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Efficient Demodulators for the Full Diversity Full Rate Golden Code

I. INTRODUCTION

It is well-known that multiple-input multiple-output (MIMO) technology can either significantly increase the robustness or boost system throughput by leveraging the inherent spatial diversity and spatial multiplexing gains in wireless systems equipped with multiple antennas [1][2]. More recently, a family of full diversity full rate (FDFR) space time block codes (STBC) has been proposed in [5][6] to provide $\min\{N_t, N_r\}$ multiplexing gain and $N_t N_r$ diversity order simultaneously, where N_t and N_r denote the number of transmit and receive antennas, respectively.

The performance advantage of the FDFR space time codes comes at the cost of higher decoding complexity at receiver. The complexity of the naive implementation of the maximum likelihood (ML) decoder for the FDFR STBC is $O(M^\kappa)$, where κ is number of independent information symbols delivered per (inner) codeword and M is the constellation size. Thus, it is quite desirable to design low complexity FDFR STBC decoding schemes for practical wireless systems. In general, a space time code is said to be fast decodable if the (worst case) complexity of the maximum-likelihood decoder scales no higher than $O(M^{\kappa-\kappa'+1})$, where $\kappa' \leq \kappa$ is the number of removable levels from the complex sphere decoding (SD) tree [7]. For example, for the 2×2 Golden code conveying four information symbols, a fast decoder must have a complexity scaling which is no greater than $O(M^3)$.

In [8], a depth-first sphere decoding algorithm and a VLSI decoder design are proposed for a 2×2 MIMO system which employs the Golden code. Sphere decoding (SD) algorithms aim at finding the ML solution and lowering the complexity of MIMO detection by analyzing only a subset of the solution space [9]. However, SD algorithms are limited by their non-deterministic (channel dependent) complexity. [3] proposes a fast (hard decision) ML decoding scheme for the 2×2 Golden code, which obtains a reduction in the decoding complexity by leveraging the

Golden code structure.

In this contribution we follow in the footsteps of [3] to obtain a fast (hard decision) ML decoding scheme for the 2×2 Golden code having a complexity of $O(M^{2.5})$. Moreover, an efficient soft-output version of $O(M^{2.5})$ complexity is also possible. A suboptimal demodulator with further reduced complexity ($O(M^{0.5})$) is also proposed, which has a better performance compared to the traditional MMSE scheme but with similar complexity.

II. MIMO SYSTEM MODEL

The complex baseband model for a MIMO link employing the Golden code is given by

$$\mathbf{Y} = [\mathbf{Y}_1, \mathbf{Y}_2] = \mathbf{H}\mathbf{X} + \mathbf{V} \quad (1)$$

where \mathbf{H} denotes the $N_r \times 2$ complex-valued channel matrix representing the fading with $N_r \geq 2$. \mathbf{X} denotes the 2×2 FDFR Golden code proposed in [4] which transmits 4 quadrature amplitude modulation (QAM) symbols every two channel uses and hence has a symbol rate of 2. In particular

$$\mathbf{X} = \begin{bmatrix} \cos(\theta)X_n + \sin(\theta)X_{n+1} & \phi^{1/2}(\cos(\theta)X_{n+2} + \sin(\theta)X_{n+3}) \\ \phi^{1/2}(\cos(\theta)X_{n+3} - \sin(\theta)X_{n+2}) & \cos(\theta)X_{n+1} - \sin(\theta)X_n \end{bmatrix}, \quad (2)$$

where $\theta = 0.5 \tan^{-1}(2)$, $\phi = -i$.

We can express (1) in the equivalent real representation as

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{G}}\tilde{\mathbf{x}} + \tilde{\mathbf{v}} \quad (3)$$

where $\tilde{\mathbf{x}} = [X_1^R, X_2^R, X_1^I, X_2^I, X_3^R, X_4^R, X_3^I, X_4^I]^T$; X_n^R and X_n^I are the real and imaginary components of X_n , $n \in \{1 : 4\}$, respectively. $\tilde{\mathbf{G}}$ is an 8×8 matrix determined by the specific structure of the Golden code. $\tilde{\mathbf{H}}$ is the $4N_r \times 8$ matrix determined solely by the fading coefficients and $\tilde{\mathbf{y}} = [\text{Re}\{\mathbf{Y}_1\}^T, \text{Im}\{\mathbf{Y}_1\}^T, \text{Re}\{\mathbf{Y}_2\}^T, \text{Im}\{\mathbf{Y}_2\}^T]^T$.

Assuming that \mathbf{H} is perfectly known at receiver, the optimal detector minimizing the error probability is the maximum likelihood (ML) detector, which solves the following least squares

minimization problem.

$$\tilde{\mathbf{x}}^* = \arg \min_{\tilde{\mathbf{x}} \in \mathcal{O}^8} \left\| \tilde{\mathbf{y}} - \tilde{\mathbf{H}}\tilde{\mathbf{G}}\tilde{\mathbf{x}} \right\|^2 \quad (4)$$

where \mathcal{O} stands for the set of underlying real-valued scalar constellation points in \sqrt{M} -PAM. It is clear that an exhaustive search of ML solution has the complexity M^4 .

III. EFFICIENT ML DEMODULATOR FOR THE 2×2 FDFR GOLDEN CODE

We now consider ML detection. We use the techniques of [3], where an efficient ML detector was derived. Moreover, an efficient soft-output demodulator, which obtains the LLRs for all the coded bits associated with the four QAM symbols with an $O(M^{2.5})$ complexity, can also be obtained.

We start by obtaining the QR decomposition of the equivalent MIMO channel $\tilde{\mathbf{H}}\tilde{\mathbf{G}}$ in (3). In particular, we have

$$\tilde{\mathbf{H}}\tilde{\mathbf{G}} = \tilde{\mathbf{\Phi}}\tilde{\mathbf{L}} \quad (5)$$

where $\tilde{\mathbf{L}} = [\tilde{\mathbf{l}}_1, \dots, \tilde{\mathbf{l}}_8]$ is an 8×8 lower triangular matrix with positive diagonal elements. $\tilde{\mathbf{\Phi}}$ is a semi-unitary matrix with $\tilde{\mathbf{\Phi}}^T \tilde{\mathbf{\Phi}} = \mathbf{I}$.

We shall first minimize over symbols $\{X_3, X_4\}$ by fixing the choice of $\{X_1, X_2\}$. For convenience, we define the real-valued vectors $\tilde{\mathbf{z}}_{12} = [X_1^R, X_2^R, X_1^I, X_2^I]^T$ for symbols $\{X_1, X_2\}$ and $\tilde{\mathbf{z}}_{34} = [X_3^R, X_4^R, X_3^I, X_4^I]^T$ for symbols $\{X_3, X_4\}$. Next, we obtain

$$\tilde{\mathbf{w}} = \tilde{\mathbf{\Phi}}^T \tilde{\mathbf{y}} = \tilde{\mathbf{L}}\tilde{\mathbf{x}} + \tilde{\mathbf{q}} \quad (6)$$

and note that the transformed noise vector $\tilde{\mathbf{q}}$ remains white. The maximum likelihood detector now chooses $\tilde{\mathbf{x}}$ so as to minimize

$$\tilde{\mathbf{x}}^* = \arg \min_{\tilde{\mathbf{x}} \in \mathcal{O}^8} \left\| \tilde{\mathbf{w}} - \tilde{\mathbf{L}}\tilde{\mathbf{x}} \right\|^2 \quad (7)$$

For any fixed choice of $\tilde{\mathbf{z}}_{12}$, i.e. $\tilde{\mathbf{z}}_{12}^0$, defining $\tilde{\mathbf{b}} = \tilde{\mathbf{w}} - [\tilde{\mathbf{l}}_1, \dots, \tilde{\mathbf{l}}_4]\tilde{\mathbf{z}}_{12}^0$, the minimization in (7) reduces to

$$\min_{\tilde{\mathbf{z}}_{34} \in \mathcal{O}^4} \left\| \tilde{\mathbf{w}} - \tilde{\mathbf{L}}\tilde{\mathbf{x}} \right\|^2 = \min_{\tilde{\mathbf{z}}_{34} \in \mathcal{O}^4} \left\| \tilde{\mathbf{b}} - [\tilde{\mathbf{l}}_5, \dots, \tilde{\mathbf{l}}_8]\tilde{\mathbf{z}}_{34} \right\|^2 \quad (8)$$

The particular structure of the Golden code of [4] ensures a nice structure of $\tilde{\mathbf{L}}$ which dramatically reduces the detection complexity. It is shown in the Appendix that the overall complexity of determining the optimal $\tilde{\mathbf{z}}_{34}^*$ in (8) for any given $\tilde{\mathbf{z}}_{12}^0$ (i.e. $\{X_1^0, X_2^0\}$) is $O(M^{0.5})$. Thus, by considering all the M^2 combinations of symbols X_1 and X_2 , we can determine the ML decision with $O(M^{2.5})$ complexity.

IV. LOW COMPLEXITY SUBOPTIMAL GROUP MMSE DEMODULATOR FOR THE 2×2 FDFR GOLDEN CODE

We shall now derive a suboptimal demodulator with further reduced complexity ($O(M^{0.5})$) by grouping the symbols and suppressing one of the groups via MMSE filtering.

Using the same real-vector model as in (3), we define $\tilde{\Psi} = \tilde{\mathbf{H}}\tilde{\mathbf{G}} = [\tilde{\Psi}_1 \quad \tilde{\Psi}_2]$. Thus, we have

$$\tilde{\mathbf{y}} = \tilde{\Psi}_1 \tilde{\mathbf{z}}_{12} + \tilde{\Psi}_2 \tilde{\mathbf{z}}_{34} + \tilde{\mathbf{v}} \quad (9)$$

To suppress $\tilde{\Psi}_2 \tilde{\mathbf{z}}_{34}$, we use the linear MMSE filter given by

$$\mathbf{W}_1 = \tilde{\Psi}_1^T (\mathbf{I} + \tilde{\Psi}_2 \tilde{\Psi}_2^T)^{-1} \quad (10)$$

Then we have

$$\tilde{\mathbf{y}}_1 = \mathbf{W}_1 \tilde{\mathbf{y}} = \tilde{\Psi}_1^T (\mathbf{I} + \tilde{\Psi}_2 \tilde{\Psi}_2^T)^{-1} \tilde{\mathbf{y}} \quad (11)$$

$$= \tilde{\Psi}_1^T (\mathbf{I} + \tilde{\Psi}_2 \tilde{\Psi}_2^T)^{-1} \tilde{\Psi}_1 \tilde{\mathbf{z}}_{12} + \tilde{\Upsilon}_1 \quad (12)$$

where $\tilde{\Upsilon}_1$ is assumed to be a Gaussian distributed vector with the distribution $\tilde{\Upsilon}_1 \sim N(\mathbf{0}, 2\tilde{\Psi}_1^T (\mathbf{I} + \tilde{\Psi}_2 \tilde{\Psi}_2^T)^{-1} \tilde{\Psi}_1)$.

Notice that $\tilde{\Psi}_1^T (\mathbf{I} + \tilde{\Psi}_2 \tilde{\Psi}_2^T)^{-1} \tilde{\Psi}_1$ has the following structure, which we can take advantage of to reduce the overall complexity,

$$\tilde{\Psi}_1^T (\mathbf{I} + \tilde{\Psi}_2 \tilde{\Psi}_2^T)^{-1} \tilde{\Psi}_1 = \begin{bmatrix} \mathbf{B}_{2 \times 2} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & \mathbf{B}_{2 \times 2} \end{bmatrix} \quad (13)$$

Recall that $\tilde{\mathbf{z}}_{12} = [X_1^R, X_2^R, X_1^I, X_2^I]^T$ so the joint demodulation of $\tilde{\mathbf{z}}_{12}$ after LMMSE filtering

can be thus split optimally into separate joint demodulation of $[X_1^R, X_2^R]^T$ and $[X_1^I, X_2^I]^T$. It can be shown that the joint demodulation of $[X_1^R, X_2^R]^T$ (or $[X_1^I, X_2^I]^T$) has the complexity of $\mathcal{O}(M^{0.5})$.

Similarly, we can apply the following MMSE filtering to $\tilde{\mathbf{y}}$ to suppress X_1 and X_2

$$\begin{aligned}\tilde{\mathbf{y}}_2 &= \tilde{\Psi}_2^T (\mathbf{I} + \tilde{\Psi}_1 \tilde{\Psi}_1^T)^{-1} \tilde{\mathbf{y}} \\ &= \tilde{\Psi}_2^T (\mathbf{I} + \tilde{\Psi}_1 \tilde{\Psi}_1^T)^{-1} \tilde{\Psi}_2 \tilde{\mathbf{z}}_{34} + \tilde{\Upsilon}_2\end{aligned}$$

where $\tilde{\Upsilon}_2$ is assumed to be a Gaussian distributed vector with the distribution $\tilde{\Upsilon}_2 \sim N(\mathbf{0}, 2\tilde{\Psi}_2^T(\mathbf{I} + \tilde{\Psi}_1 \tilde{\Psi}_1^T)^{-1}\tilde{\Psi}_2)$. The matrix $\tilde{\Psi}_2^T(\mathbf{I} + \tilde{\Psi}_1 \tilde{\Psi}_1^T)^{-1}\tilde{\Psi}_2$ also has the block-diagonal form shown in (13) and hence post-filtering the demodulation of symbols X_3 and X_4 can be performed with the complexity of $\mathcal{O}(M^{0.5})$.

V. CONCLUSION

We provided low complexity demodulators for the 2×2 FDFR Golden code. In particular, the hard decision maximum likelihood (ML) detector as well as its soft-output version were shown to have $\mathcal{O}(M^{2.5})$ complexity for M -QAM input symbols. A sub-optimal demodulator with a further reduced complexity (of $\mathcal{O}(M^{0.5})$) was also presented. The proposed demodulators are suitable for practical implementation due their low computation complexities and hence support the inclusion of the FDFR Golden code as a candidate open-loop 2 TX block code.

APPENDIX 1: FINDING THE OPTIMAL $\tilde{\mathbf{z}}_{34}^*$ FOR A GIVEN $\tilde{\mathbf{z}}_{12} = \tilde{\mathbf{z}}_{12}^0$

For the FDFR code defined in (2), the QR decomposition of $\tilde{\mathbf{H}}\tilde{\mathbf{G}}$ defined in (5) produces a special $[\tilde{\mathbf{I}}_5, \dots, \tilde{\mathbf{I}}_8]$ (an 8×4 matrix) with the following feature, which can be utilized to substantially reduce the detection complexity.

$$[\tilde{\mathbf{I}}_5, \dots, \tilde{\mathbf{I}}_8] = \begin{bmatrix} \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} \\ \mathbf{A}_{2 \times 2} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & \mathbf{A}_{2 \times 2} \end{bmatrix} \text{ and } \mathbf{A}_{2 \times 2} = \begin{bmatrix} a_1 & 0 \\ a_2 & a_3 \end{bmatrix} \quad (14)$$

where $a_i, i = 1, 2, 3$ are real-valued scalars determined by the QR decomposition of $\tilde{\mathbf{H}}\tilde{\mathbf{G}}$.

Expanding $\tilde{\mathbf{b}}$ as $\tilde{\mathbf{b}} = [b_1, b_2, \dots, b_8]^T$, the optimal $\tilde{\mathbf{z}}_{34}^*$ for any given $\tilde{\mathbf{z}}_{12} = \tilde{\mathbf{z}}_{12}^0$, can be obtained as

$$\tilde{\mathbf{z}}_{34}^* = \arg \min_{\tilde{\mathbf{z}}_{34} \in \mathcal{O}^4} \left\| \tilde{\mathbf{b}} - [\tilde{\mathbf{l}}_5, \dots, \tilde{\mathbf{l}}_8] \tilde{\mathbf{z}}_{34} \right\|^2 \quad (15)$$

$$= \arg \min_{\{X_3^R, X_4^R\} \in \mathcal{O}^2} \left\| \begin{bmatrix} b_5 \\ b_6 \end{bmatrix} - \mathbf{A}_{2 \times 2} \begin{bmatrix} X_3^R \\ X_4^R \end{bmatrix} \right\|^2 + \arg \min_{\{X_3^I, X_4^I\} \in \mathcal{O}^2} \left\| \begin{bmatrix} b_7 \\ b_8 \end{bmatrix} - \mathbf{A}_{2 \times 2} \begin{bmatrix} X_3^I \\ X_4^I \end{bmatrix} \right\|^2 \quad (16)$$

where \mathcal{O} stands for the set of underlying real-valued constellation points in \sqrt{M} -PAM. Notice that the search complexity for $\tilde{\mathbf{z}}_{34}^*$ is reduced from \mathcal{O}^4 to \mathcal{O}^2 due to the special structure of $\tilde{\mathbf{L}}$. We can further reduce the search complexity as follows.

$$\min_{X_3^R, X_4^R \in \mathcal{O}} \left\| \begin{bmatrix} b_5 \\ b_6 \end{bmatrix} - \mathbf{A}_{2 \times 2} \begin{bmatrix} X_3^R \\ X_4^R \end{bmatrix} \right\|^2 = \min_{X_3^R \in \mathcal{O}} \left\{ \left| b_5 - a_1 X_3^R \right|^2 + a_3^2 \underbrace{\min_{X_4^R \in \mathcal{O}} \left| \frac{b_6 - a_2 X_3^R}{a_3} - X_4^R \right|^2}_{\mathcal{O}(1) \text{ complexity}} \right\}$$

Thus, for each choice of $\tilde{\mathbf{z}}_{12}$ (i.e. symbols $\{X_1, X_2\}$), we can determine the best $\tilde{\mathbf{z}}_{34}$ (i.e. symbols $\{X_3, X_4\}$) with a complexity that is $\mathcal{O}(M^{0.5})$.

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PROPOSED TEXT

[Modify the text in section 11.8.2.1]

For OL SU-MIMO, the following schemes are FFS: 2 Tx rate-2 Golden code, 4Tx rate-1 SFBC + Antenna hopping, 4Tx rate-2 Double SFBC + Antenna hopping, 4Tx rate-2 SM + Antenna hopping, 4Tx rate-3 SM + Antenna hopping, 4Tx rate-3 hybrid SM + SFBC + Antenna hopping.

[Modify the text in section 11.12.2.1.1]

For OL SU-MIMO, the following schemes are FFS: rate-1 STBC/SFBC and rate-2 Double STBC/SFBC, 2 TX Golden code for rate-2, 2-DPOD for rate-1 and rate-2, rate-3 hybrid SM+STBC/SFBC, differential STBC/SFBC, Antenna hopping, and SM+Antenna hopping.