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Source	Kim Olszewski ZTE USA, Inc. 10105 Pacific Heights Blvd, Suite 250 San Diego, CA 92121 E-mail: kolszewski@zteusa.com
Re:	802.16m-08/033 - Call for Comments and Contributions on 802.16m System Description Document (SDD). Topic: PHY Text
Abstract	This contribution proposes sequences to be used for the Synchronization Channel (SCH). The sequences can be used for either a hierarchical or non-hierarchical SCH structure.
Purpose	To review and adopt the proposed text in the next revision of the SDD.
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CAZAC Sequence Codebooks for the 802.16m Synchronization Channel

Kim Olszewski
ZTE USA, Inc.

1 Introduction

The synchronization channel (SCH) is a DL physical channel used by MSs in order to synchronize with the network. It may consist of two subchannels, the P-SCH (Primary-SCH) and S-SCH (Secondary-SCH). An MS can process SCH symbols to obtain frame timing, OFDMA symbol timing, cell identifiers and other information. In choosing sequences for SCH symbols various constraints and requirements must be taken into account. For example, the following:

- SCH sequences should be as short as possible to reduce overhead.
- SCH sequences should have flat power spectrums.
- The PAPR of SCH sequences should be small to avoid clipping due to transmitter nonlinearities and to allow for maximal possible transmit power.
- SCH sequences should be robust to multipath transmission and multiple-user communications.
- SCH sequences be suitable for fast AGC adjustment which calls for small sequence amplitude variations.
- SCH sequences should have low correlation side lobes and high correlation peaks.
- For cell identification SCH sequences should serve as codewords that encode cell/sector identifiers.

Some typical sequences used for SCH symbols are PN, Walsh-Hadamard and Golay sequences. However, the perfect auto-correlation property of these sequences is extremely sensitive to Doppler shifts. CAZAC (constant amplitude zero autocorrelation) sequences provide another option. There are different constructions of CAZAC sequences resulting in different behavior with respect to Doppler and additive noise/interference.

In this contribution we describe some candidate quadratic-phase and quadratic-residue CAZAC sequences that may be used for SCH symbols. Metrics are then defined to evaluate the different candidate CAZACs. We describe how SCH codebooks may be constructed using orthogonal quadratic-phase CAZAC sequences. Codewords in the SCH codebooks correspond one-to-one with cell/sector identifiers. We describe how cell identification may be implemented using the SCH codebooks. The contribution ends with proposed text for the SCH section of the SDD draft.

2 Candidate CAZAC Sequences for the SCH

Let $\mathbb{Z}$ denote the set of integers (positive, negative or zero) and $\mathbb{Z}_K = \{0, 1, ..., K-1\}$ be the additive group of integers $\mathbb{Z}$ modulo K. A Constant Amplitude Zero Autocorrelation (CAZAC) is a K-periodic sequence $\{c[k]\}_{k=0}^{K-1}$ sequence with the following properties:

- **Constant Amplitude (CA)**: For all $k \in \mathbb{Z}_K$ the sequence's magnitude is $|c[k]| = 1$.
- **Zero Autocorrelation (ZAC)**: For all time delays $m \geq 0$ the sequence's periodic autocorrelation is

$$R_c[m] = R_c[-m] = \frac{1}{K}\sum_{k=0}^{K-1} c^*[k]c[(k+m) \bmod K] = \begin{cases} 1 & \text{if } m \bmod K = 0 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

CAZAC sequences are important SCH symbol candidates because of their defining properties: CA ensures optimal transmission efficiency. CA allows the transmission of peak power throughout the duration of an SCH symbol. This allows more power to be transmitted thereby increasing received SINR. ZAC provides tight time localization. Sharp cross-correlation peaks obviate distortion and interference in the received waveform.

If $\{c[k]\}_{k=0}^{K-1}$ is a CAZAC sequence then $\{c[k]\}_{k=0}^{K-1}$ has the following properties:

Property 1: The complex-conjugated sequence $\{c^*[k]\}_{k=0}^{K-1}$ is also a CAZAC sequence.

Property 2: For any integer m the time-shifted sequence $\{c[k+m]\}_{k=0}^{K-1}$ is also a CAZAC sequence.

Property 3: For any complex number $\kappa \in \mathbb{C}$ the sequence $\{\kappa c[k]\}_{k=0}^{K-1}$ is also a CAZAC sequence.

Property 4: The discrete Fourier transform of $\{c[k]\}_{k=0}^{K-1}$ is also a CAZAC sequence. A CAZAC sequence is a full bandwidth sequence with unity power spectrum.

Property 5: For any nth root of unity W_n and any integer m the cyclically shifted sequence $\{c[k]W_n^m\}_{k=0}^{K-1}$ is also a CAZAC sequence.

There are different types of CAZAC sequences of any given length K. The different types may be useful for different applications. The different types result in different behavior with respect to Doppler and additive noise and interference.

The different types of CAZAC sequences can be categorized into two distinct categories: quadratic-phase CAZAC sequences and quadratic-residue CAZAC sequences. Quadratic-phase CAZAC sequences are linearly swept frequency sequences. Quadratic residue CAZACs are small alphabet CAZACs since elements can be of at most three distinct values.

A quadratic-phase CAZAC sequence has elements in the form $c[k] = e^{j\frac{2\pi a}{K}P(k)}$ where $P(k)$ is a quadratic polynomial. A length K quadratic-phase CAZAC sequence $\{c[k]\}_{k=0}^{K-1}$ for $k \in \mathbb{Z}_K$ can be parametrized by writing its elements as

$$c[k] = e^{j\frac{2\pi a}{K}P(k)} = \begin{cases} e^{j\frac{2\pi a}{K}\left(\frac{k^2}{2}+bk\right)} & \text{if } K \text{ is even} \\ e^{j\frac{2\pi a}{K}\left(\frac{k^2}{2}+[2b+1]\frac{k}{2}\right)} & \text{if } K \text{ is odd} \end{cases} \tag{2}$$

Parameters a and b are integers in $\mathbb{Z}$; a and K are relatively prime meaning they have no common factor other than 1. Hence, if K is a prime number a set of $K-1$ sequences may be defined in terms of b. We use this property below to construct SCH codebooks.

Quadratic-residue sequences of prime length K are defined in terms of a Legendre symbol. Note that K must be an odd-number. For integers $k \in \mathbb{Z}_K$ and positive odd primes K the Legendre symbol is defined as

$$\left(\frac{k}{K}\right) = \begin{cases} 0 & \text{if } k \bmod K = 0 \\ +1 & \text{if } k \text{ equals a squared integer } (\text{mod } K) \\ -1 & \text{if } k \text{ does not equal a squared integer } (\text{mod } K) \end{cases} \tag{3}$$

Elements within a quadratic-residue sequence are computed from

$$c[k] = e^{j2\pi\theta(k,K)} \tag{4}$$

where

$$\theta(k,K) = \begin{cases} \arccos\left(\frac{1}{1+\sqrt{K}}\right)\left(\frac{k}{K}\right) & \text{if } K = 1 mod 4 \\ \frac{1}{2}\arccos\left(\frac{1-K}{1+K}\right)\left[(1-\delta_k)\left(\frac{k}{K}\right)+\delta_k\right] & \text{if } K = -1 mod 4 \end{cases} \tag{5}$$

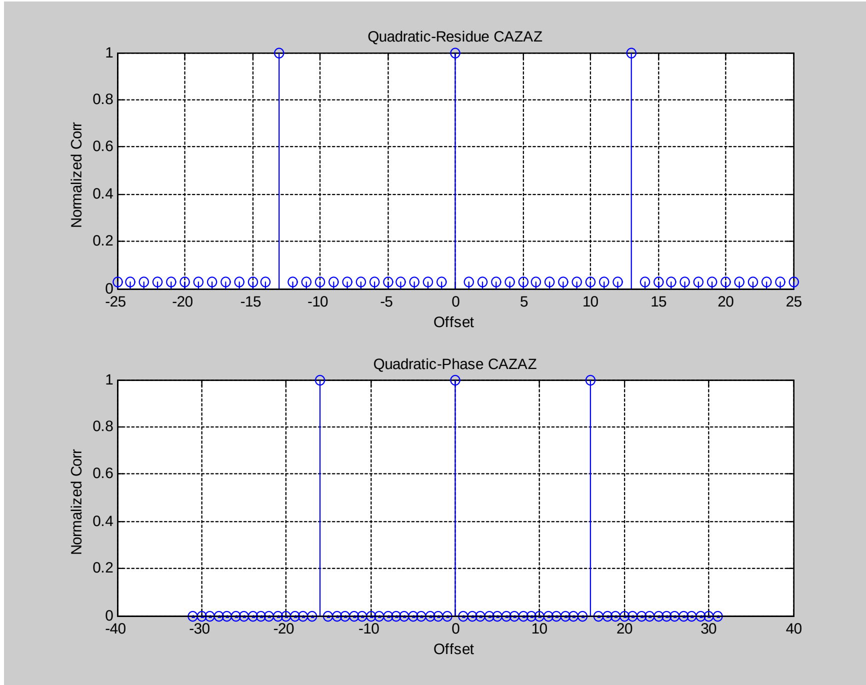

Figure 1: Example periodic auto-correlation function plots for a length-13 quadratic-residue sequence and a length-16 quadratic phase sequences. Not the increased sidelobes in the quadratic-residue CAZAC.

3 Functions for Comparing CAZAC Sequences

3.1 PSL and ISL Functions

Two important metrics for sequence performance analysis are the peak sidelobe level (PSL) function and the integrated sidelobe level (ISL) function. The PSL function of a sequence $\{c[k]\}_{k=0}^{K-1}$ is defined as

$$PSL_{dB} = 10\log_{10}\left[\frac{\max_{1\leq m\leq K-1}|\gamma_c[m]|^2}{|\gamma_c[0]|^2}\right] \quad (6)$$

and the ISL function as

$$ISL_{dB} = 10\log_{10}\left(\frac{\sum_{m=1}^{M-1}|\gamma_c[m]|^2}{|\gamma_c[0]|^2}\right) \quad (7)$$

where

$$\gamma_c[m] = \frac{1}{K}\sum_{k=0}^{K-m-1} c[k+m]c^*[k] \quad (8)$$

is defined as the aperiodic autocorrelation. The PSL function provides a measure of the largest sidelobe as compared with the peak, the ISL function provides a measure of the total power in the correlation sidelobes as compared with the peak. Note that PSL and ISL are also beneficial in determining the minimal sequence lengths to use.

Quadratic-phase CAZACs have good PSLs and ISLs. In terms of PSL and ISL quadratic-residue CAZACs are not as desirable as quadratic-phase CAZACs.

3.2 The Ambiguity Function

In signal processing an ambiguity function is a two-dimensional function of time delay and Doppler frequency. The periodic (discrete) ambiguity function of sequence $\{c[k]\}_{k=0}^{K-1}$ is defined as

$$A_c[m,n] = \frac{1}{K}\sum_{k=0}^{K-1} c^*[k]c[(k+m) \bmod K]e^{j\frac{2\pi nk}{K}} \tag{9}$$

Here integer m is associated with time delay and integer n with a frequency or Doppler shift.

The ambiguity function provides a concise way of analyzing matched filter or cross correlator operations on a received sequence. For a given constant frequency $N = n$ the ambiguity function cut $A_c[m,N]$ display's a matched filter's output as a function of time m. It is a time-delay cut with constant frequency N. Similarly, for a given constant time delay $m = M$ the ambiguity function cut $A_c[M,n]$ display's a matched filter's output as a function of Doppler frequency shift n. It is a delay cut with constant time M. Matched filter or cross correlation operations can be evaluated at regularly-spaced intervals in the delay-Doppler space of an ambiguity function plot.

As an example, consider a pseudorandom m-sequence. The ambiguity function $A_c[m,n]$ produced by a pseudorandom m-sequence is a 2-dimensional Dirac delta function. Both the zero-delay cut $A_c[0,n]$ and zero-Doppler cut $A_c[m,0]$ are unit impulses. Hence any Doppler shift in the transmitted sequence would make the correlation function theoretically disappear to zero. Clearly, this is not desirable if a MS has unknown velocity or Doppler shift since its correlation function will disappear without receiver adjustment or Doppler cancellation.

However, if knowledge of the precise Doppler frequency n is given detection can be accomplished easily without interference. Any other MS which is not moving at exactly the same velocity will produce a zero cross correlation value. Hence the computed magnitude $|A_c[m,n]|$ may be used for detecting Doppler frequency shifts and therefore MS speeds which can be derived from Doppler estimates.

For example let $\tilde{u}_n[k] = c[k]e^{-j\frac{2\pi nk}{K}}$ denote a sample from a received Doppler shifted CAZAC sequence. The periodic ambiguity function and periodic cross correlation are related as follows:

$$A_c[m,n] = R_{c,\tilde{u}_n}[m] = \frac{1}{K}\sum_{k=0}^{K-1} c^*[k]\tilde{u}_n[(k+m) \bmod K] = \frac{1}{K}\sum_{k=0}^{K-1} c^*[k]c[(k+m) \bmod K]e^{j\frac{2\pi n(k+m) \bmod K}{K}} \tag{10}$$

Hence the computed magnitude $|R_{c,\tilde{u}_n}[m]|$ may be used for detecting Doppler frequency shifts and therefore MS speeds.

Different CAZAC sequences exhibit different behavior in their ambiguity plots. The ambiguity function reveals localization properties of different CAZAC sequences. Quadratic-residue CAZAC sequences have good Doppler resolution and time-delay resolution capability (see Figure 3) but have high correlation sidelobes. On the other hand, quadratic-phase CAZAC sequences exhibit strong delay-Doppler coupling (see Figure 2) but have low correlation sidelobes.

4 The Sensitivity of CAZACS to Doppler Frequency Shifts

The perfect cross-correlation of Golay sequences is extremely sensitive to Doppler shifts. The shape of the ambiguity function plot is ideal along the zero-Doppler axis but off the zero-Doppler axis it has large sidelobes.

Sequences that can be detected in the presence of various Doppler frequency shifts belong to a class called Doppler tolerant sequences. Quadratic-phase CAZACs are examples of Doppler tolerant sequences. Quadratic-phase CAZACs have good PSLs and ISLs but their ambiguity functions exhibit strong delay-Doppler coupling, see Figure 2.

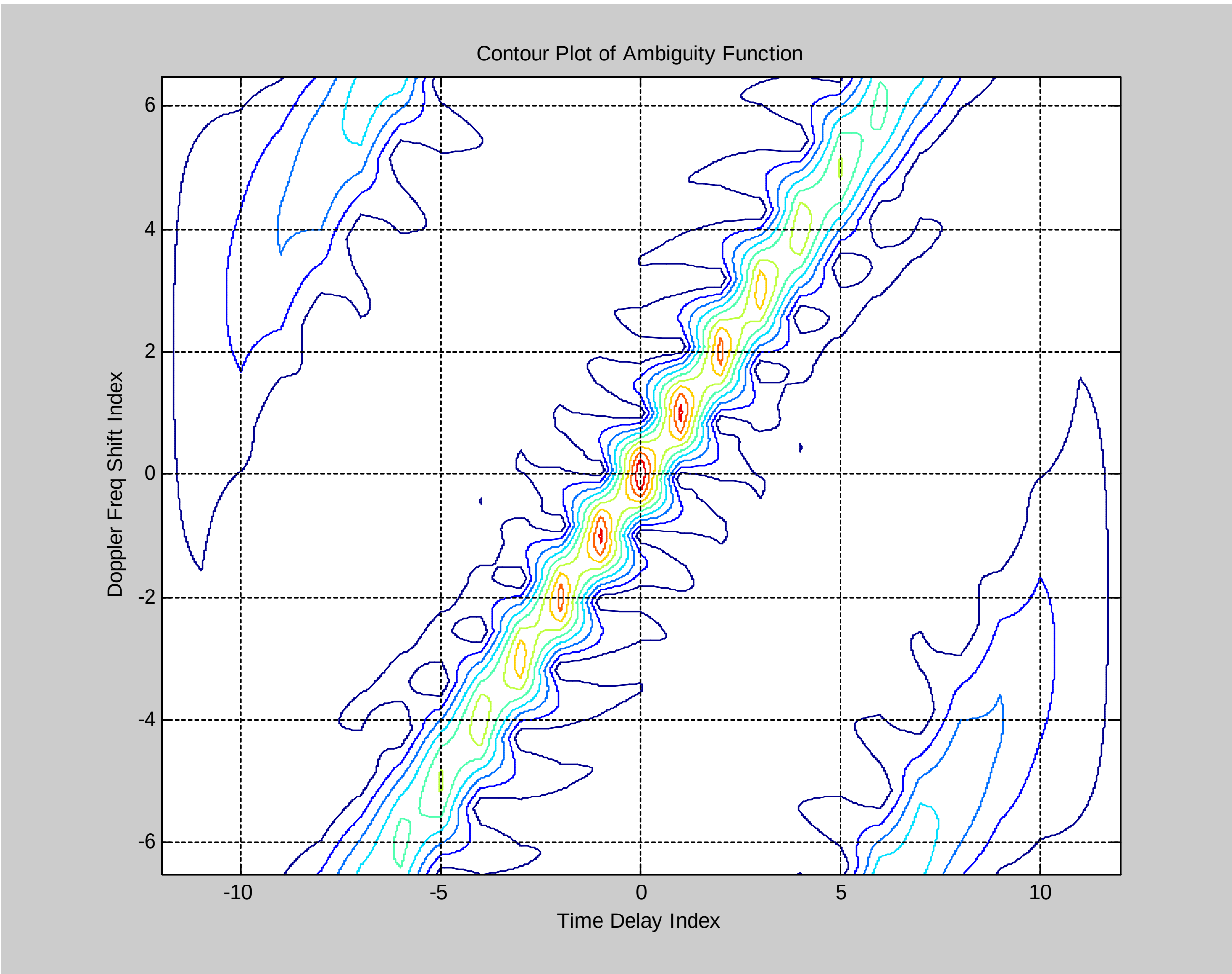

Figure 2: Eaxmple contour plot of the ambiguity function of quadratic-phase CAZAC.

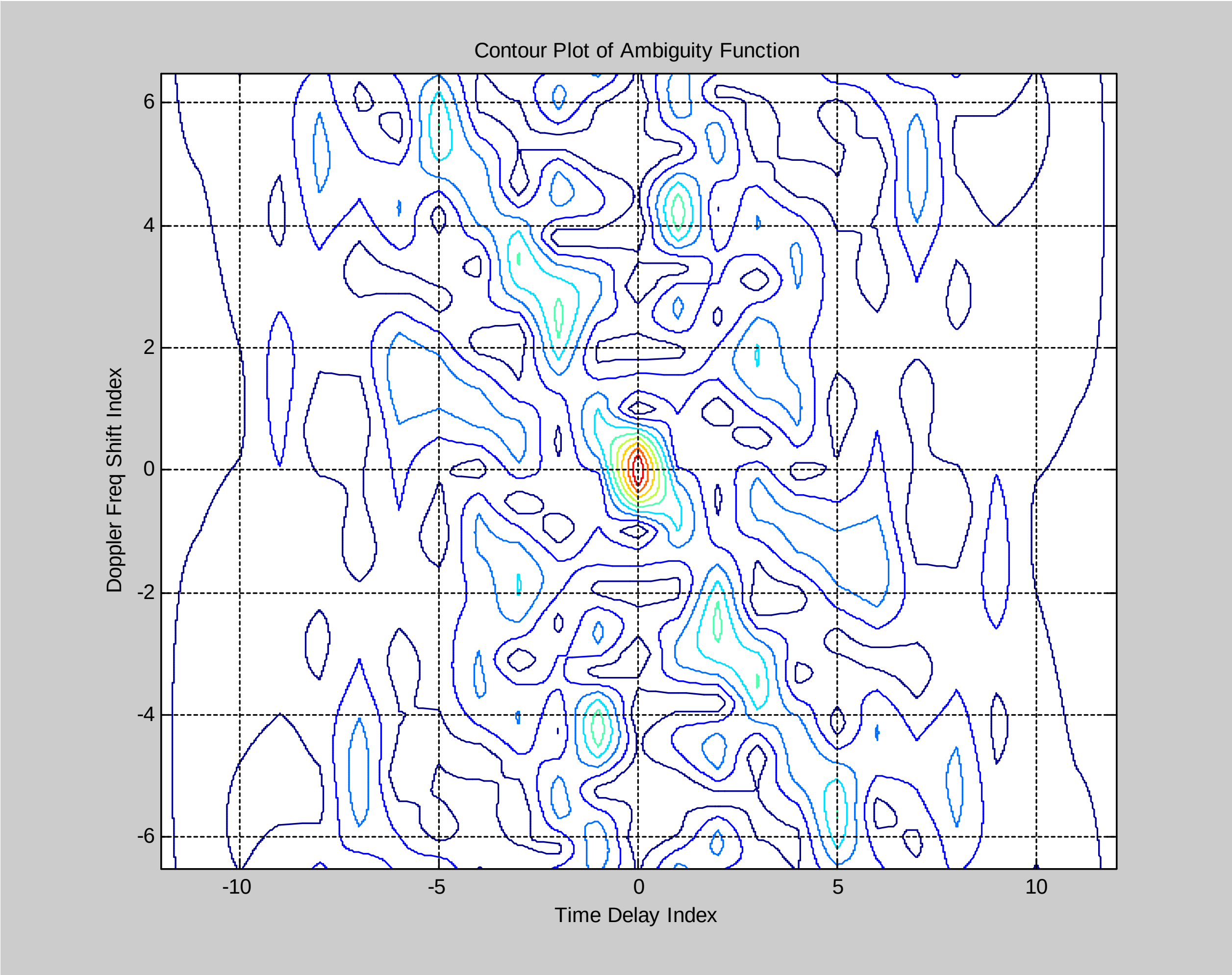

Figure 3: Example contour plot of the ambiguity function of quadratic-residue CAZAC.

If a sequence is Doppler tolerant the output of a cross-correlator will not be significantly degraded when the received sequence is Doppler frequency shifted. However, the frequency shift translates into a time shift of the correlator output, a decrease in the output amplitude, and an increase in the sidelobes. Therefore, the cost for improved behavior under a frequency shift is decreased time delay accuracy. However, these degradations are usually not significant. Therefore, even if an MS signal processor is not matched to Doppler its output will have a peak that can be detected.

In contrast, when using a quadratic-residue sequence we would need to compensate for Doppler shifts or have a bank of cross-correlators matched to the range of expected MS Doppler frequencies. This is because the ambiguity function of a quadratic-residue sequence has a peak only at the zero Doppler cut.

5 CAZAC Sequence Codebooks for the Synchronization Channel

Based on the above described CAZAC sequence characteristics and plots we propose the use of quadratic-phase CAZAC sequences for the SCH. We now describe how SCH codebooks can be defined by exploiting the properties of these sequences.

As shown above a length K quadratic-phase CAZAC sequence $\{c[k]\}_{k=0}^{K-1}$ for $k \in \mathbb{Z}_K$ can be defined in terms of parameters a and b. We set $a = 1$ and for K odd we define the sequences

$$\begin{aligned} C_1 &= \left\{e^{j\frac{\pi}{K}k(k-1)}\right\}_{k=0}^{K-1} && \text{if } b=-1 \\ C_2 &= \left\{e^{j\frac{\pi}{K}k(k-3)}\right\}_{k=0}^{K-1} && \text{if } b=-2 \\ &\vdots && \vdots \\ C_K &= \left\{e^{j\frac{\pi}{K}k(k-2K+1)}\right\}_{k=0}^{K-1} && \text{if } b=-K \end{aligned} \tag{11}$$

Similarly, for K even we define the sequences

$$\begin{aligned} C_1 &= \left\{e^{j\frac{\pi}{K}k(k-2)}\right\}_{k=0}^{K-1} && \text{if } b=-1 \\ C_2 &= \left\{e^{j\frac{\pi}{K}k(k-4)}\right\}_{k=0}^{K-1} && \text{if } b=-2 \\ &\vdots && \vdots \\ C_K &= \left\{e^{j\frac{\pi}{K}k(k-2K)}\right\}_{k=0}^{K-1} && \text{if } b=-K \end{aligned} \tag{12}$$

Using these sequences we construct the CAZAC sequence codebook

$$\mathcal{C} = \left\{C_1 = \{c_1[k]\}_{k=0}^{K-1}, \ldots, C_M = \{c_M[k]\}_{k=0}^{K-1}\right\} \tag{13}$$

where $M \leq K$ and

$$c_i[k] = \begin{cases} e^{j\frac{\pi}{K}k(k-2i)} & \text{if } K \text{ is even} \\ e^{j\frac{\pi}{K}k(k-2i+1)} & \text{if } K \text{ is odd} \end{cases} \tag{14}$$

Each codeword in C_i in $\mathcal{C}$ is a unique quadratic-phase CAZAC sequence. In addition, as shown in the next section the CAZAC sequence codewords are orthogonal. Note that other codebooks can be constructed by changing the value of parameter a, a and K must be relatively prime.

6 Some Properties of CAZAC Sequence Codebooks

Let K be even and

$$c_{l_i}[k] = e^{j\frac{\pi}{K}k(k-2l_i)} = e^{j\frac{\pi}{K}\left(k^2-2l_ik\right)} \tag{15}$$

For any l_i and l_j in $\{0, 1, 2, \ldots, M\}$ the periodic cross-correlation between $\{c_{l_i}[k]\}_{k=0}^{K-1}$ and $\left\{c_{l_j}[k]\right\}_{k=0}^{K-1}$ is

$$\begin{aligned}
R_{c_{l_i}c_{l_j}}[m] &= \frac{1}{K}\sum_{k=0}^{K-1} c_{l_i}[k]\left(c_{l_j}[(k+m) \bmod K]\right)^* &(16)\\
&= \frac{1}{K}\sum_{k=0}^{K-m-1} c_{l_i}[k]\left(c_{l_j}[k+m]\right)^* + \frac{1}{K}\sum_{k=K-m}^{K-1} c_{l_i}[k]\left(c_{l_j}[k+m-K]\right)^* &(17)\\
&= \frac{1}{K}\sum_{k=0}^{K-1} e^{-j\frac{\pi}{K}\left(k^2-2kl_i\right)}e^{j\frac{\pi}{K}\left((k+m)^2-2(k+m)l_j\right)} &(18)\\
&= e^{j\frac{\pi m}{K}(m-2l_j)}\left[\frac{1}{K}\sum_{k=0}^{K-1} e^{j\frac{2\pi k}{K}(m+l_i-l_j)}\right] &(19)
\end{aligned}$$

For the summation we have

$$\frac{1}{K}\sum_{k=0}^{K-1} e^{j\frac{2\pi k}{K}(m+l_i-l_j)} = \begin{cases} 1 & \text{if } (m+|l_i-l_j|) \bmod K = 0 \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

Hence we have

$$R_{c_{l_i}c_{l_j}}[m] = \begin{cases} e^{j\frac{\pi m}{K}(m-2l_j)} & \text{if } (m+|l_i-l_j|) \bmod K = 0 \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

and

$$\left|R_{c_{l_i}c_{l_j}}[m]\right| = \begin{cases} 1 & \text{if } (m+|l_i-l_j|) \bmod K = 0 \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

A sequence set C_{CS} is categorized as being orthogonal if for any $m \geq 0$ and $i < M$ we have

$$\left|R_{c_{l_i}c_{l_j}}[m]\right| = \begin{cases} 1 & \text{if } l_i = l_j \text{ and } m = 0 \\ 0 & \text{if } l_i \neq l_j \text{ and } m = 0 \end{cases} \tag{23}$$

A sequence set C_{CS} is categorized as being Z-orthogonal or a zero-correlation zone (ZCZ) sequence set of size Z if for any $m \geq 0$ and $i < M$ we have

$$\left|R_{c_{l_i}c_{l_j}}[m]\right| = \begin{cases} 1 & \text{if } l_i = l_j \text{ and } m = 0 \\ 0 & \text{if } l_i \neq l_j \text{ and } m = 0 \\ 0 & \text{if } l_i \neq l_2 \text{ and } 1 \leq |m| \leq Z \end{cases} \tag{24}$$

where $Z \leq N/M - 1$. Hence, for any C_i and C_j $(i \neq j)$ in C_{CS} we have

$$\begin{aligned}
\left|R_{c_{l_i}c_{l_i}}[m]\right| &= 0 \text{ if } 1 \leq |m| \leq Z &(25)\\
\left|R_{c_{l_i}c_{l_j}}[m]\right| &= 0 \text{ if } 0 \leq |m| \leq Z &(26)
\end{aligned}$$

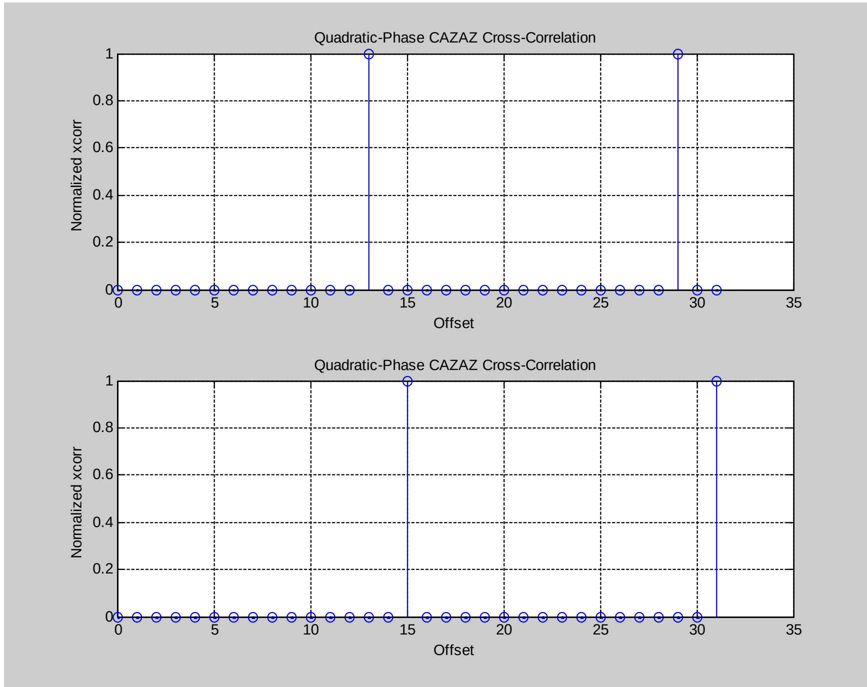

Figure 4: Example periodic cross-correlations for quadratic-phase sequences.

7 Cyclically Shifted CAZAC Sequence Codebooks

Let $\Psi = \{\psi_1, \psi_2, \ldots, \psi_M\}$ denote a set of positive integers where for all $i, j \in \{1, 2, \ldots, M\}$ we have $\psi_i \neq \psi_j$ and $0 \leq \psi_i < K - 1$. Using these values and $c_i[k]$ as defined above we define the cyclically shifted codebook

$$\mathcal{S} = \{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{C}_M\} \tag{27}$$

where $M \leq K$ and

$$\mathcal{S}_i = \left\{ C_{i1} = \{c_i[k + \psi_1 \bmod K]\}_{k=0}^{K-1}, \ldots, C_{iM} = \{c_i[k + \psi_M \bmod K]\}_{k=0}^{K-1} \right\} \tag{28}$$

Hence by Property 5 each $\mathcal{S}_i$ in $\mathcal{S}$ is a CAZAC sequence cyclically shifted by ψ_i samples. The cyclically shifted CAZAC sequence codewords are orthogonal.

8 Cell Identification

Figure 5 shows how a SCH symbols may be generated using a cell-specific codebook. The outputs of the DL-MIMO precoder are mapped by a Subcarrier Mapper to an allocated set of P-SCH or S-SCH subcarriers. In the frequency domain, each SCH symbol may be mapped to contiguous or non-contiguous equally-spaced subcarriers that may comprise a subcarrier set. The specific mapping is for future study. Depending on the allocated bandwidth the length of the IFFT may be different. The P-SCH may use the same set of contiguous or non-contiguous subcarriers for all possible 802.16m bandwidths. For example, SCH symbols may be mapped to the top portion or subcarriers in all allowed 802.16m channel bandwidths. To enable repetition encoding and improved synchronization SCH symbols may be transmitted several times during one transmit time interval. The SCH symbols may be transmitted at equal or unequal intervals. The number of symbols is a design parameter whose value is for future study.

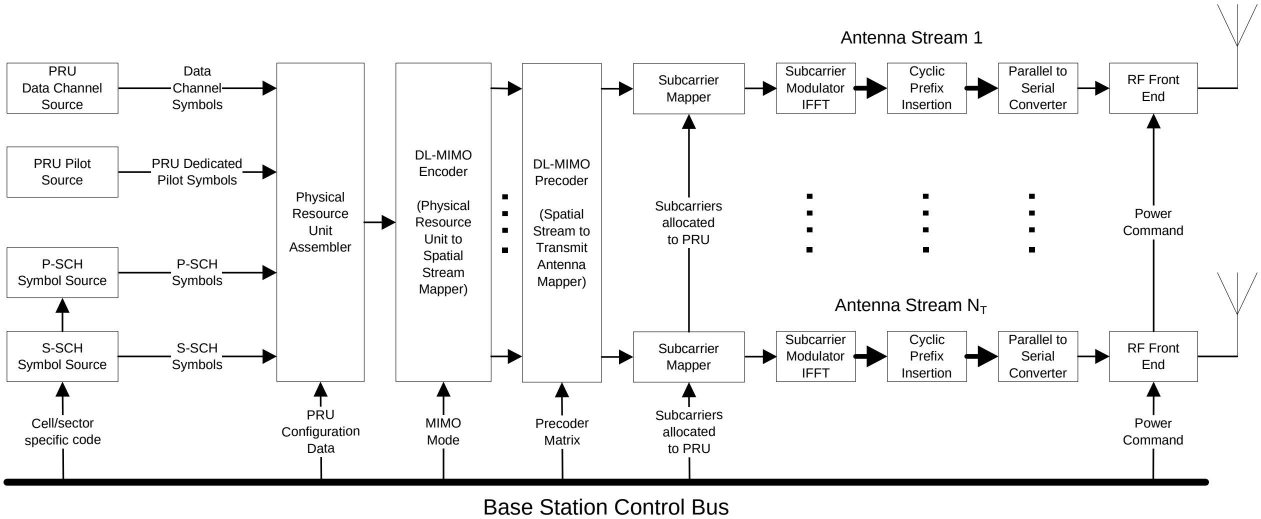

Figure 5: Conceptual block diagram showing base station SCH construction and signal processing.

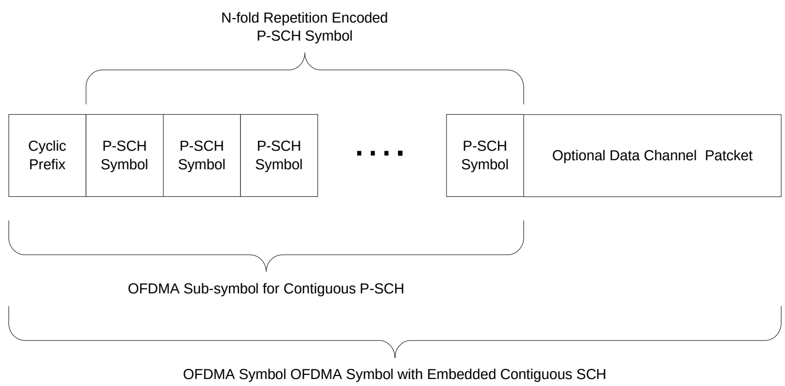

Figure 6: Example showing locations of N identical P-SCH or preamble symbols within an OFDMA symbol. Preamble or P-SCH should contain at least 4 repeated (6 is desirable) CAZAC sequence symbols for noise/interference averaging and rapid gain acquisition.

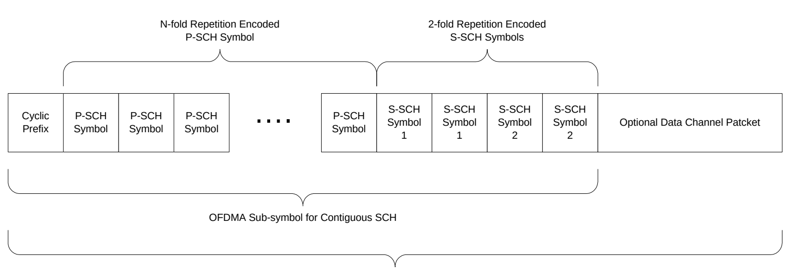

Figure 7: Example showing locations of N identical P-SCH symbols and two repeated S-SCH symbols within an OFDMA symbol

8.1 Cell Identification via the P-SCH

If only a primary SCH is used a single-stage detection procedure may be used for cell/sector identification. Let

$$\mathcal{C} = \{C_1, C_2, \ldots, C_M\} \quad (29)$$

denote an SCH codebook of length $M \leq K$. As a function of $\mathcal{C}$ the cell/sector identifier for the pth cell/sector may be defined as

$$Cell\_ID_p(\mathcal{C}) = f(C_1, C_2, ..., C_M) \quad (30)$$

$$p \in \{1, 2, \ldots, N_{Cells}\} \quad (31)$$

The one-to-one function f may be a simple look-up table that maps the indices of detected codewords $C_1, C_2, \ldots, C_M$ to a unique cell identifier. From combinatorics the maximum number of cells/sectors N_{Cells} that can supported by the codebook $\mathcal{C}$ is defined as

$$N_{Cells} = M^M \quad (32)$$

Note that N_{Cells} equals the number of permutations (with repetition) of the codewords in $\mathcal{C}$. For example, if $M = 2$ the following four unique cell/sector IDs are possible

$$Cell\_ID_1 = f(C_1, C_1) \quad (33)$$

$$Cell\_ID_2 = f(C_1, C_2) \quad (34)$$

$$Cell\_ID_3 = f(C_2, C_1) \quad (35)$$

$$Cell\_ID_4 = f(C_2, C_2) \quad (36)$$

If $M = 4$, 5 and 6 and we have $N_{Cells} = 256$, 3125 and 46656 which shows the exponential increase. Hence a small codebook size is sufficient. This allows the codewords can be n-fold repetition encoded to increase detection probability.

8.2 Cell Identification via the P-SCH and S-SCH

If both a primary and secondary SCH are used the number of supported cells/sectors can be increased without increasing the number of cross-correlation operations for detection. In this approach a two-stage detection procedure may be used.

In the first-stage detection one of the $M \leq K$ codewords in the codebook $\mathcal{C}$ is detected from received P-SCH symbols. As shown in Figure 6 and 7 the codeword may be n-fold repetition encoded to increase detection probability.

In the second-stage detection, one or more cyclically-shifted versions of the detected codeword are detected from received S-SCH symbols. As shown in Figure 7 the cyclically-shifted codeword may also be n-fold repetition encoded to increase detection probability.

A cell/sector identifier is defined by the combined indices of the detected codeword and the detected cyclic-shift of the codeword. As a function of the SCH codebook $\mathcal{C}$ and cyclically shifted codebook $\mathcal{S}$ the identifier for the pth cell/sector may be defined as

$$Cell\_ID_p(\mathcal{C}, \mathcal{S}) = f(C_i, S_{i1}, S_{i2}, ..., S_{iM}) \quad (37)$$

$$p \in \{1, 2, \ldots, N_{Cells}\} \quad (38)$$

The one-to-one function f may be a look-up table that maps the detected indices i and $i_1, i_2, \ldots, i_M$ to a cell identifier $Cell\_ID_p$. The number of cells N_{Cells} that can supported by the codebook $\mathcal{C}$ is defined as

$$N_{Cells} = M \cdot M^M = M^{M+1} \tag{39}$$

As an example let the CAZAC sequence length K be an even number and let the codebook size be $M = 2$. Let $\Psi = \{\psi_1, \psi_2\}$ denote a pair of positive cyclic shift integers where for $i, j \in \{1, 2\}$ we have $\psi_i \neq \psi_j$ and $0 \leq \psi_i < K - 1$. The SCH codebook is then

$$\mathcal{C} = \{C_1, C_2\} \tag{40}$$

where the orthogonal codewords are defined as the CAZAC sequences

$$C_1 = \{c_1[k]\}_{k=0}^{K-1} = \left\{e^{j\frac{\pi}{K}k(k-2)}\right\}_{k=0}^{K-1} \tag{41}$$

$$C_2 = \{c_2[k]\}_{k=0}^{K-1} = \left\{e^{j\frac{\pi}{K}k(k-4)}\right\}_{k=0}^{K-1} \tag{42}$$

The corresponding cyclically shifted codewords are defined as

$$\mathcal{S}_1 = \{S_{11}, S_{12}\} \tag{43}$$

$$\mathcal{S}_2 = \{S_{21}, S_{22}\} \tag{44}$$

where

$$S_{ij} = \left\{c_i[k + \psi_j \bmod K]\right\}_{k=0}^{K-1}, \ i, j \in \{1, 2\} \tag{45}$$

Note that the first index i denotes the codeword and the second index j the cyclic shift of the codeword. Hence the following eight unique cell/sector IDs are possible

$$\text{Group 1:} \begin{cases} Cell\_ID_1 = f(C_1, S_{11}, S_{11}) \\ Cell\_ID_2 = f(C_1, S_{11}, S_{12}) \\ Cell\_ID_3 = f(C_1, S_{12}, S_{11}) \\ Cell\_ID_4 = f(C_1, S_{12}, S_{12}) \end{cases} \tag{46}$$

$$\text{Group 2:} \begin{cases} Cell\_ID_5 = f(C_2, S_{21}, S_{21}) \\ Cell\_ID_6 = f(C_2, S_{21}, S_{22}) \\ Cell\_ID_7 = f(C_2, S_{22}, S_{21}) \\ Cell\_ID_8 = f(C_2, S_{22}, S_{22}) \end{cases} \tag{47}$$

The first group are the possible choices when C_1 is first-stage detected from the P-SCH. The second group are the possible choices when C_2 is first-stage detected from the P-SCH. The second-stage detection then uses C_1 or C_2 (whichever is detected in first-stage) to detect the values S_{ij} from the S-SCH symbol. Once the CAZAC sequence codeword C_i and the cyclically time-shifted codeword S_{ij} are known the unique cell identifier $Cell\_ID_i$ is known via the one-to-one function f (e.g. a lookup table).

The two-stage process does not increase the number of cross-correlation operations. More specifically, using the example above it can be seen that the first stage requires 2 cross-correlations to detect either C_1 or C_2. Once C_1 or C_2 is detected the second stage requires 4 cross-correlations to detect the cyclic shifts of the detected codeword C_1 or C_2. Hence the total number of cross-correlations for cell identification is 6. To support the same number of identifiers using the single-stage approach we would have

$$
\begin{aligned}
Cell\_ID_1 &= f(C_1, C_1, C_1) && (48)\\
Cell\_ID_2 &= f(C_1, C_1, C_2) && (49)\\
Cell\_ID_3 &= f(C_1, C_2, C_1) && (50)\\
Cell\_ID_4 &= f(C_1, C_2, C_2) && (51)\\
Cell\_ID_5 &= f(C_2, C_1, C_1) && (52)\\
Cell\_ID_6 &= f(C_2, C_1, C_2) && (53)\\
Cell\_ID_7 &= f(C_2, C_2, C_1) && (54)\\
Cell\_ID_8 &= f(C_2, C_2, C_2) && (55)
\end{aligned}
$$

If only a single-stage detection is used a total of 6 cross-correlations are required, two for each of the values in the function arguments.

9 References

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10 Proposed Text

11.7.2.1.3 Synchroziation Channel Sequences
11.7.2.1.3.1 Sequence Properties
11.7.2.1.3.1 Cell-Specific Sequence Codebooks
11.7.2.1.3.1 Sequence Generation
11.7.2.1.3.1 Sequence to P-SCH and S-SCH Mapping