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Abstract	This document proposes a technique to use relays in conjunction with ARQ/HARQ
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I. INTRODUCTION

In "conventional" wireless multihop packet networks packets are transported from a single transmitter to a single receiver, typically along a predetermined route. No attempt is made to take advantage of the broadcast nature of the wireless medium. Lately, various techniques [1] have been proposed to take advantage of the ability to transmit to multiple destinations without having to pay any penalty in terms of network resources. Furthermore, in presence of fading, which is usually mostly statistically independent over the network links, these techniques provide diversity gain. Diversity obtained this way is usually referred to as cooperative diversity [1]. In situations where other forms of diversity are precluded, cooperative diversity relaying can be very effective to mitigate random fading.

Ideally, routing in an ad-hoc network requires complete knowledge of the state of the network. However, in reality, even medium size networks do not have sufficient resources to discover and track all possible paths in presence of mobility. Hence, we focus on a more modest two-hop relaying protocol, which requires only single-hop average channel state information (CSI). The protocol can be implemented as an add-on feature to an existing routing protocol. The proposed protocol aims to take advantage of other nodes in the vicinity in an opportunistic fashion.

Other technique to improve the packet delivery ratio are ARQ and HARQ. Various forms of these techniques has been thoroughly studied [5]-[7]. However, applying both ARQ and cooperative diversity in a multihop networks raises issues and possibilities that have not been thoroughly investigated before. We propose a simple protocol utilizing two-hop cooperative diversity augmented with ARQ, and study analytically and through simulation its expected system-wide benefits in an ad-hoc wireless network, where the node location follows a homogeneous Poisson point process. When a packet transmitted by a source node is not received by the destination node, the protocol makes an optimal choice whether the source node itself re-transmits the packet, or whether another node that received the packet will serve as a relay and re-transmit it. In the later case, the protocol chooses the "best" relay to re-transmit based on the information available to it.

III. SYSTEM MODEL

A. Nodes location model

Nodes locations follow a homogeneous Poisson point process with density λ.

The number of nodes in a region A is Poisson distributed r.v. with mean $\lambda \times \text{area}(A)$:

$$P\left\{N(A)=k\right\}=e^{-\lambda\times\text{area(A)}}\frac{\left(\lambda\times\text{area}(A)\right)^k}{k!}$$

The number of nodes in two disjoint regions are independent.

Given the number of nodes in a region A the location of each node is uniform in A.

B. Propagation model:

We assume distance related loss and Rayleigh fading. The SNR

at node j due to transmission from node i located d_{ij} units away, is:

$$\gamma_{ij}=\frac{KP_T}{d_{ij}^{\delta}}X_{ij}^2 \qquad (1)$$

where K represents all the constant gains such as antenna gains, P_T is the transmit power, P_N is the noise power and δ is the path loss exponent. We assume that the antennas are omnidirectional and K, P_T and P_N are common for all nodes. Fading components of all the channel gains, i.e. X_{ij}'s are independent and identically distributed. X_{ij}^2 is an exponential random variable where $\mathbb{E}[X_{ij}^2] = \mu = 1$, and hence its cdf is equal to $F_{X_{ij}^2}(y) = 1 - \exp(-y)$ for $y \geq 0$. We assume that the channel coherence time is sufficiently long, such that the channel does not change during the delivery time of a packet, which is at most two timeslots. We do not consider interference in this paper.

We assume that transmission by node i is successfully received by node j, if and only if the received SNR, denoted by γ_{ij}, is larger than a given threshold value γ_t. We define

$$r_N(P_T) = \left(K \frac{P_T}{P_N \gamma_t}\right)^{1/\delta},$$

which is the transmission range of a node transmitting with power P_T in the absence of fading. Then,

$$\gamma_{ij} > \gamma_t \longrightarrow R < r_N(P_T) X_{ij}^{2/\delta}. \tag{2}$$

The expectation of γ_{ij} is denoted by $\bar{\gamma}_{ij}$ and $\bar{\gamma}_{ij}$ normalized by the threshold SNR γ_t is denoted by g_{ij}.

$$g_{ij} = \frac{\bar{\gamma}_{ij}}{\gamma_t} = \left(\frac{r_N}{d_{ij}}\right)^{\delta} \mathbb{E}[(X_{ij})^2] = \left(\frac{r_N}{d_{ij}}\right)^{\delta}$$

Hence,

$$g_{sd} = \left(\frac{r_{Ns}}{D}\right)^{\delta},$$

where node S is the source node, node D is the destination node, and $d_{sd} = D$.

IV. Description of the Relay-Assisted ARQ Protocol

A pre-requisite of this protocol is that each network node knows the mean (the local average) path loss to all its "neighbors". The neighbors of a node are defined as nodes that might possibly receive transmissions from the node under favorable propagation conditions. The possibility that a non-neighboring node receives a message is considered unlikely and the protocol ignores it. The destination node is always a neighbor of the source node. The protocol has two stages. In stage I, the source transmits a packet with transmit power P_S, specifying the intended destination. If the destination receives the packet successfully, it sends a short acknowledgement message (ACK) to the source. To account for propagation and processing delays, the source uses a time-out counter defining a time window for ACK to arrive from the destination. In case ACK arrives in time, the protocol cycle is terminated. If the source does not receive an ACK from the destination before its timer expires, it assumes that its transmission is not successful and stage II starts. In stage II the source broadcasts a message requesting that all nodes that have correctly received the transmitted packet identify themselves. Nodes that satisfy this condition are called "potential relays". Each potential relay sends a short ACK to the source, which also includes its mean channel gain to the destination. We assume that the ACK messages from potential relays are always received correctly by the source.

In scenarios where each relay is able to track the instantaneous gain of the channel between itself and the destination, the source can pick the relay with the largest gain as in [15]. However, in this paper we do not assume that every node can readily provide its instantaneous channel gain to any node that one of its neighbors wants to send data to. Instead, we assume that nodes can provide only local average channel gains to their one-hop neighbors (This is also a pre-requisite for routing). This feature allows the protocol to perform also in mobile scenarios. We assume that the source also knows its average channel gain to the destination.

If there are more than one potential relays with respect to the source, the source selects the the one with the best average channel gain to the destination. The source then either retransmits itself or asks this relay to transmit. In Section V-B we derive the threshold $g_{min}(g_{sd})$ for average relay-destination SNR below which source transmission should be preferred over relay transmission. The source re-transmits also when no potential relay is available. After the second stage is completed, the destination combines the two received signals using Maximum Ratio Combining (MRC).

We note that concurrent transmissions of the ACKs from multiple potential relays can cause collisions if not managed by a separate protocol. A simple protocol for this purpose is given below. The source includes the value of $g_{min}(g_{sd})$ in the transmitted packet, then potential relays whose g_{rd} is smaller than $g_{min}(g_{sd})$ do not respond. Each potential relay is required to wait a certain time interval before responding. This time interval is related to the g_{rd} of the relay through a properly chosen monotonically decreasing function $G(g_{sd}, g_{rd})$. Let the longest waiting time interval, the one corresponding to $g_{min}(g_{sd})$ be Δt_{max} (See Fig 1). Then the potential relay with the highest channel gain to the destination will transmit the ACK first, and the rest, sensing the transmission of the "best" relay, will withdraw. Note that even if some other potential relays fail to sense the signal of the "best" relay, with high probability no collision will occur, because the ACK messages are short compared to Δt_{max}. The source then waits to the end of the Δt_{max} interval (no potential relay will be on the air beyond this point) and then sends a message to the "best" relay instructing it to transmit to the destination. The "best" relay then transmits the data packet terminating the cycle. Having to wait for a specific instruction from the source prevents the possibility that the "best" relay, not receiving an ACK from the destination, will transmit even though the destination may have issued an ACK that was received by the source.

In the propagation model described in Section III-B, the average channel gain is uniquely determined by the length of the link. To simplify the analysis we do not consider shadow fading. However, the model and the analysis can be generalized to include lognormal shadowing. In Section VI we present simulation results, which include the effect of shadowing on the performance.

V. Analysis of the Relay-Assisted ARQ protocol

In this section we analyze the performance of the relay-assisted ARQ protocol for given P_S and P_r. We use r_{Nr} to denote $r_N(P_r)$ - the transmission range of a relay in absence of fading. The average SNR and average normalized SNR of an arbitrary potential relay are denoted by $\bar{\gamma}_{rd,a}$ and $g_{rd,a}$, respectively. The average SNR and average normalized SNR of the best potential relay are denoted by $\bar{\gamma}_{rd}$ and g_{rd}. The probability of failure in the first transmission by the source is denoted by P_1.

$$P_1 = \mathbb{P}\{\gamma_{sd} < \gamma_t\} = 1 - \exp(-\gamma_t/\bar{\gamma}_{sd}) = 1 - \exp(-1/g_{sd}) \tag{3}$$

As a reference we also consider the protocol where the source employs an ARQ protocol without the relays, i.e. it retransmits if the first transmission is not successful. We call this protocol source ARQ and denote its outage probability by P_{ARQ}. This probability is equal to

$$P_{ARQ} = 1 - \exp\left(-1/(2g_{sd})\right). \tag{4}$$

A. Minimum Average SNR for Relay Transmission: $g_{min}(g_{sd})$

Let P_{s2} denote the probability that the destination cannot receive successfully if the source re-transmits given that the first transmission by the source has failed. Recall that the destination node uses MRC, and therefore the SNR at the MRC combiner output is the sum of the SNRs obtained in the two attempts. Hence,

$$\begin{aligned} P_{s2} &= \mathbb{P}\{2\gamma_{sd} < \gamma_t \,|\, \bar{\gamma}_{sd}, \gamma_{sd} < \gamma_t\} && (5) \\ &= \frac{\mathbb{P}\{\gamma_{sd} < \gamma_t/2 \,|\, \bar{\gamma}_{sd}\}}{\mathbb{P}\{\gamma_{sd} < \gamma_t \,|\, \bar{\gamma}_{sd}\}} && (6) \\ &= \frac{1 - \exp\left(-\frac{\gamma_t}{2\bar{\gamma}_{sd}}\right)}{1 - \exp\left(-\frac{\gamma_t}{\bar{\gamma}_{sd}}\right)} = \frac{1 - \exp\left(-\frac{1}{2g_{sd}}\right)}{1 - \exp\left(-\frac{1}{g_{sd}}\right)}. && (7) \end{aligned}$$

Let P_{sr} be the probability that the destination cannot receive successfully if the best relay transmits following a failed transmission by the source. Then,

$$\begin{aligned} P_{sr} &= \mathbb{P}\{\gamma_{sd} + \gamma_{rd} < \gamma_t \,|\, \bar{\gamma}_{sd}, \bar{\gamma}_{rd}, \gamma_{sd} < \gamma_t\} && (8) \\ &= \frac{\mathbb{P}\{\gamma_{sd} + \gamma_{rd} < \gamma_t, \gamma_{sd} < \gamma_t \,|\, \bar{\gamma}_{sd}, \bar{\gamma}_{rd}\}}{\mathbb{P}\{\gamma_{sd} < \gamma_t \,|\, \bar{\gamma}_{sd}\}} && (9) \\ &= \frac{1}{1 - \exp\left(-\frac{\gamma_t}{\bar{\gamma}_{sd}}\right)} \int_0^{\gamma_t} \int_0^{\gamma_t - \gamma_{sd}} \frac{1}{\bar{\gamma}_{rd}} e^{-\gamma_{rd}/\bar{\gamma}_{rd}} \frac{1}{\bar{\gamma}_{sd}} e^{-\gamma_{sd}/\bar{\gamma}_{sd}} d\gamma_{rd}\, d\gamma_{sd} && (10) \\ &= 1 - \frac{\bar{\gamma}_{rd}}{\bar{\gamma}_{rd} - \bar{\gamma}_{sd}} \frac{\exp\left(-\frac{\gamma_t}{\bar{\gamma}_{rd}}\right) - \exp\left(-\frac{\gamma_t}{\bar{\gamma}_{sd}}\right)}{1 - \exp\left(-\frac{\gamma_t}{\bar{\gamma}_{sd}}\right)} && (11) \\ &= 1 - \frac{g_{rd}}{g_{rd} - g_{sd}} \frac{\exp\left(-\frac{1}{g_{rd}}\right) - \exp\left(-\frac{1}{g_{sd}}\right)}{1 - \exp\left(-\frac{1}{g_{sd}}\right)}. && (12) \end{aligned}$$

Let a_s and a_r denote the two possible actions the source can take: a_s is the retransmission by the source and a_r is the transmission by the best relay. The optimal decision can be summarized as:

$$P_{sr} \underset{a_r}{\overset{a_s}{\gtrless}} P_{s2} \tag{13}$$

$$1 - \frac{g_{rd}}{g_{rd} - g_{sd}} \frac{\exp\left(-\frac{1}{g_{rd}}\right) - \exp\left(-\frac{1}{g_{sd}}\right)}{1 - \exp\left(-\frac{1}{g_{sd}}\right)} \underset{a_r}{\overset{a_s}{\gtrless}} \frac{1 - \exp\left(-\frac{1}{2g_{sd}}\right)}{1 - \exp\left(-\frac{1}{g_{sd}}\right)}. \tag{14}$$

Note that P_{sr} monotonically decreases with g_{rd}. After some arithmetic (14) simplifies to the following form:

$$g_{rd} \underset{a_s}{\overset{a_r}{\gtrless}} g_{min}(g_{sd}),$$

where g_{min} denotes the minimum g_{rd} required for the relay transmission to be advantageous over the source retransmission. The function g_{min} is given by

$$g_{min}(g_{sd}) = g_{sd}\left(1 - \exp\left(-\frac{1}{2g_{sd}}\right)\right)\left[g_{sd}\left(1 - \exp\left(-\frac{1}{2g_{sd}}\right)\right)W(f(g_{sd})) + 1\right]^{-1}, \tag{15}$$

where we define $f()$ as

$$f(x) = \frac{\exp\left(\frac{1+\exp\left(-\frac{1}{2x}\right)}{2x\left(-1+\exp\left(-\frac{1}{2x}\right)\right)}\right)}{x\left(-1+\exp\left(-\frac{1}{2x}\right)\right)}, \tag{16}$$

and W is the Omega function or Lambert's W-function [16]. $W(x) = w$ if x and w satisfy $x = we^w$. Fig. 2 shows g_{min} for a wide range of g_{sd} values. We note that g_{min} has a limit as g_{sd} goes to infinity:

$$\lim_{g_{sd}\to\infty} g_{min}(g_{sd}) = \frac{1}{W(-2e^{-2}) + 2} \approx 0.6275.$$

B. Outage Probability

Let P_{RARQ} be the outage probability, and P_2 the probability that the second transmission coming after a failed first attempt did not help. Then, the outage probability is equal to

$$P_{RARQ} = P_1 P_2(g_{sd}). \tag{17}$$

The pdf and cdf of the normalized average SNR received at the destination from the best relay are denoted by $f_{g_{rd}}$ and $F_{g_{rd}}$, respectively. According to the protocol, the source retransmits if either there is no potential relay or the g_{rd} is less than $g_{min}(g_{sd})$. For analytical convenience, instead of treating the case of no potential relays separately, we modify g_{rd} as follows: If the there is no potential relay, we say g_{rd} is equal to zero.

We can express P_2 as

$$\begin{aligned}
P_2(g_{sd}) &= \int_0^{\infty} \min\{P_{s2}(g_{sd}), P_{sr}(x, g_{sd})\} f_{g_{rd}}(x) \mathrm{d}x && (18)\\
&= \int_0^{g_{min}} P_{s2}(g_{sd}) f_{g_{rd}}(x) \mathrm{d}x + \int_{g_{min}}^{\infty} P_{sr}(x, g_{sd}) f_{g_{rd}}(x) \mathrm{d}x && (19)\\
&= F_{g_{rd}}(g_{min}) P_{s2}(g_{sd}) + \int_{g_{min}}^{\infty} P_{sr}(x, g_{sd}) f_{g_{rd}}(x) \mathrm{d}x && (20)\\
&= F_{g_{rd}}(g_{min}) P_{s2}(g_{sd}) + \int_{g_{min}}^{\infty} \left(1 - \frac{x}{x - g_{sd}} \frac{\exp\left(-\frac{1}{x}\right) - \exp\left(-\frac{1}{g_{sd}}\right)}{1 - \exp\left(-\frac{1}{g_{sd}}\right)}\right) f_{g_{rd}}(x) \mathrm{d}x && (21)
\end{aligned}$$

By substituting P_{s2}, P_{sr}, and P_1 from (7), (12) and (3) in (17), we obtain

$$\begin{aligned}
P_{RARQ} &= \underbrace{F_{g_{rd}}(g_{min})(1 - \exp(-1/(2g_{sd})))}_{P_{out,1}}\\
&+ \underbrace{\int_{g_{min}}^{\infty} \left((1 - \exp(-1/(g_{sd}))) - \frac{x}{x - g_{sd}}\left(\exp\left(-\frac{1}{x}\right) - \exp\left(-\frac{1}{g_{sd}}\right)\right)\right) f_{g_{rd}}(x) \mathrm{d}x}_{P_{out,2}} && (22)
\end{aligned}$$

The integral in the term $P_{out,2}$ can be converted to an expression with $F_{g_{rd}}$ using integration by parts.

$$\begin{aligned}
P_{out,2} &= (1 - \exp(-1/(g_{sd})))(1 - F_{g_{rd}}(g_{min})) - \int_{g_{min}}^{\infty} \frac{x}{x - g_{sd}}\left(\exp\left(-\frac{1}{x}\right) - \exp\left(-\frac{1}{g_{sd}}\right)\right) f_{g_{rd}}(x) \mathrm{d}x\\
&= (1 - \exp(-1/(g_{sd})))(1 - F_{g_{rd}}(g_{min})) - \left|\frac{x}{x - g_{sd}}(\exp(-1/x) - \exp(-1/g_{sd})) F_{g_{rd}}(x)\right|_{g_{min}}^{\infty}\\
&- \int_{g_{min}}^{\infty} \left(\frac{g_{sd}(\exp(-1/x) - \exp(-1/g_{sd}))}{(x - g_{sd})^2} - \frac{\exp(-1/x)/x}{x - g_{sd}}\right) F_{g_{rd}}(x) \mathrm{d}x\\
&= (1 - \exp(-1/(g_{sd})))(1 - F_{g_{rd}}(g_{min})) - (1 - \exp(-1/g_{sd}))\\
&+ \frac{g_{min}}{g_{min} - g_{sd}}(\exp(-1/g_{min}) - \exp(-1/g_{sd})) F_{g_{rd}}(g_{min})\\
&\underbrace{- \int_{g_{min}}^{\infty} \left(\frac{g_{sd}(\exp(-1/x) - \exp(-1/g_{sd}))}{(x - g_{sd})^2} - \frac{\exp(-1/x)/x}{x - g_{sd}}\right) F_{g_{rd}}(x) \mathrm{d}x}_{I} && (23)
\end{aligned}$$

Then

$$P_{out,2} = F_{g_{rd}}(g_{min}) \left(\frac{g_{min}}{g_{min} - g_{sd}} \left(\exp(-1/g_{min}) - \exp(-1/g_{sd})\right) - \left(1 - \exp(-1/g_{sd})\right) \right) - I. \quad (24)$$

To continue, we need to find $F_{g_{rd}}$. The analysis given below follows the same approach as [17].

C. Number of Potential Relays

We make use of a well-known result on Poisson processes [18], which is also used in [19], to calculate the probability mass function (pmf) of the number of potential relays.

Theorem 1: Let the number of objects N in a given region be a Poisson random variable with mean μ. Let ε_i be the event that object i has a certain property. If all the events are independent and have the same probability of occurrence $p = \mathbb{P}\{\varepsilon_i | N = n\}$, for all n, then the number of objects out of N objects having the defined property is also Poisson random variable with mean $p\mu$.

Let $B(a, b; r)$ denote a disc with radius r centered at point (a, b). Suppose that the objects are the nodes in $B(0, 0; r_0)$ and the desired property is having a reliable link to the source at $(0, 0)$. Then the number of potential relays within r_0 of the source are

$$N_r(B(0, 0; r_0)) \sim Poiss(\mu_r(r_0)), \quad (25)$$

where

$$\mu_r(r_0) = \lambda \pi r_0^2 p_r \quad (26)$$

and p_r is the probability that an arbitrary node in $B(0, 0; r_0)$ is a potential relay.

All the nodes in $B(0, 0; r_0)$ are uniformly distributed in the region. Hence, R, the distance from an arbitrary node to the source at $(0, 0)$ has the pdf $f_R(r) = 2r/r_0^2, \quad 0 \le r \le r_0$. Using (2), we then calculate the limit of $r_0^2 p_r$ as $r_0 \to \infty$, covering the whole plane.

$$
\begin{aligned}
\lim_{r_0^2 \to \infty} r_0^2\, p_r &= \lim_{r_0^2 \to \infty} r_0^2\, \mathbb{P}\{R < r_{Ns} X^{2/\delta}\} \\
&= \lim_{r_0^2 \to \infty} r_0^2 \int_0^\infty f_X(x) \int_0^{\min(r_0, r_{Ns} x^{2/\delta})} \frac{2r}{r_0^2} dr\, dx \\
&= \lim_{r_0^2 \to \infty} \int_0^\infty f_X(x) \min(r_0^2, r_{Ns}^2 x^{4/\delta}) dx \\
&= r_{Ns}^2 \mathbb{E}_X[X^{4/\delta}] \qquad (27)
\end{aligned}
$$

As given in [19], $\mathbb{E}_X[X^{4/\delta}] = \Gamma(1 + 2/\delta)$ for Rayleigh distributed X, where the gamma function $\Gamma(.)$ is defined as

$$
\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt.
$$

Then, $\mu_r = \lim_{r_0 \to \infty} \mu_r(r_0)$, the average number of potential relays in the entire plane, is found by substituting (27) in (26),

$$
\mu_r = \lambda \pi r_{Ns}^2 \Gamma\left(1 + \frac{2}{\delta}\right) \qquad (28)
$$

and the pmf of the number of potential relays of the source is

$$
N_r \sim Poiss\left(\lambda \pi r_{Ns}^2 \Gamma\left(1 + 2/\delta\right)\right).
$$

D. Distance of a Potential Relay to the Source

Let R denote the distance of an arbitrary node in $B(0,0;r_0)$ to the source at $(0,0)$ and $\mathcal{A}(r_0)$ denote the event that this node has a direct connection with the source. In Section 9A of [19, eqn (47)], the pdf of R given $\mathcal{A}(r_0)$ as $r_0 \to \infty$ is calculated:

$$
f_{R|\mathcal{A}}(r) \;=\; \frac{2r\, \exp\left(-(r/r_{Ns})^\delta\right)}{r_{Ns}^2\, \Gamma(1 + 2/\delta)}. \qquad (29)
$$

E. Distance of a Potential Relay to the Destination

Consider an arbitrary potential relay, node i, which has connection to the source. Let L_i denote the distance of this node to the destination at $(D, 0)$. Since the locations of such nodes are i.i.d., so are their distances to the destination, $L_i \sim f_L(l)$, where the distribution is defined by

$$F_L(l) \;=\; \int_0^\infty \mathbb{P}\{L \leq l \,|\, R = r\} f_{R|\mathcal{A}}(r) dr.$$

We first calculate the probability that the distance of a potential relay to the destination is below l given that R, its distance from the source, is r. Since the angular distribution of all potential relays around the source is uniform in $[0, 2\pi]$, such nodes are located uniformly on the circle $C(0,0;r)$, where $C(u,v;a)$ is a circle centered at the point (u,v) with radius a. If the circle $C(0,0;r)$ and the disk $B(D,0;l)$ intersect partially, i.e. $|l-D| < r < l+D$, then the probability that $L \leq l$ is equal to the fraction of the length of $C(0,0;r)$ that is within $B(D,0;l)$. In the illustration of Fig. 3, this fraction is equal to θ/π. Using the law of cosines, we can express θ as:

$$\theta = \arccos\left(\frac{D^2 - l^2 + r^2}{2Dr}\right)$$

If $l > r + D$, then the circle is within the disk and all the points on the circle are within l of the point $(D,0)$. However, if $r > l - D$, none of the nodes on the circle are closer to D than l.

Hence, we obtain the conditional cdf $F_{L|R}(l|r)$ as

$$F_{L|R}(l|r) \;=\; \begin{cases} \frac{1}{\pi}\arccos\left(\frac{D^2-l^2+r^2}{2Dr}\right), & \\ \qquad\qquad |l-D| < r < l+D; & \\ 1, \qquad\qquad 0 < r < l-D; & \\ 0, \qquad\qquad \text{otherwise}. & \end{cases} \tag{30}$$

where $l, D, r > 0$. Then, averaging over R, the cdf of L is obtained from

$$\begin{aligned} F_L(l) &= \int_0^\infty F_{L|R}(l|r) f_{R|\mathcal{A}}(r) dr \\ &= \underbrace{\int_0^{\max\{0,l-D\}} f_{R|\mathcal{A}}(r) dr}_{I_1} + \underbrace{\frac{1}{\pi}\int_{|l-D|}^{l+D} \arccos\left(\frac{D^2 - l^2 + r^2}{2Dr}\right) f_{R|\mathcal{A}}(r) dr}_{I_2}. \end{aligned} \tag{31}$$

We substitute (29) in the first integral I_1, use the change of variable $u = r/r_{Ns}$ and obtain

$$\begin{aligned} I_1 &= \int_0^{\max\{0,l-D\}} f_{R|\mathcal{A}}(r)dr = \frac{2}{\Gamma(1+2/\delta)}\int_0^{\max\{0,l-D\}} \frac{r}{r_{Ns}^2}\ \exp\left(-(r/r_{Ns})^{\delta}\right)dr \\ &= \frac{2}{\Gamma(1+2/\delta)}\int_0^{\max\{0,l/r_{Ns}-D/r_{Ns}\}} u\exp\left(-u^{\delta}\right)du \end{aligned} \tag{32}$$

Note that for $\delta = 2$,

$$I_1 = 2 - \exp(-\max\{0, l/r_{Ns} - D/r_{Ns}\}^2),$$

and for $\delta = 4$,

$$I_1 = \frac{\sqrt{\pi}\,\Gamma(3/2)}{4}\text{erf}(\max\{0, l/r_{Ns} - D/r_{Ns}\}^2),$$

where erf denotes the error function. I_2 has no closed form expression but it can be expressed in terms of u, D/r_{Ns} and l/r_{Ns}.

$$I_2 = \frac{2}{\Gamma(1+2/\delta)}\int_{|l/r_{Ns}-D/r_{Ns}|}^{l/r_{Ns}+D/r_{Ns}} u\exp\left(-u^{\delta}\right)\arccos\left(\frac{(D/r_{Ns})^2-(l/r_{Ns})^2+u^2}{2(D/r_{Ns})u}\right)du \tag{33}$$

Since both (32) and (33) are functions of l/r_{Ns}, d/r_{Ns} and δ only, we denote $F_L(l)$ as

$$F_L(l) = h(l/r_{Ns}, D/r_{Ns}, \delta). \tag{34}$$

F. Distribution of Averaged Normalized SNR Received at the Destination from an Arbitrary Potential Relay

Since i is arbitrary, $g_{rd,i}$ is i.i.d. over the nodes, and we can drop the index and denote this cdf by $F_{g_{rd,a}}$. Then, using (2) we obtain

$$F_{g_{rd,a}}(g) = 1 - F_L\left(r_{Nr}g^{-1/\delta}\right) \tag{35}$$

$$\begin{aligned} &= 1 - \frac{2}{\Gamma(1+2/\delta)}\Bigg\{\int_0^{\max\{0,g^{-1/\delta}r_{Nr}/r_{Ns}-D/r_{Ns}\}} u\exp\left(-u^{\delta}\right)du \\ &\quad + \int_{|g^{-1/\delta}r_{Nr}/r_{Ns}-D/r_{Ns}|}^{g^{-1/\delta}r_{Nr}/r_{Ns}+D/r_{Ns}} u\exp\left(-u^{\delta}\right)\arccos\left(\frac{(D/r_{Ns})^2-(g^{-1/\delta}r_{Nr}/r_{Ns})^2+u^2}{2(D/r_{Ns})u}\right)du\Bigg\} \\ &= 1 - \frac{2}{\Gamma(1+2/\delta)}\Bigg\{\int_0^{\max\{0,(\eta/g)^{1/\delta}-\tilde{D}\}} u\exp\left(-u^{\delta}\right)du \\ &\quad + \int_{|(\eta/g)^{1/\delta}-\tilde{D}|}^{(\eta/g)^{1/\delta}+\tilde{D}}\left(u\exp\left(-u^{\delta}\right)\frac{1}{\pi}\arccos\left(\frac{\tilde{D}^2-(\eta/g)^{2/\delta}+u^2}{2\tilde{D}u}\right)du\right)\Bigg\}, \end{aligned} \tag{36}$$

where we defined η and $\tilde{D}$ as

$$\eta = (r_{Nr}/r_{Ns})^{\delta} \quad \text{and} \quad \tilde{D} = D/r_{Ns}.$$

G. Distribution of the Averaged Normalized SNR of the Selected Potential Relays

Let us consider the case where we have exactly k potential relays ($k \geq 1$) and we choose the relay with the largest average SNR. The joint cdf of the largest of k IID random variables is equal to

$$F^{(k)}_{g_{rd}}(g) = (F_{g_{rd,a}}(g))^k. \tag{37}$$

Note that the above distribution is a function of k.

H. Average SNR of the Best Potential Relay at the Destination

The number of potential relays is Poisson distributed with parameter μ_r, given by (28). When we average (37) over N_r the number of potential relays, we obtain:

$$\begin{aligned} F_{g_{rd}}(g) &= \mathbb{P}\{g_{rd} < g\} = \mathbb{P}\{N_r = 0\} + \sum_{k=1}^{\infty} \mathbb{P}\{N_r = k\}\mathbb{P}\{g_{rd} < g | N_r = k\} & (38) \\ &= \mathbb{P}\{N_r = 0\} + \sum_{k=1}^{\infty} \mathbb{P}\{N_r = k\} F^{(k)}_{g_{rd}}(g) \\ &= \exp(-\mu_r) + \sum_{k=1}^{\infty} \exp(-\mu_r) \frac{\mu_r^k}{k!} (F_{g_{rd,a}}(g))^k \\ &= \exp(-\mu_r) \left(1 + \sum_{k=1}^{\infty} \frac{(\mu_r\, F_{g_{rd,a}}(g))^k}{k!}\right) \\ &= \exp(-\mu_r) \exp(\mu_r (F_{g_{rd,a}}(g))) = \exp(-\mu_r (1 - F_{g_{rd,a}}(g))), & (39) \end{aligned}$$

where $F_{g_{rd,a}}(g)$ is given by (36). In Fig. 4 we plot $F_{g_{rd,a}}$ and $F_{g_{rd}}$ for different μ_r values using (36) and (39). It is observed that the average SNR received at the destination from the best relay increases with μ_r, i.e., with the density of nodes in the network. P_{RARQ} can now be computed numerically from (22) using (24), (36), and (39).

VI. Numerical Results

In this section we validate the analytical results of this paper with simulations. In our simulation study, for each data point 10000 topologies are generated where the source is placed at position $(-D/2, 0)$ and the destination is placed at position $(+D/2, 0)$ on a $K \times K$ square, where K is chosen depending on the node density λ. $N = 600$ other nodes are placed randomly and uniformly on the region. The source and relays are assumed to have identical transmission ranges ($r_{Nr} = r_{Ns}$, $\eta = 1$). The path loss exponent δ is 4. The Rayleigh fading is generated i.i.d. across all the links. In Fig. 5 we plot the outage probability as a function of $\tilde{D}$ at $\mu_r = 3$ for single hop transmission, ARQ protocol, and the relay assisted ARQ protocol. Fig. 6 shows the same curve for $\mu_r = 8$. The curves of P_{RARQ} from the simulations and the analysis (from (22)) agree completely, which validates both our analysis and simulation setup. We see that our protocol can decrease the outage probability significantly.

To observe the effect of the node density on the outage probability of relay RARQ, we vary the number of nodes in the area and in Fig. 7, we plot the outage probability of relay RARQ as a function of μ_r. It is seen that as μ_r increases, the outage probability of RARQ decreases rapidly.

Finally, to study the effect of shadowing on the performance of RARQ as well as ARQ and the direct transmission, we include lognormal shadowing in the link model of our simulation. The instantaneous SNR expression given in (1) is modified as

$$\gamma_{ij} = \frac{KP_T}{d_{ij}^{\delta} P_N} 10^{Z_{ij}/10} X_{ij}^2, \tag{40}$$

where Z_{ij} is a Gaussian random variable with zero mean and variance equal to σ^2 and Z_{ij}s are independent and identically distributed across all links. Fig. 8 shows the outage probability as a function of σ. It is seen that as σ increases the performance of RARQ improves. This could be attributed to the fact that μ_r increases with σ, as seen in [19, eqn (12)]:

$$\mu_r = \lambda \pi r_{Ns}^2 \exp(2\alpha^2)\Gamma(1 + 2/\delta), \qquad \alpha = \frac{\ln 10}{10} \frac{\sigma}{\delta}. \tag{41}$$

The performance of direct transmission and ARQ, in contrast does not change significantly.

VII. CONCLUSIONS

We have proposed a simple protocol implementing two-hop cooperative diversity augmented with ARQ. The protocol requires only limited and slow changing information about the channels between a node and each of its "neighbors". We have studied analytically and through simulations the improvement in packet outage probability obtained when the protocol is implemented over a large ad-hoc network with node locations following a homogeneous Poisson point process. We have validated our analytic derivations by comparing them to the simulations results. We have shown graphically how increasing the intensity of the nodes reduces the outage probability, or to put it differently, allows a further destination to be reached with acceptable packet delivery ratio.

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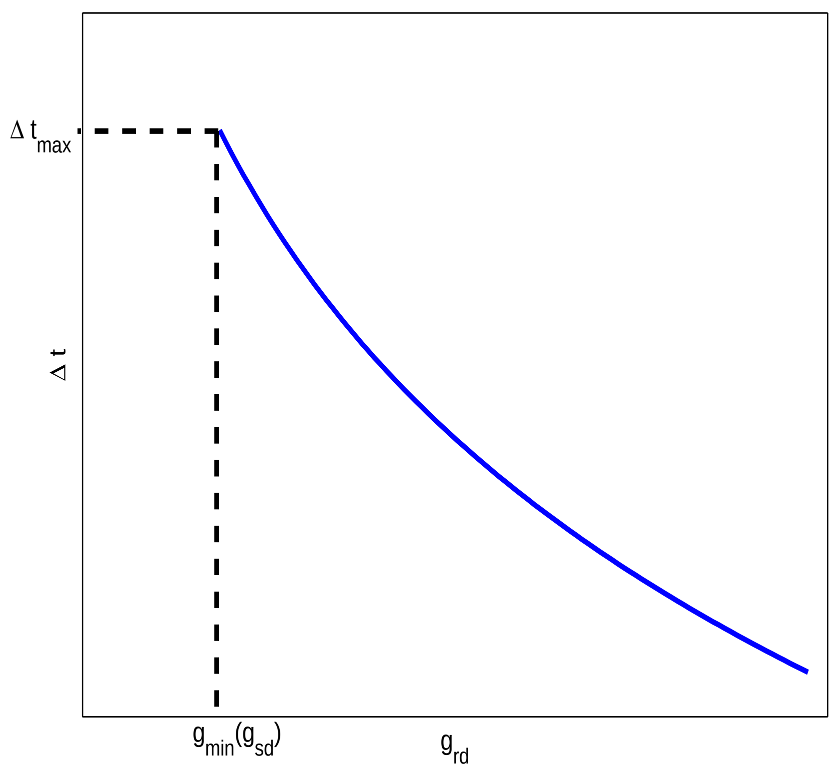

Fig. 1. $G(g_{rd}, g_{sd})$.

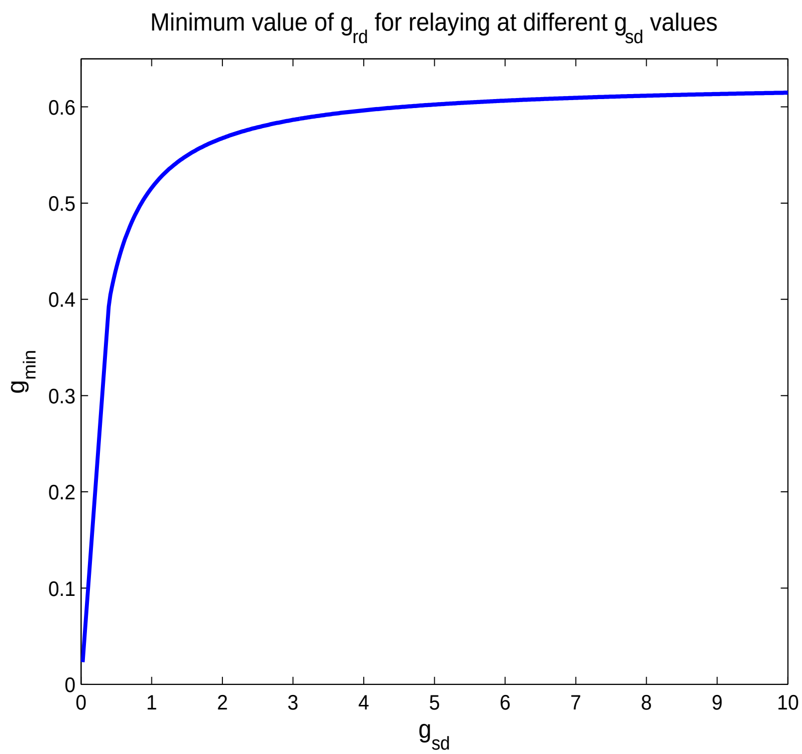

Fig. 2. g_{min} as a function of g_{sd}.

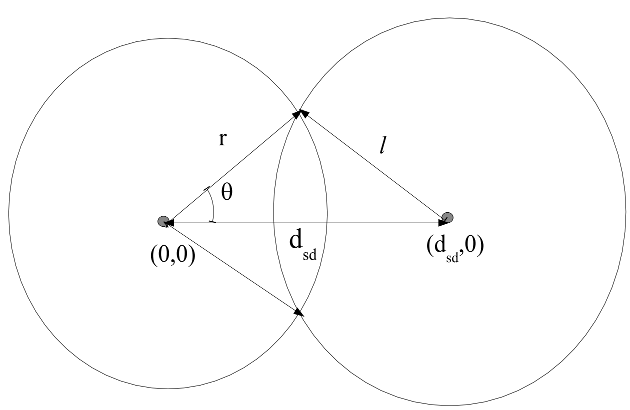

Fig. 3. Illustration for the calculation of $F_{L|R}(l|r)$ for $|l-D|<r<|l+D|$

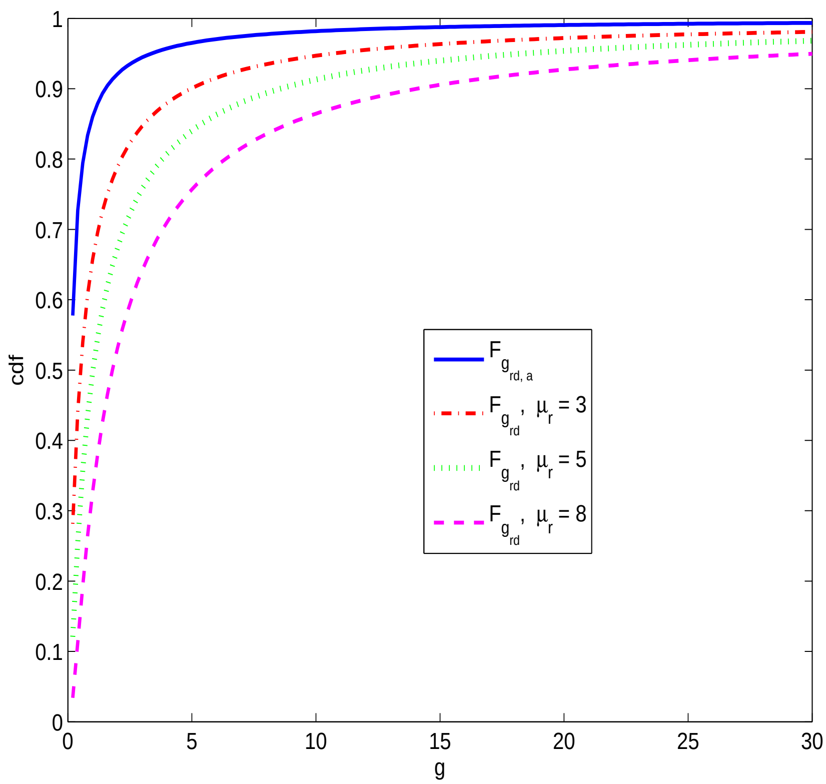

Fig. 4. The cdf of the average SNR of an arbitrary potential relay to the destination ($F_{g_{rd,a}}$) and the average SNR of the best potential relay to the destination ($F_{g_{rd}}$) for different μ_r values ($\mu_r = \{3, 5, 8\}$). $r_{Ns} = r_{Nr}(\eta = 1)$, $D/r_{Ns} = 1.5$ and $\delta = 4$.

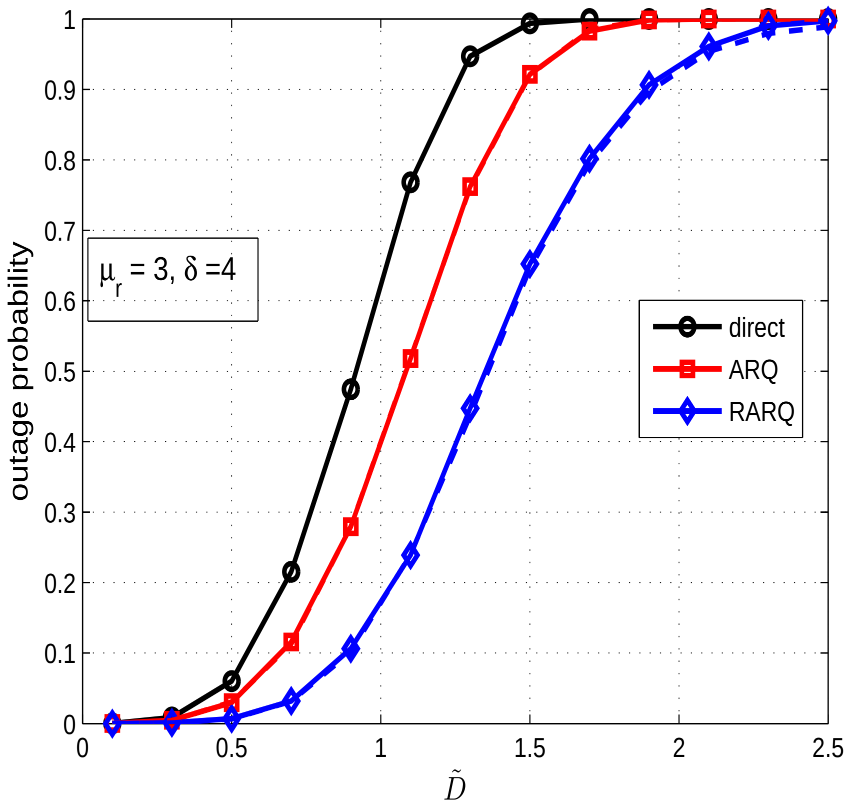

Fig. 5. The outage probabilities of ARQ and relay-assisted ARQ as a function of $\tilde{D}$ for $\mu_r = 3$, $\delta = 4$, and $r_{Nr} = r_{Ns} = 1$. Analytical curve for RARQ is shown by dotted lines.

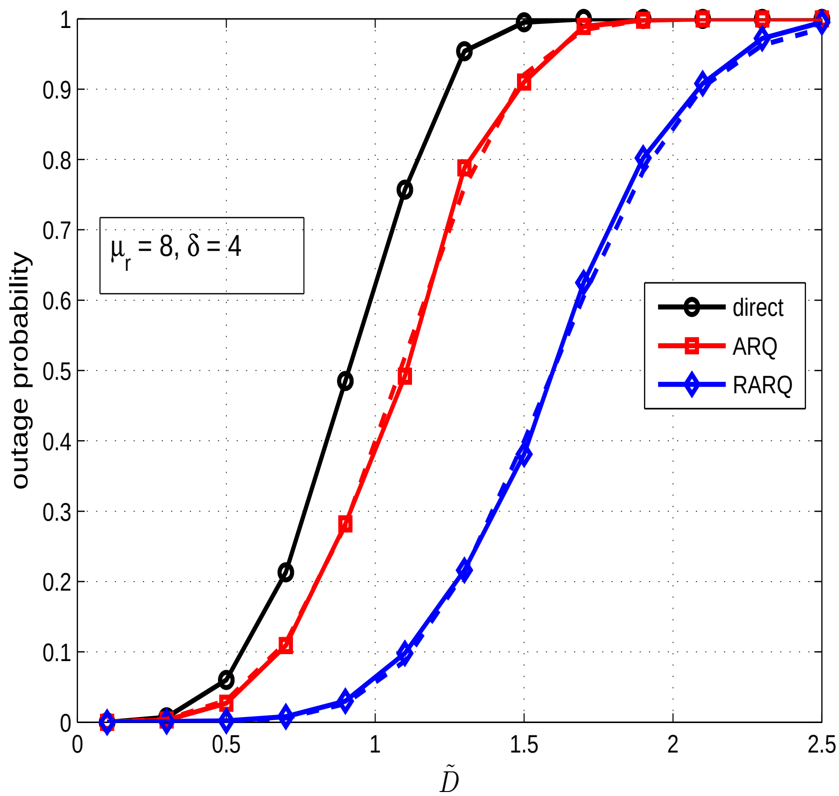

Fig. 6. The outage probabilities of ARQ and relay-assisted ARQ as a function of $\tilde{D}$ for $\mu_r = 8$, $\delta = 4$, and $r_{Nr} = r_{Ns} = 1$. Analytical curve for RARQ is shown by dotted lines.

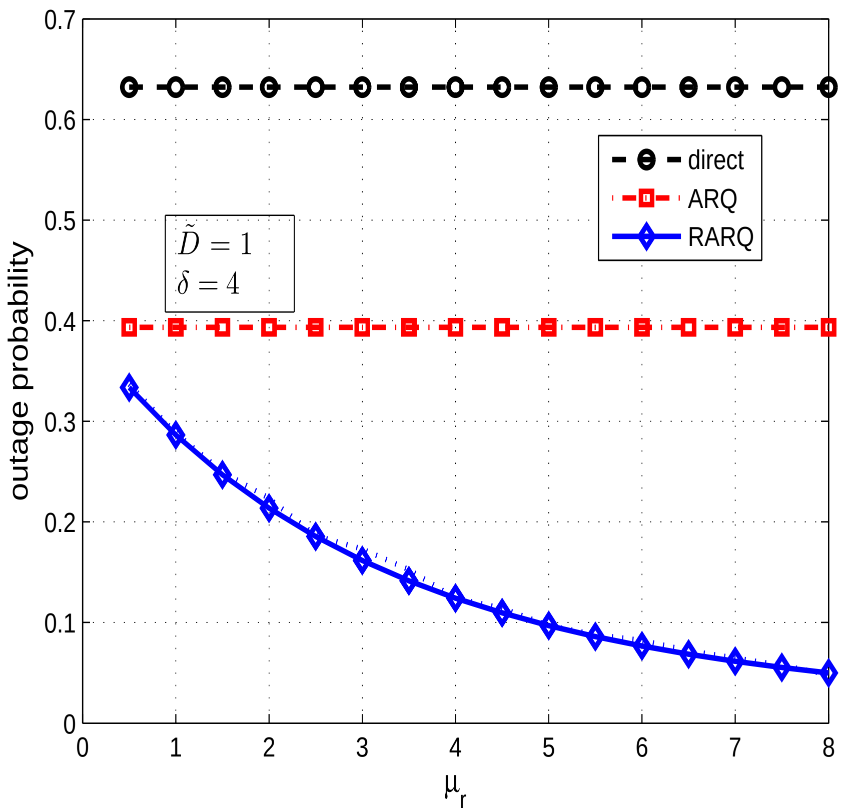

Fig. 7. The outage probabilities of ARQ and relay-assisted ARQ as a function of μ_r for $\tilde{D} = 1$, $\delta = 4$, and $r_{Nr} = r_{Ns} = 1$. Analytical curve for RARQ is shown by dotted lines.

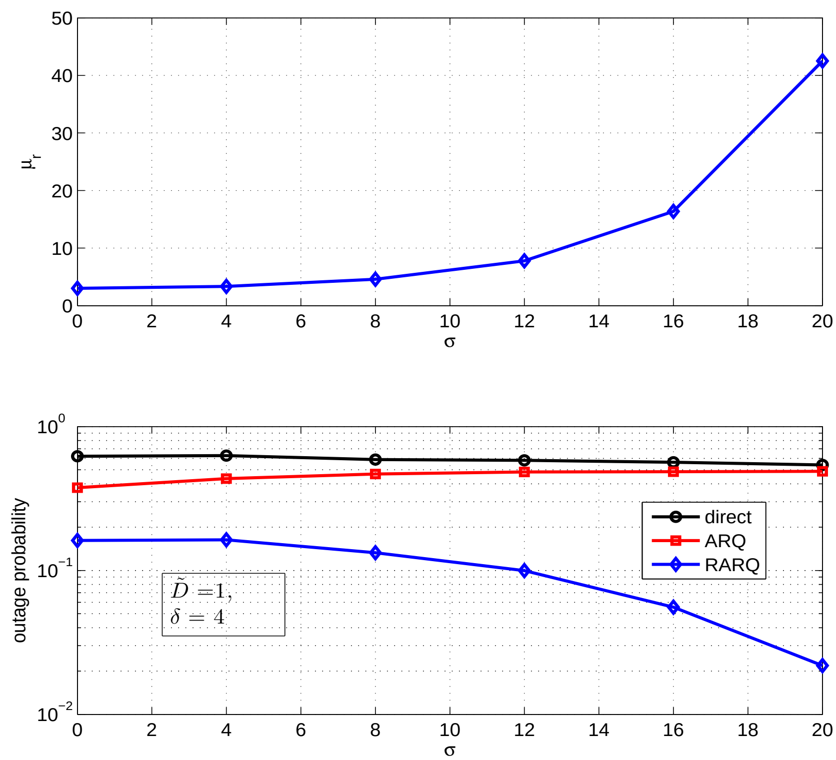

Fig. 8. The outage probabilities of ARQ and relay-assisted ARQ as a function of σ for $\tilde{D} = 1$, $\delta = 4$, and $r_{Nr} = r_{Ns} = 1$.