<table>
<thead>
<tr>
<th>Project</th>
<th>IEEE 802.16 Broadband Wireless Access Working Group [<a href="http://ieee802.org/16">http://ieee802.org/16</a>]</th>
</tr>
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<tbody>
<tr>
<td>Title</td>
<td>LDPC coding for OFDMA PHY</td>
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<tr>
<td>Date Submitted</td>
<td>2004-08-24</td>
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<tr>
<td>Source(s)</td>
<td>Brian Classon, Yufei Blankenship, Jerry Kim, Panyuh Joo, Gyubum Kyung, Hongsil Jeong, Sang-Hyo Kim, Hanju Kim, DS Park, Jaeho Jeon, Samsung, Eric Jacobsen, Bo Xia, Intel, Kyuhyuk Chung, Min-seok Oh, LG Electronics, Brian Johnson, Alek Purkovic, Nina Burns, Sergey Sukobok, Nortel Networks, Eli Sasha, Zion Hadad, Yigal Leiba, Itzik Kitroser, Yossi Segal</td>
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<td><a href="mailto:brian.classon@motorola.com">brian.classon@motorola.com</a>, <a href="mailto:yufei.blankenship@motorola.com">yufei.blankenship@motorola.com</a>, <a href="mailto:kimjy@samsung.com">kimjy@samsung.com</a>, <a href="mailto:panyuh@samsung.com">panyuh@samsung.com</a>, <a href="mailto:gyubum.kyung@samsung.com">gyubum.kyung@samsung.com</a>, <a href="mailto:hongsil.jeong@samsung.com">hongsil.jeong@samsung.com</a>, <a href="mailto:sanghyo7.kim@samsung.com">sanghyo7.kim@samsung.com</a>, <a href="mailto:hanju.kim@samsung.com">hanju.kim@samsung.com</a>, <a href="mailto:dspark@samsung.com">dspark@samsung.com</a>, <a href="mailto:jhjeon@samsung.com">jhjeon@samsung.com</a>, <a href="mailto:eric.a.jacobsen@intel.com">eric.a.jacobsen@intel.com</a>, <a href="mailto:bo.xia@intel.com">bo.xia@intel.com</a>, <a href="mailto:kyuhyuk@lge.com">kyuhyuk@lge.com</a>, <a href="mailto:minoh@lge.com">minoh@lge.com</a>, <a href="mailto:brjohnso@nortelnetworks.com">brjohnso@nortelnetworks.com</a>, <a href="mailto:apurkovi@nortelnetworks.com">apurkovi@nortelnetworks.com</a>, <a href="mailto:nburns@nortelnetworks.com">nburns@nortelnetworks.com</a>, <a href="mailto:ssukobok@nortelnetworks.com">ssukobok@nortelnetworks.com</a>, <a href="mailto:shasha@runcom.co.il">shasha@runcom.co.il</a>, <a href="mailto:zionh@runcom.co.il">zionh@runcom.co.il</a>, <a href="mailto:yigall@runcom.co.il">yigall@runcom.co.il</a>, <a href="mailto:itzikk@runcom.co.il">itzikk@runcom.co.il</a>, <a href="mailto:yossis@runcom.co.il">yossis@runcom.co.il</a></td>
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Re: IEEE P802.16-REVe/D4-2004, ballot #14c

Abstract This contribution provides common LDPC text agreed to by an informal LDPC group.

Purpose Complete the LDPC specification text.

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Overview

An informal LDPC group has been working on the goal of achieving consensus on a proposed LDPC code design as an optional advanced code for the OFDMA PHY. Many excellent code designs have been submitted. The codes have been qualitatively and quantitatively characterized, and it is clear that a LDPC code with excellent flexibility and performance, as well as low encoding and decoding complexity, can be defined for 802.16e.

Eight companies (Intel, LG, Motorola, Nokia, Nortel, Runcom, Samsung, and TI) provided detailed proposals with code descriptions and simulation results to the group on 13 August 2004. Contribution 278r1 provided specification text for three of the proposals that share a large amount of commonality (Intel, Motorola, Samsung). This contribution is a harmonized reply from all eight companies.

References


Features

The LDPC codes have excellent performance, and contain features that provide flexibility and low encoding/decoding complexity.

- **Structured block LDPC for low complexity decoding.** The entire matrix (i.e., both the sections that correspond to the information and the parity) is composed of the same style of blocks, which reduces decoder implementation complexity and allows structured decoding.

- **Shortening for low-complexity block size flexibility.** The information portion of the matrix is defined such that excellent performance is achieved through shortening.

- **Low-complexity encoding.** The encoding can be performed directly from the H matrix for reduced complexity.

- **Designed to match the OFDMA subchannel structure.** No puncturing or rate-matching operations are required to provide exact code rates for many different block sizes.

- **No channel interleaver required.**

- **Compatible with hybrid ARQ** (Chase or Incremental Redundancy).
Simulation Results

Simulation results for the Motorola code of the rate 1/2, 2/3, and 3/4 code families are shown in Figure 1, Figure 2, Figure 3, respectively. The code sizes considered are $n = 48z$, where $z$ is the expansion factor. For rate 1/2, 2/3, and 3/4, 19 $z$ values ranging from 12 to 48 are shown, which correspond to 19 code sizes with $n$ ranging from 576 to 2304. The simulation conditions are: AWGN channel, BPSK modulation, maximum of 50 iterations using generic floating-point belief propagation.

Figure 1. FER performance of 19 $R = 1/2$ structured codes. Base matrix size: $m_b = 24$, $n_b = 48$. AWGN, BPSK.
Figure 2. FER performance of 19 $R = 2/3$ structured codes. Base matrix size: $m_b = 16, n_b = 48$. AWGN, BPSK.

Figure 3. FER performance of 19 $R = 3/4$ structured codes. Base matrix size: $m_b = 12, n_b = 48$. AWGN, BPSK.
Recommended Text Changes:

Add/Modify the following text to 802.16e_D4, adjusting the numbering as required:

8.4.9.2 Encoding

<add text to the end of the ‘Concatenation’ paragraph starting at line 39>, and for the LDPC encoding scheme (see 8.4.9.2.5) the concatenation rule is defined in 8.4.9.5.4.

8.4.9.2.5 Low Density Parity Check Code (optional)

8.4.9.2.5.1 Code Description

The LDPC code is based on a set of one or more fundamental LDPC codes. Each of the fundamental codes is a systematic linear block code. Using the described methods of scaling and shortening in 8.4.9.2.5.3 Code Rate and Block Size Adjustment, the fundamental codes can accommodate various code rates and packet sizes. The code set can be applied to packets from \[40\] bytes up to \(~200\) bytes.

Each LDPC code in the set of LDPC codes is defined by a matrix \(H\) of size \(m\)-by-\(n\), where \(n\) is the length of the code and \(m\) is the number of parity check bits in the code. The number of systematic bits is \(k=n-m\).

The matrix \(H\) is defined as

\[
H = \begin{bmatrix}
P_{0,0} & P_{0,1} & P_{0,2} & \cdots & P_{0,n_b-2} & P_{0,n_b-1} \\
P_{1,0} & P_{1,1} & P_{1,2} & \cdots & P_{1,n_b-2} & P_{1,n_b-1} \\
P_{2,0} & P_{2,1} & P_{2,2} & \cdots & P_{2,n_b-2} & P_{2,n_b-1} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
P_{m_b-1,0} & P_{m_b-1,1} & P_{m_b-1,2} & \cdots & P_{m_b-1,n_b-2} & P_{m_b-1,n_b-1}
\end{bmatrix} = P^{H_b}
\]

where \(P_{i,j}\) is one of a set of \(z\)-by-\(z\) permutation matrices or a \(z\)-by-\(z\) zero matrix. The matrix \(H\) is expanded from a binary base matrix \(H_b\) of size \(m_b\)-by-\(n_b\), where \(n = z \cdot n_b\) and \(m = z \cdot m_b\), with \(z\) an integer \(\geq 1\). The base matrix is expanded by replacing each 1 in the base matrix with a \(z\)-by-\(z\) permutation matrix, and each 0 with a \(z\)-by-\(z\) zero matrix. The base matrix \(n_b\) is an integer is an integer multiple of 24.

\(H_b\) is partitioned into two sections, where \(H_{b1}\) corresponds to the systematic bits and \(H_{b2}\) corresponds to the parity-check bits, such that \(H_b = \begin{bmatrix} (H_{b1})_{m \times n_b} & (H_{b2})_{m \times n_b} \end{bmatrix}\).
8.4.9.2.5.2 LDPC encoding

The code is flexible in that it can accommodate various code rates as well as packet sizes. Since LDPCs are block-oriented codes, some restrictions are necessary on the combinations of available code rates and codeword sizes in order to control complexity.

The encoding of a packet at the transmitter generates parity-check bits \( p = (p_0, \ldots, p_{m-1}) \) based on an information block \( s = (s_0, \ldots, s_{k-1}) \), and transmits the parity-check bits along with the information block. Because the current symbol set to be encoded and transmitted is contained in the transmitted codeword, the information block is also known as systematic bits. The encoder receives the information block \( s = (s_0, \ldots, s_{k-1}) \) and uses the matrix \( H_{bm} \) to determine the parity-check bits. The expanded matrix \( H \) is determined from the model matrix \( H_{bm} \). Since the expanded matrix \( H \) is a binary matrix, encoding of a packet can be performed with vector or matrix operations conducted over GF(2).

One method of encoding is to determine a generator matrix \( G \) from \( H \) such that \( G H^T = 0 \). A \( k \)-bit information block \( s_{1 \times k} \) can be encoded by the code generator matrix \( G_{k \times n} \) via the operation \( x = s G \) to become an \( n \)-bit codeword \( x_{1 \times n} \), with codeword \( x = [s \ p] = [s_0, s_1, \ldots, s_{k-1}, p_0, p_1, \ldots, p_{m-1}] \), where \( p_0, \ldots, p_{m-1} \) are the parity-check bits; and \( s_0, \ldots, s_{k-1} \) are the systematic bits.

Encoding an LDPC code from \( G \) can be quite complex. The LDPC codes are defined such that very low complexity encoding directly from \( H \) is possible.

8.4.9.2.5.3 Code Rate and Block Size Adjustment

The code design will be flexible to support a range of code rates and block sizes through code rate and block size adjustment of the one or more \( H \) matrices of the fundamental code set. The exact methods for supporting code rate and block size adjustment will depend on the final design. For each supported rate and block size, there will be some combination of matrix selection, shortening, repetition, matrix expansion, and/or concatenation will be used.

Different block sizes and code rates are supported through using a variable \( z \) expansion factor. In each case, the number of information bits is equal to the code rate times the coded block size \( n \). In addition to matrix expansion, shortening is used and puncturing may be used to support some coded block sizes and code rates.

<table>
<thead>
<tr>
<th>( n ) (bits)</th>
<th>( n ) (bytes)</th>
<th>( k ) (bytes)</th>
<th>Number of subchannels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( R=1/2 )</td>
<td>( R=2/3 )</td>
</tr>
<tr>
<td>576</td>
<td>72</td>
<td>36</td>
<td>48</td>
</tr>
<tr>
<td>672</td>
<td>84</td>
<td>42</td>
<td>56</td>
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</tbody>
</table>
Shortening may be applied to any expanded H matrix by reducing the number of subchannels available for the codeword. The number of bit corresponding to the reduced number of subchannels is equal to the number of shortened bits $L$. The matrix $H$ is designed such that excellent performance is achieved under shortening, with different column weights interlaced between the first $L$ columns of $H_1$ and the rest of $H_1$. Encoding with shortening is similar to encoding without shortening, except that the current symbol set has only $k-L$ systematic bits in the information block, $s'= (s_0, \ldots, s_{k-L-1})$. When encoding, the encoder first prepends $L$ zeros to $s'$ of length $(k-L)$. Then the zero-padded information vector $s=[0_L s']$ is encoded using $H$ as if unshortened to generate parity bit vector $p$ (length $m$). After removing the prepended zeros, the code bit vector $x=[s' \ p]$ is transmitted over the channel. This encoding procedure is equivalent to encoding $s'$ using the last $(n-L)$ columns of matrix $H$ to determine the parity-check vector $p$.

The $z$ expansion factors are determined by the target block size $n$ and the base matrix size $n_b$. Examples of the $z$ expansion factors are given in the tables below. The base matrix $n_b$ is an integer is an integer multiple of 24.

TBD description of code adjustment.
8.4.9.2.5.4 Packet Encoding

After harmonization, this section will be replaced with something equivalent to what is in section 8.2.1.2.4.1 which is the concatenation/packet encoding scheme for the CTC.

Since transported data packets can be any size from typically about 40 bytes up to 12000 bits and larger, the system must be able to encode variable length packets in a consistent manner. This consistency is required to ensure that the receiver always knows how to reconstruct the information field from the encoded transmitted data.

The encoding block size \(k\) shall depend on the number of subchannels allocated and the modulation specified for the current transmission. Concatenation of a number of subchannels shall be performed in order to make larger blocks of coding where it is possible, with the limitation of not passing the largest block under the same coding rate (the block defined by the 64-QAM modulation). The table below specifies the concatenation of subchannels for different allocations and modulations. The concatenation rule follows the subchannel concatenation rule for CC (Table 315) except that for LDPC the concatenation does not depend on the code rate.

For any modulation and FEC rate, given an allocation of \(N_{sch}\) subchannels, we define the following parameters:

- \(j\) parameter dependent on the modulation and FEC rate
- \(N_{sch}\) number of allocated subchannels
- \(F\) floor\((N_{sch}/j)\)
- \(M\) \(N_{sch}\) mod \(j\)

The subchannel concatenation rule for CC in Table 315 is applied, noting that in Table 315 the parameter \(n\) is equal to \(N_{sch}\), the parameter \(k\) is equal to \(F\), and the parameter \(m\) is equal to \(M\). The parameter \(j\) for LDPC is determined as shown in the table below.

<table>
<thead>
<tr>
<th>Modulation</th>
<th>(j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QPSK</td>
<td>(j=24)</td>
</tr>
<tr>
<td>16-QAM</td>
<td>(j=12)</td>
</tr>
<tr>
<td>64-QAM</td>
<td>(j=8)</td>
</tr>
</tbody>
</table>

Each packet is encoded as an entity. In other words, the data boundary of a packet is respected by the encoder. Control information and packets that result in a codeword size \(n\) of less than 576 bits smaller than 40 bytes are encoded using convolutional coding (CC) with appropriate code rates and modulation orders, as described in section 8.4.9.2.1.

The length and required rate of the packet that is to be encoded is all that is needed to encode or decode the packet using the following rules:
If $\text{Length} \leq N_i$ bits, then TBD.
If $\text{Length} > N_i$ bits and $\leq 2 N_i$ bits, then TBD
If $\text{Length} > 2 N_i$ bits, then compute $N_r = \text{modulo}(\text{Length}, N_i)$ (in bits), then TBD.

Combination of shortening and concatenation when TBD

The intent of the above rule set is to provide a means for data transmission without the need for additional information beyond the packet field length. This scheme does so with a simple rule set that reduces the rate of the last codewords in order to reduce the number of iterations (and therefore the latency) that must be performed on the last portion of the data. The length and position of the shortened codewords and erased bits are deterministic when the above rules are followed.

For all packets the codeword bits can be indexed using the corresponding column indices of the $H$ matrix. Using this convention the systematic codeword bits comprise the leftmost bits starting at bit location zero, and fill the codeword to bit $k-1$. The remaining $N-k$ bits of the codeword, from indices $k$ to $N-1$, are the parity bits. The codeword systematic bits are filled in an order consistent with the indices, so that the first bits of the packet fill the codeword from the lowest indices linearly to the highest indices. The codeword is then transmitted in a linear fashion starting from the lowest indices so that the systematic bits are transmitted first, followed by the parity bits. For shortened codewords the zeros are padded in the low order bits, so that the final codeword starting at the lowest indices contains first zero-padded bits and then the systematic data bits followed by the parity bits. The zero-padded bits are not typically sent over the channel.