Project	**IEEE 802.16 Broadband Wireless Access Working Group** <http://ieee802.org/16>
Title	**Comments to 16m EVM document: SINR Computation for Multi-Codeword Chase-Combining Hybrid ARQ Systems**
Date Submitted	**2008-05-05**
Source(s)	Narayan Prasad NEC Laboratories America, Inc. Xiaodong Wang NEC Laboratories America, Inc Yuefeng Zhou NEC Europe E-mail: yuefeng.zhou@eu.nec.com
Re:	IEEE 802.16m-08/016r1, Call for SDD Comments and Contributions (2008-03-20)
Abstract	This contribution proposes a post-processing SINR calculation method for multi-codeword MIMO systems with HARQ for 16m EVM document
Purpose	To be discussed and approved by TGm.
Notice	*This document does not represent the agreed views of the IEEE 802.16 Working Group or any of its subgroups*. It represents only the views of the participants listed in the "Source(s)" field above. It is offered as a basis for discussion. It is not binding on the contributor(s), who reserve(s) the right to add, amend or withdraw material contained herein.
Release	The contributor grants a free, irrevocable license to the IEEE to incorporate material contained in this contribution, and any modifications thereof, in the creation of an IEEE Standards publication; to copyright in the IEEE's name any IEEE Standards publication even though it may include portions of this contribution; and at the IEEE's sole discretion to permit others to reproduce in whole or in part the resulting IEEE Standards publication. The contributor also acknowledges and accepts that this contribution may be made public by IEEE 802.16.
Patent Policy	The contributor is familiar with the IEEE-SA Patent Policy and Procedures: <http://standards.ieee.org/guides/bylaws/sect6-7.html#6> and <http://standards.ieee.org/guides/opman/sect6.html#6.3>. Further information is located at <http://standards.ieee.org/board/pat/pat-material.html> and <http://standards.ieee.org/board/pat>.

SINR Computation for Multi-Codeword Chase-Combining Hybrid ARQ Systems

Narayan Prasad & Xiaodong Wang

NEC

INTRODUCTION

In this contribution, we consider MIMO systems with Chase Combining HARQ (CC-HARQ). The current evaluation methodology document [1] considers the PHY abstraction methodology for HARQ systems which is applicable to bit-level combining. Recently, [2] has provided the PHY abstraction methodology for the optimal (pre-equalization) symbol level combining in single-codeword (SCW) MIMO CC-HARQ systems. We complete the picture by providing the PHY abstraction methodology for symbol level combining in CC-HARQ multi-codeword (MCW) MIMO systems.

CC-HARQ WITH BLANKING

We first consider an MCW CC-HARQ system with blanking. In such a system, $K \geq 1$ codewords are first simultaneously transmitted using K physical (or virtual) antennas, which are then decoded by the receiver. The receiver sends a separate ACK/NACK for every codeword. Upon receiving the per-codeword ACK/NACK, the transmitter re-transmits only those codewords for which a NACK was received. A new set of K codewords can be transmitted only if no NACKs are received or if the maximum re-transmission limit is reached without the successful decoding of at-least one codeword. Thus, the discrete-time model of the signal received during a symbol interval on the n^{th} sub-carrier during the i^{th} round can be written as[1]

$$\boldsymbol{y}_i = \boldsymbol{H}_i \boldsymbol{s}_i + \boldsymbol{n}_i, \tag{1}$$

[1]For notational convenience we do not indicate the tone index n.

where $\boldsymbol{H}_i = [\boldsymbol{h}_{i,1}, \cdots, \boldsymbol{h}_{i,K}]$ is an $N_r \times K$ matrix denoting the channel response seen at the n^{th} sub-carrier. $\boldsymbol{n}_i$ models the noise plus interference and is assumed to have a covariance matrix $\boldsymbol{\Sigma}_i$. Let $\mathcal{U}_i \subseteq \{1, \cdots, K\}$ denote the set containing the indices of the undecoded codewords at the start of the i^{th} round so that $\mathcal{U}_1 = K$. The symbol vector $\boldsymbol{s}_i = [s_{i,1}, \cdots, s_{i,K}]^T$ contains the re-transmitted symbols from the codewords having indices in $\mathcal{U}_i$, i.e., $s_{i,j} = s_j, \ \forall j \in \mathcal{U}_i$ and $s_{i,j} = 0, \ \forall j \notin \mathcal{U}_i$. Thus, without loss of optimality, we can set $\boldsymbol{h}_{i,j} = \mathbf{0}, \ \forall j \notin \mathcal{U}_i$. Post whitening the model in (1) can be written as:

$$\tilde{\boldsymbol{y}}_i = \boldsymbol{\Sigma}_i^{-1/2} \boldsymbol{y}_i = \tilde{\boldsymbol{H}}_i \boldsymbol{s}_i + \tilde{\boldsymbol{n}}_i, \tag{2}$$

where $\tilde{\boldsymbol{H}}_i = \boldsymbol{\Sigma}_i^{-1/2} \boldsymbol{H}_i = [\tilde{\boldsymbol{h}}_{i,1}, \cdots, \tilde{\boldsymbol{h}}_{i,K}]$.

We first consider the symbol-level combining before MIMO equalization, in the case when the correctly decoded codewords after any round are not subtracted from the received observations. Under this restriction, the concatenated received observations for the i^{th} round can be modeled as

$$\tilde{\boldsymbol{y}}_{(i)} = \tilde{\boldsymbol{H}}_{(i)} \boldsymbol{s} + \tilde{\boldsymbol{n}}_{(i)}, \tag{3}$$

where $\boldsymbol{s} = [s_1, \cdots, s_K]^T$ and $\tilde{\boldsymbol{H}}_{(i)} = [\tilde{\boldsymbol{H}}_1^T, \cdots, \tilde{\boldsymbol{H}}_i^T]^T$. Consequently, the resulting SINR at the LMMSE equalizer output can be be written as

$$\begin{aligned} \text{SINR}_{i,k}^{\text{eff}} &= \frac{1}{(\boldsymbol{I} + \tilde{\boldsymbol{H}}_{(i)}^{\dagger} \tilde{\boldsymbol{H}}_{(i)})_{k,k}^{-1}} - 1 \\ &= \frac{1}{\left(\boldsymbol{I} + \sum_{j=1}^{i} \tilde{\boldsymbol{H}}_j^{\dagger} \tilde{\boldsymbol{H}}_j\right)_{k,k}^{-1}} - 1 \\ &= \frac{1}{\left(\boldsymbol{I} + \tilde{\boldsymbol{A}}_i\right)_{k,k}^{-1}} - 1, \ \forall k \in \mathcal{U}_i, \end{aligned} \tag{4}$$

where $\tilde{\boldsymbol{A}}_i = \sum_{j=1}^{i} \tilde{\boldsymbol{H}}_j^{\dagger} \tilde{\boldsymbol{H}}_j = \sum_{j=1}^{i} \boldsymbol{H}_j^{\dagger} \boldsymbol{\Sigma}_j^{-1} \boldsymbol{H}_j$.

On the other hand, if the correctly decoded codewords can be subtracted from the received observations during the time interval between two consecutive rounds, then the concatenated received observations post-subtractions for the i^{th} round (which also represent the sufficient statistics) can be written as

$$\tilde{\boldsymbol{z}}_{(i)} = \tilde{\boldsymbol{G}}_{(i)}\boldsymbol{s}_i + \tilde{\boldsymbol{n}}_{(i)}, \tag{5}$$

where $\tilde{\boldsymbol{z}}_{(i)} = [\tilde{\boldsymbol{z}}_1, \tilde{\boldsymbol{z}}_2 \cdots, \tilde{\boldsymbol{z}}_i]$ with $\tilde{\boldsymbol{z}}_j = \tilde{\boldsymbol{y}}_j - \sum_{k \in \mathcal{U}_i^c} \tilde{\boldsymbol{h}}_{j,k} s_k, \;\; 1 \leq j \leq i-1$ and $\tilde{\boldsymbol{G}}_{(i)} = [\tilde{\boldsymbol{G}}_1^T, \cdots, \tilde{\boldsymbol{G}}_i^T]^T$ with $\tilde{\boldsymbol{G}}_j = [\tilde{\boldsymbol{g}}_{j,1}, \cdots, \tilde{\boldsymbol{g}}_{j,K}]$ with $\tilde{\boldsymbol{g}}_{j,k} = \tilde{\boldsymbol{h}}_{j,k}, \; \forall k \in \mathcal{U}_i$ and $\tilde{\boldsymbol{g}}_{j,k} = \boldsymbol{0}, \; \forall k \notin \mathcal{U}_i$ for $1 \leq j \leq i$. Thus, the SINR at the LMMSE equalizer output can be be now be written as

$$\begin{aligned} \text{SINR}_{i,k}^{\text{eff}} &= \frac{1}{(\boldsymbol{I} + \tilde{\boldsymbol{G}}_{(i)}^{\dagger}\tilde{\boldsymbol{G}}_{(i)})_{k,k}^{-1}} - 1 \\ &= \frac{1}{(\boldsymbol{I} + \sum_{j=1}^{i} \tilde{\boldsymbol{G}}_j^{\dagger}\tilde{\boldsymbol{G}}_j)_{k,k}^{-1}} \\ &= \frac{1}{(\boldsymbol{I} + \tilde{\boldsymbol{B}}_i)_{k,k}^{-1}} - 1, \; \forall k \in \mathcal{U}_i \end{aligned} \tag{6}$$

where $\tilde{\boldsymbol{B}}_i = \sum_{j=1}^{i} \tilde{\boldsymbol{G}}_j^{\dagger}\tilde{\boldsymbol{G}}_j$. Note that the matrix $\tilde{\boldsymbol{B}}_i$ can be obtained from the matrix $\tilde{\boldsymbol{A}}_i$ by setting the rows and columns of the matrix $\tilde{\boldsymbol{A}}_i$ with indices not in $\mathcal{U}_i$ to be the vector of all zeros.

Finally, note that we can use other decoders (such as the MMSE-SIC) as well on the models in (3) and (5).

CC-HARQ without Blanking

We now consider an MCW CC-HARQ system without blanking. In such a system, $K \geq 1$ codewords are first simultaneously transmitted using K physical (or virtual) antennas, which are then decoded by the receiver. The receiver sends a separate ACK/NACK for every codeword. Upon receiving the per-codeword ACK/NACK, the transmitter re-transmits those codewords for which a NACK was received along with a new codeword for every ACK. A packet error is declared if a codeword cannot be successfully decoded after a maximum number, say Q, of transmissions. Thus, K codewords are transmitted in every round. The discrete-time model of the signal received during a symbol interval on the n^{th} sub-carrier during the i^{th} round after whitening, can be written as in (2) but where $\boldsymbol{s}_i$ is a vector containing K non-zero modulated symbols and hence $\tilde{\boldsymbol{H}}_i$ is a matrix with all non-zero columns. Hence the effective SINR obtained at the i^{th} round after q transmissions of the k^{th} codeword and symbol-level combining after

MMSE equalization is given by

$$\mathrm{SINR}_{i,k}^{\mathrm{eff}} = \sum_{j=i-q+1}^{i} \left(\frac{1}{\left(\boldsymbol{I} + \tilde{\boldsymbol{H}}_j^{\dagger} \tilde{\boldsymbol{H}}_j\right)_{k,k}^{-1}} - 1 \right). \tag{7}$$

Now let us consider the the effective SINR for the k^{th} codeword obtained with symbol-level combining before MIMO equalization. We can adopt the approach used in the previous section and determine the SINR using the concatenated received statistics. Towards this end, we refer to a round as a regeneration round if K new codewords are transmitted during that round. Then consider the i^{th} round and let $1 \leq t \leq i$ be the index of the recent most regeneration round. Supposing that subtraction of codewords from the received observations is not allowed, we can write the concatenated received sufficient statistics as,

$$\tilde{\boldsymbol{y}}_{(i)} = \underbrace{[\tilde{\boldsymbol{H}}_{(i)}^{\mathrm{eff}}, \tilde{\boldsymbol{H}}_{(i)}^{\mathrm{int}}]}_{\tilde{\boldsymbol{F}}_{(i)}} [\boldsymbol{s}_i; \boldsymbol{s}_i^{\mathrm{int}}] + \tilde{\boldsymbol{n}}_{(i)}, \tag{8}$$

where $\boldsymbol{s}_i = [s_{i,1}, \cdots, s_{i,K}]^T$ is the symbol vector transmitted during the symbol interval of the i^{th} round, $\tilde{\boldsymbol{y}}_{(i)} = [\tilde{\boldsymbol{y}}_t, \cdots, \tilde{\boldsymbol{y}}_i]$. The symbols not corresponding to the K codewords of the i^{th} round are collected in the vector $\boldsymbol{s}_i^{\mathrm{int}}$. Consequently, the resulting SINR at the LMMSE equalizer output can be be written as

$$\mathrm{SINR}_{i,k}^{\mathrm{eff}} = \frac{1}{(\boldsymbol{I} + [\tilde{\boldsymbol{H}}_{(i)}^{\mathrm{eff}}, \tilde{\boldsymbol{H}}_{(i)}^{\mathrm{int}}]^{\dagger} [\tilde{\boldsymbol{H}}_{(i)}^{\mathrm{eff}}, \tilde{\boldsymbol{H}}_{(i)}^{\mathrm{int}}])_{k,k}^{-1}} - 1, \ \forall k \in \{1, \cdots, K\}. \tag{9}$$

On the other hand if the correctly decoded codewords can be subtracted from the received observations during the time interval between two consecutive rounds, we can still expand the concatenated received statistics post-subtraction, as in the RHS of (8) but where $\boldsymbol{s}_i^{\mathrm{int}}$ now contains the symbols corresponding to codewords that could not be correctly decoded even after the maximum number of transmissions. Thus, the SINR is given by (9) after using the appropriate $\tilde{\boldsymbol{H}}_{(i)}^{\mathrm{eff}}, \tilde{\boldsymbol{H}}_{(i)}^{\mathrm{int}}$.

The SINR computation based on the concatenated received statistics in (8) (with or without codeword subtraction) has the drawback that in order to perform this type of combining, the receiver must store all the received observations starting from the recent-most regeneration round. Moreover, for any positive maximum re-transmission limit, the memory required may grow unbounded, making such a combining scheme impractical. Thus, practical schemes will use some combination of post-equalization and pre-equalization combining schemes. One such scheme, where correctly decoded codewords are subtracted from the received observations between two consecutive rounds, is suggested in the following. The main underlying idea is that we reset the buffers whenever one or more codewords cannot be correctly decoded even after the maximum number of transmissions, after storing the SINRs computed for the remaining erroneously decoded codewords as base SINRs. The effective SINRs for such codewords are then obtained as the sum of these base SINRs and the SINRs obtained via pre equalization symbol level combining which is done using only the subsequent received observations. An efficient combining scheme is presented in the algorithm below.

Algorithm 1 Efficient CC-HARQ for MCW Systems without Blanking: SINR Computation

Initialize $\{\text{SINR}_k^{\text{base}} = 0, q_k = 0\}_{k=1}^{K}, t = 0, \tilde{\boldsymbol{S}} = []$.
while $t \leq T_{\text{finish}}$ **do**
  Update $t \to t+1,\ \tilde{\boldsymbol{S}} \to \tilde{\boldsymbol{S}} + \tilde{\mathbf{H}}_t^{\dagger}\tilde{\mathbf{H}}_t$ and $\{q_k \to q_k + 1\}_{k=1}^{K}$.
  Compute $\text{SINR}_k = \frac{1}{(\boldsymbol{I}+\tilde{\boldsymbol{S}})^{-1}_{k,k}} - 1$.
  Update $\{\text{SINR}_{t,k}^{\text{eff}} \to \text{SINR}_k^{\text{base}} + \text{SINR}_k\}_{k=1}^{K}$ and use them for PHY abstraction and simulating codeword error events.
  Let $\mathcal{U}$ denote the set of users for which codeword errors occur and let $\mathcal{C} = \mathcal{U}^c$ denote its complement.
  For each $i \in \mathcal{C}$, set $q_i = 0$ & $\text{SINR}_i^{\text{base}} = 0$ and replace the i^{th} row and column of $\tilde{\boldsymbol{S}}$ with the K-length vector of all zeros.
  if there is a user $p \in \mathcal{U}$ such that $q_p = Q$ **then**
    For each user $p \in \mathcal{U}$ such that $q_p = Q$, declare a packet error and set $q_p = 0$ & $\text{SINR}_p^{\text{base}} = 0$.
    For each user $p \in \mathcal{U}$ such that $q_p < Q$, set $\text{SINR}_p^{\text{base}} = \text{SINR}_{t,p}^{\text{eff}}$.
    Set $\tilde{\boldsymbol{S}} = []$.
  end if
end while

CONCLUSION

The current draft of the evaluation methodology document provides the PHY abstraction methodology for bit-level combining and post-equalization symbol level combining. Recently, a method for pre-equalization symbol level combining in MIMO single-codeword CC HARQ systems has been proposed. We have proposed pre-equalization symbol level combining in MIMO CC HARQ multi-codeword systems with and without blanking. We have also proposed an efficient symbol level combining method for MIMO CC HARQ multi-codeword systems without blanking, that is a combination of pre and post equalization combining.

REFERENCES

[1] IEEE 802.16m 08/004r1. IEEE 802.16m evaluation methodology document. March 2008.

[2] IEEE 802.16m 08/054r1. Post-processing SINR calculation for MIMO H-ARQ. March 2008.

PROPOSED TEXT

[Insert new sections 4.7.1. and 4.7.2. after 4.7.]

4.7.1. Symbol-Level Combining before MIMO Equalization in Multi-Codeword CC HARQ Systems with Blanking

For symbol level combining before MIMO equalization in multi-codeword CC HARQ systems with blanking, the post-processing SINR can be obtained by computing the the post-processing SINR of the combined received signal.

As in Section 4.4.4., for the i^{th} transmission, the received signal at the n^{th} subcarrier is given by

$$\underline{Y}_i^{(0)}(n) = \sqrt{P_{tx,i}^{(0)} P_{loss,i}^{(0)}} \underline{H}_i^{(0)}(n) \underline{X}_i^{(0)}(n) + \sum_{j=1}^{N_I} \sqrt{P_{tx,i}^{(j)} P_{loss,i}^{(j)}} \underline{H}_i^{(j)}(n) \underline{X}_i^{(j)}(n) + \underline{U}_i^{(0)}(n), \tag{10}$$

where the subscript i denotes the transmission index. In the i^{th} transmission $\underline{X}_i^{(0)}(n)$ is a $K \times 1$ vector. The non-zero elements of $\underline{X}_i^{(0)}(n)$ are the re-transmitted symbols which correspond to the codewords which have not yet been correctly decoded. Let $\mathcal{U}_i$ denote the set containing the indices of such codewords and let $\underline{D}_i = \mathrm{diag}\{d_1, \cdots, d_K\}$ be a diagonal matrix such that $d_k = 1,\ k \in \mathcal{U}_i$ and $d_k = 0,\ k \notin \mathcal{U}_i$. Define $\underline{\eta}_i(n) = \sum_{j=1}^{N_I} \sqrt{P_{tx,i}^{(j)} P_{loss,i}^{(j)}} \underline{H}_i^{(j)}(n) \underline{X}_i^{(j)}(n) + \underline{U}_i^{(0)}(n)$ and let the covariance matrix of $\underline{\eta}_i(n)$ be denoted by $\underline{\Sigma}_i(n)$.

In the case when correctly decoded codewords are not subtracted from the received observations, the post-processing SINR for the k^{th} codeword on the n^{th} subcarrier using the linear MMSE equalizer on the combined received signal is given by

$$\frac{1}{(\underline{I} + \sum_{m=1}^{i} P_{tx,m}^{(0)} P_{loss,m}^{(0)} \sigma_0^2 \underline{D}_m \underline{H}_m^{(0)\,*}(n) \underline{\Sigma}_m^{-1}(n) \underline{H}_m^{(0)}(n) \underline{D}_m^{(0)}(n))_{k,k}^{-1}} - 1,\ k \in \mathcal{U}_i. \tag{11}$$

Next, in the case when correctly decoded codewords can be subtracted from the received observations during the time interval between two consecutive re-transmissions, the post-processing SINR for the k^{th} codeword on the n^{th} subcarrier using the linear MMSE equalizer on the combined signal, is given by

$$\frac{1}{(\underline{I} + \sum_{m=1}^{i} P_{tx,m}^{(0)} P_{loss,m}^{(0)} \sigma_0^2 \underline{D}_i \underline{H}_m^{(0)\,*}(n) \underline{\Sigma}_m^{-1}(n) \underline{H}_m^{(0)}(n) \underline{D}_i^{(0)}(n))_{k,k}^{-1}} - 1,\ k \in \mathcal{U}_i. \tag{12}$$

4.7.2. Symbol-Level Combining in Multi-Codeword CC HARQ Systems without Blanking

The received signal at the n^{th} subcarrier, for the i^{th} transmission is given by Equation (10). Notice that now the $K \times 1$ vector $\underline{X}_i^{(0)}(n)$ contains re-transmitted symbols for the codewords which have not yet been correctly decoded and its remaining elements are symbols corresponding to new codewords that are being transmitted for the first time. Then, the post-processing SINR obtained after q transmissions of the k^{th} codeword on the n^{th} subcarrier, by symbol level combining after linear MMSE equalization, is given by

$$\sum_{m=i-q+1}^{i} \left(\frac{1}{(\underline{I} + P_{tx,m}^{(0)} P_{loss,m}^{(0)} \sigma_0^2 \underline{H}_m^{(0)\,*}(n) \underline{\Sigma}_m^{-1}(n) \underline{H}_m^{(0)}(n))_{k,k}^{-1}} - 1 \right). \tag{13}$$

The post-processing SINR for pre-equalization symbol-level combining can be obtained using the concatenated received signal model given by

$$\underline{Y}_{(i)}^{(0)}(n) = \underline{H}_{(i)}^{(0)}(n) \underline{X}_{(i)}^{(0)}(n) + \underline{\eta}_{(i)}(n), \tag{14}$$

where

$$\underline{Y}_{(i)}^{(0)}(n) = [\underline{Y}_t^{(0)}(n); \cdots ; \underline{Y}_i^{(0)}(n)],\ \underline{X}_{(i)}^{(0)}(n) = [\underline{X}_t^{(0)}(n); \cdots ; \underline{X}_i^{(0)}(n)], \tag{15}$$

and $t : 1 \leq t \leq i$, is the index of the recent-most transmission when K new codewords are transmitted. Further, when codeword subtraction during the time between transmissions is possible, the SINR can be computed using the modified concatenated received signal model obtained after subtracting all the correctly decoded codewords from the received observations in (14).