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Source(s)	Kim Olszewski ZTE USA, Inc 10105 Pacific Heights Blvd, Suite 250 San Diego, CA 92121 E-mail: kolszewski@zteusa.com *<http://standards.ieee.org/faqs/affiliationFAQ.html>
Re:	IEEE 802.16m-07/031 - Call for Comments on Draft 802.16m Evaluation Methodology Document
Abstract	This document contains proposed text for the IEEE 802.16m-07/080r3 Draft Evaluation Methodology Document. The text addresses modifications in per SINR computations to account for antenna array implementations with Cyclic Delay Diversity (CDD)
Purpose	To review and adopt proposed text in next revision of the Evaluation Methodology Document
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The Impact of Cyclic Delay Diversity on Per-tone SINR Computations

Kim Olszewski
ZTE USA, Inc

Introduction

Cyclic delay diversity (CDD) is a spatial diversity technique that may be implemented within an OFDM-based system equipped with two or more transmit antennas. The basic idea of a CDD implementation is to transform transmit diversity into frequency diversity. A CDD implementation will result in an increase in a channel's frequency selectivity or equivalently a decrease in a channel's coherence bandwidth. This contribution shows CDD's impact on computing per-tone SINR for OFDM-based systems. Receiver per-tone SINR computations should be modified to account for artificially induced frequency selective associated with a CDD implementation. The final section of this contribution proposes a new section for the current evaluation document (IEEE C802.16m-07/080r3) that accounts for CDD implementations.

Transmitted Base Station Signal

Figure 1 illustrates a simplified baseband model of a BS transmitter with a CDD implementation and a mobile station receiver. All baseband signal processing operations including the IFFT operation are subsumed by the OFDM Signal Source block. The time-domain OFDM signal is split into N_T antenna branches. The top branch carries the original time-domain signal, the other branches carry cyclically shifted versions of the original time-domain signal. For the N_T transmit antennas the cyclic shift values used are

$$\delta_m = m, \; m = 0,1,2,\ldots,N_T - 1$$

samples. Other shift values may be used but the maximal cyclic shift is bounded by the size of the IFFT. Note that CDD is independent of the cyclic prefix and allows an increase in channel frequency selectivity without increasing the overall channel delay spread. For this reason it can accommodate different channel delays and signal propagation distances or cell sizes.

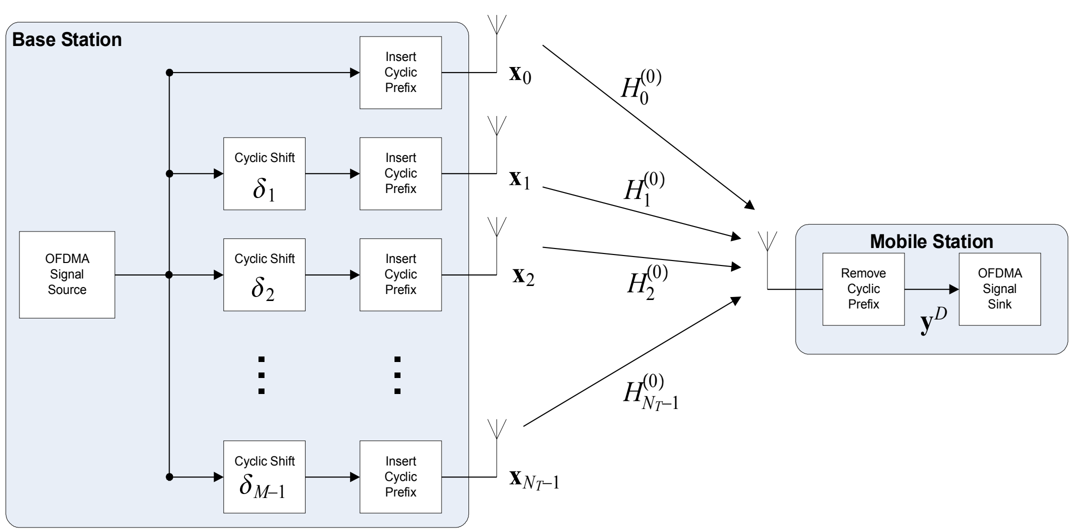

Figure 1. Simple conceptual block diagram of CDD implementation.

Let k denote discrete time, n a discrete frequency or sub-carrier index, complex-number $x(k)$ a time-domain sample, complex-number $X(n)$ a frequency-domain sample, and N_g the length of the cyclic prefix. The time-domain representation of a baseband OFDM symbol transmitted from the m th antenna branch is the length $N = N_{FFT} + N_g$ vector

$$\mathbf{x}_m = \begin{bmatrix} \mathbf{x}_m^{CP} & \mathbf{x}_m^{D} \end{bmatrix},\ m = 0,1,\ldots,N_T - 1$$

where

$$\mathbf{x}_m^{D} = \underbrace{\begin{bmatrix} x(-\delta_m) & x(1-\delta_m) & \ldots & x(N-1-\delta_m) \end{bmatrix}}_{\text{OFDM Data Symbol}}$$

and

$$\begin{aligned} \mathbf{x}_m^{CP} &= \begin{bmatrix} x(-\delta_m - N_g) & \ldots & x(-\delta_m - 1) \end{bmatrix} \\ &= \underbrace{\begin{bmatrix} x(N-\delta_m - N_g) & \ldots & x(N-1-\delta_m) \end{bmatrix}}_{\text{Cyclic Prefix}} \end{aligned}$$

The length N_{FFT} vector $\mathbf{x}_m^{D}$ contains OFDM data symbols and the length N_g vector $\mathbf{x}_m^{CP}$ the OFDM symbol's cyclic prefix. Elements in $\mathbf{x}_m^{D}$ and $\mathbf{x}_m^{CP}$ are defined as

$$x(k-\delta_m) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} e^{-j2\pi n\delta_m/N} X(n) e^{j2\pi nk/N}$$
$$k = 0,1,\ldots,N-1 \text{ and } m = 0,1,\ldots,N_T - 1$$

Note that we have used the IFFT time shift property

$$x(k-\delta_m) \overset{IFFT}{\leftrightarrow} e^{-j2\pi n\delta_m/N} X(n)$$

for $x(k-\delta_m)$ and the IFFT periodicity property $x[-k] = x[N-k]$ to write the cyclic prefix in two forms.

The baseband OFDM symbol transmitted by all N_T antenna branches may be written as the vector summation

$$\mathbf{x} = \sum_{m=0}^{N_T-1} \mathbf{x}_m = \begin{bmatrix} \mathbf{x}^{CP} & \mathbf{x}^{D} \end{bmatrix}$$

where

$$\mathbf{x}^D = \underbrace{\left[\sum_{m=0}^{N_T-1} x(-\delta_m) \quad \sum_{m=0}^{N_T-1} x(1-\delta_m) \quad \ldots \quad \sum_{m=0}^{N_T-1} x(N-1-\delta_m)\right]}_{\text{OFDM Data Symbol}}$$

and

$$\mathbf{x}^{CP} = \left[\sum_{m=0}^{N_T-1} x(-\delta_m - N_g) \quad \ldots \quad \sum_{m=0}^{N_T-1} x(-\delta_m - 1)\right]$$

$$= \underbrace{\left[\sum_{m=0}^{N_T-1} x(N-\delta_m - N_g) \quad \ldots \quad \sum_{m=0}^{N_T-1} x(N-1-\delta_m)\right]}_{\text{Cyclic Prefix}}$$

The k th element of $\mathbf{x}$ is defined as

$$\begin{aligned} x(k) &= \sum_{m=0}^{N_T-1} x(k-\delta_m) \\ &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X(n) e^{j2\pi nk/N} + \frac{1}{\sqrt{N}} \sum_{m=1}^{N_T-1} \sum_{n=0}^{N-1} e^{-j2\pi n \delta_m/N} X(n) e^{j2\pi nk/N} \\ &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \left(1 + \sum_{m=1}^{N_T-1} e^{-j2\pi n\delta_m/N}\right) X(n) e^{j2\pi nk/N} \\ &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} D(n) X(n) e^{j2\pi nk/N}, \; k = 0,\ldots,N-1 \end{aligned}$$

where

$$D(n) = 1 + \sum_{m=1}^{N_T-1} e^{-j2\pi n\delta_m/N}, \; n = 0,1,\ldots,N-1$$

From $D(n)$ it is seen that CDD will increase the channel's frequency selectivity but will not increase the overall channel delay spread because cyclic shifts are performed prior to cyclic prefix insertions.

Received Mobile Subscriber Signal

Assume that the guard interval of an OFDM symbol has been removed and that the cyclic prefix is greater than the channel delay spread. The time-domain representation of the received data part of the OFDM symbol (see Figure 1) is then

$$\mathbf{y}^D = \left[y^{(0)}(0) \quad y^{(0)}(1) \quad \ldots \quad y^{(0)}(N-1)\right]$$

where

$$y^{(0)}(k) = \sqrt{P_{tx}^{(0)}P_{loss}^{(0)}}\sum_{m=0}^{N_T-1} H_m^{(0)}x^{(0)}(k-\delta_m) + \sum_{j=1}^{N_I}\sqrt{P_{tx}^{(j)}P_{loss}^{(j)}}H^{(j)}x^{(j)}(k) + u^{(0)}(k)$$
$$k = 0,1,\ldots,N-1$$

The notation used is that given in the current evaluation document and is also given below for completeness.

Applying an FFT to $\mathbf{y}^D$ gives the frequency-domain representation of received OFDM symbol

$$\mathbf{Y}^D = \left[Y^{(0)}(0) \quad Y^{(0)}(1) \quad \ldots \quad Y^{(0)}(N-1)\right]$$

where

$$\begin{aligned}Y^{(0)}(n) = {} & \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} y^{(0)}(k)e^{-j2\pi nk/N} \\ & + \sum_{k=0}^{N-1}\left(\sum_{j=1}^{N_I}\sqrt{P_{tx}^{(j)}P_{loss}^{(j)}}H^{(j)}x^{(j)}(k)\right)e^{-j2\pi nk/N} \\ & + \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} u^{(0)}(k)e^{-j2\pi nk/N}\end{aligned}$$

The three FFTs in the above equation can be written as

$$\begin{aligned}\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} y^{(0)}(k)e^{-j2\pi nk/N} &= \frac{\sqrt{P_{tx}^{(0)}P_{loss}^{(0)}}}{\sqrt{N}}\sum_{k=0}^{N-1}\left(\sum_{m=0}^{N_T-1} H_m^{(0)}x^{(0)}(k-\delta_m)\right)e^{-j2\pi nk/N} \\ &= \sqrt{P_{tx}^{(0)}P_{loss}^{(0)}}\left(\sum_{m=0}^{N_T-1} H_m^{(0)}e^{-j2\pi n\delta_m/N}\right)X^{(0)}(n) \\ &= \sqrt{P_{tx}^{(0)}P_{loss}^{(0)}}\tilde{H}^{(0)}(n)X^{(0)}(n)\end{aligned}$$

$$\sum_{k=0}^{N-1}\left(\sum_{j=1}^{N_I}\sqrt{P_{tx}^{(j)}P_{loss}^{(j)}}H^{(j)}x^{(j)}(k)\right)e^{-j2\pi nk/N} = \sum_{j=1}^{N_I}\sqrt{P_{tx}^{(j)}P_{loss}^{(j)}}H^{(j)}(n)X^{(j)}(n)$$

and

$$\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} u^{(0)}(k)e^{-j2\pi nk/N} = U^{(0)}(n)$$

Hence by substitution we have

$$Y^{(0)}(n) = \sqrt{P_{tx}^{(0)} P_{loss}^{(0)}}\,\widetilde{H}^{(0)}(n) X^{(0)}(n) + \sum_{j=1}^{N_I} \sqrt{P_{tx}^{(j)} P_{loss}^{(j)}}\, H^{(j)}(n) X^{(j)}(n) + U^{(0)}(n)$$

where

$$\widetilde{H}^{(0)}(n) = \sum_{m=0}^{N_T-1} H_m^{(0)} e^{-j2\pi n\delta_m/N}$$

denotes the *effective or composite channel gain* that subsumes both CDD transmit diversity and the physical channel. N_I is the number of interferers. $P_{tx}^{(j)}$ is the total transmit power from the j th BS (per sector) or MS. $P_{loss}^{(j)}$ is the distance dependent path loss including shadowing, antenna gain/loss and cable losses from the j th sector or MS. $H^{(j)}(n)$ is the channel gain for the desired MS for the n th sub-carrier and j th user/sector. $X^{(j)}(n)$ is the symbol transmitted by the j th user/sector on the n th sub-carrier. $U^{(0)}(n)$ is the receiver thermal noise, modeled as AWGN noise with zero mean and variance σ^2.

If a proposal contains a CDD implementation the effective or composite channel frequency response $\widetilde{H}^{(0)}(n)$ should be used for per tone SINR computations. Using $\widetilde{H}^{(0)}(n)$ the n th tone post processing SINR for a MISO (multi-input, single-output) system with CDD is defined as

$$\begin{aligned} SINR(n) &= \frac{\left|\widetilde{H}^{(0)}(n)\right|^2 P_{xx}^{(0)}(n)}{P_{uu}^{(0)}(n)} \\ &= \frac{P_{tx}^{(0)} P_{loss}^{(0)} \left|\widetilde{H}^{(0)}(n)\right|^2}{\sigma^2 + \sum_{j=1}^{N_I} P_{tx}^{(j)} P_{loss}^{(j)} \left|H^{(j)}(n)\right|^2} \end{aligned}$$

To better see the artificially induced frequency selective associated with CDD we give an simple example for a two-element antenna array. For simplicity assume $P_{tx}P_{loss} = 1$ and an ideal channel with $H_m = 1$ for all m so $\widetilde{H}(n) = 1 + e^{-j2\pi n\delta_m/N}$. The magnitude-squared channel gain for a two-element antenna with one antenna shifted δ_m is then

$$\begin{aligned} |\widetilde{H}(n)|^2 &= (1 + e^{-j2\pi n\delta_m/N})(1 + e^{j2\pi n\delta_m/N}) \\ &= 2\left[1 + \cos\left(\frac{2\pi n\delta_m}{N}\right)\right],\ n = 0,1,\ldots,N-1 \end{aligned}$$

Figure 2 shows plots for $\delta_m = 1,2,4$ and 6 and $N_{FFT} = 256$. As shown in the plots a larger value of δ_m results in higher frequency selectivity.

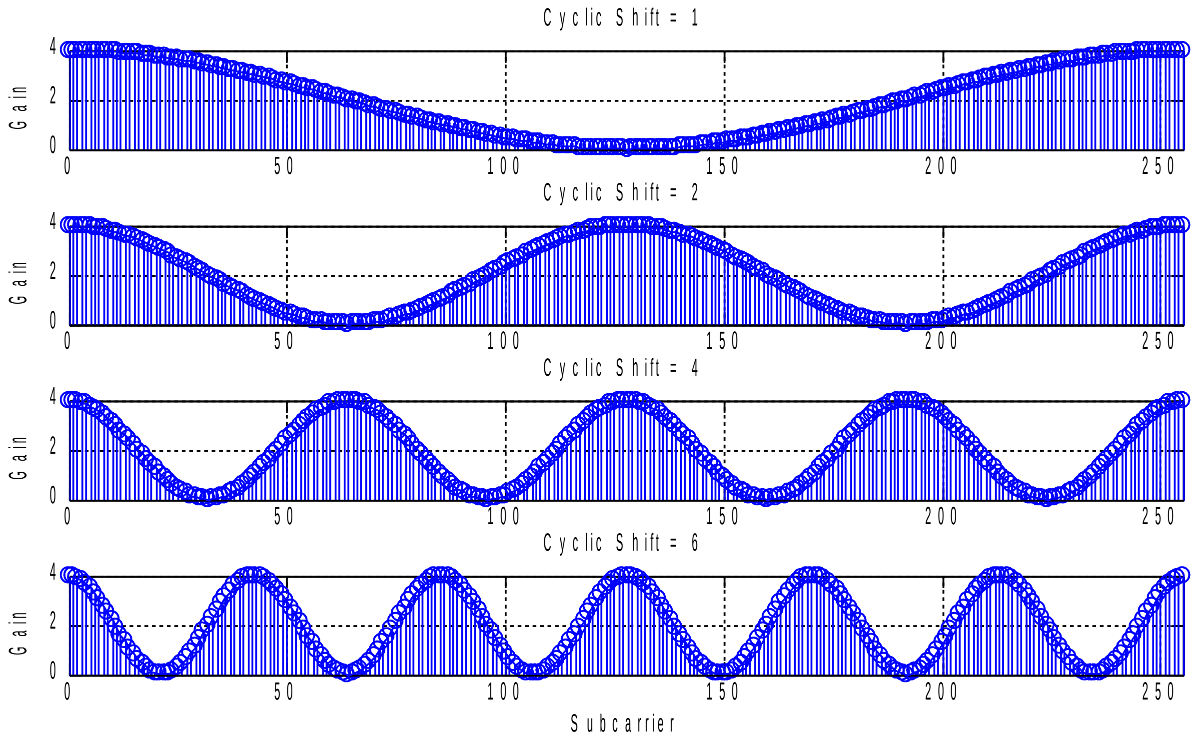

Figure 2. Composite magnitude-squared channel gain for a two antenna CDD implementation with ideal physical channel.

As another more general example let $\lambda = 2\pi n / N$ then using N_T antennas with N_T *equally* spaced delays $\delta_m = 0,1,\ldots,N_T - 1$ we have

$$
\begin{aligned}
|\widetilde{H}(n)|^2 &= \left(\sum_{m=0}^{N_T-1} e^{-j\delta_m\lambda}\right)\left(\sum_{k=0}^{N_T-1} e^{j\delta_k\lambda}\right) \\
&= \left(\frac{1-e^{-j\lambda M}}{1-e^{-j\lambda}}\right)\left(\frac{1-e^{j\lambda M}}{1-e^{j\lambda}}\right) \\
&= \frac{1-\cos(M\lambda)}{1-\cos(\lambda)}
\end{aligned}
$$

whose limit and maximum at $\lambda = 0$ is N_T^2 . Figure 2 shows plots for $N_T = 3$ and 4 . As N_T increases the peaks dominate and the other frequencies become negligible. Hence only low and high frequency spectral components will be passed by $|\widetilde{H}(n)|^2$ as the number of antennas increases.

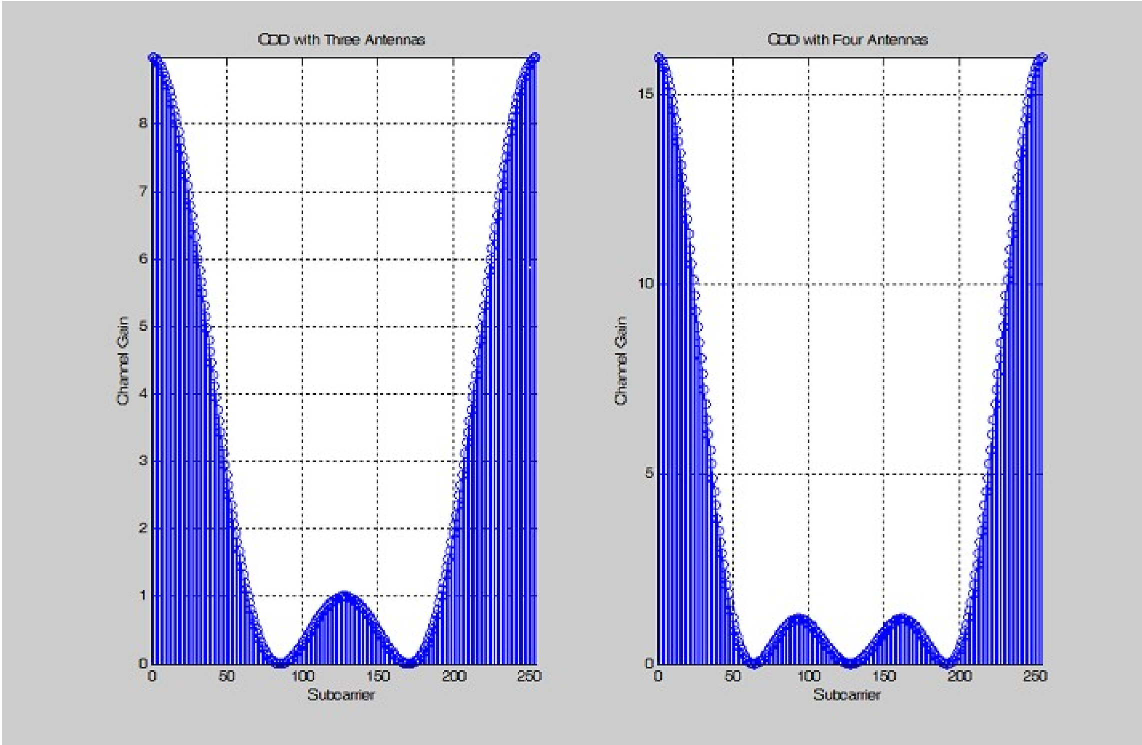

Figure 3. Composite magnitude-squared channel gain for three and four antenna CDD implementations with ideal physical channel and equal cyclic shifts.

Proposed Text

Add the following subsection within section 4.5 of IEEE C802.16m-07/089r3:

4.5.7 Per-tone Post Processing SINR for MISO and MIMO with CDD

For MISO (multi-input, single-output) and MIMO proposals with CDD implementations the effective CDD or composite channel gains should be used for per tone SINR computations. For example, the n th tone post processing SINR for a MISO system with a CDD implementation may be defined as

$$SINR^{(0)}(n) = \frac{P_{tx}^{(0)} P_{loss}^{(0)} \left|\widetilde{H}^{(0)}(n)\right|^2}{\sigma^2 + \sum_{j=1}^{N_I} P_{tx}^{(j)} P_{loss}^{(j)} \left|H^{(j)}(n)\right|^2}$$

where

$$\widetilde{H}^{(0)}(n) = \sum_{m=0}^{N_T - 1} H_m^{(0)} e^{-j2\pi n \delta_m / N}$$

is defined as the effective CDD or composite channel gain that incorporates the physical channel gains $H_m^{(0)}$ and the artificially induced frequency selective associated with a CDD cyclic shifts $e^{-j2\pi n \delta_m / N}$. Example cyclic shift values are

$$\delta_m = m,\ m = 0,1,2,\ldots,N_T - 1$$

samples where N_T is the number of transmit antennas. The cyclic shift $\delta_0 = 0$ is assumed to be the reference antenna in a CDD implementation.