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Re:	Call for comments on SDD text
Abstract	Evaluation of CL SU and MU MIMO codebooks
Purpose	To discuss and adopt in SDD text
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Release	
Patent Policy	

Evaluation of CL SU and MU-MIMO codebooks

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Introduction

The codebook(s) for 2 and 4 transmit antennas should be designed based on performance and complexity. Some desirable properties of the codebook are:

1. *Low complexity* codebook design can be attained by choosing the elements of each matrix/vector from a small set. An example of the small set is the 4-alphabet size {±1, ±j}. This avoids the need for computing matrix multiplication. Additionally a nested property can further reduce the complexity of CQI calculation when rank adaptation is performed.
2. *Rank overriding*: Base station may perform rank overriding which will result in significant CQI mismatch if the codebook structure cannot cope with this. A nested property w.r.t. rank overriding can be exploited to limit the impact of such issues.
3. *Power amplifier balance*: Codebook design with constant modulus (CM) property is beneficial for avoiding unnecessary increase in PAPR.
4. Good performance for a wide range of propagation scenarios: *uncorrelated, correlated, and dual-polarized channels*. A DFT-like codebook is optimal for linear array with small antenna spacing since the vectors match with the structure of the transmit array response. Additionally, with a clever choice of the matrices and the entries of the codebook (rotated block diagonal structure), significant gains can be obtained in dual-polarized scenarios.
5. Low feedback and signaling overhead. We believe that a 3 bit codebook for 2Tx and 4bit codebook for 4Tx is a reasonable compromise. Additional gain can be obtained by the use of a differential codebook.
6. Low memory requirement

16e codebooks have been optimized exclusively for uncorrelated scenarios and the performance loss in other scenarios (correlated antennas, dual-polarized antennas) is quite severe. In order to achieve significant performance gain in many deployment scenarios and for several CL MIMO schemes, we propose not to use 16e codebooks in 16m but recommend the use of a unique codebook composed of a combination of DFT-based matrices and precoding matrices optimized for cross-polarized channels. Those codebooks are referred to as standard codebooks.

Additionally, as it was explained in contribution IEEE C80216m-08_850r1, we propose to also use a differential codebook. The standard and differential codebooks are two different modes. Standard codebook is the default mode and the differential codebook would be used as an optional mode to boost performance. The BS decides whether the use of the differential codebook is necessary, depending on the cell load, QoS, etc.

In this contribution, we provide comparisons of the codebooks submitted to the MIMO RG. The simulation results are compliant with EMD and the simulation assumptions agreed in doc C80216m-MIMO-08_033r2.

Differential codebooks are not discussed in this contribution.

Performance Analysis

We compare the performance of the following “basic” codebooks in SU and MU MIMO:

Contribution Number(s)/Author	2Tx codebook size	4Tx codebook size
16e 6bits (C80216m-MIMO-08_066) Qinghua Li, Intel	3 bits	6 bits
CB1 (C80216m-MIMO-08_063, C80216m-08_1187) Bruno Clerckx, Samsung Electronics	3 bits	4 bits
CB2 (C80216m-MIMO-08_070, C80216m-08_1109) Yang Tang, Huawei	3 bits	4 bits
CB3 (C80216m-MIMO-08_069, C80216m-08_983, C80216m-08_973) Jaewan Kim, LGE	3 bits	4 bits
CB4 (C80216m-MIMO-08_074, C80216m-08_1101) Bishwarup Mondal, Motorola	3 bits	4 bits
CB5 (C80216m-MIMO-08_072r1, C80216m-08_916) Ron Porat, Nextwave	No proposal	4 bits
CB6 (C80216m-MIMO-08_067, C80216m-MIMO-08_1156) Shaohua li, NSN	No proposal	6 bits

Note that for 2Tx, CB1, CB2, CB3 and CB4 are the same.

SU MIMO

2x2

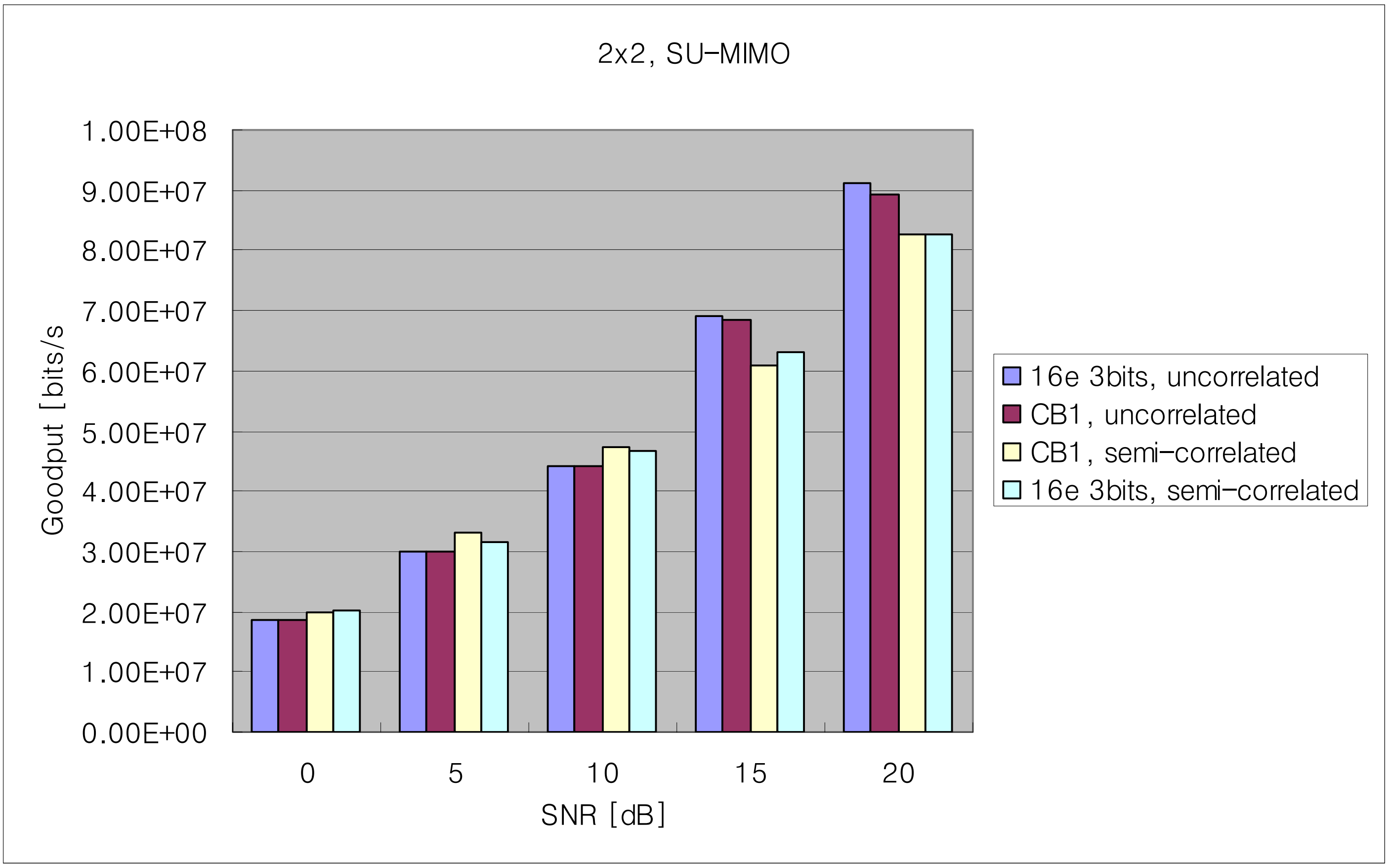

Figure 1. Performance of submitted codebooks in SU MIMO 2x2 uncorrelated and semi-correlated channels

4x2

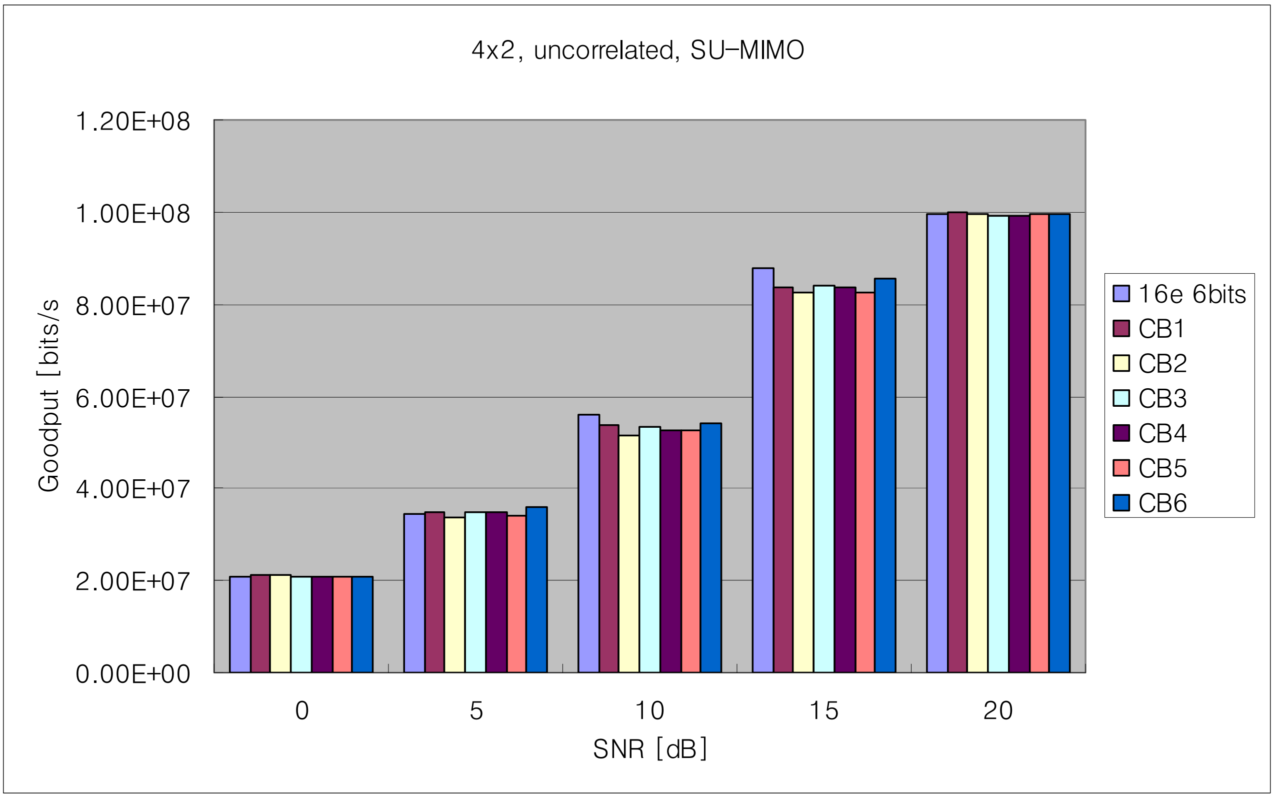

Figure 2. Performance of submitted codebooks in SU MIMO 4x2 uncorrelated channels

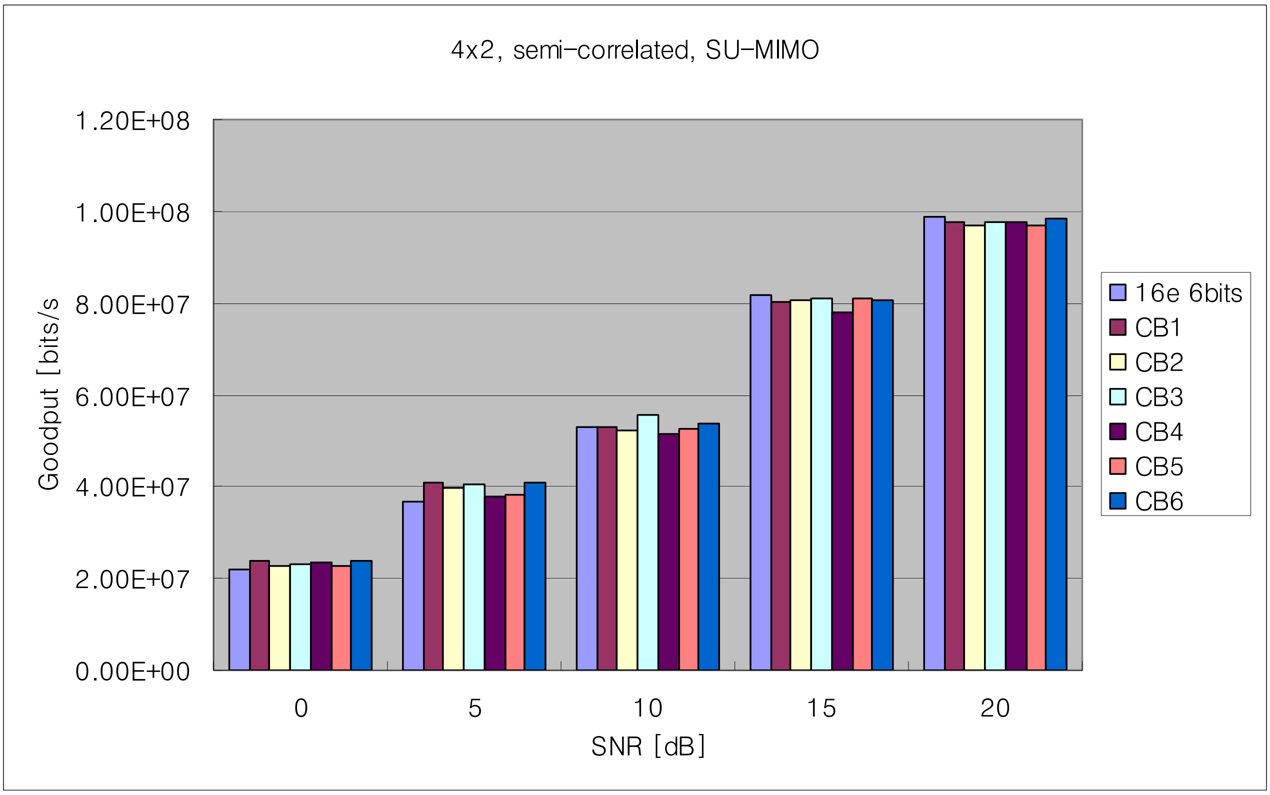

Figure 3. Performance of submitted codebooks in SU MIMO 4x2 semi-correlated channels

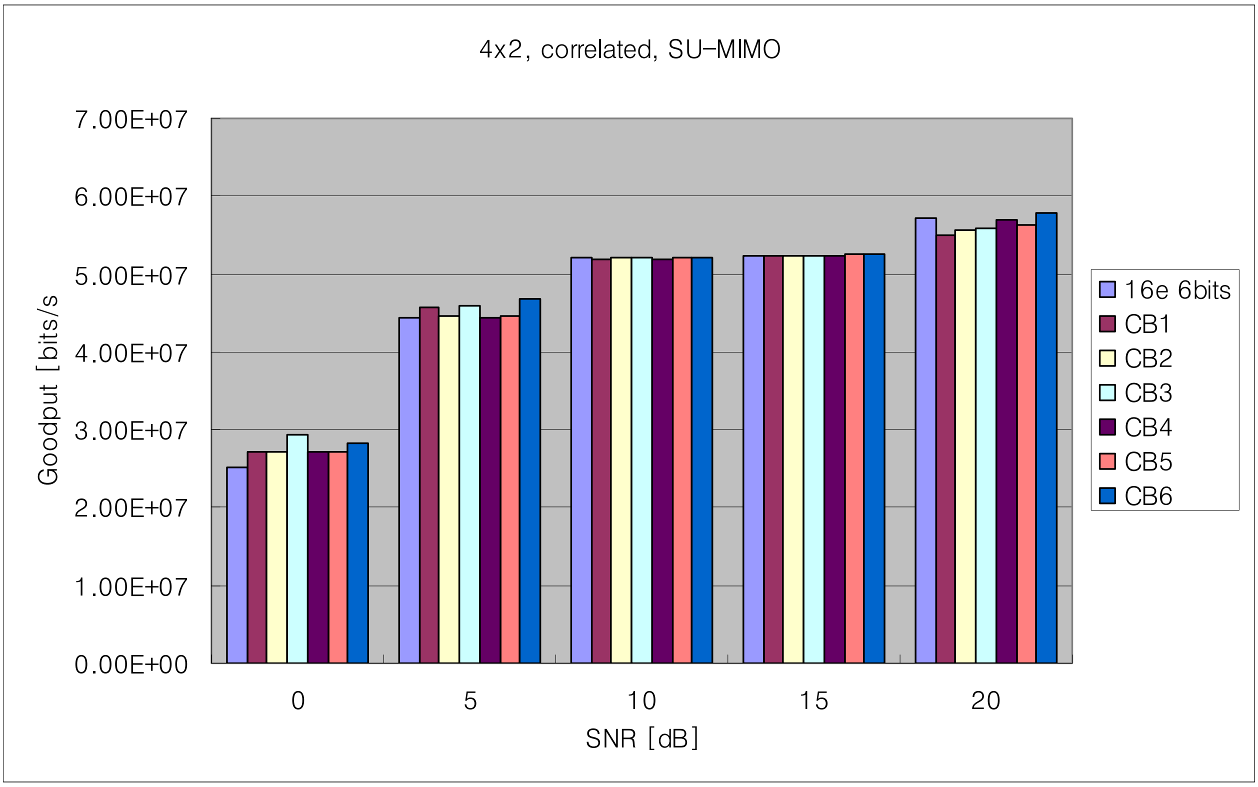

Figure 4. Performance of submitted codebooks in SU MIMO 4x2 correlated channels

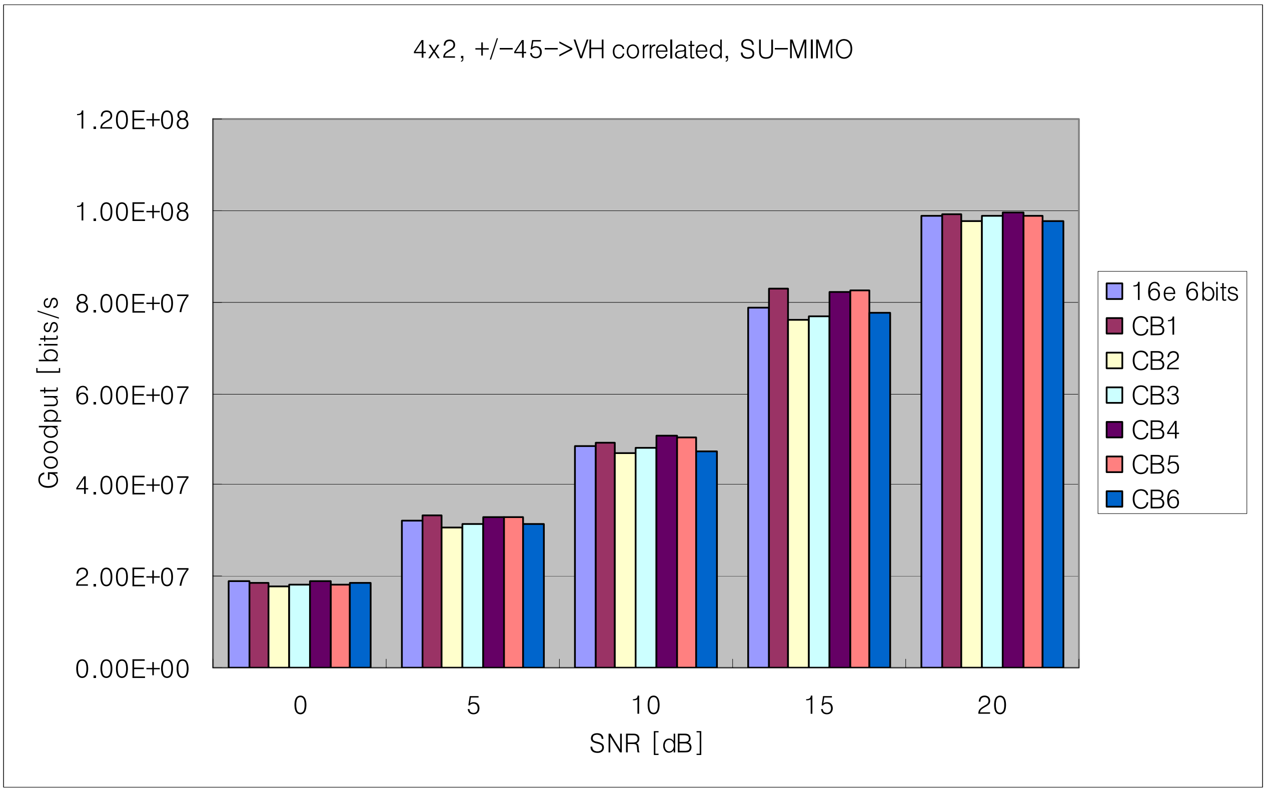

Figure 5. Performance of submitted codebooks in SU MIMO 4x2 dual-polarized correlated channel

4x4

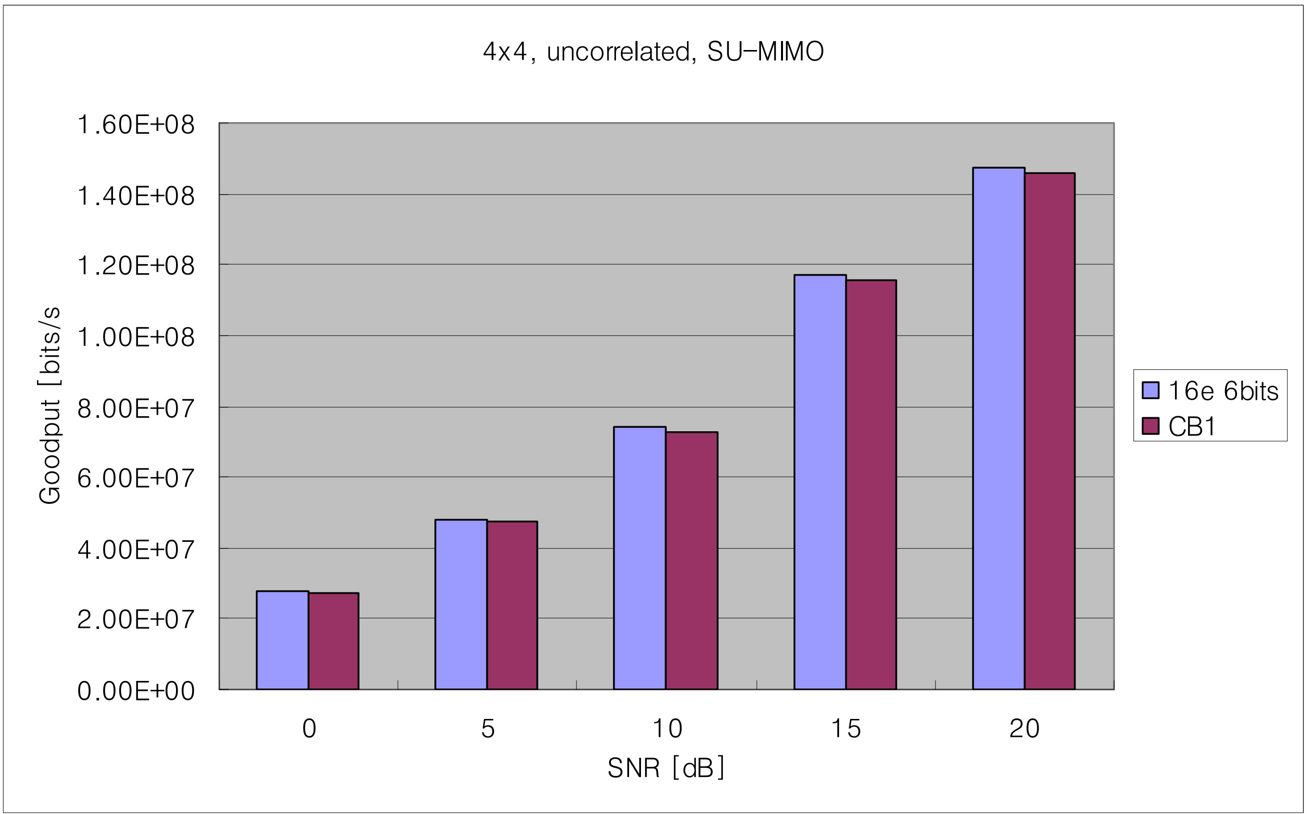

Figure 6. Performance of 16e 6bits codebook vs. CB1 in SU MIMO 4x4 uncorrelated channels

SU MIMO Conclusions

2Tx:

- Slight gain for 16e codebook in uncorrelated channels
- Slight gain for CB1 in correlated channels

4Tx

- Slight gain for 16e codebook over all others in uncorrelated channels at high SNR
- Slight gain for CB1, CB3 and CB6 as correlation increases
- Slight gain for CB1, CB4 and CB5 in dual-polarized channels

MU MIMO (ZFBF is performed)

4x2

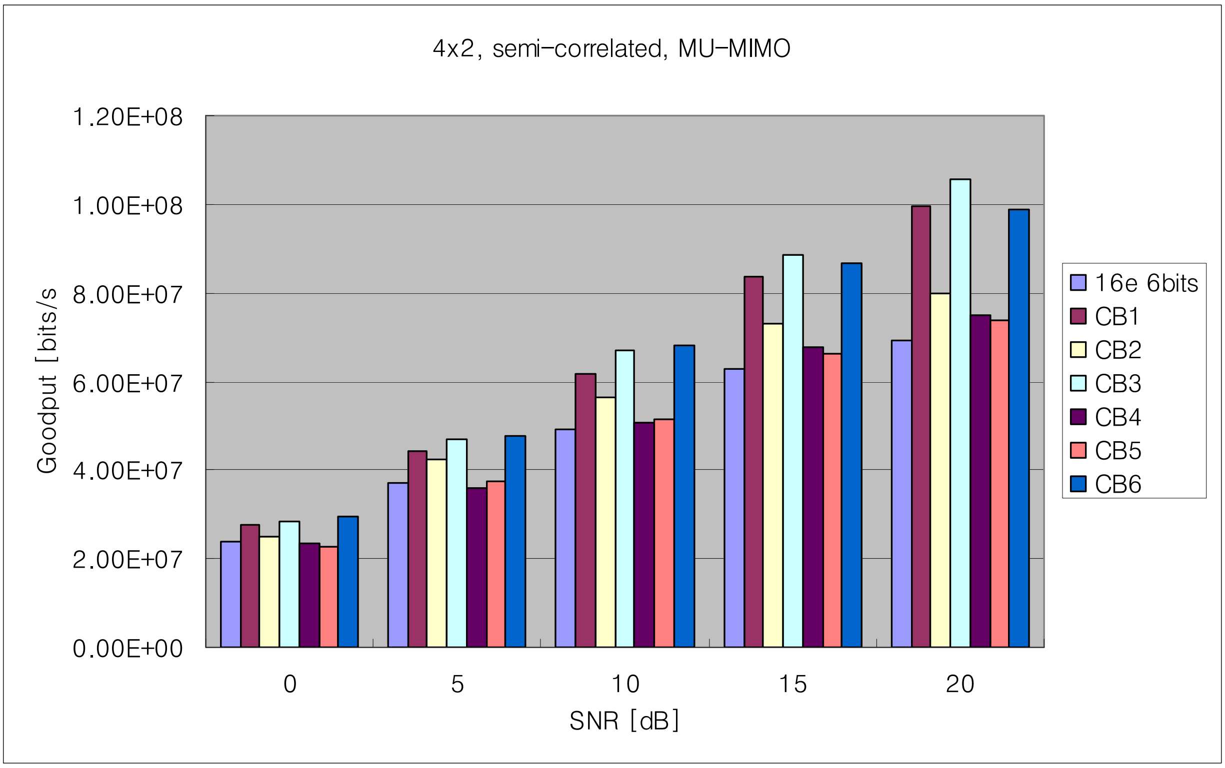

Figure 7. Performance of submitted codebooks in MU MIMO 4x2 semi-correlated channels

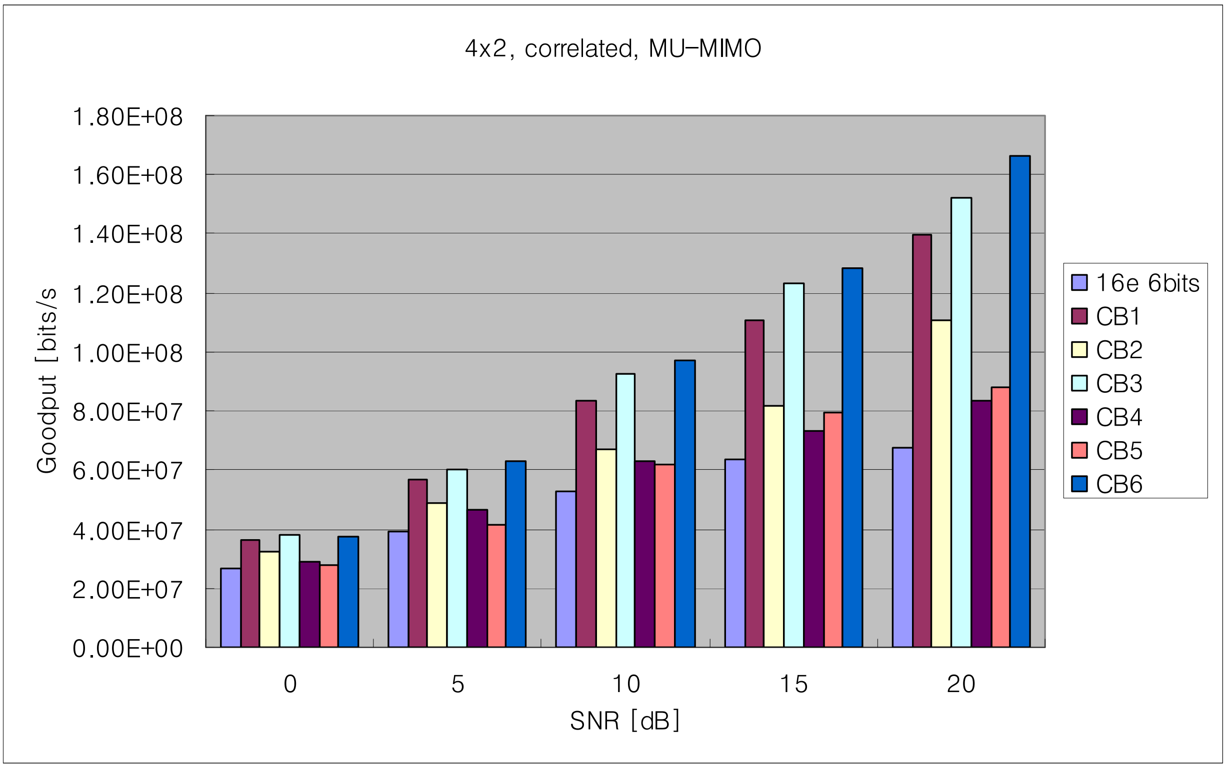

Figure 8. Performance of submitted codebooks in MU MIMO 4x2 correlated channels

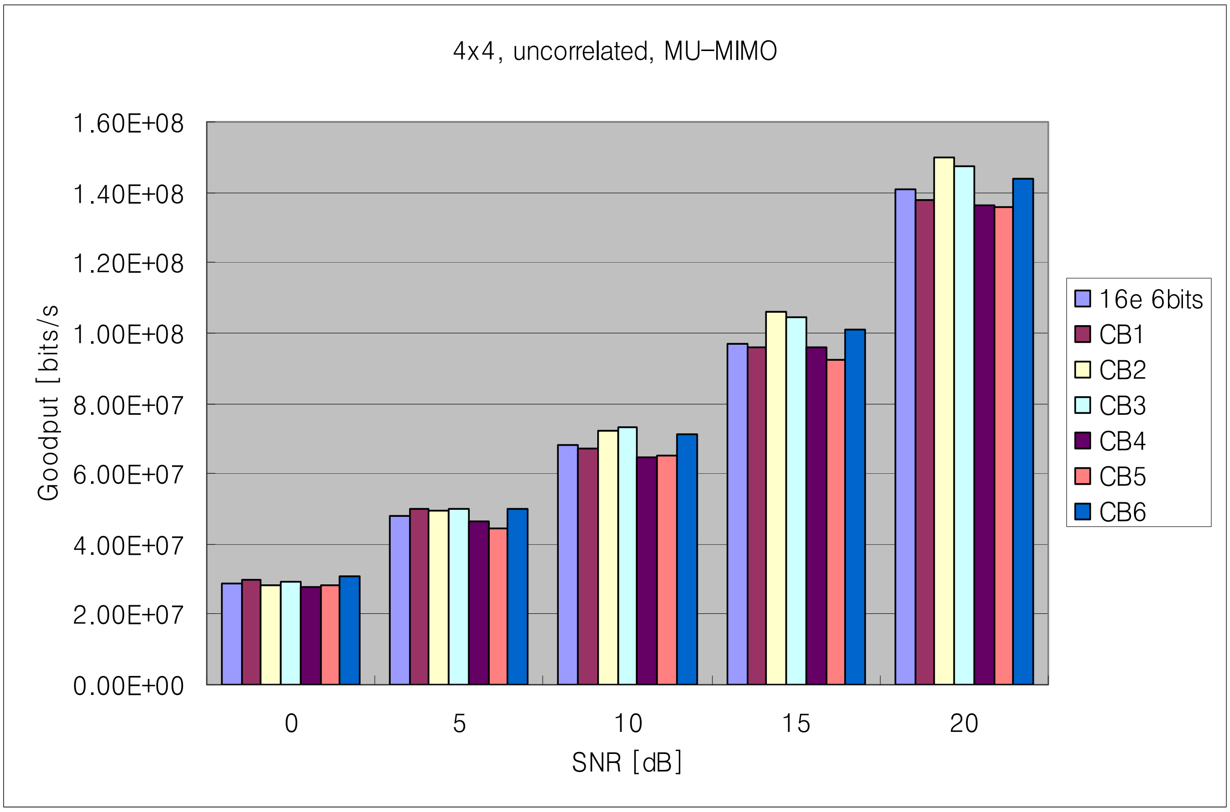

Figure 9. Performance of submitted codebooks in MU MIMO 4x4 uncorrelated channels

MU MIMO Conclusions

- CB1, CB3 and CB6 achieve significant gains over 16e, CB2, CB4 and CB5 in correlated and semi-correlated channels
- In uncorrelated channels with the same number of Tx as Rx, performance gap reduces and CB2 and CB3 have slight advantages over other codebooks at high SNR.

Simulation conditions

Transmission bandwidth	10 MHz
Centre frequency	2.5 GHz
Subframe duration	0.6171 ms
Subcarrier spacing	10. 938 kHz
FFT size	1024
Number of occupied subcarriers	1008
Number of OFDM symbols per subframe	6
Number of subcarriers per Resource Unit	18
Spatial channel environment	Modified PedB channel, 3 km/h, uncorrelated at MS - Uncorrelated case: 4 wavelengths spacing and 15 degree angular spread at the base station - Semi-Correlated case: 4 wavelength spacing and 3 degree angular spread at the base station - Correlated case: 0.5 wavelength spacing and 3 degree angular spread at the base station - Dual polarized case: +/-45 → V-H, 4 wavelength spacing and 3 degree angular spread at the base station
CQI feedback	6 subframes delay, error-free (~4ms delay)
Subchannelization and frequency granularity of feedback	LLRU (4 adjacent PRU)
Feedback load	Full feedback (for every resource unit), 10 users
pilot pattern	16m
Channel estimation	- Ideal - Real channel estimation on midamble and dedicated pilots using 16m pilot pattern
MIMO detection method	Linear MMSE
Modulation and coding	10 MCS levels
HARQ	Chase Combining, non-adaptive, 8 subframes retransmission delay, maximum 4 retransmissions

Complexity Analysis, nested property, rank overriding and PAPR

Those issues were detailed already in contribution 851r1.

	16e	CB1	CB2	CB3	CB4	CB5	CB6
Nested property for CQI calculation	No	Yes	Yes	Yes	Yes	Yes	No
Constrained alphabet	No	Mainly QPSK-1 8PSK matrix	No	No	QPSK	QPSK	No
Nested property for rank overriding	No	Yes	Yes	No	Yes	Yes	No
Constant modulus	No	Yes	Yes	Yes	Yes	Yes	No

Classification of codebooks by increasing order of complexity:

CB4 ~ CB5 ~< CB1 < CB2 < CB3 < CB6 < 16e

Conclusions

Overall, considering performance in various propagation conditions for both SU and MU-MIMO and the complexity, we propose to include CB1 in SDD text.

Appendix: CB1 Codebooks

Codebook for 2-antenna BS in DL

The Matrix codebook is a DFT codebook with QPSK and 8PSK alphabet.
The codebook matrices are W_1, W_2, W_3, and W_4.

$$W_1 = \frac{1}{\sqrt{2}} * \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (1)$$

$$W_2 = \frac{1}{\sqrt{2}} * \begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix} \quad (2)$$

$$W_3 = \begin{bmatrix} 1 & \\ & \frac{(1+j)}{\sqrt{2}} \end{bmatrix} * W_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ \frac{(1+j)}{\sqrt{2}} & \frac{(-1-j)}{\sqrt{2}} \end{bmatrix} \quad (3)$$

$$W_4 = \begin{bmatrix} 1 & \\ & \frac{(1+j)}{\sqrt{2}} \end{bmatrix} * W_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ \frac{(-1+j)}{\sqrt{2}} & \frac{(1-j)}{\sqrt{2}} \end{bmatrix} \quad (4)$$

Mapping from Matrix codebook to transmit SU MIMO codebook is specified in Table 1.

Table 1 – SU MIMO Codebook for 2-antenna BS in DL

Transmit Codebook Index	Rank 1	Rank 2
1	$C_{1,1}$ = W1(:,1)	$C_{1,2}$ = W1(:,12)
2	$C_{2,1}$ = W1(:,2)	$C_{2,2}$ = W2(:,12)
3	$C_{1,1}$ = W2(:,1)	$C_{3,2}$ = W3(:,12)
4	$C_{4,1}$ = W2(:,2)	$C_{4,2}$ = W4(:,12)
5	$C_{5,1}$ = W3(:,1)	n/a
6	$C_{6,1}$ = W3(:,2)	n/a
7	$C_{7,1}$ = W4(:,1)	n/a
8	$C_{8,1}$ = W4(:,2)	n/a

The MU MIMO codebook is given by the subset of the matrix codebook defined by the 2 matrices

$$W_1 = \frac{1}{\sqrt{2}} * \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (5)$$

$$W_2 = \frac{1}{\sqrt{2}} * \begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix} \quad (6)$$

The MU MIMO feedback codebook is specified in Table 2.

Table 2 – MU MIMO Feedback Codebook for 2-antenna BS in DL

Transmit Codebook Index	Rank 1
1	W1(:,1)
2	W1(:,2)
3	W2(:,1)
4	W2(:,2)

The MU-MIMO transmit codebook is {W1,W2}.

Codebook for 4-antenna BS in DL

The Matrix codebook is a DFT codebook with QPSK and 8PSK alphabet and a block diagonal structure. Let us define the rotation matrix and QPSK DFT matrix as

$$U_{rot} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \qquad (7)$$

and $$DFT = 0.5*\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \qquad (8)$$

The codebook matrices are W_1, W_2, W_3, W_4, W_5 and W_6.

$$W_1 = \frac{1}{\sqrt{2}}*U_{rot}*\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \qquad (9)$$

$$W_2 = \frac{1}{\sqrt{2}}*U_{rot}*\begin{bmatrix} 1 & 1 & 0 & 0 \\ j & -j & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & j & -j \end{bmatrix} \qquad (10)$$

$$W_3 = \frac{1}{\sqrt{2}}*U_{rot}*\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & j & -j \end{bmatrix} \qquad (11)$$

$$W_4 = \frac{1}{\sqrt{2}}*U_{rot}*\begin{bmatrix} 1 & 1 & 0 & 0 \\ j & -j & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \qquad (12)$$

$$W_5 = diag(1,1,1,-1)*DFT = 0.5*\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ -1 & j & 1 & -j \end{bmatrix} \qquad (13)$$

$$W_6 = diag(1, \frac{(1+j)}{\sqrt{2}}, j, \frac{(-1+j)}{\sqrt{2}}) * DFT = 0.5 * \begin{bmatrix} 1 & 1 & 1 & 1 \\ \frac{(1+j)}{\sqrt{2}} & \frac{(-1+j)}{\sqrt{2}} & \frac{(-1-j)}{\sqrt{2}} & \frac{(1-j)}{\sqrt{2}} \\ j & -j & j & -j \\ \frac{(-1+j)}{\sqrt{2}} & \frac{(1+j)}{\sqrt{2}} & \frac{(1-j)}{\sqrt{2}} & \frac{(-1-j)}{\sqrt{2}} \end{bmatrix} \quad (14)$$

Mapping from Matrix codebook to transmit SU MIMO codebook is specified in Table 3Table 31.

Table 31 – SU MIMO Codebook for 4-antenna BS in DL

Transmit Codebook Index	Rank 1	Rank 2	Rank 3	Rank 4
1	$C_{1,1}$ =W1(:,2)	$C_{1,2}$ =W1(:,12)	$C_{1,3}$ =W1(:,123)	$C_{1,4}$ =W1(:,1234)
2	$C_{2,1}$ =W1(:,3)	$C_{2,2}$ =W1(:,13)	$C_{2,3}$ =W1(:,124)	$C_{2,4}$ =W2(:,1234)
3	$C_{3,1}$ =W1(:,4)	$C_{3,2}$ =W1(:,14)	$C_{3,3}$ =W1(:,134)	$C_{3,4}$ =W3(:,1234)
4	$C_{4,1}$ =W2(:,2)	$C_{4,2}$ =W1(:,23)	$C_{4,3}$ =W1(:,234)	$C_{4,4}$ =W4(:,1234)
5	$C_{5,1}$ =W2(:,3)	$C_{5,2}$ =W1(:,24)	$C_{5,3}$ =W2(:,123)	$C_{5,4}$ =W5(:,1234)
6	$C_{6,1}$ =W2(:,4)	$C_{6,2}$ =W1(:,34)	$C_{6,3}$ =W2(:,124)	$C_{6,4}$ =W6(:,1234)
7	$C_{7,1}$ =W3(:,1)	$C_{7,2}$ =W2(:,13)	$C_{7,3}$ =W2(:,134)	n/a
8	$C_{8,1}$ =W4(:,1)	$C_{8,2}$ =W2(:,14)	$C_{8,3}$ =W2(:,234)	n/a
9	$C_{9,1}$ =W5(:,1)	$C_{9,2}$ =W2(:,23)	$C_{9,3}$ =W3(:,123)	n/a
10	$C_{10,1}$ =W5(:,2)	$C_{10,2}$ =W2(:,24)	$C_{10,3}$ =W3(:,134)	n/a
11	$C_{11,1}$ =W5(:,3)	$C_{11,2}$ =W3(:,13)	$C_{11,3}$ =W4(:,123)	n/a
12	$C_{12,1}$ =W5(:,4)	$C_{12,2}$ =W3(:,14)	$C_{12,3}$ =W4(:,134)	n/a
13	$C_{13,1}$ =W6(:,1)	$C_{13,2}$ =W4(:,13)	$C_{13,3}$ =W5(:,123)	n/a
14	$C_{14,1}$ =W6(:,2)	$C_{14,2}$ =W4(:,14)	$C_{14,3}$ =W5(:,134)	n/a
15	$C_{15,1}$ =W6(:,3)	$C_{15,2}$ =W5(:,13)	$C_{15,3}$ =W6(:,124)	n/a
16	$C_{16,1}$ =W6(:,4)	$C_{16,2}$ =W6(:,24)	$C_{16,3}$ =W6(:,234)	n/a

The MU-MIMO codebook is given by the subset of the matrix codebook defined by the 2 matrices

$$W_3 = \frac{1}{\sqrt{2}} * U_{rot} * \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & j & -j \end{bmatrix} \quad (15)$$

$$W_6 = diag(1, \frac{(1+j)}{\sqrt{2}}, j, \frac{(-1+j)}{\sqrt{2}}) * DFT = 0.5 * \begin{bmatrix} 1 & 1 & 1 & 1 \\ \frac{(1+j)}{\sqrt{2}} & \frac{(-1+j)}{\sqrt{2}} & \frac{(-1-j)}{\sqrt{2}} & \frac{(1-j)}{\sqrt{2}} \\ j & -j & j & -j \\ \frac{(-1+j)}{\sqrt{2}} & \frac{(1+j)}{\sqrt{2}} & \frac{(1-j)}{\sqrt{2}} & \frac{(-1-j)}{\sqrt{2}} \end{bmatrix} \quad (16)$$

The MU MIMO feedback codebook is specified in Table 4.

Table 4 – MU MIMO Feedback Codebook for 4-antenna BS in DL

Transmit Codebook Index	Rank 1
1	W3(:,1)
2	W3(:,2)
3	W3(:,3)
4	W3(:,4)
5	W6(:,1)
6	W6(:,2)
7	W6(:,3)
8	W6(:,4)

For dual-polarized scenarios, the labeling of the B physical antennas is assumed to be such that rows 1 and 2 of the matrices W_k correspond to one polarization and rows 3 and 4 correspond to the other polarization. This is further clarified by the following example where antenna ports 0 and 1 are V-polarized and antenna ports 2 and 3 are H-polarized. The same applies to +/−45-degree polarized as well as other dual-polarized setups.

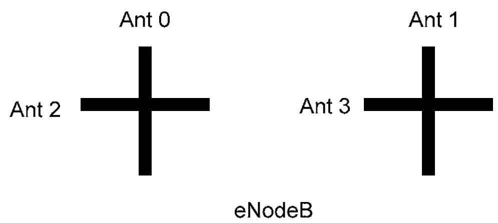

Figure 8 – Mapping of codebook to the physical antenna ports

Differential Codebook for CL SU and MU MIMO

The general procedure of differential feedback scheme can be described as below
At time instant $\tau = 0$, the MS choose the appropriate precoder and rank from the transmit SU MIMO codebook presented in previous sections for 2, 4 Tx. Denote this precoder as $\mathbf{F}_0$.
For time instant $\tau = 1, 2, \cdots T_{\max}$, the MS differentially rotates the previous codeword as;

$$\mathbf{F}_\tau = \tilde{\mathbf{\Theta}}_i \mathbf{F}_{\tau-1} \qquad (17)$$

At time $T_{\max}+1$, the process is reset and τ is fixed to 0 again.

The rotation and differential codebook are constructed according to the following 2-step procedure.
Firstly, we construct an equally spaced finite set differential codebook, $\{\theta\} = \{\mathbf{\Theta}_1, \ldots, \mathbf{\Theta}_{2^B}\}$, where each codeword is a $N_t \times N_t$ unitary matrix.
For four transmit antennas at BS, we recommend the following structure of those unitary matrices

$$\mathbf{\Theta}_l = \mathbf{\Phi}^l \mathbf{D}, \quad l = 1, \cdots, 2^B, \qquad (18)$$

where $\mathbf{D}$ is the $N_t \times N_t$ DFT matrix and

$$\mathbf{\Phi} = \begin{bmatrix} e^{j2\pi\phi_1/2^B} & 0 & \cdots & 0 \\ 0 & e^{j2\pi\phi_2/2^B} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & e^{j2\pi\phi_{N_t}/2^B} \end{bmatrix}, \ \phi_i \in \{1, 2, \cdots, 2^B\} \qquad (19)$$

For example when $N_t = 4$, the following values for $[\phi_1, \phi_2, \phi_3, \phi_4]$ are specified in Table 5.

Table 5 – Cardinality of Differential Codebook with 4 Antennas at BS

Cardinality of $\{\theta\}$	$[\phi_1, \phi_2, \phi_3, \phi_4]$
B=4	$[1, 3, 4, 8]$

Secondly, the rotation codebook is then build using the following operations,

- Generate the matrix for $i = 1, 2, \ldots, 2^B$

  $$\mathbf{\Psi}_i(\rho, \mathbf{\Theta}_i) = \rho \mathbf{I} + \sqrt{1-\rho^2}\,\mathbf{\Theta}_i \qquad (20)$$

  where ρ is an effective time correlation coefficient.

- Then, take $\tilde{\mathbf{\Theta}}_i$ such that

  $$\tilde{\mathbf{\Theta}}_i = \arg\min_{\tilde{\mathbf{\Theta}}_i} \left\| \mathbf{\Psi}_i(\rho, \mathbf{\Theta}_i) - \tilde{\mathbf{\Theta}}_i \right\|_F \quad \text{for} \quad i = 1, 2, \ldots, 2^B \qquad (21)$$

- Let us define the SVD of $\mathbf{\Psi}_i(\rho, \mathbf{\Theta}_i)$ as $\mathbf{\Phi}_i \mathbf{\Lambda}_i \mathbf{B}_i^*$, then the solution of the above optimization problem can be given as

  $$\tilde{\mathbf{\Theta}}_i = \mathbf{\Phi}_i \mathbf{B}_i^* \qquad (22)$$

- Construct the rotation codebook (adapted to current ρ value)

$$\left\{ \tilde{\theta} \right\} = \left\{ \tilde{\mathbf{\Theta}}_1, \ldots, \tilde{\mathbf{\Theta}}_{2^B} \right\} \tag{23}$$

It is recommended to use ρ=0.9 for 4Tx and ρ=0.95 for 2Tx.

The unitary matrices for four transmit antennas at BS with ρ=0.9 are given below.

$\tilde{\mathbf{\Theta}}_1$ =

0.9567 + 0.0417i	0.0684 + 0.1542i	0.1036 + 0.1320i	0.1613 + 0.0201i
-0.0607 + 0.1574i	0.9322 + 0.0029i	-0.0257 - 0.2653i	0.1414 + 0.1076i
-0.0823 + 0.1462i	0.0777 - 0.2549i	0.9183 + 0.2037i	-0.0831 - 0.0956i
-0.1567 + 0.0431i	-0.1535 + 0.0894i	0.0956 - 0.0831i	0.9337 - 0.2327i

$\tilde{\mathbf{\Theta}}_2$ =

0.9396 + 0.1527i	0.1496 + 0.1411i	0.2002 + 0.0990i	0.0269 + 0.0316i
-0.1411 + 0.1496i	0.9601 - 0.1555i	-0.0529 - 0.0251i	0.0827 - 0.0380i
-0.2116 + 0.0715i	-0.0196 - 0.0552i	0.9374 + 0.0301i	0.2570 + 0.0326i
0.0414 + 0.0033i	-0.0853 - 0.0316i	-0.2570 - 0.0326i	0.9309 + 0.2374i

$\tilde{\mathbf{\Theta}}_3$ =

0.9270 + 0.2411i	0.1542 + 0.0992i	0.0476 + 0.0346i	0.1618 + 0.1392i
-0.1791 + 0.0389i	0.9221 - 0.1792i	0.1375 + 0.1618i	-0.0412 + 0.1927i
-0.0308 - 0.0502i	-0.0969 + 0.1890i	0.9140 - 0.2683i	-0.1277 + 0.1667i
-0.1905 + 0.0962i	0.0356 + 0.1938i	0.1667 + 0.1277i	0.9076 - 0.2188i

$\tilde{\mathbf{\Theta}}_4$ =

0.9604 + 0.1704i	-0.0114 + 0.0036i	-0.1093 + 0.0930i	-0.1359 + 0.0969i
0.0114 - 0.0036i	0.9839 - 0.0091i	0.0741 + 0.0750i	-0.0930 - 0.1093i
0.0930 + 0.1093i	-0.0750 + 0.0741i	0.9839 - 0.0091i	-0.0036 - 0.0114i
0.0969 + 0.1359i	0.1093 - 0.0930i	-0.0036 - 0.0114i	0.9604 + 0.1704i

$\tilde{\mathbf{\Theta}}_5$ =

0.9301 + 0.2281i	-0.1266 + 0.0364i	-0.0870 + 0.2160i	0.0718 + 0.0782i
0.1153 + 0.0638i	0.9717 + 0.1710i	-0.0708 - 0.0456i	-0.0410 + 0.0214i
0.0023 + 0.2329i	0.0692 - 0.0479i	0.9301 + 0.2341i	-0.1037 - 0.0895i
-0.0448 + 0.0963i	0.0460 - 0.0041i	0.0895 - 0.1037i	0.9681 - 0.1749i

$\tilde{\mathbf{\Theta}}_6$ =

0.9239 + 0.2135i	-0.1679 + 0.1484i	0.0788 + 0.1302i	-0.1501 + 0.0700i
0.1484 + 0.1679i	0.8870 + 0.2301i	-0.2150 - 0.0813i	0.1032 - 0.2157i
-0.0364 + 0.1478i	0.2095 - 0.0946i	0.9337 + 0.0799i	0.2139 - 0.0078i
0.1556 + 0.0567i	-0.0796 - 0.2255i	-0.2139 + 0.0078i	0.9046 + 0.2265i

$\tilde{\mathbf{\Theta}}_7$ =

0.9319 + 0.1518i	-0.1110 + 0.2491i	-0.1561 - 0.0682i	-0.0041 + 0.0721i

0.0976 + 0.2546i	0.9190 - 0.1100i	0.0859 - 0.0053i	0.1625 + 0.1876i
0.1227 - 0.1181i	-0.0773 + 0.0378i	0.9244 - 0.2914i	-0.0789 + 0.1342i
-0.0313 + 0.0650i	-0.1111 + 0.2219i	0.1342 + 0.0789i	0.9192 - 0.2529i

$\tilde{\mathbf{\Theta}}_8$ =

0.9222 - 0.0386i	0.0068 - 0.0794i	-0.2353 - 0.0095i	-0.2918 - 0.0321i
0.0068 - 0.0794i	0.9475 - 0.2510i	0.1789 - 0.0132i	-0.0229 + 0.0158i
0.2353 + 0.0095i	-0.1789 + 0.0132i	0.9538 + 0.0073i	-0.0384 + 0.0335i
0.2918 + 0.0321i	0.0229 - 0.0158i	-0.0384 + 0.0335i	0.9286 + 0.2197i

$\tilde{\mathbf{\Theta}}_9$ =

0.9446 - 0.1610i	-0.0810 - 0.1798i	-0.1478 + 0.0594i	0.0299 - 0.1293i
0.0699 - 0.1844i	0.9625 - 0.0688i	0.0910 + 0.0229i	-0.0815 + 0.1209i
0.1114 + 0.1138i	-0.0753 - 0.0560i	0.9201 + 0.2943i	-0.1084 - 0.1446i
-0.0218 - 0.1309i	0.0805 + 0.1215i	0.1446 - 0.1084i	0.9396 - 0.2135i

$\tilde{\mathbf{\Theta}}_{10}$ =

0.9250 - 0.2107i	-0.1449 - 0.1273i	0.0807 - 0.1338i	-0.1800 - 0.0778i
0.1273 - 0.1449i	0.9360 + 0.1128i	-0.1854 + 0.0814i	-0.0832 - 0.1614i
-0.0375 - 0.1517i	0.1886 + 0.0735i	0.9310 - 0.0770i	0.2485 + 0.0158i
0.1823 - 0.0723i	0.0553 - 0.1730i	-0.2485 - 0.0158i	0.9047 + 0.2194i

$\tilde{\mathbf{\Theta}}_{11}$ =

0.9006 - 0.2267i	-0.1948 - 0.1068i	-0.0646 - 0.2283i	0.0718 - 0.1632i
0.2133 - 0.0622i	0.9135 + 0.2858i	-0.1600 + 0.0722i	0.0549 - 0.0275i
0.0277 - 0.2357i	0.1279 + 0.1202i	0.9104 - 0.2221i	-0.1069 + 0.1526i
-0.1234 - 0.1288i	-0.0402 + 0.0464i	0.1526 + 0.1069i	0.9197 - 0.2895i

$\tilde{\mathbf{\Theta}}_{12}$ =

0.9550 - 0.2020i	0.0700 + 0.0209i	-0.0886 - 0.0854i	-0.1024 - 0.1278i
-0.0700 - 0.0209i	0.9582 - 0.0280i	0.1337 - 0.1239i	0.1703 - 0.1161i
0.0854 - 0.0886i	-0.1239 - 0.1337i	0.9739 - 0.0005i	-0.0366 + 0.0425i
0.1278 - 0.1024i	-0.1161 - 0.1703i	-0.0366 + 0.0425i	0.9197 + 0.2859i

$\tilde{\mathbf{\Theta}}_{13}$ =

0.9274 - 0.2418i	0.1685 - 0.1020i	0.0488 - 0.0343i	0.1475 - 0.1320i
-0.1913 - 0.0470i	0.8936 - 0.2315i	0.1478 - 0.1782i	0.0584 + 0.2282i
-0.0320 + 0.0504i	-0.1081 - 0.2047i	0.9136 + 0.2671i	-0.1133 - 0.1548i
-0.1784 - 0.0857i	-0.0333 + 0.2332i	0.1548 - 0.1133i	0.9054 - 0.2207i

$\tilde{\mathbf{\Theta}}_{14}$ =

0.9423 - 0.1401i	0.1080 - 0.1272i	0.2190 - 0.0917i	-0.0019 - 0.0903i
-0.1272 - 0.1080i	0.9538 - 0.1788i	-0.0429 + 0.1382i	-0.0972 - 0.0097i
-0.2197 - 0.0900i	0.0674 + 0.1280i	0.9094 - 0.0778i	0.2982 + 0.0256i

0.0625 - 0.0652i	0.0756 - 0.0619i	-0.2982 - 0.0256i	0.9202 + 0.2141i

$\tilde{\mathbf{\Theta}}_{15}$ =

0.9469 - 0.0481i	0.0945 - 0.1362i	0.1041 - 0.1137i	0.2171 - 0.0514i
-0.0295 - 0.1632i	0.9480 + 0.1088i	-0.0542 + 0.2172i	-0.1063 + 0.0254i
-0.0652 - 0.1397i	0.0330 + 0.2214i	0.9082 - 0.2293i	-0.1367 + 0.1734i
-0.2203 - 0.0356i	0.0641 + 0.0885i	0.1734 + 0.1367i	0.9103 - 0.2466i

$\tilde{\mathbf{\Theta}}_{16}$ =

1.0000 + 0.0000i	-0.0000 - 0.0000i	0.0000 - 0.0000i	0.0000 - 0.0000i
0 - 0.0000i	0.9500 + 0.2179i	0.0000 + 0.0000i	0.0500 - 0.2179i
0 - 0.0000i	0.0000 + 0.0000i	1.0000 + 0.0000i	-0.0000 + 0.0000i
-0.0000 - 0.0000i	0.0500 - 0.2179i	0.0000 + 0.0000i	0.9500 + 0.2179i

The unitary matrices for two transmit antennas at BS with ρ=0.95 are given below.

$\tilde{\mathbf{\Theta}}_{1}$ =

0.9498 + 0.2064i	0.2347 + 0.0128i
-0.2345 + 0.0164i	0.9465 - 0.2211i

$\tilde{\mathbf{\Theta}}_{2}$ =

0.9505 + 0.2699i	0.0165 + 0.1528i
-0.0142 + 0.1530i	0.9463 - 0.2844i

$\tilde{\mathbf{\Theta}}_{3}$ =

0.9485 - 0.2861i	0.1357 + 0.0067i
-0.1357 + 0.0083i	0.9519 + 0.2747i

$\tilde{\mathbf{\Theta}}_{4}$ =

0.9699 + 0.1789i	-0.0374 - 0.1609i
0.0269 - 0.1630i	0.9561 - 0.2419i

$\tilde{\mathbf{\Theta}}_{5}$ =

0.9454 + 0.0369i	-0.2368 + 0.2209i
0.2358 + 0.2220i	0.9456 - 0.0325i

$\tilde{\mathbf{\Theta}}_{6}$ =

0.9744 + 0.0209i	0.1499 + 0.1665i
-0.1702 + 0.1457i	0.9688 + 0.1056i

$\tilde{\mathbf{\Theta}}_{7}$ =

0.9560 - 0.2786i	-0.0767 + 0.0503i
0.0894 + 0.0201i	0.9929 - 0.0752i

$\tilde{\mathbf{\Theta}}_8$ =

$$\begin{matrix} 0.9508 + 0.3097i & -0.0117 + 0.0012i \\ 0.0087 + 0.0079i & 0.9478 + 0.3187i \end{matrix}$$