric that maximizes post-processed signal-to-noise ratio (SNR), the receiver determines a precoding rotation matrix from the codebook for each OFDM sub-carrier. The index of this matrix is then sent to the transmitter via a feedback path, where exactly the same matrix is reconstructed and used to precode the transmitted symbols. As shown in Figure 1, this operation requires only \( \log_2 N \) bits to be fed back from the transmitter to the receiver per OFDM sub-carrier. For example, if the set has eight rotation matrices, then three bits per sub-carrier need to be sent back.

For the sake of clarity, we will treat the simpler 2×2 case first. This will be followed by the generalized \( P \times Q \) case, where \( P = Q > 2 \). We will also show that 2×2 is a special case of the generalized \( P \times Q \) MIMO, which allows us to treat all the MIMO cases in a single unified framework. We will then discuss the design of 4×2 MIMO system with 2 transmit streams and 4 transmit antennas. For all the schemes, we will address the design of basis codebook and the impact of its size on the performance gain of closed-loop schemes.

3.1. 2×2 MIMO

For 2×2 MIMO, we define the codebook with a set of \( N \) rotation matrices denoted by \( \mathbf{V} \)

\[
\mathbf{V}_{N_1N_2} = \begin{bmatrix}
e^{j\phi_{n_2}} \cos \theta_{n_1} & -e^{j\phi_{n_2}} \sin \theta_{n_1} \\
\sin \theta_{n_1} & \cos \theta_{n_1}
\end{bmatrix},
\]

where

\[
\phi_{n_2} = \frac{2\pi n_2}{N_2}, n_2 = 0,1,\ldots,N_2 - 1
\]

\[
\theta_{n_1} = \frac{\pi n_1}{2N_1}, n_1 = 0,1,\ldots,N_1 - 1
\]

and \( N = N_1N_2 \).

Note that for each sub-carrier, the index of the rotation matrix may be sent from the receiver to the transmitter only once per frame. This is assuming that the channel stays static over the frame duration.
Figure 1: Proposed closed-loop transmission scheme for a $P$-input $Q$-output system.

### 3.2. $P \times Q$ ($P = Q$) MIMO

Let us now consider the general $P \times Q$ case, where $P = Q > 2$. We generate the real unitary rotation by applying a sequence of $P(P-1)/2$ Givens rotations [3] to the channel matrix as follows

$$
V(\theta) = \prod_{i=1}^{P-1} \prod_{k=i+1}^{P} G(i, k, \theta),
$$

where the Givens rotation matrix is given is

$$
G(i, k, \theta) =
\begin{bmatrix}
1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & c & \cdots & s & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \ddots & \ddots & \vdots \\
0 & \cdots & -s & \cdots & c & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & \cdots & 0 & \cdots & 1
\end{bmatrix}
$$

with $c = \cos(\theta)$ and $s = \sin(\theta)$. Since $G(i, k, \theta)$ is orthogonal, it is clear that the resulting rotation matrix $V(\theta)$ is unitary.

Note that each Givens rotation in the above product can be associated with a different rotation angle. For example, for $P = Q = 3$, $V(\theta_1, \theta_2, \theta_3)$ is the product of three Givens rotations

$$
V(\theta_1, \theta_2, \theta_3) = G(1, 2, \theta_1)G(1, 3, \theta_2)G(2, 3, \theta_3).
$$

As in the $2 \times 2$ case, we quantize the Givens rotation angles and form a codebook of unitary matrices. For instance, for $3 \times 3$, the quantized set of $N$ rotation matrices is given by

$$
V_{N_1N_2N_3+N_0} = G(1, 2, \theta_{n_1})G(1, 3, \theta_{n_2})G(2, 3, \theta_{n_3}),
$$

where
\[ \theta_n = \frac{\pi n_1}{2N_1}, n_1 = 0,1,\ldots,N_1-1, \]
\[ \theta_n = \frac{\pi n_2}{2N_2}, n_2 = 0,1,\ldots,N_2-1, \]
\[ \theta_n = \frac{\pi n_3}{2N_3}, n_3 = 0,1,\ldots,N_3-1, \]
and \( N = N_1N_2N_3 \).

The feedback requirement in this case is again \( \log_2 N \) bits. If each rotation is quantized to four angles, then \((N_1, N_2, N_3) = (4, 4, 4)\), which results in a total of \( N = 64 \) unitary rotation matrices. This implies a feedback of 6 bits per OFDM sub-carrier. The selection of optimum rotation matrix is similar to the \( 2 \times 2 \) case and will be discussed in Section 3.4.

From the above discussion, we can easily appreciate that the Givens rotation approach to the generation of \( P \times Q \) unitary matrices can as well be extended to higher MIMO configurations. For example, for \( 4 \times 4 \), the matrix \( V \) is a product of \( P(P-1)/2 = 4 \) Givens rotations. Moreover, we also note that \( 2 \times 2 \) is a special case of Givens rotation, where only one rotation is employed.

### 3.3. \( 4 \times 2 \) MIMO

For 4 transmit antennas with 2 transmit streams, we split the transmitter into two 2-transmit antenna units. Each unit then transmits one data stream. We associate a \( 2 \times 1 \) precoding vector with each data stream. The two resulting vectors are combined to form the precoding matrix \( V \) as follows

\[ V_{N_1 N_2 + n_1} = \begin{bmatrix} w_{n_1} & 0 \\ 0 & w_{n_2} \end{bmatrix}, \]

where

\[ w_{n_1} = \begin{bmatrix} 1 \\ e^{j(\pi/4 + 2\pi n_1/N_1)} \end{bmatrix}, n_1 = 0,\ldots,N_1-1, \]
\[ w_{n_2} = \begin{bmatrix} 1 \\ e^{j(\pi/4 + 2\pi n_2/N_2)} \end{bmatrix}, n_2 = 0,\ldots,N_2-1, \]

and \( N = N_1N_2 \). Note that this scheme does transmit beamforming by combining data from two antennas and forming one transmit stream.

### 3.4. Selection of Rotation Matrix

The selection of rotation matrix depends on the type of receiver employed to recover the transmitted source symbols. In this contribution, we will consider the iterative minimum-mean-squared error (MMSE) receiver, which detects the transmitted symbols in the order of decreasing post-processed SNR; i.e., the most “reliable” symbols are detected first and removed from the received signal followed by estimating symbols of decreasing reliability.

The MMSE post-processed SNR of the \( P \) received symbol streams is given by
\[ \text{SNR}_i = h_i^H \left( \sum_{j=1, j \neq i}^{P} h_j h_j^H + \sigma^2 I \right)^{-1} h_i, \quad i = 1, \ldots, P, \]

where \( h \) is the \( i \)th column of the channel matrix \( H \) and \( I \) is the \( P \times P \) identity matrix. Note that the above SNR value is computed for the open-loop transmission.

In order to pick the best rotation matrix for each tone in the OFDM symbol, we compute the post-processed SNR for each unitary rotation matrix in the basis set. If we define the rotated channel matrix as

\[ H'_n = HV_n, \quad n = 0, 1, \ldots, N - 1, \]

then the post-processed SNR for each case is given by

\[ \text{SNR}_{n,j}^r = h_{n,i}^r H \left( \sum_{j=1, j \neq i}^{P} h_{n,j}^r h_{n,j}^r H + \sigma^2 I \right)^{-1} h_{n,i}^r, \quad i = 1, \ldots, P; n = 0, \ldots, N - 1. \]

Of the \( P \) received streams, we pick up the smallest SNR value and maximize it over all possibilities of the rotation matrices. Mathematically, the selection of rotation matrix can be stated as

\[ V_{n}^{\text{opt}} = \arg \max_n \left( \min_i \left( \text{SNR}_{n,i}^r \right) \right). \]

In other words, the above operation guarantees the maximization of the minimum post-processed SNR over all the possible choices. Note that for IMMSE processing, the interference term \( \sum_{j=1, j \neq i}^{P} h_{n,j}^r h_{n,j}^r H \) deflates each time a signal is estimated and subtracted from the received signal.

4. Simulation Results

To verify the potential of the proposed closed-loop method, we carried out numerical simulations for various baseband MIMO OFDM system configurations employing IMMSE receiver.

For the simulations, we considered 768 data tones in the OFDM symbol, which employed 1024-point IFFT/FFT at the transmitter/receiver. The frame duration is set to 5ms and a delay of 2 frames is used for the feedback of channel-state information. We used convolutional coding for forward-error correction and employed iterative minimum mean squared error (IMMSE) receiver for decoding of transmitted symbols.

In the simulations to follow, we used ITU OIP-B outdoor-to-indoor pedestrian channels with vehicular speeds of 3 km/hr. Transmit antenna correlation of \( \rho = 0.2 \) or \( \rho = 0.7 \) is used in the experiments. For all the simulations, ideal channel knowledge is assumed at the receiver.

The frame-error rate (FER) results are discussed below for each MIMO configuration, where the open-loop performance is compared against the closed-loop performance to gauge the gain.
4.1. 2×2

Various simulation results for 2×2 MIMO using different modulation modes are shown in Figure 2 to Figure 7. Note that \((N_1, N_2) = (4,1)\) corresponds to a feedback of 2 bits per sub-carrier.

Figure 2: Performance comparison of 2×2 open-loop MIMO against closed-loop MIMO; QPSK, Rate 3/4, \(\rho = 0.7\).

Figure 3: Performance comparison of 2×2 open-loop MIMO against closed-loop MIMO; 16-QAM, Rate 3/4, \(\rho = 0.7\).
Figure 4: Performance comparison of 2×2 open-loop MIMO against closed-loop MIMO; 64-QAM, Rate 3/4, $\rho = 0.7$.

Figure 5: Performance comparison of 2×2 open-loop MIMO against closed-loop MIMO; QPSK, Rate 3/4, $\rho = 0.2$. 
Figure 6: Performance comparison of 2×2 open-loop MIMO against closed-loop MIMO; 16-QAM, Rate 3/4, $\rho = 0.2$.

Figure 7: Performance comparison of 2×2 open-loop MIMO against closed-loop MIMO; 16-QAM, Rate 1/2, $\rho = 0.2$.

4.2. 4×4

For the 4×4 simulation results depicted below, the feedback requirement is 6 bits per sub-carrier.
Figure 8: Performance comparison of 4×4 open-loop MIMO against closed-loop MIMO; QPSK, Rate 3/4, $\rho = 0.7$.

Figure 9: Performance comparison of 4×4 open-loop MIMO against closed-loop MIMO; 16-QAM, Rate 3/4, $\rho = 0.2$.

4.3. 4×2

We compare the performance of 4×2 closed-loop MIMO against the 2×2 open-loop mode. The parameter set $(N_1, N_2) = (2, 2)$ implies a feedback of 2 bits per sub-carrier, whereas $(N_1, N_2) = (4, 4)$ corresponds to 4 bits feedback per sub-carrier.
Figure 10: Performance comparison of 2×2 open-loop MIMO against 4×2 closed-loop MIMO; QPSK, Rate 3/4, $\rho = 0.7$.

Figure 11: Performance comparison of 2×2 open-loop MIMO against 4×2 closed-loop MIMO; 16-QAM, Rate 3/4, $\rho = 0.7$. 
The closed-loop performance of different MIMO modes considered above is summarized in Table 1. The table also lists the feedback bits requirement for each case.

Table 1: Summary of the closed-loop performance for various MIMO modes.

<table>
<thead>
<tr>
<th>MIMO Mode</th>
<th>Modulation</th>
<th>Code Rate</th>
<th>$\rho$</th>
<th>Feedback bits requirement per sub-carrier</th>
<th>Gain over open loop mode (dB) at 1% FER</th>
</tr>
</thead>
<tbody>
<tr>
<td>2×2</td>
<td>QPSK</td>
<td>$\frac{3}{4}$</td>
<td>0.7</td>
<td>2</td>
<td>2.5</td>
</tr>
<tr>
<td>2×2</td>
<td>16-QAM</td>
<td>$\frac{3}{4}$</td>
<td>0.7</td>
<td>2</td>
<td>3.7</td>
</tr>
<tr>
<td>2×2</td>
<td>64-QAM</td>
<td>$\frac{3}{4}$</td>
<td>0.7</td>
<td>2</td>
<td>3.5</td>
</tr>
<tr>
<td>2×2</td>
<td>QPSK</td>
<td>$\frac{3}{4}$</td>
<td>0.2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2×2</td>
<td>16-QAM</td>
<td>$\frac{3}{4}$</td>
<td>0.2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2×2</td>
<td>16-QAM</td>
<td>$\frac{1}{2}$</td>
<td>0.2</td>
<td>2</td>
<td>0.8</td>
</tr>
<tr>
<td>4×4</td>
<td>QPSK</td>
<td>$\frac{3}{4}$</td>
<td>0.7</td>
<td>6</td>
<td>2 (at 10% FER)</td>
</tr>
<tr>
<td>4×4</td>
<td>16-QAM</td>
<td>$\frac{3}{4}$</td>
<td>0.2</td>
<td>6</td>
<td>3.5</td>
</tr>
<tr>
<td>4×2</td>
<td>QPSK</td>
<td>$\frac{3}{4}$</td>
<td>0.7</td>
<td>4</td>
<td>6.7</td>
</tr>
<tr>
<td>4×2</td>
<td>QPSK</td>
<td>$\frac{3}{4}$</td>
<td>0.7</td>
<td>2</td>
<td>6.2</td>
</tr>
<tr>
<td>4×2</td>
<td>16-QAM</td>
<td>$\frac{3}{4}$</td>
<td>0.7</td>
<td>2</td>
<td>7.8</td>
</tr>
<tr>
<td>4×2</td>
<td>64-QAM</td>
<td>$\frac{3}{4}$</td>
<td>0.7</td>
<td>2</td>
<td>6.5</td>
</tr>
</tbody>
</table>

5. Conclusions

The proposed MIMO closed-loop scheme requires minimal feedback and results in appreciable gain over the corresponding MIMO open-loop mode. As expected, we observe larger gain for higher antenna correlation. Moreover, the gain increases with more transmit/receive antennas.
Note that interpolation across frequency can be employed to further reduce the feedback requirement in our proposed methodology. However, interpolation works only when the OFDMA sub-carriers assigned to a user are arranged contiguously over the frequency band. Therefore, its application is limited only to certain frame structures.

6. Proposed Text Changes

To be determined by the IEEE 802.16e working group.

7. References