**Project**  

**Title**  
Improved Space-Time Codes for the OFDMA PHY with four transmission antennas

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**Re:**  
Contribution supporting TGe WG ballot #14c

**Abstract**  
Improved full-rate full-diversity space-time code for 4-Tx antennas

**Purpose**  
Adoption of proposed changes into P802.16e  
*Crossed-out indicates deleted text, underlined blue indicates new text change to the Standard, and underlined green indicates newly added text from the original contribution*

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An Improved Space-Time Code for 4-Tx antennas

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1. Introduction

We propose a space-time code (STC) with full rate full diversity (FDFR) for four transmission antenna configuration. This code has lower encoding and decoding complexity than the exiting FDFR space-time codes [1]. The proposed method does not change matrix A itself, but need to apply different mapping rules for input symbols of existing matrix A. Therefore, no additional encoding complexity and trivial additional decoding complexity are required to provide low power consumption. The performance gain, however, is higher than that of the STC with simple use of existing matrix A.

2. Proposed space-time code

We propose to replace the existing transmission matrix

\[
\begin{pmatrix}
  1 & 2 \\
  2 & 1 \\
  3 & 4 \\
  4 & 3 \\
\end{pmatrix}
\]

with the new the transmission matrix \(A_1\) which is defined as follows:

Let the complex symbols to be transmitted be \(x_1, x_2, x_3, x_4\) which take values from a Gray-mapped square QAM constellation (8.4.9.4.2 in [1]). Let \(s_i = x_i e^{j\theta}\) for \(i = 1, 2, 3, 4\), where \(\theta = \tan^{-1}(2)\) for QPSK, \(\theta = \tan^{-1}(1/4)\) for 16QAM, and \(\theta = \tan^{-1}(1/8)\) for 64 QAM and let

\[
\begin{align*}
\bar{s}_1 &= s_{1I} + js_{1Q} \\
\bar{s}_2 &= s_{2I} + js_{2Q} \\
\bar{s}_3 &= s_{3I} + js_{3Q} \\
\bar{s}_4 &= s_{4I} + js_{4Q}
\end{align*}
\]

where \(s_i = s_{ii} + js_{iQ}\).

The proposed Space-Time-Frequency code for 4Tx-Rate 1 configuration is
The first two columns correspond to the two OFDM symbols and one subcarrier. Similarly the last two columns correspond to the same two OFDM symbols, but for the next subcarrier. Let $A_i = \begin{bmatrix} \bar{s}_1 & -\bar{s}_2^* & 0 & 0 \\ \bar{s}_2 & \bar{s}_1^* & 0 & 0 \\ 0 & 0 & \bar{s}_3 & -\bar{s}_4^* \\ 0 & 0 & \bar{s}_4 & \bar{s}_3^* \end{bmatrix}$ be the channel coefficients for the first subcarrier. The channel is assumed to be quasi-static for two OFDM symbols, but could be varying across the subcarriers. Let $A_i = \begin{bmatrix} H_1(l) & H_2(l) & H_3(l) & H_4(l) \end{bmatrix}$ be the channel coefficients for the second subcarrier.

The above encoding process results in the bit mapping rules between the binary bits and the regular QAM constellation symbols. The bit mapping rules are as follows:

Let the binary information vectors to be transmitted as $u_1 = [u_{10} \ u_{11} \ L \ u_{1,B-1}]^T$, $u_2 = [u_{20} \ u_{21} \ L \ u_{2,B-1}]^T$, $u_3 = [u_{30} \ u_{31} \ L \ u_{3,B-1}]^T$, $u_4 = [u_{40} \ u_{41} \ L \ u_{4,B-1}]^T$, where $B$ is the number of bits to be transmitted within a symbol duration. For example, $B = 2, 4, \text{and} 6$ for QPSK, 16QAM, and 64QAM, respectively. Let $\bar{s}_i = m(b_i)$, where $b_i = [u_1^Tu_1^T] \ L \ b_2 = [u_1^Tu_2^T] \ L \ b_3 = [u_1^Tu_4^T] \ L \ b_4 = [u_1^Tu_2^T]$, and $m(\cdot)$ is the square QAM modulation function whose input and output are defined as:

for QPSK ($B=2$)

<table>
<thead>
<tr>
<th>$b_{10}$</th>
<th>$b_{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\cdot$</td>
<td>$00$</td>
</tr>
<tr>
<td>$\cdot$</td>
<td>$01$</td>
</tr>
<tr>
<td>$\cdot$</td>
<td>$10$</td>
</tr>
<tr>
<td>$\cdot$</td>
<td>$11$</td>
</tr>
</tbody>
</table>

for 16QAM ($B=4$),
and finally for 64QAM ($B=6$), $m(\cdot)$ can be similarly defined as the $2^B$QAM constellations where the mapping rule between the input bits and output symbols are expressed as follows:

- $b_{i1} b_{i0} b_{i9} b_{i8} b_{i7} b_{i6}$ (real-axis: from left to right)
  (111011) (111010) (111000) (110001) (111101) (111100) (111110) (111111)
  (110011) (110010) (110000) (110001) (110101) (110100) (110110) (110111)
  (100011) (100010) (100000) (100001) (100101) (100100) (100110) (100111)
  (101011) (101010) (101000) (101001) (101101) (101100) (101110) (101111)
  (001011) (001010) (001000) (001001) (001101) (001100) (001110) (001111)
  (000011) (000010) (000000) (000001) (000101) (000100) (000110) (000111)
  (010011) (010010) (010000) (010001) (010101) (010100) (010110) (010111)
  (011011) (011010) (011000) (011001) (011101) (011100) (011110) (011111)

- $b_{i5} b_{i4} b_{i3} b_{i2} b_{i1} b_{i0}$ (imaginary-axis: from bottom to top)
  (011011) (010011) (000011) (001011) (101011) (100011) (110011) (111011)
  (011010) (010010) (000010) (001010) (101010) (100010) (110010) (111010)
  (011000) (010000) (000000) (001000) (101000) (100000) (110000) (111000)
  (011001) (010001) (000001) (001001) (101001) (100001) (110001) (111001)
  (011101) (011100) (011000) (011001) (111001) (111010) (111100) (111110)
  (011110) (011110) (011010) (011011) (111010) (111011) (111100) (111110)
  (011111) (011111) (011101) (011110) (011011) (011010) (011001) (011000)
Thus, the encoding process requires simple bit mapping between the binary information and modulated symbols. The proposed space-time code can employ the decoder of the STC in [1].

The phase rotation and I-Q coordinate interleaving based implementation and the direct bit mapping based implementation are equivalent.

3. Decoder and soft bit metric calculation

We assume that the channel is quasi-static and the receiver adopts one receive antenna. Then, the received signals can be expressed as

\[
\begin{bmatrix}
  y_1^1 \\
  y_1^2 \\
  y_2^1 \\
  y_2^2
\end{bmatrix} = \begin{bmatrix}
  \hat{s}_1 - \hat{s}_2^* \\
  \hat{s}_2 \hat{s}_1^* \\
  0 \\
  0
\end{bmatrix} \begin{bmatrix}
  h_1^n \ h_2^i \\
  h_1^i \ h_2^j
\end{bmatrix} + \text{noise}
\]

where \( y_k^i \) denotes the received signal on the \( k \)-th subcarrier at time \( i \) and \( h_k^i \) denotes the channel response between the \( j \)-th transmit antenna and the receive antenna on the \( k \)-th subcarrier. The estimates of the transmit symbols \( \hat{s}_k \) are obtained by

\[
\begin{bmatrix}
  \hat{z}_1 \\
  \hat{z}_2 \\
  \hat{z}_3 \\
  \hat{z}_4
\end{bmatrix} = \begin{bmatrix}
  \alpha^{-1} & 0 & 0 & 0 \\
  0 & \alpha^{-1} & 0 & 0 \\
  0 & 0 & \beta^{-1} & 0 \\
  0 & 0 & 0 & \beta^{-1}
\end{bmatrix} \begin{bmatrix}
  h_1^n \ h_2^i \\
  -h_2^i h_1^n \\
  0 \\
  0
\end{bmatrix} \begin{bmatrix}
  y_1^1 \\
  y_1^2 \\
  y_2^1 \\
  y_2^2
\end{bmatrix}
\]

where \( \alpha = |h_1|^2 + |h_2|^2 \), \( \beta = |h_3|^2 + |h_4|^2 \).

The soft bit metric for the channel decoder input is calculated as the log likelihood ratio of each bit:

\[
\text{LLR}_{s_k} = \log \frac{P(x_{ib} = 1 | y_1^i, y_2^i)}{P(x_{ib} = 0 | y_1^i, y_2^i)} = \log \sum_{\hat{s}_k \in S^1} P(y_1^i, y_2^i | \hat{s}_k) P(\hat{s}_k) / \sum_{\hat{s}_k \in S^0} P(y_1^i, y_2^i | \hat{s}_k) P(\hat{s}_k)
\]

Applying max log algorithm, the LLR is further simplified as

\[
\text{LLR}_{s_k} = \log \frac{\max_{\hat{s}_k \in S^1} P(y_1^i, y_2^i | \hat{s}_k) P(\hat{s}_k)}{\max_{\hat{s}_k \in S^0} P(y_1^i, y_2^i | \hat{s}_k) P(\hat{s}_k)}
\]

\[
= \min_{\hat{s}_k \in S^1} \left( \frac{\alpha | z_1 - \hat{s}_1 |^2 + \beta | z_3 - \hat{s}_3 |^2}{2\sigma^2} \right) - \min_{\hat{s}_k \in S^0} \left( \frac{\alpha | z_1 - \hat{s}_1 |^2 + \beta | z_3 - \hat{s}_3 |^2}{2\sigma^2} \right)
\]

Since the bit mapping of the proposed scheme can be separated into real and imaginary axes, i.e., each bit is related to only the real or the imaginary part of the modulated symbol, the decoder needs to apply \( 2^{n-1} \) candidate constellation points in order to find the minimum Euclidean distance in the LLR equation. Note that the size of the searching space is NOT the whole constellation of size \( 2^{2n} \). Thus, the increase in the decoding complexity of the proposed scheme is moderate compared with the original matrix \( A \).
Specifically, for example, the LLRs when $B = 2$ can be calculated as

$$LLR_{u0} = \min(2\alpha z_m + \beta (6z_{ai} + 8), \alpha (6z_m + 8) - 2\beta z_{ai}) / 0.5\sigma_w^2$$

$$- \min(-2\alpha z_m + \beta (-6z_{ai} + 8), \alpha(-6z_m + 8) + 2\beta z_{ai}) / 0.5\sigma_w^2$$

$$LLR_{ui} = \min(\alpha(6z_m + 8) - 2\beta z_{ai}, -2\alpha z_m + \beta(-6z_{ai} + 8)) / 0.5\sigma_w^2$$

$$- \min(2\alpha z_m + \beta (6z_{ai} + 8), \alpha(-6z_m + 8) + 2\beta z_{ai}) / 0.5\sigma_w^2$$

$$n = \text{mod}(m+1,4) + 1$$

$$(\alpha, \beta) = \begin{cases} (|h_2|^2 + |h_3|^2, |h_1|^2 + |h_1|^2), & m = 1,2 \\ (|h_3|^2 + |h_1|^2, |h_2|^2 + |h_2|^2), & m = 3,4 \end{cases}$$

where the subscript $r$ and $i$ denote the real and the imaginary parts of a complex variable, respectively.

### 4. Performance of the proposed space-time code

We compare the proposed FDFR STC scheme with the existing FDFR STC method [1] existing matrix $A$ in section 8.4.8.3.4 [3] for several scenarios.

![Performance comparison](image)

**Figure 1:** Performance comparison of the proposed method with the existing method [1] matrix $A$ for 16QAM, 3/4 rate, Vehicular A channel of 60 kmph
5. Response to the reply comments

- About backward compatibility
  This contribution deals with an enhanced multiple antenna transmission scheme for IEEE 802.16e as an option. Therefore, this scheme has no confliction with the methods of IEEE 802.16d.

- About the small performance difference between the proposed scheme and original method based on matrix $A$ in Sec. 8.4.8.3.4.

In the band AMC mode, performance improvement is observed compared with the existing scheme of matrix $A$ (see figures 1, 2). Specifically, 1.2dB SNR gain is seen for FER$=1.4\times10^{-2}$ for 16QAM, 3/4 rate in Vehicular A channel of 60 km/h. We observe this gain over the existing matrix $A$ increases with higher coding rate and/or larger constellation. This gain is more prominent in less frequency selective channels such as ITU Ped A channels. The authors of [4] claims there is little difference between the proposed scheme and the existing one, but their simulation environment only considers the diversity subchannel with PUSC, smaller constellation (QPSK) under the relatively high frequency selective channel (Ped B). In the band AMC mode, however, we cannot have enough diversity gain from frequency domain due to insufficient frequency selectivity, and the diversity gain from the spatial domain is much more feasible. Therefore, FDFR STC is highly required for four transmit antennas to achieve maximum performance.

For the complexity issues, the proposed method does not change matrix $A$ itself, but need to apply phase rotation-coordinate interleaving method or direct bit mapping rules for input symbols of existing matrix $A$. Therefore, no additional encoding complexity is needed in the proposed method. This is because...
binary bits are directly mapped into the modulated symbol. In addition, as discussed in Section 3, the 
additional decoding complexity is trivial (the complexity is comparable with existing three antenna case 
[5]). In order to maximize overall performance of IEEE802.16e, FDFR STC for four transmit antennas is 
strongly desirable.

- About the decoding complexity to compute soft bit metric.

The additional decoding complexity is trivial. In Section 3, one example of the simple decoding 
procedure is discussed in detail. In addition, the complexity is comparable with existing STC for three 
antenna case [5].

6. Specific text changes

[Replace matrix A in the section 8.4.8.3.3 (p.164) and the following sentence with the following]

Replacing the document in Sec. 8.4.8.3.4 “Transmission schemes for 4-antenna BS” with the new transmission matrix $A^*$ as:

8.4.8.3.4. Transmission schemes for 4-antenna BS

Replace the existing transmission matrix A:

$A_1 = \begin{bmatrix}
    \bar{s}_1 & -\bar{s}_2^* & 0 & 0 \\
    \bar{s}_2 & \bar{s}_1^* & 0 & 0 \\
    0 & 0 & \bar{s}_3 & -\bar{s}_4^* \\
    0 & 0 & \bar{s}_4 & \bar{s}_3^*
\end{bmatrix}$

With $A_1$ shown below:

where the complex symbols to be transmitted are $x_2, x_3, x_4$, which take values from a square QAM 
constellation and $s_i = x_i e^{i\theta}$ for $i=1, 2, 3, 4$, where $\theta = \tan^{-1}(2)$ for QPSK, $\theta = \tan^{-1}(1/4)$ for 16QAM, and 
$\theta = \tan^{-1}(1/8)$ for 64 QAM and let

$\bar{s}_1 = s_{31} + js_{30}; \bar{s}_2 = s_{21} + js_{20}; \bar{s}_3 = s_{31} + js_{30}; \bar{s}_4 = s_{41} + js_{40}$ where $s_i = s_d + js_d$.

Equivalently, $\bar{s}_i$ can be obtained by the direct bit mapping defined as

$\bar{s}_i = m(b_i)$. 


where \( \mathbf{b}_1 = [u_0^T u_1^T]^T \), \( \mathbf{b}_2 = [u_2^T u_3^T]^T \), \( \mathbf{b}_3 = [u_4^T u_5^T]^T \), \( \mathbf{b}_4 = [u_6^T u_7^T]^T \), \( \mathbf{u}_1 = [u_{10} u_{11} L u_{1B-1}]^T \), \( \mathbf{u}_2 = [u_{20} u_{21} L u_{2B-1}]^T \), \( \mathbf{u}_3 = [u_{30} u_{31} L u_{3B-1}]^T \), \( \mathbf{u}_4 = [u_{40} u_{41} L u_{4B-1}]^T \). \( B \) denotes the number of bits to be transmitted within a symbol duration \((B=2, 4, 6)\). \( m() \) denotes the square QAM modulation function whose input and output are defined as for QPSK \((B=2)\):

![QPSK Diagram]

and for 16QAM \((B=4)\):
for 64QAM ($B=6$), $m(\cdot)$ is defined as:

- $b_{11} b_{10} b_9 b_8 b_7 b_6$. (real-axis: from left to right)
  (111011) (111010) (111000) (111001) (111011) (111100) (111101) (111110) (111111)
  (110011) (110010) (110000) (110001) (110011) (110100) (110101) (110110) (110111)
  (100011) (100010) (100000) (100001) (100011) (100100) (100101) (100110) (100111)
  (101011) (101010) (101000) (101001) (101011) (101100) (101101) (101110) (101111)
  (001011) (001010) (001000) (001001) (001011) (001100) (001101) (001110) (001111)
  (000011) (000010) (000000) (000001) (000011) (000101) (000100) (000110) (000111)
  (010011) (010010) (010000) (010001) (010011) (010100) (010101) (010110) (010111)
  (011011) (011010) (011000) (011001) (011011) (011100) (011101) (011110) (011111)

- $b_{15} b_{14} b_{13} b_{12} b_{11} b_{10}$. (imaginary-axis: from bottom to top)
  (011011) (010011) (000011) (001011) (101011) (100011) (110011) (111011) (111111)
  (011010) (010010) (000010) (001010) (101010) (100010) (110010) (111010) (111110)
  (011000) (010000) (000000) (001000) (101000) (100000) (110000) (111000) (111100)
  (011001) (010001) (000001) (001001) (101001) (100001) (110001) (111001) (111101)
  (011101) (010101) (001101) (000101) (100101) (101101) (110101) (111101) (111111)
  (011100) (010100) (001100) (000100) (100100) (101100) (110100) (111100) (111110)
  (011110) (010110) (001110) (000110) (100110) (101110) (110110) (111110) (111111)
  (011111) (010111) (001111) (000111) (100111) (101111) (110111) (111111) (111111)
References


