Project	**IEEE 802.16 Broadband Wireless Access Working Group** <http://ieee802.org/16>
Title	**Doppler Spectrum Approximation Using Sum of Sinusoids**
Date Submitted	**2007-07-09**
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Re:	Call for contributions for 802.16m Evaluation Methodology, IEEE 802.16m-07/023
Abstract	This contribution provides a simple analytical method to approximate the specific non-Jakes Doppler spectrum that arises due to the Laplacian Angular Power Profile, instead of omni-directional arrival angles that result in the classical Jakes spectrum
Purpose	For discussion and approval by 802.16 TGm
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Doppler Spectrum Approximation Using Sum of Sinusoids

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Purpose

This contribution provides a simple closed-form solution to approximate the specific non-Jakes Doppler spectrum that arises due to the Laplacian angular power profile, as opposed to the classical Jakes spectrum resulted from omni-directional arrival angles.

Similar to the simulation of Jakes spectrum, which is often approximated using the sum of a few sinusoids with pre-determined sinusoidal frequencies (also called "Doppler frequencies") , the simulation of any arbitrary Doppler spectrum can rely on the same "sum of sinusoids" approach, as described in step 6 and 7 of section 3.2.7. However, the calculation of those Doppler frequencies often has to resort to numerical methods such as Method of Exact Doppler Spread (MEDS) or L2-Norm method. Since a numerical method can only be applied after the exact shape of the spectrum is known, these numerically-obtained Doppler frequencies will have to be re-calculated if the exact Doppler spectrum changes, for example, due to a different velocity. It is undesirable in a system simulation to numerically re-calculate the sinusoidal frequencies on the fly, as opposed to Jakes spectrum for which the Doppler frequencies can be easily obtained that are proportional to speed.

The method provided here will obtain the Doppler frequencies easily that are also proportional to the velocity. It can be pre-determined and pre-stored for each quantized angle (i.e., the angle between travel direction and a mean AOA). The difference from Jakes spectrum is just that the non-Jakes spectrum still depends on the mean AOA, while for Jakes spectrum the same set of Doppler frequencies can be used that does not depend on AOA due to omni-direction assumption.

Non-Jakes Doppler Spectrum

The well-known classical Jakes spectrum was derived assuming omni-directional angle of arrival of a large number of rays with the same arrival time (i.e., at a channel tap). As the bandwidth increases and the temporal resolution gets finer, each resolvable delay tap may attribute to fewer rays or often a cluster of rays bounced off the same scatterer.

In the current evaluation methodology document, the Cluster Delayed Line or CDL channel models from WINNER project were adopted, where each channel tap attributes to a single cluster of rays with a mean AOA/AOD and a Laplacian power angular profile. As a result, the step 6 of section 3.2.7 defines the following Doppler spectrum

$$S_n(f) \propto \begin{cases} \dfrac{1}{\sqrt{f_{\max}^2 - f^2}} \exp\left\{ -\dfrac{\sqrt{2}\left|\cos^{-1}(f/f_{\max}) + \vartheta_n\right|}{AS_{MS,Path}} \right\}, & |f| \le f_{\max} = f_c v/c \\ 0, & \text{otherwise} \end{cases} \qquad (1.1)$$

where ϑ_n is the angle between the traveling direction and the mean AOA for the *n*-th tap, and $AS_{MS,\ Path}$ is the angular spread of that tap (i.e., path). Also, f_{max} is the maximal Doppler frequency defined by the carrier frequency f_c and speed v. It is obviously not a Jakes spectrum, but rather a combination of Jakes and Laplacian.

To simulate such a temporarily correlated process, one could use the filtering approach with a filter frequency response as defined by the Doppler (the filter is driven by i.i.d. Gaussian process). Of course, the equivalent time domain filter response is not trivial to obtain at all. Even if possible, the resulting long FIR filter will certainly not of convenience to use for generating the correlated process.

Alternatively, a more typical method is to use the summation of a moderate number of sinusoids with random phases but unit amplitude. The frequencies of those sinusoids (i.e., Doppler frequencies) are chosen carefully so that the resulting Doppler spectrum matches the desired one. A well-known example of that sum-of-sinusoid method is its usage in the generation of Jakes spectrum.

For any arbitrary Doppler spectrum, one can find the Doppler frequencies using either Method of Exact Doppler Spread (MEDS) or L_2-Norm Method (LNPM). Since a numerical method can only be applied after the exact shape of the spectrum is known, these numerically-obtained Doppler frequencies will have to be re-calculated if the exact Doppler spectrum changes even due to, for example, a slightly different velocity. It is undesirable in a system simulation to numerically re-calculate the sinusoidal frequencies on the fly for each set of speed v, mean AOA ϑ_n, and angular spread $AS_{MS, Path}$. Note that for Jakes spectrum, the Doppler frequencies can be easily obtained that are proportional to speed only since Jakes spectrum is not parameterized by mean AOA or angular spread.

Closed-Form Solution for Doppler Frequencies to Approximate Specific Non-Jakes Doppler Spectrum

For system simulation, it is important to reduce simulation run-time while not sacrificing the modeling accuracy. Using a numerical method to compute Doppler frequencies on the fly according to a speed v, mean AOA ϑ_n, and angular spread $AS_{MS, Path}$ is not desirable, given there are 10-20 taps for each link.

This section will describe an easy way to obtain the Doppler frequencies according to the mean AOA, but scalable to angular spread and velocity. The method will allow for the pre-calculation and pre-storing of the Doppler frequencies for each quantized mean AOA. Similar to the derivation of spatial correlation which can also be pre-calculated as a function of mean AOA and AS, the pre-calculation method may benefit the system simulation significant.

In order to derive the closed form solution, we first observed that the Doppler frequencies for Jakes spectrum, which are essentially $\cos(2\pi/N)$ with N being the number of sinusoids, are really coming from assumption of omni-directional arrival process. We also observed that in a ray-based implementation, the time-variation of the channel is really realized through summing over the 20 equal-power rays in each path. Hence, if we approximate the Doppler spectrum with N rays (sinusoids) whose frequencies are derived from the angles between their arrival direction and traveling direction, we should achieve the Doppler spectrum as specified previously.

For that purpose, we use N sinusoids that are evenly distributed around the mean AOA, the offset angles from the mean AOA is determined in the same way as in SCM or WINNER, i.e., choosing the angle spacing between equal power sub-rays to make sure that area under the Laplacian PDF (i.e., separated by the sub-rays) equal to 1/(N+1) where N is the number of sub-rays, i.e., for the positive side

$$\frac{1}{2}\left[\exp\left\{-\frac{\sqrt{2}\,|\alpha_1|}{AS}\right\}-\exp\left\{-\frac{\sqrt{2}\,|\alpha_2|}{AS}\right\}\right]=\frac{1}{N+1} \qquad (1.2)$$

where α_1 and α_2 are two adjacent angles with an increasing order and for the first angle on the positive side assuming an even N is

$$\frac{1}{2}\left[1-\exp\left\{-\frac{\sqrt{2}\,|\alpha_1|}{AS}\right\}\right]=\frac{0.5}{N+1} \tag{1.3}$$

For N=10 and AS=1, the angles are [±1.2054 ±0.7153 ±0.4286 ±0.2252 ±0.0674]. Note that due to finite quantization, the standard deviation of all the ten angles is not "1" any more, it is C=0.6639 instead. So scaling of 1/C must be used to compensate for the finite quantization. For any AS, the N=10 angles around mean AOA can be obtained as:

$$\phi_{n,i}=\vartheta_n+AS_{MS,Path}*[-1.8157,-1.0775,-0.6456,-0.3392,-0.1015, 0.1015, 0.3392, 0.6456, 1.0775, 1.8157] \tag{1.4}$$

The N=10 sinusoid frequencies used to approximate the non-Jakes spectrum is then

$$f_{n,i}=f_{\max}\cos(\phi_{n,i}) \tag{1.5}$$

Proposed Text

Insert the following text at the end of line 18 page 49 (section 3.2.7)

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For example, the non-Jakes Doppler spectrum can be simulated using the summation of 10 equal-power sinusoids with random phases, but their frequencies are defined as

$$f_{n,i}=f_{\max}\cos(\phi_{n,i})$$

where $\phi_{n,i}=\vartheta_n+AS_{MS,Path}*[-1.8157,-1.0775,-0.6456,-0.3392,-0.1015, 0.1015, 0.3392, 0.6456, 1.0775, 1.8157]$

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