

|                |  |   |
|----------------|--|---|
| Project        | <b>IEEE 802.16 Broadband Wireless Access Working Group</b> < <a href="http://ieee802.org/16">http://ieee802.org/16</a> >   |   |
| Title          | <b>Comparison of Link Performance Abstraction for ML Receivers with MMIB and RBIR Metrics</b>  |   |
| Date Submitted | <b>2007-01-16</b>  |   |
| Source(s)      | Krishna Sayana<br>Jeff Zhuang<br>Ken Stewart   | E-mail: <a href="mailto:KrishnaKamal@motorola.com">KrishnaKamal@motorola.com</a><br><a href="mailto:Jeff.Zhuang@motorola.com">Jeff.Zhuang@motorola.com</a><br><a href="mailto:Ken.Stewart@motorola.com">Ken.Stewart@motorola.com</a><br><br>* <a href="http://standards.ieee.org/faqs/affiliationFAQ.html">http://standards.ieee.org/faqs/affiliationFAQ.html</a> > |
|                | Motorola Inc,<br>600 N US Hwy 45,<br>Libertyville, IL-60048  |   |
| Re:            | Call for contributions for 802.16m Evaluation Methodology, IEEE 802.16m-07/048, 12/17/07   |   |
| Abstract       | This contribution compares link abstraction methodologies proposed for ML receivers based on MMIB metrics developed in C802.16m-07/142 and RBIR metrics in C802.16m-07/187   |   |
| Purpose        | For discussion and approval by 802.16 TGM  |   |
| Notice         | <i>This document does not represent the agreed views of the IEEE 802.16 Working Group or any of its subgroups. It represents only the views of the participants listed in the "Source(s)" field above. It is offered as a basis for discussion. It is not binding on the contributor(s), who reserve(s) the right to add, amend or withdraw material contained herein.</i>   |   |
| Release        | The contributor grants a free, irrevocable license to the IEEE to incorporate material contained in this contribution, and any modifications thereof, in the creation of an IEEE Standards publication; to copyright in the IEEE's name any IEEE Standards publication even though it may include portions of this contribution; and at the IEEE's sole discretion to permit others to reproduce in whole or in part the resulting IEEE Standards publication. The contributor also acknowledges and accepts that this contribution may be made public by IEEE 802.16.   |   |
| Patent Policy  | The contributor is familiar with the IEEE-SA Patent Policy and Procedures:<br>< <a href="http://standards.ieee.org/guides/bylaws/sect6-7.html#6">http://standards.ieee.org/guides/bylaws/sect6-7.html#6</a> > and<br>< <a href="http://standards.ieee.org/guides/opman/sect6.html#6.3">http://standards.ieee.org/guides/opman/sect6.html#6.3</a> >.<br>Further information is located at < <a href="http://standards.ieee.org/board/pat/pat-material.html">http://standards.ieee.org/board/pat/pat-material.html</a> > and<br>< <a href="http://standards.ieee.org/board/pat">http://standards.ieee.org/board/pat</a> >. |   |

# Link Performance Abstraction Comparison for ML Receivers

*Krishna Sayana, Jeff Zhuang and Ken Stewart*  
*Motorola Inc*

## I. Introduction

This contribution provides the comparison of two link evaluation methodologies as defined in the current draft of Evaluation Methodology Document, namely the MMIB and RBIR methods, with focus on MIMO Maximum Likelihood (ML) receivers. Link performance abstraction using MMIB metrics for ML receivers was proposed for adoption in a previous contribution C802.16m-07/142 [2]. This contribution developed an approach that is based on a) defining the mutual information between each transmitted bit and its associated LLR before turbo decoding, 2) approximating the PDFs of LLR by a mixture Gaussian distributions 3) identifying dominant Gaussian distributions 4) calculating the means and variances of those dominant Gaussian PDF from the channel matrix (i.e., the Eigen values and matrix condition numbers)

Contribution C802.16m-07/187 derives RBIR metrics as symbol based mutual information normalized to bit level by dividing with the modulation order. Again for ML abstraction, this approach also uses Gaussian distribution approximation approach similar to MMIB, with means and variances derived from the channel matrix.

However, there are some key differences in how these metrics are derived that should be looked at when down-selecting and choosing one approach over the other.

Mainly we find that RBIR metrics proposed for ML receivers do not satisfy some properties that are expected/assumed about mutual information metric in the evaluation methodology document. These mappings are developed using adjustment parameters aided by link simulations for each MCS, as opposed to numerically obtaining true value of MI by Monte-Carlo methods.

The study below is based on our best understanding of RBIR implementation and uses the functions, approximations and tables currently provided in the RBIR section 4.3.1.3 of the EVM document. Since it is expected that numerical approximations are not always completely accurate, we only focus here on aspects that are particularly important and not isolated cases, and cannot be attributed to approximation errors alone.

We suggest that these technical issues be addressed and clarified in the working group before accepting the EVM document.

## II. Brief Overview of Implementation

### MMIB Implementation Steps for ML

- Based on bit-level MI between transmitted bit and post processing LLR
- Compute three parameters based on Eigen-values of H and the “Eigen-spread”. Quality of each data stream fundamentally depends on the Eigen-values and the “Eigen-spread”

$$H^H H = V D V^H, D = \begin{pmatrix} \lambda_{\max} & 0 \\ 0 & \lambda_{\min} \end{pmatrix}, p_a = \min\{p, 1-p\}, \text{ where } |\mathbf{V}| \cdot |\mathbf{V}| = \begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix}, 0 \leq p \leq 1$$

- Obtain set of dominant Gaussian PDFs and define MIB as sum of the J(.) functions based on these PDFs

$$[\gamma(1), \gamma(2), \gamma(3)] = \text{sort}_{asc} \{ \lambda_{\max} p_a + \lambda_{\min} (1 - p_a), \lambda_{\min} p_a + \lambda_{\max} (1 - p_a), \\ \lambda_{\max} (1 - 2\sqrt{p_a(1-p_a)}) + \lambda_{\min} (1 + 2\sqrt{p_a(1-p_a)}) \}$$

$$QPSK : I_2^{(2 \times 2)}(\lambda_{\min}, \lambda_{\max}, p_a) = \frac{1}{2} J(a\sqrt{\gamma(1)}) + \frac{1}{2} J(b\sqrt{\gamma(2)}), \quad a = 0.85, b = 1.19$$

$$16QAM / 64QAM : I_m^{(2 \times 2)}(\lambda_{\min}, \lambda_{\max}, p_a) = \frac{1}{3} \left[ J(a\sqrt{\gamma(1)}) + J(b\sqrt{\gamma(2)}) + J(c\sqrt{\gamma(3)}) \right]$$

- Parameter a,b,c depend on the condition number of the channel matrix and modulation order only, but not on code rate. To clarify, note that the in SISO MIB functions are expressed as a mapping from channel quality as represented by SNR to MIB. For MIMO matrix channel, it is a three-dimensional mapping from these three parameters (Eigen values and spread) to MIB. The actual three dimensional mapping is obtained by generating numerical LLR PDFs of the matrix channel and a modulation constellation and evaluating MIB with this LLR PDF. To avoid a cumbersome lookup table, the following convenient functional approximation table has been provided.

| 16 QAM                         | $1 < \kappa \leq 10$               | $10 < \kappa \leq 100$             | $\kappa > 100$                     |
|--------------------------------|------------------------------------|------------------------------------|------------------------------------|
| $-10dB < \lambda_{\min} < 8dB$ | $a = 0.48, b = 0.27$<br>$c = 0.69$ | $a = 0.40, b = 0.21$<br>$c = 0.56$ | $a = 0.32, b = 0.13$<br>$c = 0.37$ |
| $\lambda_{\min} > 8dB$         | $a = 0.35, b = 0.43$<br>$c = 0.59$ | $a = 0.37, b = 0.33$<br>$c = 100$  | $a = 0.42, b = 0.11$<br>$c = 100$  |

| 64 QAM                         | $1 < \kappa \leq 10$               | $10 < \kappa \leq 100$             | $\kappa > 100$                     |
|--------------------------------|------------------------------------|------------------------------------|------------------------------------|
| $-10dB < \lambda_{\min} < 8dB$ | $a = 0.23, b = 0.16$<br>$c = 0.59$ | $a = 0.12, b = 0.12$<br>$c = 0.38$ | $a = 0.08, b = 0.07$<br>$c = 0.17$ |
| $\lambda_{\min} > 8dB$         | $a = 0.20, b = 0.21$<br>$c = 0.62$ | $a = 0.22, b = 0.13$<br>$c = 100$  | $a = 0.24, b = 0.08$<br>$c = 100$  |

### RBIR Implementation Steps for ML

- Define “symbol-level” LLR (i.e., ratio of the conditional probability of a certain symbol to the summed probability of other symbols), which is approximated as a Gaussian PDFs with a mean AVE and variance VAR. Define the mutual information per symbol or SI as the mutual information between symbol and its symbol-level LLR

$$SI = \frac{1}{M} \sum_{i=1}^M \int_{-\infty}^{\infty} p(LLR_i) \log_2 \frac{M}{1 + e^{-LLR_i}} dLLR_i, \quad SI \approx \log_2 M - \frac{1}{\log_e 2} \left[ \begin{array}{l} \frac{2}{3} f_1(AVE) \\ + \frac{f_1(AVE + \sqrt{3VAR})}{6} \\ + \frac{f_1(AVE - \sqrt{3VAR})}{6} \end{array} \right],$$

$$f_1(x) = \log_e(1 + e^{-x})$$

- Approximation is used because there is no closed-form solution to the integration
- Two “hypothetical” data streams with the associated SNR defined by the associated column norm of H (2x2).
- Calculate AVE and VAR of SI for each “hypothetical” stream
  - The calculation of AVE and VAR of symbol-level LLR uses 3-4 nearest neighboring

constellation in an approximation, and AVE and VAR must resort to some numerical integration. Hence a look-up table based on the column norm of H is provided in the following

$$\gamma_{dB} = 10 \log_{10} \left( \frac{d^2 |H_k|^2}{\sigma^2} \right), d = \begin{cases} \sqrt{2}, & \text{for QPSK} \\ 2/\sqrt{10}, & \text{for 16QAM} \\ 2/\sqrt{42}, & \text{for 64QAM} \end{cases}$$

$H_k$  is the  $k$ -th column vector of the channel matrix  $\mathbf{H} = [H_1 \ H_2]$ , and  $\sigma^2$  is the variance of noise

| $\gamma_{dB}$ (dB) | [-20:0.5:30] |          |           |          |          |          |          |          |          |         |         |
|--------------------|--------------|----------|-----------|----------|----------|----------|----------|----------|----------|---------|---------|
| AVE                | [            | -0.4016  | -0.4123   | -0.4233  | -0.4344  | -0.4457  | -0.4571  | -0.4687  | -0.4804  |         |         |
|                    |              | -0.4922  | -0.5041   | -0.5160  | -0.5279  | -0.5397  | -0.5515  | -0.5631  | -0.5745  |         |         |
|                    |              | -0.5856  | -0.5962   | -0.6065  | -0.6161  | -0.6249  | -0.6329  | -0.6399  | -0.6456  |         |         |
|                    |              | -0.6499  | -0.6524   | -0.6530  | -0.6513  | -0.6470  | -0.6396  | -0.6287  | -0.6139  |         |         |
|                    |              | -0.5944  | -0.5697   | -0.5391  | -0.5018  | -0.4567  | -0.4031  | -0.3396  | -0.2650  |         |         |
|                    |              | -0.1780  | -0.0770   | 0.0398   | 0.1743   | 0.3286   | 0.5051   | 0.7063   | 0.9352   | 1.1949  |         |
|                    |              | 1.4889   | 1.8211    | 2.1959   | 2.6179   | 3.0926   | 3.6259   | 4.2245   | 4.8961   | 5.6491  |         |
|                    |              | 6.4933   | 7.4396    | 8.5006   | 9.6904   | 11.0251  | 12.5229  | 14.2045  | 16.0930  | 18.2146 |         |
|                    |              | 20.5989  | 23.2784   | 26.2897  | 29.6733  | 33.4750  | 37.7458  | 42.5431  | 47.9314  |         |         |
|                    |              | 53.9830  | 60.7788   | 68.4100  | 76.9786  | 86.5992  | 97.4004  | 109.5263 | 123.1389 |         |         |
| VAR                |              | 138.4197 | 155.5725  | 174.8260 | 196.4366 | 220.6922 | 247.9159 | 278.4700 | 312.7611 |         |         |
|                    |              | 351.2455 | 394.4351  | 442.9043 | 497.2976 | 558.3381 | 626.8372 | 703.7054 | 789.9640 |         |         |
|                    |              | 886.7593 | 995.3772] |          |          |          |          |          |          |         |         |
|                    |              | [        | 0.2952    | 0.3003   | 0.3055   | 0.3108   | 0.3162   | 0.3218   | 0.3276   | 0.3336  | 0.3400  |
|                    |              |          | 0.3541    | 0.3620   | 0.3705   | 0.3800   | 0.3904   | 0.4021   | 0.4152   | 0.4301  | 0.4471  |
|                    |              |          | 0.4887    | 0.5143   | 0.5438   | 0.5779   | 0.6175   | 0.6633   | 0.7164   | 0.7779  | 0.8491  |
|                    |              |          | 1.0270    | 1.1373   | 1.2645   | 1.4112   | 1.5801   | 1.7741   | 1.9967   | 2.2516  | 2.5430  |
|                    |              |          | 3.2542    | 3.6849   | 4.1737   | 4.7277   | 5.3548   | 6.0636   | 6.8644   | 7.7686  | 8.7895  |
|                    |              |          | 9.9429    | 11.2474  | 12.7253  | 14.4033  | 16.3140  | 18.4964  | 20.9982  | 23.8761 | 27.1982 |
|                    |              |          | 31.0450   | 35.5109  | 40.7058  | 46.7560  | 53.8056  | 62.0176  | 71.5751  | 82.6815 | 95.5627 |
|                    |              | 110.4754 | 127.720   | 147      | 170      | 197      | 228      | 263      | 304      | 352     |         |
|                    |              | 406      | 470       | 543      | 627      | 725      | 838      | 968      | 1118     | 1292    |         |
|                    |              | 1492     | 1724      | 1991     | 2300     | 2657     | 3069     | 3545     | 4095     | 4729    |         |
|                    |              | 5462     | 6309      | 7287     | 8416     | 9720     | 11226    | 12965];  |          |         |         |

- Modify the AVE and VAR of the SI for each data stream, since the channel quality of actual data streams cannot be represented by the column norm of H
  - Heuristic parameters ( $a_1, a_2$ ) are introduced with a provided table (MCS-dependent, also condition number dependent, derived from link simulations and optimized with AWGN reference). The scaling factor of “2” for 64QAM seems to be unexplained.

$$\begin{aligned} AVE_{Stream,i} &= a_i \times AVE, & VAR_{Stream} &= VAR & \text{for QPSK, 16QAM} \\ AVE_{Stream,i} &= a_i \times AVE, & VAR_{Stream} &= 2 \times VAR & \text{for 64QAM} \end{aligned}$$

| Parameter a                                 |                        | QPSK<br>1/2 | QPSK<br>3/4 | 16QAM<br>1/2 | 16QAM<br>3/4 | 64QAM<br>1/2 | 64QAM<br>2/3 | 64QAM<br>3/4 | 64QAM<br>5/6 |
|---|------------------------|-------------|-------------|--------------|--------------|--------------|--------------|--------------|--------------|
| $k < 10$<br>$\lambda_{min} dB \leq -10$     | 1 <sup>st</sup> Stream | 0.9000      | 1.0000      | 1.0000       | 1.0000       | 1.0000       | 1.0000       | 1.0000       | 1.0000       |
|   | 2 <sup>nd</sup> Stream | 0.9000      | 1.0000      | 1.0000       | 1.0000       | 1.0000       | 1.0000       | 1.0000       | 1.0000       |
| $k < 10$<br>$-10 < \lambda_{min} dB \leq 8$ | 1 <sup>st</sup> Stream | 2.8372      | 1.4444      | 0.4343       | 1.5737       | 0.7872       | 1.0000       | 1.0000       | 1.0000       |
|   | 2 <sup>nd</sup> Stream | 1.4801      | 1.4859      | 0.6389       | 1.1526       | 1.1000       | 1.0000       | 1.1000       | 1.0000       |

|   |                        |        |        |        |        |        |        |        |        |
|---|------------------------|--------|--------|--------|--------|--------|--------|--------|--------|
| $k < 10$<br>$\lambda_{\min} dB > 8$                   | 1 <sup>st</sup> Stream | 1.2000 | 1.0000 | 0.6000 | 0.9889 | 0.4695 | 1.5889 | 1.5000 | 0.9222 |
|   | 2 <sup>nd</sup> Stream | 1.2000 | 1.2000 | 0.6000 | 1.3632 | 0.3111 | 2.0667 | 1.0667 | 0.9333 |
| $10 \leq k < 100$<br>$\lambda_{\min} dB \leq -10$     | 1 <sup>st</sup> Stream | 1.9264 | 1.1731 | 1.0000 | 1.0000 | 2.0000 | 1.0000 | 1.0000 | 1.0000 |
|   | 2 <sup>nd</sup> Stream | 1.6172 | 1.3444 | 1.0000 | 1.0000 | 2.0000 | 1.0000 | 1.0000 | 1.0000 |
| $10 \leq k < 100$<br>$-10 < \lambda_{\min} dB \leq 8$ | 1 <sup>st</sup> Stream | 0.8833 | 1.1900 | 0.5000 | 1.1246 | 0.6611 | 0.8556 | 1.0111 | 1.0000 |
|   | 2 <sup>nd</sup> Stream | 0.8857 | 1.3000 | 0.5000 | 0.8532 | 0.6500 | 0.8333 | 1.1556 | 1.0111 |
| $10 \leq k < 100$<br>$\lambda_{\min} dB > 8$          | 1 <sup>st</sup> Stream | 1.1000 | 1.0000 | 0.5500 | 1.0000 | 0.7310 | 1.0778 | 1.1111 | 0.8333 |
|   | 2 <sup>nd</sup> Stream | 1.1000 | 1.1000 | 0.5500 | 1.0000 | 0.9111 | 1.0778 | 1.1667 | 0.8333 |
| $k \geq 100$<br>$\lambda_{\min} dB \leq -10$          | 1 <sup>st</sup> Stream | 0.8000 | 0.9737 | 0.4000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.6889 |
|   | 2 <sup>nd</sup> Stream | 0.8111 | 1.2456 | 0.4000 | 0.7479 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $k \geq 100$<br>$-10 < \lambda_{\min} dB \leq 8$      | 1 <sup>st</sup> Stream | 0.9736 | 0.9573 | 1.7303 | 0.8532 | 1.4895 | 0.8889 | 0.8889 | 0.7556 |
|   | 2 <sup>nd</sup> Stream | 2.6241 | 1.0222 | 0.4667 | 0.8310 | 0.6444 | 0.9445 | 1.0555 | 0.8445 |
| $k \geq 100$<br>$\lambda_{\min} dB > 8$               | 1 <sup>st</sup> Stream | 0.9000 | 1.0000 | 0.4500 | 1.0000 | 0.9000 | 0.9000 | 0.8889 | 0.7556 |
|   | 2 <sup>nd</sup> Stream | 0.9000 | 1.0000 | 0.4500 | 1.0000 | 0.9000 | 1.0000 | 1.0000 | 0.7667 |

- For vertical MIMO, combining SI of both stream is necessary
  - Another set of heuristic parameters ( $p_1, p_2$ ) are introduced with a provided table (again MCS-dependent, also condition number dependent, derived from link simulations and optimized with AWGN reference).

$$SI = p_1 \cdot SI_{stream1} + p_2 \cdot SI_{stream2}$$

|   |       | QPSK   | 16QAM  | 64QAM  |
|---|-------|--------|--------|--------|
|   |       | 1/2    | 1/2    | 1/2    |
| $k < 10$<br>$\lambda_{\min} dB \leq -10$              | $p_1$ | 0.5000 | 0.5000 | 0.5000 |
|   | $p_2$ | 0.5000 | 0.5000 | 0.5000 |
| $k < 10$<br>$-10 < \lambda_{\min} dB \leq 8$          | $p_1$ | 0.5445 | 0.6667 | 1.0000 |
|   | $p_2$ | 0.6556 | 0.4444 | 0.3333 |
| $k < 10$<br>$\lambda_{\min} dB > 8$                   | $p_1$ | 0.5000 | 0.5000 | 0.5667 |
|   | $p_2$ | 0.5000 | 0.5000 | 0.5223 |
| $10 \leq k < 100$<br>$\lambda_{\min} dB \leq -10$     | $p_1$ | 1.0000 | 0.5000 | 0.5000 |
|   | $p_2$ | 0.1111 | 0.5000 | 0.5000 |
| $10 \leq k < 100$<br>$-10 < \lambda_{\min} dB \leq 8$ | $p_1$ | 0.6667 | 0.1111 | 0.5333 |
|   | $p_2$ | 0.3333 | 0.8889 | 0.5111 |
| $10 \leq k < 100$<br>$\lambda_{\min} dB > 8$          | $p_1$ | 0.5000 | 0.5000 | 0.8889 |
|   | $p_2$ | 0.5000 | 0.5000 | 0.1111 |
| $k \geq 100$<br>$\lambda_{\min} dB \leq -10$          | $p_1$ | 0.2222 | 0.0000 | 0.5000 |
|   | $p_2$ | 0.7778 | 0.5556 | 0.5000 |
| $k \geq 100$<br>$-10 < \lambda_{\min} dB \leq 8$      | $p_1$ | 0.6667 | 0.4778 | 0.0000 |
|   | $p_2$ | 0.0000 | 0.6556 | 1.0000 |
| $k \geq 100$<br>$\lambda_{\min} dB > 8$               | $p_1$ | 0.5000 | 0.5000 | 0.5000 |
|   | $p_2$ | 0.5000 | 0.5000 | 0.5000 |

### III. RBIR for Some Typical Channels

In this section, a few simple channel cases are used for the purpose of sanity check, in an effort to analyzing the behavior of the two methods.

#### Comment 1: RBIR for diagonal channels

$$H = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$$

where x is any value. (assuming  $\sigma^2 = 1$  in all the examples for simplicity.)

The above channel must give approximately same value of MI for both streams. However, since RBIR tables have different values of ‘a’ for each stream, the above results in difference values of RBIR for the two streams

*Specific example:*

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

RBIR for QPSK-1/2 for the above channel is 0.627 for stream1 and 0.4322 for stream2. In the domain of effective SNR, such a different in RBIR maps to a large difference (**2.8 dB vs 0.25 dB**) in post-ML quality of the two stream, but we know that actually the post-ML quality of the two stream should be same.

#### Comment 2: RBIR Symmetry

Consider two channels  $\mathbf{H}_1 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ ,  $\mathbf{H}_2 = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$ , which are just column permutations of each other. The performance of both the above channels is expected to be same with vertical encoding

For almost all channels tested it is found that a channel and its column permutation give different values of RBIR for vertical encoding.

*Specific Example:*

$$H = \begin{bmatrix} 0.5955 + 2.0955i & -1.8932 + 0.4440i \\ -1.1083 + 4.5946i & -0.8652 - 0.5163i \end{bmatrix}$$

The RBIR (for 16QAM R-1/2) of the above randomly generated channel and its permutation are given by 0.61 and 0.58 respectively.

Note that the SI of both hypothetical streams are the same because of the same column norms. Given that the actual SI is the result of weighted sum of the two SIs, the only way that will results in same RBIR after column permutation is by setting  $p_1 = p_2$ , which is not the case in the provided table. A wild guess is that index 1 and 2 are tied to any properties of the matrix, but currently this is not clear at all.

### Comment 3: RBIR Properties

$$\mathbf{a)} \quad H = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, \quad \text{RBIR}(16\text{QAM-R1/2}) = \mathbf{0.64}, \quad \text{RBIR}(64\text{QAM-R1/2}) = \mathbf{0.71}$$

$$H = [ 2.4458 + 1.5065i \quad 0.4224 + 1.8827i \\ 0.9998 + 2.5964i \quad -3.0534 - 2.3936i ] \quad , \quad \text{RBIR}(16\text{QAM-R1/2}) = \mathbf{0.65}, \quad \text{RBIR}(64\text{QAM R1/2}) = \mathbf{0.73}$$

RBIR does not satisfy the basic property of for any bit-level information metric that it should *decrease with increase in modulation level* (Note: This is satisfied for SISO RBIR/MMIB and ML MMIB functions)

$$\mathbf{b)} \quad H = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, \quad \text{RBIR}_{ML}(H, 64\text{QAM-R1/2}) = \mathbf{0.55}, \quad \text{RBIR}_{SISO}(h=10, 64\text{QAM}) = \mathbf{0.94}$$

As shown above or the above diagonal channel it is expected that RBIR metric for ML is ‘approximately’ same as that of the SISO, since a diagonal channel is essentially two parallel SISO channels. However, the above result shows that they are very different.

**c) Note also in the above example, the channel in b) has higher SNR than that in a), and but the former gives a smaller RBIR metric for 64QAM-R1/2.** It is not correct intuitively.

**d)** Note further that the parameters p1 and p2 do not add up to one, which results in an RBIR value of greater than one in certain cases.

### Comment 4: Negative Values of SI and RBIR for high SNR

$$H = \begin{bmatrix} 25 & 5 \\ 5 & 25 \end{bmatrix}$$

For the above channel  $\text{RBIR}(\text{stream 1}) = \mathbf{0.166}$ ,  $\text{RBIR}(\text{stream2}) = \mathbf{-0.212}$  for 64QAM R-1/2

RBIR for vertical encoding is  $\mathbf{-0.0166}$ , which is too small and negative, and RBIR for stream 2 is also negative. An information metric cannot be negative and it is also expected that bit-information rate must be much larger at these SNRs.

Note that the SNR for the above channel (defined by column norm is around 28 dB). This behavior is observed for typical channels at high SNRs.

One way to explain this is by looking at the approximation in Eq (43) of the EVM document. The second term in SI expression is dominated by  $f1(\text{AVE-sqrt}(\text{VAR}))$ , which is large for large values of VAR, values which are obtained well within operating SNR regions. We have also noted that substituting the approximation with the actual numerical integration results in the similar inconsistency.

### Comment 5: RBIR for different code rates – Applicability to IR/HARQ

According to the current development of RBIR for ML receivers, different values of RBIR are obtained for different code rates, for the same modulation (due to code-rate dependent heuristic adjustment parameters sets).

This is different from ‘RBIR/MMIB for SISO and MMIB for ML receivers’, which have a unique value for a given modulation, i.e., code rate independent. Hence, the development for IR in section 4.6.3 Eqs 92-95, which is based on this assumption for a MI metric no longer applies to RBIR as defined for ML.

Since RBIR is dependent on code rate (similar to effective SNR in EESM), new RBIR tables would be required to cover the new possible code rates with IR.

### **Comment 6: Approximation used in RBIR integral**

The approximation in (43) is suggested for avoiding computation of RBIR function in real-time. We have seen that this approximation works approximately well when  $AVE/\sqrt{VAR} < 0.5$  (empirical observation)

- 1)  $AVE = 5, VAR = 40$  (Obtained for  $H = \text{diag}[5 \ 5]$ , 16QAM-R  $\frac{1}{2}$ )  
Num. Integral = 0.69, Num. Approx = 0.64
- 2)  $AVE = 5, VAR = 20$   
Num. Integral = 0.86, Num. Approx = 0.83

From the Tables 25 and 26, it can be verified that situations in case (a) arise often. It would be good to clarify under what conditions this approximation is good, and if there are alternate approximations that can be used in general.

### **Comment 7: Search for Optimal Adjustment Parameters for RBIR**

In the current version of EVM, Appendix Q provides an approach to derive adjustment parameters for each MCS. There are a total of 18 'a' parameters and 18 'p' parameters (defined for each stream and partition) to be obtained for *each MCS*.

How is the optimization in Q2 Step 3 performed? Does it involve a simultaneous search over 9 parameters and does expression in Q.3 Step 4 require a search over 18 dimensions simultaneously to minimize the expression?

The current text says '*for all H which belongs to a particular range of k and lambda*'. So, if this implies obtaining parameters (a,p) for a partition by selecting H in that partition, what does H mean here? It does not make sense, since the data here is effective SNRs which are computed for a given fading channel realization (described by many different H on different subcarriers, which can belong to multiple partitions).

Further detail needs to be provided in this section.



## IV. Comparison Table

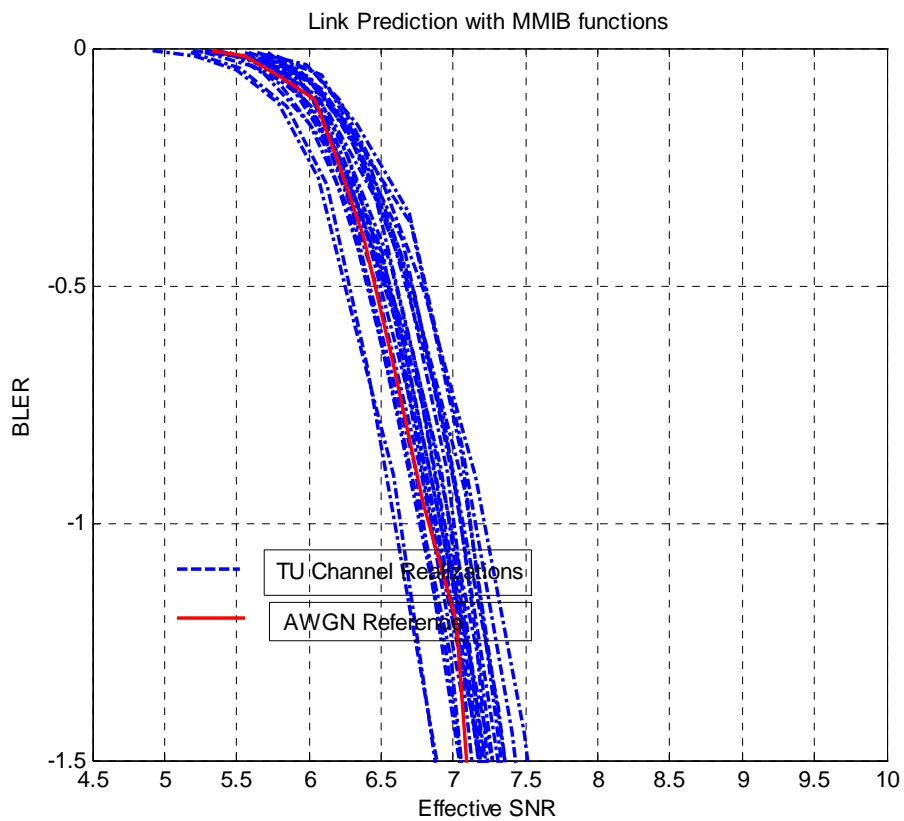
|  | MMIB   | RBIR  | Comment  |
|--|--|---|--|
| 1) SISO Systems and MIMO Systems with Linear Receivers | A simple SNR to MMIB mapping is used which is defined by a function unique to each modulation (or a lookup table can also be used)   | A simple SNR to RBIR mapping is used which is defined by a lookup table unique to each modulation   | These SNR to MMIB/RBIR mapping are found to be close and hence no difference is expected in these simulations. <i>The rest of the comparison deals with ML receivers</i>   |
| 2) Physical Interpretation and HARQ                    | MMIB functions correspond to the true observed bit mutual information of the matrix channel.<br>They satisfy the properties of bit mutual information a) Fall in the range [0.0 1.0] b) Decreasing with increase in modulation order   | It was seen that RBIR does not meet these constraints. See Sections above.  | It cannot be claimed that the RBIR metrics can be combined across modulations etc., since RBIR metric generated does not meet the requirements of mutual information. In that sense, does not conform to the definitions used in previous studies and the remaining sections of EVM. |
| 3) Channel Dependency                                  | MMIB functions are generated by<br>a) Numerically obtaining (by <i>Monte-Carlo simulation and numerical integration of LLR PDFs obtained from a Gaussian matrix channel and a constellation mapping model</i> ), the theoretical true bit-level MI of a matrix channel<br>b) Approximating this function with numerical approximations<br>c) NO AWGN or link simulations are used at any point, as these functions are independent of the underlying system<br>(Note: Compare to SISO MI functions, which are defined for a SISO Gaussian channel defined by an SNR) | RBIR functions are obtained by a)<br>Performing Link Simulations<br>b) Optimizing parameters $a_i$ and $p_i$ for each MCS<br>c) A different value of RBIR is obtained for each code rate, even though the modulation is fixed. Does not conform to the definition of information metric | For RBIR, the parameter tables must be optimized with link simulations and synchronized between companies. The primary objective of using MI metrics is to avoid parameter tables which must be updated with link simulations  |
| 3) Heuristic Adjustment for codes                      | MMIB functions are independent of system parameters, code rates etc.,  | Same as above   | Same as above  |

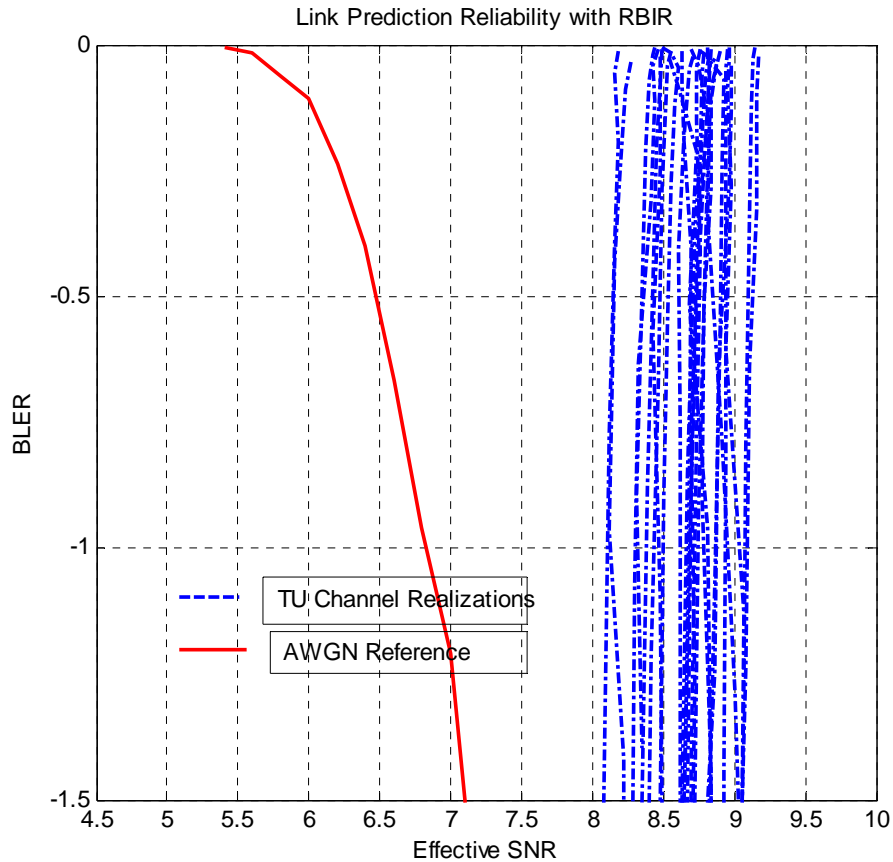
|  |  |  |  |
|--|--|--|--|
|  | For characterizing performance with new binary codes or additional code rates, only AWGN simulations are required to generate AWGN reference curve |  |  |
|--|--|--|--|

## V. Numerical Results

The numerical results are generated to check the reliability of performance prediction. 25 different channel realizations are simulated. For each realization, mean SNR is swept so that the entire BLER range is obtained. PB3, IID MIMO channel model is used. MCS used is 16QAM Rate  $\frac{1}{2}$  with PUSC.

The red curve in the figure represents AWGN reference, where x-axis is the AWGN SNR and the y-axis is the BLER in AWGN simulation condition. The blue curves are obtained from a fading simulation. For each BLER point, the MI of the corresponding channel realization is computed. This MI is mapped to an effective SNR using the SISO MI to SNR relationship of the corresponding modulation. Same set of data from link results are used in both figures below.





## VI. Conclusions

RBIR modeling for ML requires four “fudge” parameters ( $a_1, a_2, p_1, p_2$ ) by link simulations for each MCS level. Our numerical results with RBIR for ML do not show good prediction reliability. Multiple simple examples also suggest that the RBIR approach as defined cannot pass even simple sanity check or basic properties as expected from a mutual information metric. Furthermore, due to dependency on heuristic adjustment required for different MCS levels, it is not clear that this method can be applied to model post-IR performance since IR can be deemed as generating an additional code rates after combing.

On the other hand, MMIB functions for ML are capacity measures defined for the underlying bit channel induced by a modulation constellation and the matrix channel. Because the framework for MMIB is well-grounded based on information theory, much like their SISO counterparts (both MMIB, which is defined as bit channel capacity for SISO for and RBIR defined as normalized symbol capacity for SISO), MMIB method does not require any heuristic parameters that depends on MCS and is also independent of the actual code, system parameters etc. In addition they satisfy all the properties of a bit-level mutual information metric like the SISO MI metrics and not surprisingly show good prediction for ML.

Based on this study, it is suggested to adopt the MMIB as the baseline link to system mapping methodology, especially for ML-MIMO receiver modeling.

## References

- [1] Krishna Sayana, Jeff Zhuang, Ken Stewart, IEEE C80216m-07/097, "Link Performance Abstraction based on Mean Mutual Information per Bit (MMIB) of the LLR Channel", IEEE 802.16 TGm contribution, May 04, 2007.
- [2] Krishna Sayana, Jeff Zhuang, Ken Stewart, IEEE C80216m-07/0142, "Link Performance Abstraction for ML Receivers based on Mean Mutual Information per Bit (MMIB) Metrics", IEEE 802.16 TGm contribution, July 10, 2007.
- [3] IEEE 802.16m Evaluation Methodology 037r2.
- [4] Hongmin Zheng et.al, IEEE C802.16m-07/187, "Link Performance Abstraction for ML Receivers based on RBIR Metrics", IEEE 802.16 TGm contribution, Sept 10, 2007.