New CINR Formula for ML Decoded SM

Document Number: IEEE C802.16m-08/359

Date Submitted: 2008-05-05

Source:

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Re:

IEEE 802.16m-08/016r1 Call for Contributions on Project 802.16m SDD: **Downlink MIMO Schemes**

Purpose:

To suggest a new CINR formula for ML decoded SM

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Outline

- Introduction
- The 802.16e Solution
- Problem Examples
- Proposed Method
- Implementation Point of View
- Simulation Results

Introduction

- Per tone post processing CINR (ppCINR) is a crucial concept in MIMO techniques, that underlies link adaptation and mode selection.
- The expression for the ppCINR is elementary in linearly decoded systems (e.g. SISO, MRC, STC, ZF decoded SM).
- However, the situation is different in the case of ML decoded SM – no simple expression.

The 802.16e Solution

- We consider the following SM model

$$\mathbf{y} = \mathbf{Hs} + \rho\,\mathbf{n}, \tag{1}$$

- The 802.16e standard adopted an information theoretic metric

$$C = \frac{1}{N}\log\det\left(\mathbf{I} + \frac{\mathbf{H}^*\mathbf{H}}{\rho^2}\right), \qquad \mathrm{ppCINR} = e^C - 1, \tag{2}$$

- Here N is the number of spatial streams and ρ^2 is the noise and interference variance.

The 802.16e Solution – cont.

- The metric (2) coincides with the elementary ppCINR in the case of single stream linearly decoded transmission (MRC, STC, etc.)
- However, Eq. (2) also implies two serious drawbacks:

  - It leads to inconsistent results in many situations and thus suffers from limited accuracy.
  - It may not be extended to the horizontal case where more than one CINR measure is needed.

Example for the problem with (1)

- Consider the channel matrix H, given by

$$\mathrm{H} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

- The CINR of stream #0 is $2/\rho^2$, while that of stream #1 is 0, on a linear scale.
- Eq. (2) produces approx. half the CINR of the strong stream (in dB)

$$\frac{1}{2} 10 \log_{10}\left(\frac{2}{\rho^2}\right)[dB] \tag{3}$$

Example for the problem with (1) – cont.

- For an orthogonal channel matrix of the form

$$H = \begin{bmatrix} 1 & .5 \\ 1 & -.5 \end{bmatrix}$$

eq. (2) produces the logarithmic average of the two streams.

- There is an inconsistency in the CINR calculation and averaging method between streams.

Proposed Method

- We start by looking at the per-stream error probability.
- The CINR estimate should satisfy the following (in QPSK):

$$\Pr\left(\text{error in } s_i\right) \approx \exp\left(-\frac{CINR_i(\mathbf{H})}{2}\right) \qquad (4)$$

- Thus, in order to find the CINR, we need to evaluate the error probability per stream.

Proposed Method – cont.

- The asymptotic expression (in the high CINR regime) that symbol $\mathbf{s}$ is transmitted and $\hat{\mathbf{s}}$ is detected is given in [1] by

$$\Pr(\mathbf{s} \rightarrow \hat{\mathbf{s}} | \mathbf{s}) = \exp\left(-\frac{\left\| \mathbf{H}(\mathbf{s} - \hat{\mathbf{s}}) \right\|^2}{4\rho^2} \right) \qquad (5)$$

- We define an error in a specific stream such that

$$\Pr(\text{error in } \mathbf{s}_i) = \Pr(\hat{\mathbf{s}} \text{ in } B_i) \qquad (6)$$

where B_i does not contain $\mathbf{s}$.

[1]Tarokh, V., Seshadri, N., and , Calderbank,, A., R. SpaceTime Codes for High Data Rate Wireless Communication: Performance Criterion and Code Construction. *IEEE transactions on information theory,*, 44(2):744765, March 1998.

Proposed Method – cont.

- Combining eq. (5) and (6) we arrive at

$$\Pr\text{ (error in } \mathbf{s}_i) = \sum_{\mathbf{p}\in B_i} \exp\left(-\frac{\left\| \mathbf{H}(\mathbf{s}-\mathbf{p}) \right\|^2}{4\rho^2} \right) \qquad (7)$$

$$= \sum_{\mathbf{e}\in A_i} \exp\left(-\frac{\left\| \mathbf{He} \right\|^2}{4\rho^2} \right)$$

Max log approx.

$$\xrightarrow{\hphantom{xxxxxx}} \approx \exp\left(-\min_{\mathbf{e}\in A_i} \frac{\left\| \mathbf{He} \right\|^2}{4\rho^2} \right) \qquad (8)$$

where **e=s-p** is an error vector and A_i is the set of all vectors **e** that correspond to set B_i.

Proposed Method – cont.

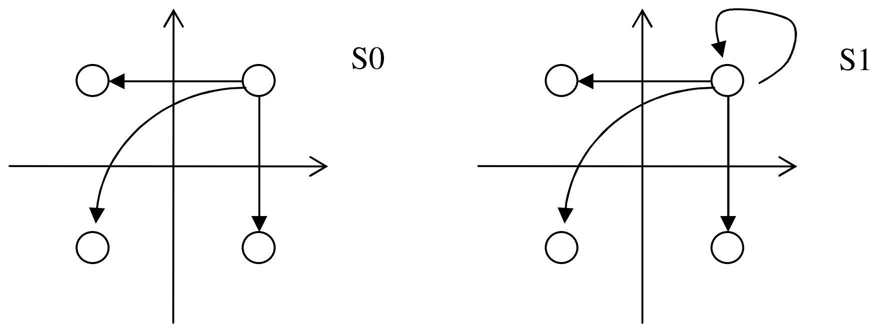
- Equating the exponential presented in Eq. (4) and (8) we arrive at

$$\mathrm{CINR}_i(\mathrm{H}) \approx \min_{\mathrm{e} \in A_i} \frac{\|\mathbf{He}\|^2}{2\rho^2} \tag{9}$$

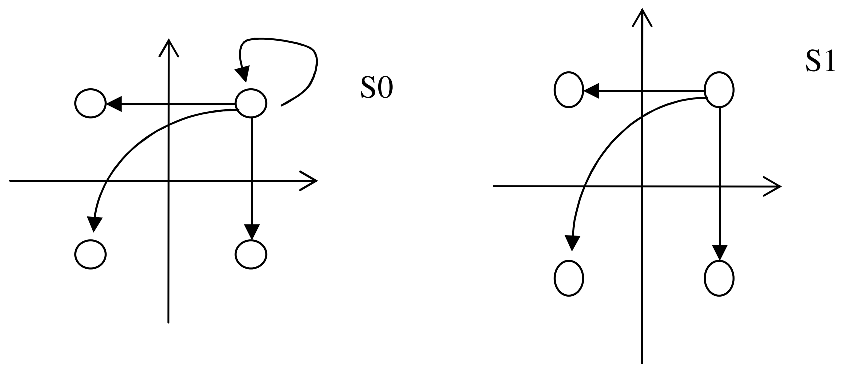
- Let us look at what the set A_i consists of. For simplicity we will look at the case with 2 streams.
- An error in stream s_0 means that the first component in **e** is non zero.

Proposed Method – cont.

- Transitions corresponding to set A_0 when symbol (1,1) is transmitted in s_0:



- Transitions corresponding to set A_1 when symbol (1,1) is transmitted in s_1:



Proposed Method – cont.

- We can now form the entire sets A_0 and A_1 that consist of all possible transition vectors.
- Removing redundant elements and ignoring vectors that correspond to far transitions, sets A_0 and A_1 may be approximated as

$$\hat{A}_0 = \sqrt{2}\left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}-1\\1\end{bmatrix}, \begin{bmatrix}j\\1\end{bmatrix}, \begin{bmatrix}-j\\1\end{bmatrix}, \begin{bmatrix}1+j\\1\end{bmatrix}, \begin{bmatrix}1-j\\1\end{bmatrix}, \begin{bmatrix}-1+j\\1\end{bmatrix}, \begin{bmatrix}-1-j\\1\end{bmatrix}\right\}$$

$$\hat{A}_1 = \sqrt{2}\left\{\begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}-1\\1\end{bmatrix}, \begin{bmatrix}j\\1\end{bmatrix}, \begin{bmatrix}-j\\1\end{bmatrix}, \begin{bmatrix}1+j\\1\end{bmatrix}, \begin{bmatrix}1-j\\1\end{bmatrix}, \begin{bmatrix}-1+j\\1\end{bmatrix}, \begin{bmatrix}-1-j\\1\end{bmatrix}\right\}.$$

Proposed Method – cont.

- The per stream CINR estimation takes the form

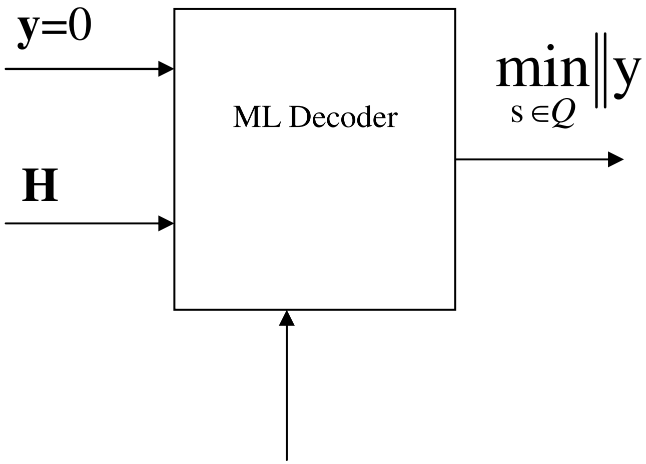
$$CINR_0(\mathrm{H}) \cong \min_{\mathrm{e} \in \hat{\mathrm{A}}_0} \frac{\|\mathrm{He}\|^2}{2\rho^2}$$

$$CINR_1(\mathrm{H}) \cong \min_{\mathrm{e} \in \hat{\mathrm{A}}_1} \frac{\|\mathrm{He}\|^2}{2\rho^2}$$

- If vertical encoding is being used, any averaging method can be applied to the individual stream CINR (linear, logarithmic, input to ECINR etc.)

Implementation Point of View

- From an implementation point of view, the suggested CINR estimation method is best implemented using the ML decoder itself, as shown in the following figure.



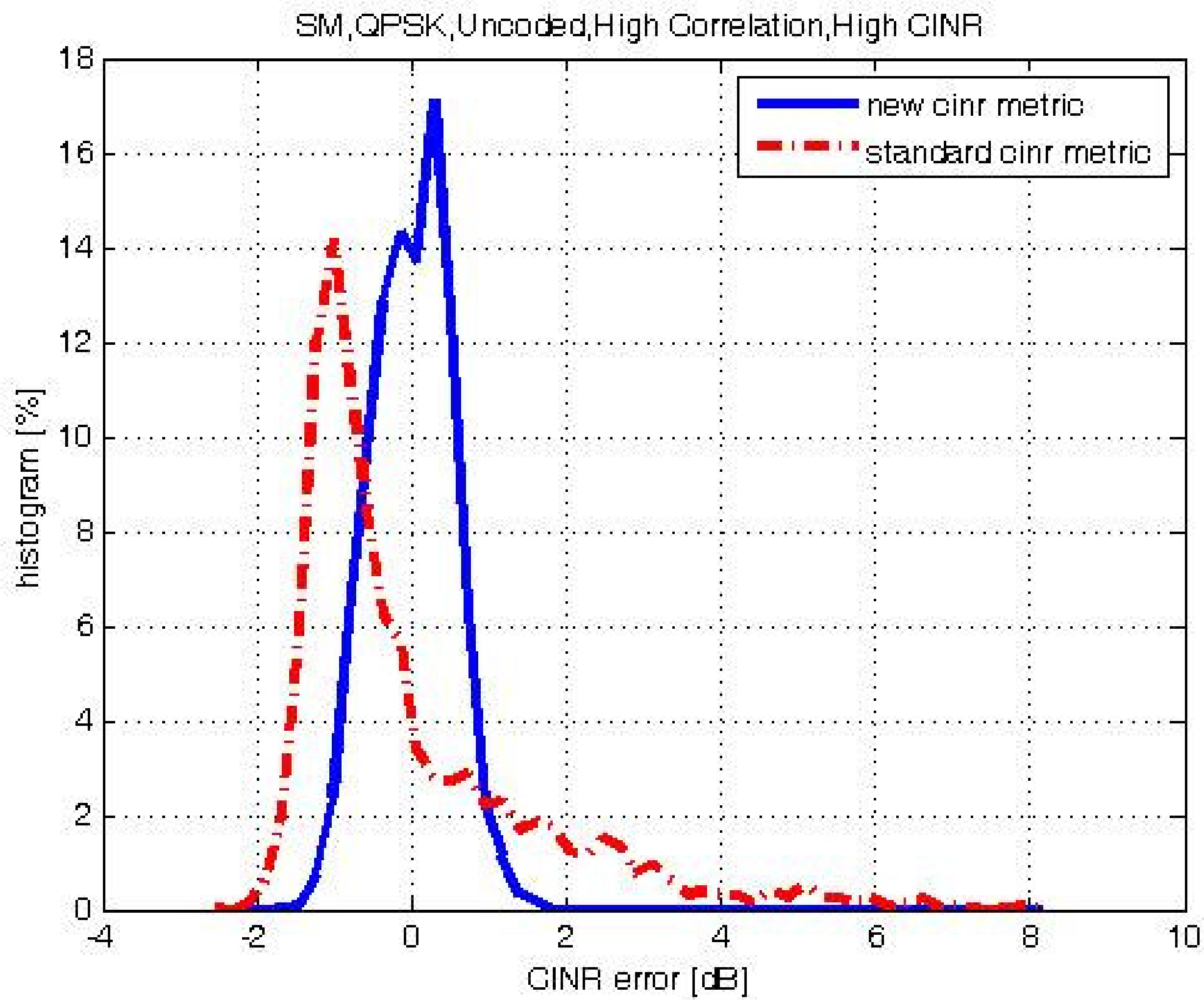
$$\min_{s \in Q} \|y - Hs\|^2 = \min_{e \in A_i} \|He\|^2$$

Search space Q (usually subset of QAM^M) = A_i

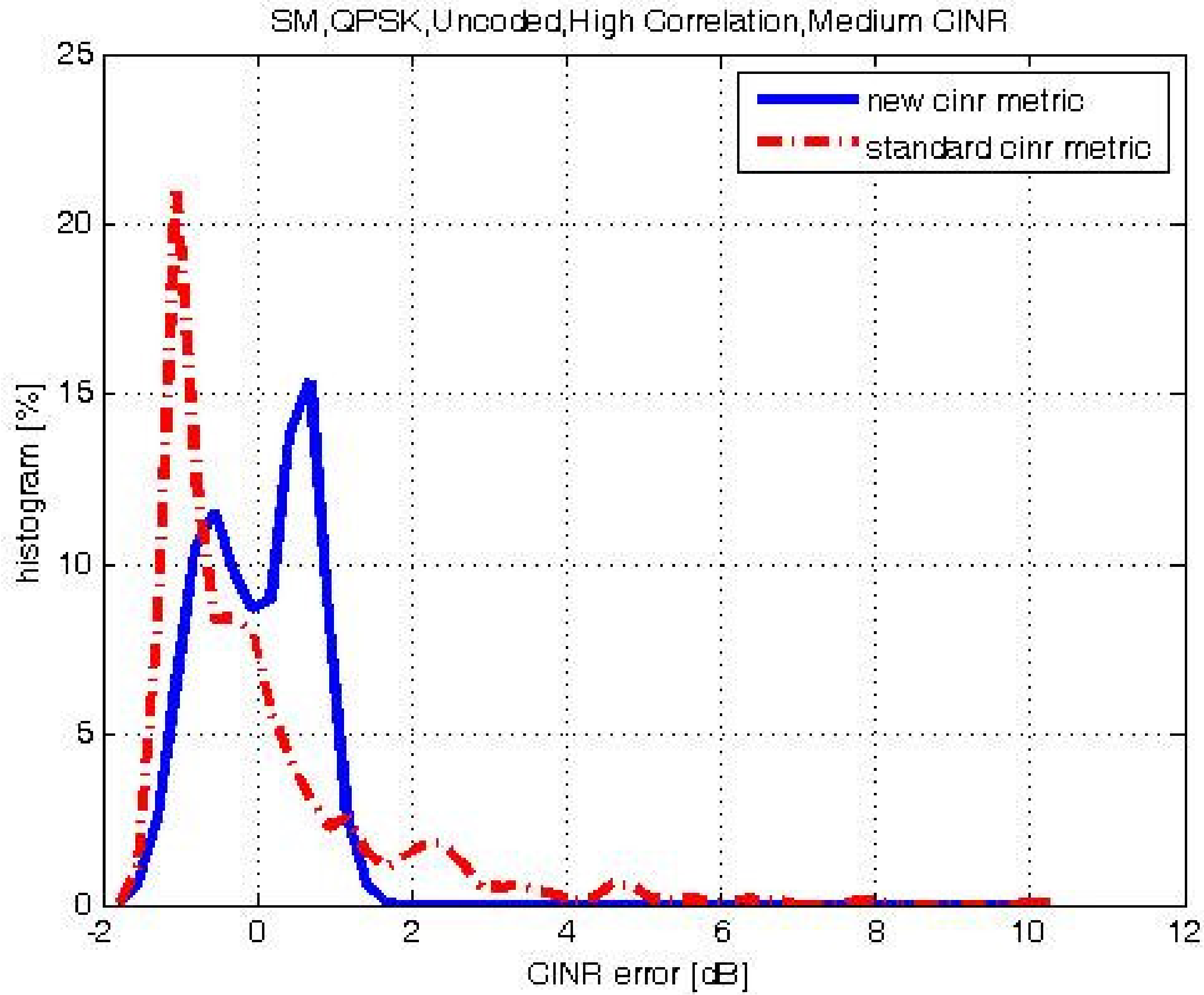
Simulation Results

- We give here simulation results focusing on the accuracy of the new and standard (802.16e) CINR formulas.
- The simulations consider the following:
  - Per tone CINR (Rayleigh fading matrix)
  - 2X2 channel matrix with Matrix B transmission
  - Single CINR estimate per tone (logarithmic averaging with the new formula)
  - Various (standard) spatial correlation
  - Several SNR regimes
  - The performance metric is BER

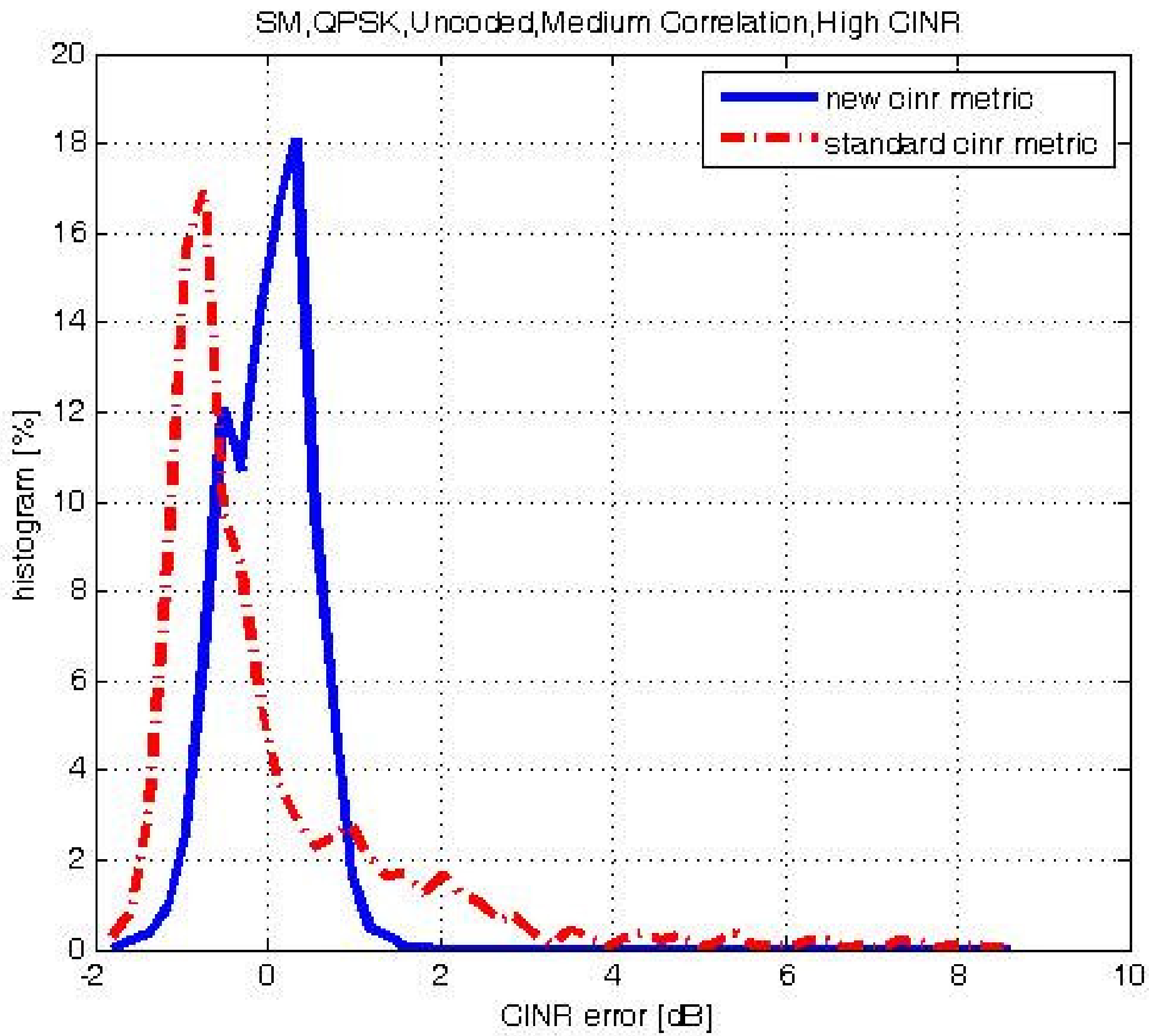
Simulation Results



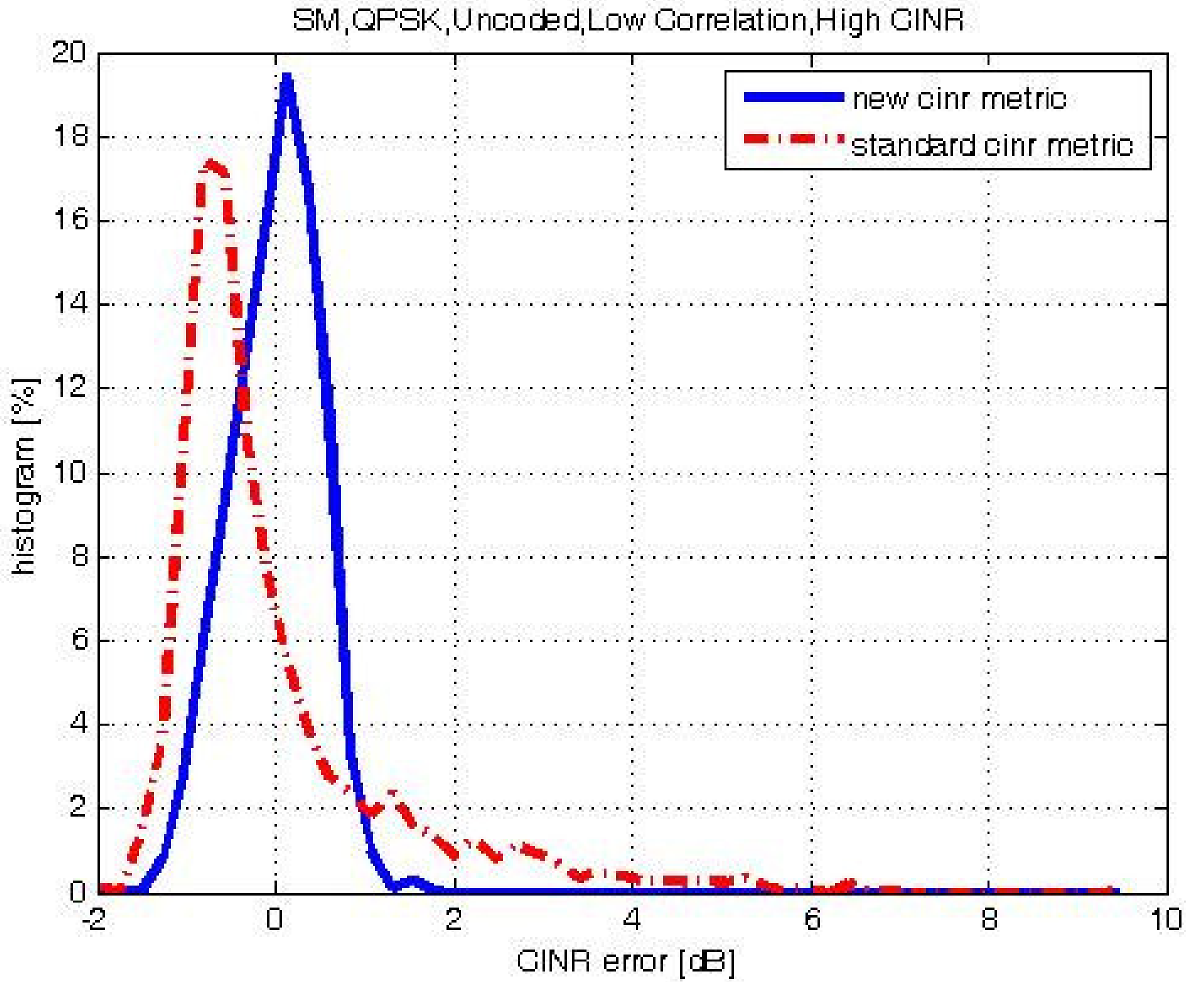
Simulation Results



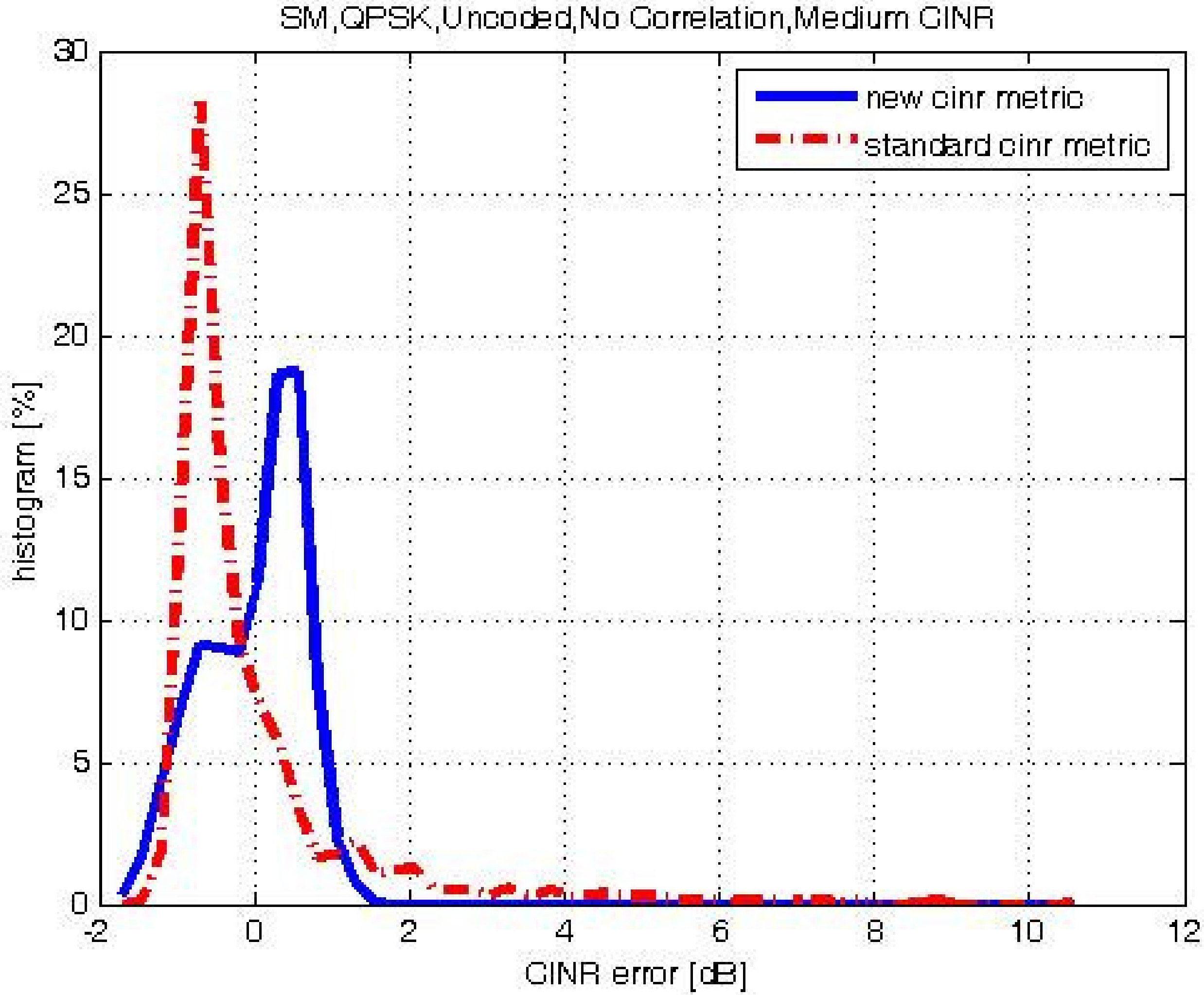
Simulation Results



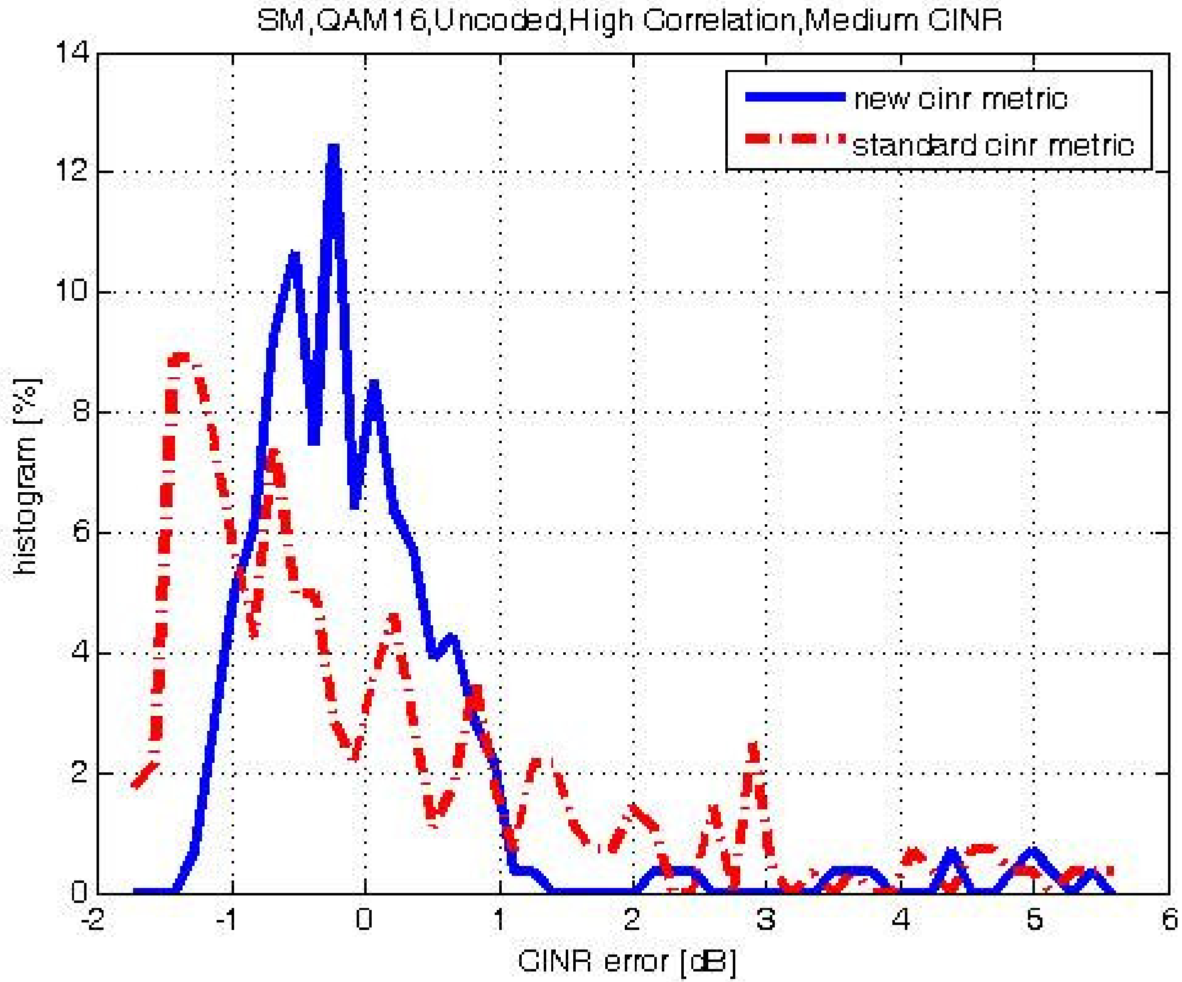
Simulation Results



Simulation Results



Simulation Results



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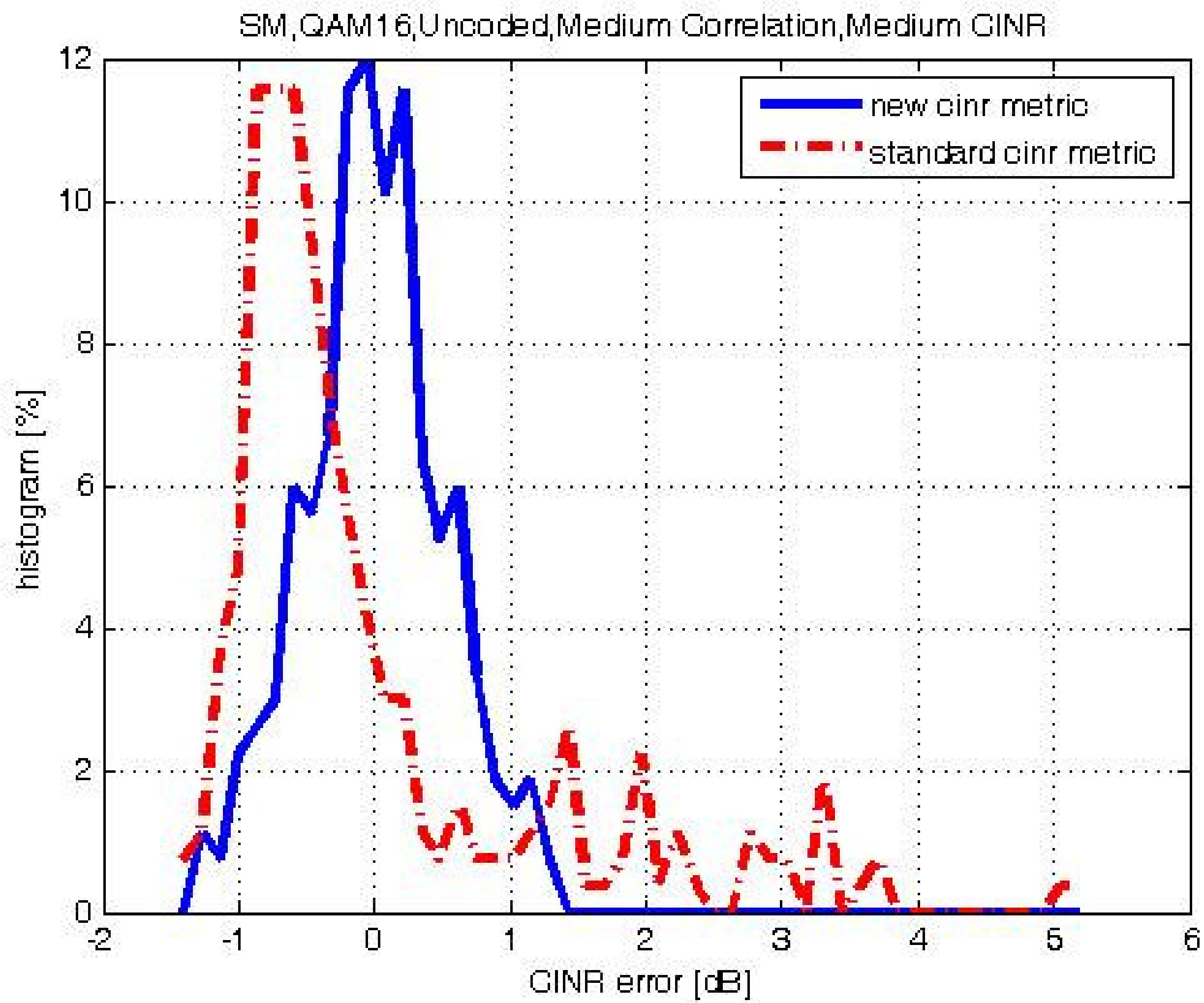
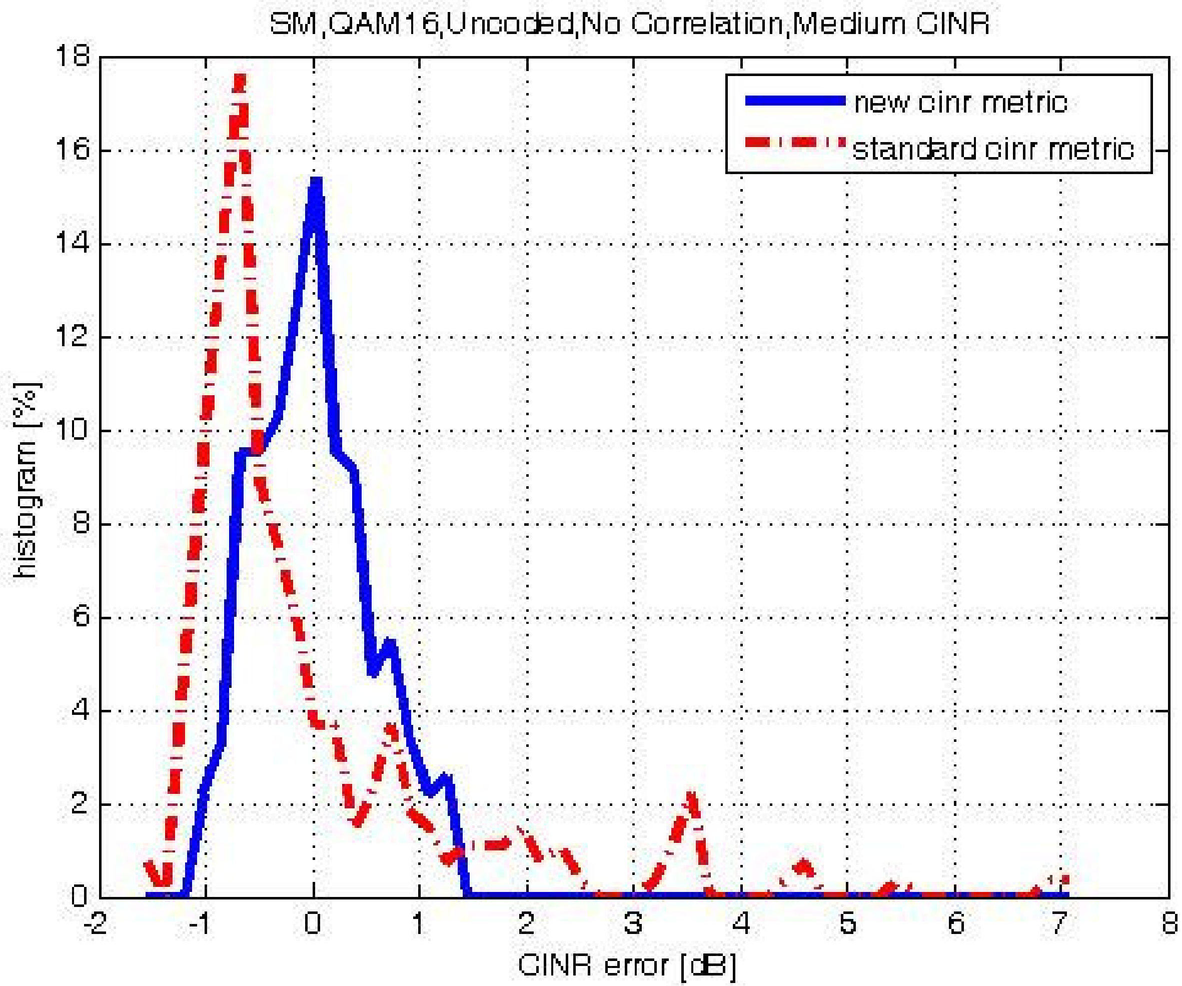


Simulation Results



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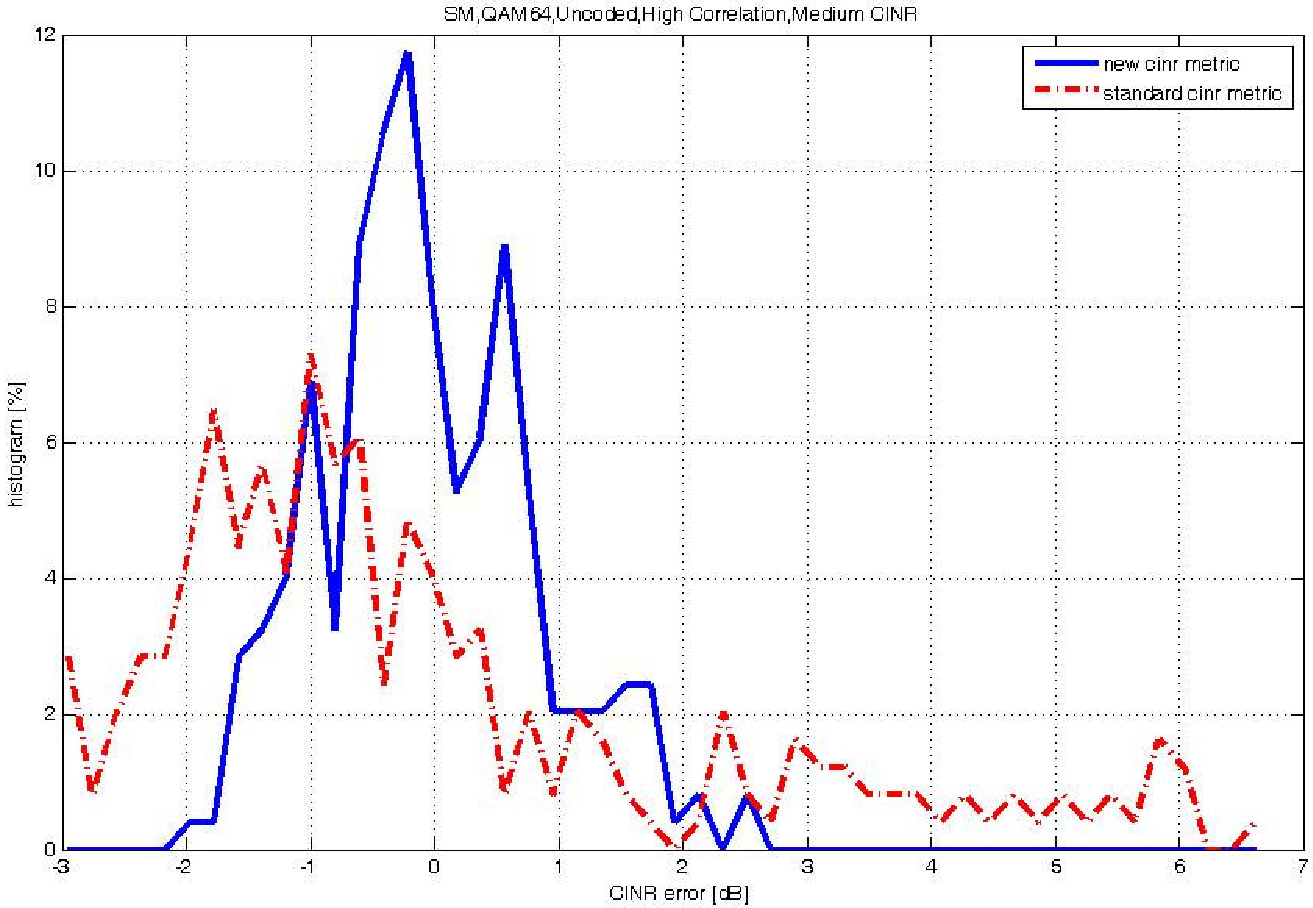

Conclusions and Suggestions

- The new formula is significantly more accurate than the 802.16e formula (especially in high correlation).
- The QPSK based computation is also adequate for 16QAM and 64QAM.
- Moreover, the new formula allows:
  - Flexibility in the type of averaging between layers (or 2 inputs to ECINR)
  - "Built in" horizontal MIMO capabilities
  - Simple implementation through the ML decoder
- Thus, we suggest to incorporate the new formula into the 802.16m standard.