Project	**IEEE 802.16 Broadband Wireless Access Working Group** <http://ieee802.org/16>
Title	**Codebook based pre-coding MIMO**
Date Submitted	**2008-05-05**
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Re:	802.16m-08/005 Call for contributions on project 802.16m system description document (SDD). Specific topic : Codebook structure for DL closed-loop MIMO
Abstract	This contribution provides the codebook structure for DL closed-loop MIMO
Purpose	For discussion and approval by TGM
Notice	
Release	
Patent Policy	

Codebook based pre-coding MIMO

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1. Introduction

Various MIMO schemes using multi antennas have been introduced in the standardization for the IEEE802.16e communication system in order to increase the received SINR and enhance the spectral efficiency. In particular, it has been known that a codebook based pre-coding MIMO could bring a significant gain in the spectral efficiency aspect in the downlink CL-MIMO.

In this contribution, we propose a DFT based codebook scheme as a prominent pre-coding MIMO for the IEEE802.16m downlink system in the aspect of spectral efficiency [1] and feedback signaling overhead reduction. The performance of the proposed scheme has been compared with that of the IEEE802.16e codebook through computer simulation.

2. Considerations for codebook design

It is a very practical issue how to design a codebook scheme with some criteria like performance and complexity. Some criteria have been known for the design as below described.

- Performance and feedback signaling overhead

In the downlink closed-loop MIMO, downlink channel state information could be exploited for the BS throughput enhancement through the uplink feedback channel of MS. But the exact full channel state information is not obtainable at the BS side because of the limited overhead of feedback signaling. Its quantized information is only available for the downlink closed-loop MIMO. Codebook is a kind of good way to utilize the obtained limited channel state information. As an alternative, there is a method for using a sounding channel of uplink, which needs an extra calibration work.

The trade-off between performance and feedback signaling overhead must be always considered for codebook design. Because a larger size of codebook gives better performance but its attainable gain gets smaller gradually with the increase of codebook size as it reaches to the saturation point.

- Nested structure

It is convenient for operation if all the other lower rank matrices could be extracted from the highest rank matrix. It enables that all the column vector elements of the other rank matrices to be simply constituted by the combination of the column vector elements of the highest rank matrix. Such a structure would reduce the complexity of the implementation for the PMI (Pre-coding Matrix Index) searching and decoding operation.

- Computational complexity

The usage of small sets of antenna weight elements, for example $\pm 1, \pm j$, can reduce the computational complexity at the terminal side because the equivalent estimated channel is created just by simple manipulation of conjugation, negation or addition.

- Channel characteristics

Most MIMO schemes work well under rich scattering channels but not well under the highly correlated channel environments like a rural area. Even in the urban area, the performance is degraded as the propagation channel is partly correlated. Therefore, a desirable codebook scheme is the one having robust performance characteristics even in the correlated channel environments.

DFT based codebook shows robust performance even in correlated channel environments so that it can be easily applied to any deployment scenarios.

- PAPR(Peak to Average Power Ratio) reduction

The current IEEE802.16e codebook which is not constant modulus scheme has a weak point in PAPR. Since it does not always guarantee the equal power transmission per antenna, it causes antenna power imbalance problem. Therefore, a codebook scheme with constant modulus characteristics should be considered for the IEEE802.16m codebook.

3. Proposed Codebook

We propose two step codebook design; basic matrix and its extension. In this section, we propose basic matrix based on DFT matrix and in the next section, we propose its extension with phase shift matrix. Basic matrix for codebook design of 4 Tx antenna pre-coding is as follows;

$$\mathbf{B} = \left[\mathbf{b}_0\, \mathbf{b}_1\, \mathbf{b}_2\, \mathbf{b}_3\right] = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \quad (1)$$

Using the basic matrix of equation (1), a simple DFT codebook of (4, 6, 4, 1) structure can be created by the combination of column vector elements like equation (2).

$$\mathbf{C}_{15} = \left\{ \begin{matrix} [\mathbf{b}_0],[\mathbf{b}_1],[\mathbf{b}_2],[\mathbf{b}_3],[\mathbf{b}_0\mathbf{b}_1],[\mathbf{b}_0\mathbf{b}_2],[\mathbf{b}_0\mathbf{b}_3],[\mathbf{b}_1\mathbf{b}_2],[\mathbf{b}_1\mathbf{b}_3], \\ [\mathbf{b}_2\mathbf{b}_3],[\mathbf{b}_0\mathbf{b}_1\mathbf{b}_2],[\mathbf{b}_0\mathbf{b}_1\mathbf{b}_3],[\mathbf{b}_0\mathbf{b}_2\mathbf{b}_3],[\mathbf{b}_1\mathbf{b}_2\mathbf{b}_3],[\mathbf{b}_0\mathbf{b}_1\mathbf{b}_2\mathbf{b}_3] \end{matrix} \right\} \quad (2)$$

4. Extension of Codebook Size

We use a phase shift matrix of equation (3) for the extension of codebook size.

$$\mathbf{P}(\phi_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{j\phi_i} & 0 & 0 \\ 0 & 0 & e^{j2\phi_i} & 0 \\ 0 & 0 & 0 & e^{j3\phi_i} \end{bmatrix} \quad (3)$$

As an example, from the equation (2), ϕ_i values for a 2-bit codebook extension become '0', '$\frac{\pi}{8}$', '$\frac{\pi}{4}$' and '$\frac{3\pi}{8}$', respectively. For the case of n extension bits, $\phi_i = \frac{\pi * i}{2N}$ $(i = 0, ..., N-1, N = 2^n)$.

Therefore, an extended codebook can be represented as $\mathbf{P}(\phi_i)\mathbf{B}$. We will call it as 'Phase-adapted DFT (P-DFT) codebook' from now on.

5. Simulation results

In this section, we have evaluated the system level throughput with five types of codebook under the simulation assumption defined in IEEE802.16m EMD[2]. Phase-adapted DFT codebook (Max. 5bit : Phase shift 2bit, basic matrix (4,6)), IEEE802.16e codebook (6bit) [3], LTE codebook (4bit) [4], IEEE802.16e codebook (3bit) [3] and DFT + AS (Max. 5bit : (32, 24), refer to Appendix) have been simulated for comparison.

Following table 1 and 2 include system level simulation assumptions.

Table 1. Basic simulation assumption

Parameter	Assumption
OFDM parameters	10 MHz (1024 subcarriers)
Subframe length	2.5 ms
Channelization	Band AMC
Number of total RB	48
Number of bands for PMI and CQI calculation	4
Channel Models	Extended Ped-B
Mobile Speed (km/h)	3 km/h
Number of MCS level	10
Channel Code	CTC
Antenna configuration	4 transmitter, 2 receiver => [4Tx, 2Rx]
MIMO receiver	Linear Minimum Mean Squared Error (LMMSE)
Channel Estimation	Perfect channel estimation

Table 2. System parameter assumption

Parameter		Assumption
Cellular Layout		Hexagonal grid, 19 cell sites, wrap-around, 3 sectors per site
Distance-dependent path loss		L=130.19 + 37.6log$_{10}$(.R), R in kilometers
Inter site distance		1.5km
Penetration loss		10dB
Shadowing standard deviation		8 dB
Shadowing correlation	Between cells	0.5
	Between sectors	1.0
Antenna pattern (horizontal) (For 3-sector cell sites with fixed antenna patterns)		$A(\theta) = -\min\left[12\left(\frac{\theta}{\theta_{3dB}}\right)^2, A_m\right]$ θ_{3dB} = 70 degrees, A_m = 20 dB
Total BS TX power		46dBm
Minimum distance between MS and cell		35 meters
Target block error rate		10 %
HARQ		Chase combining with maximum retransmission 4, Asynchronous
OFDM symbols (Data symbols) per subframe		24 (18)
Scheduling Criterion		Proportional Fair
Users per sector		10
Link Mapping		RBIR
Other Cell interference		8 dominant interferers
CQI feedback delay		5 ms
CQI feedback method		Full band CQI
PMI feedback period		Every subframe
Rank feedback period		12 subframes
Channel Scenario		Mandatory Scenario (Spatially correlated and zero correlated channel : 4 lambda antenna spacing)

Figure 1 shows average throughput of each codebook scheme for spatially correlated channel. As shown in figure 1, the phase-adapted DFT codebook performs better than other codebook schemes such as IEEE802.16e codebook (6bit)[3] and the LTE codebook (4bit) [4] and etc .

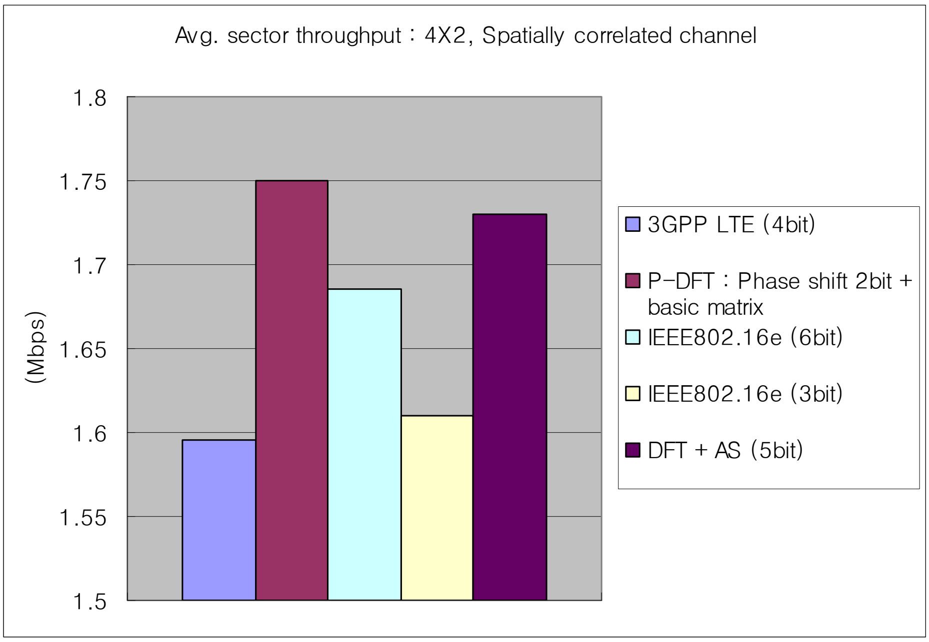

Figure 1. Comparison of avg. sector throughput

In table 3, the same system-level throughput is specifically presented according to each codebook scheme. As seen in the table, the phase-adapted DFT codebook performs better than the others due to the fact that it is more suitable for various correlation spatial channel conditions. DFT + AS codebook also shows similar performance.

Table 3. Avg. sector throughput of each codebook scheme

(For spatially correlated channel)

Codebook type	IEEE802.16e (6bit)	Phase-adapted DFT (Max. 5bit : Phase shift 2 bit + basic matrix (4, 6))	3GPP LTE (4bit)	IEEE802.16e (3bit)	DFT + AS(Max.5bit(32,24))
Avg. sector throughput	1.685 Mbps	1.75 Mbps	1.595 Mbps	1.61Mbps	1.73Mbps
Relative gain	0%	3.9%	-7%	-4%	2.7%

Figure 2 shows average throughput of each codebook scheme for zero correlated channel model. As shown in figure 2, IEEE802.16e codebook (6bit) performs better than other codebooks .

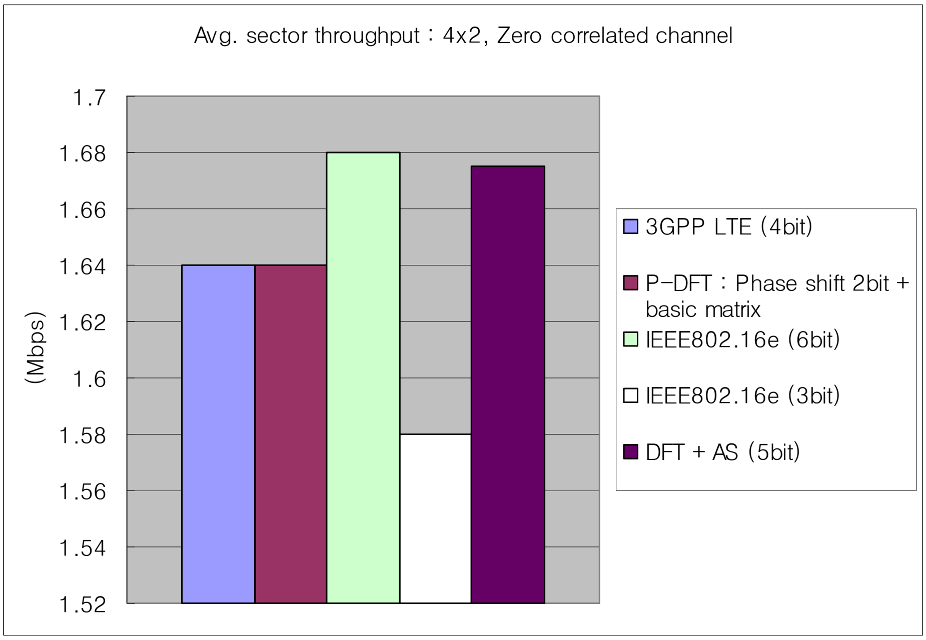

Figure 2. Comparison of avg. sector throughput

In table 4, the same system-level throughput is specifically presented according to each codebook scheme. As seen in the table, IEEE802.16e codebook performs better than the others due to the fact that it is more suitable for zero correlated channel conditions and its codebook size is larger than that of other codebooks. DFT + AS codebook shows similar performance.

Table 4. Avg. sector throughput of each codebook scheme
(For zero correlated channel model)

Codebook type	IEEE802.16e (6bit)	Phase-adapted DFT (Max. 5bit : Phase shift 2 bit + basic matrix (4, 6))	3GPP LTE (4bit)	IEEE802.16e (3bit)	DFT + AS (Max.5bit(32,24))
Avg. sector throughput	1.68 Mbps	1.64 Mbps	1.64 Mbps	1.58Mbps	1.675Mbps
Relative gain	0%	-2.4%	-2.4%	-6%	-0.3%

From the above results, phase-adapted DFT shows the best performance in spatially correlated channel model and equal performance with that of 3GPP LTE in zero correlated channel model. It also meets all the considerations for codebook design.

In addition, we can find that DFT + AS codebook also shows good performance regardless of the channel correlation characteristics in both spatially correlated channel and zero correlated channel when compared with other codebooks. But it may not be an appropriate scheme for pre-coding MIMO in the aspect of PAPR since antenna selection may cause the power imbalance problem between each transmit antenna. It can be a good solution for IEEE802.16m codebook when we see performance vs codebook size if PAPR can be alleviated through band scheduling.

6. Conclusion

In this contribution, we proposed the codebook based pre-coding based on phase-adapted DFT matrix for the single user downlink MIMO of IEEE802.16m. From the simulation results, we can summarize and conclude as follows:

- The phase-adapted DFT codebook (Max. 5bit) provides excellent performance under AoD based spatial channel condition. Even in ‘0’ correlated channel environment, its relative gain loss is only 2.4% when compared with IEEE802.16e codebook (6bit).
- The proposed codebook can be easily extended to larger codebook by adding another phase ϕ_i in the equation (3).
- The proposed codebook meets all of aforementioned consideration points.

Therefore, we propose to employ the phase-adapted DFT matrix for the codebook-based pre-coding of the downlink single user MIMO of IEEE802.16m.

Text Proposal for the 802.16m SDDs

============================= *Start of Proposed Text* ================================

11.x.2.2.1 SU-MIMO

DFT matrix of equation (1) shall be used for the basic matrix for the codebook design of 4 Tx antenna pre-coding MIMO.

$$\mathbf{B} = \left[\mathbf{b}_0\,\mathbf{b}_1\,\mathbf{b}_2\,\mathbf{b}_3\right] = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \quad (1)$$

Then, the complete codebook is given as follows:

Codebook Index	Rank 1	Rank 2	Rank 3	Rank 4
0	$[\mathbf{b}_0]$	$[\mathbf{b}_0\mathbf{b}_1]$	$[\mathbf{b}_0\mathbf{b}_1\mathbf{b}_2]$	$[\mathbf{b}_0\mathbf{b}_1\mathbf{b}_2\mathbf{b}_3]$
1	$[\mathbf{b}_1]$	$[\mathbf{b}_0\mathbf{b}_2]$	$[\mathbf{b}_0\mathbf{b}_1\mathbf{b}_3]$	
2	$[\mathbf{b}_2]$	$[\mathbf{b}_0\mathbf{b}_3]$	$[\mathbf{b}_0\mathbf{b}_2\mathbf{b}_3]$	

3	$[\mathbf{b}_3]$	$[\mathbf{b}_1\mathbf{b}_2]$	$[\mathbf{b}_1\mathbf{b}_2\mathbf{b}_3]$	
4		$[\mathbf{b}_1\mathbf{b}_3]$		
5		$[\mathbf{b}_2\mathbf{b}_3]$		

Phase shift matrix of equation (2) shall be also used for the extension of codebook size.

$$\mathbf{P}(\phi_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{j\phi_i} & 0 & 0 \\ 0 & 0 & e^{j2\phi_i} & 0 \\ 0 & 0 & 0 & e^{j3\phi_i} \end{bmatrix} \qquad (2)$$

As an example, from the equation (2), ϕ_i values for a 2-bit codebook extension become '0', 'π/8', 'π/4'and '3π/8', respectively. For the case of *n* extension bits, $\phi_i = \frac{\pi * i}{2N}$ $(i = 0,...., N-1, N = 2^n)$.

So, combined codebook can be represented as $\mathbf{P}(\phi_i)\mathbf{B}$.

============================== *End of Text Proposal* ==============================

Appendix

Codebook structure of DFT + AS (32, 24, 16, 1) for all ranks is as follows. In this structure, full antenna selection is made for rank 4, full and 3 antenna selection for rank 3, full, 3 and 2 antenna selection for rank 2, full, 3, 2 and 1 antenna selection for rank 1, respectively. And the same size of DFT matrix as the number of selected antenna is applied for each rank.

(a) Rank 4:

$$\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \qquad (1)$$

(b) Rank 3:

$$\frac{1}{2}\begin{bmatrix}1 & 1 & 1\\ 1 & j & -1\\ 1 & -1 & 1\\ 1 & -j & -1\end{bmatrix},\frac{1}{2}\begin{bmatrix}1 & 1 & 1\\ 1 & j & -j\\ 1 & -1 & -1\\ 1 & -j & j\end{bmatrix},\frac{1}{2}\begin{bmatrix}1 & 1 & 1\\ 1 & -1 & -j\\ 1 & 1 & -1\\ 1 & -1 & j\end{bmatrix},\frac{1}{2}\begin{bmatrix}1 & 1 & 1\\ j & -1 & -j\\ -1 & 1 & -1\\ -j & -1 & j\end{bmatrix},$$

$$\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 1 & \text{-0.5000 + j0.8660} & \text{-0.5000 - j0.8660}\\ 1 & \text{-0.5000 - j0.8660} & \text{-0.5000 + j0.8660}\\ 0 & 0 & 0\end{bmatrix},$$

$$\frac{1}{\sqrt{3}}\begin{bmatrix}0 & 0 & 0\\ 1 & 1 & 1\\ 1 & \text{-0.5000 + j0.8660} & \text{-0.5000 - j0.8660}\\ 1 & \text{-0.5000 - j0.8660} & \text{-0.5000 + j0.8660}\end{bmatrix},$$

$$\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 1 & \text{-0.5000 + j0.8660} & \text{-0.5000 - j0.8660}\\ 0 & 0 & 0\\ 1 & \text{-0.5000 - j0.8660} & \text{-0.5000 + j0.8660}\end{bmatrix},$$

$$\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 0 & 0 & 0\\ 1 & \text{-0.5000 + j0.8660} & \text{-0.5000 - j0.8660}\\ 1 & \text{-0.5000 - j0.8660} & \text{-0.5000 + j0.8660}\end{bmatrix} \qquad (2)\backslash$$

(b) Rank 2:

$$\frac{1}{2}\begin{bmatrix}1&1\\1&j\\1&-1\\1&-j\end{bmatrix},\frac{1}{2}\begin{bmatrix}1&1\\1&-1\\1&1\\1&-1\end{bmatrix},\frac{1}{2}\begin{bmatrix}1&1\\1&-j\\1&-1\\1&j\end{bmatrix},\frac{1}{2}\begin{bmatrix}1&1\\j&-1\\-1&1\\-j&-1\end{bmatrix},\frac{1}{2}\begin{bmatrix}1&1\\j&-j\\-1&-1\\-j&j\end{bmatrix},\frac{1}{2}\begin{bmatrix}1&1\\-1&-j\\1&-1\\-1&j\end{bmatrix},$$

$$\frac{1}{\sqrt{3}}\begin{bmatrix}1&1\\1&-0.5000+j0.8660\\1&-0.5000-j0.8660\\0&0\end{bmatrix},\frac{1}{\sqrt{3}}\begin{bmatrix}1&1\\1&-0.5000-j0.8660\\1&-0.5000+j0.8660\\0&0\end{bmatrix},\frac{1}{\sqrt{3}}\begin{bmatrix}1&1\\-0.5000+j0.8660&-0.5000-j0.8660\\-0.5000-j0.8660&-0.5000+j0.8660\\0&0\end{bmatrix},$$

$$\frac{1}{\sqrt{3}}\begin{bmatrix}0&0\\1&1\\1&-0.5000+j0.8660\\1&-0.5000-j0.8660\end{bmatrix},\frac{1}{\sqrt{3}}\begin{bmatrix}0&0\\1&1\\1&-0.5000-j0.8660\\1&-0.5000+j0.8660\end{bmatrix},\frac{1}{\sqrt{3}}\begin{bmatrix}0&0\\1&1\\-0.5000+j0.8660&-0.5000-j0.8660\\-0.5000-j0.8660&-0.5000+j0.8660\end{bmatrix},$$

$$\frac{1}{\sqrt{3}}\begin{bmatrix}1&1\\1&-0.5000+j0.8660\\0&0\\1&-0.5000-j0.8660\end{bmatrix},\frac{1}{\sqrt{3}}\begin{bmatrix}1&1\\1&-0.5000-j0.8660\\0&0\\1&-0.5000+j0.8660\end{bmatrix},\frac{1}{\sqrt{3}}\begin{bmatrix}1&1\\-0.5000+j0.8660&-0.5000-j0.8660\\0&0\\-0.5000-j0.8660&-0.5000+j0.8660\end{bmatrix},$$

$$\frac{1}{\sqrt{3}}\begin{bmatrix}1&1\\0&0\\1&-0.5000+j0.8660\\1&-0.5000-j0.8660\end{bmatrix},\frac{1}{\sqrt{3}}\begin{bmatrix}1&1\\0&0\\1&-0.5000-j0.8660\\1&-0.5000+j0.8660\end{bmatrix},\frac{1}{\sqrt{3}}\begin{bmatrix}1&1\\0&0\\-0.5000+j0.8660&-0.5000-j0.8660\\-0.5000-j0.8660&-0.5000+j0.8660\end{bmatrix}, \quad (3)$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\1&-1\\0&0\\0&0\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}0&0\\1&1\\1&-1\\0&0\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}0&0\\0&0\\1&1\\1&-1\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\0&0\\1&-1\\0&0\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\0&0\\0&0\\1&-1\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}0&0\\1&1\\0&0\\1&-1\end{bmatrix}$$

(d) Rank 1:

$$\frac{1}{2}\begin{bmatrix}1\\1\\1\\1\end{bmatrix},\frac{1}{2}\begin{bmatrix}1\\j\\-1\\-j\end{bmatrix},\frac{1}{2}\begin{bmatrix}1\\-1\\1\\-1\end{bmatrix},\frac{1}{2}\begin{bmatrix}1\\-j\\-1\\j\end{bmatrix},$$

$$\frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\\0\end{bmatrix},\frac{1}{\sqrt{3}}\begin{bmatrix}1\\-0.5000+j0.8660\\-0.5000-j0.8660\\0\end{bmatrix},\frac{1}{\sqrt{3}}\begin{bmatrix}1\\-0.5000-j0.8660\\-0.5000+j0.8660\\0\end{bmatrix},$$

$$\frac{1}{\sqrt{3}}\begin{bmatrix}0\\1\\1\\1\end{bmatrix},\frac{1}{\sqrt{3}}\begin{bmatrix}0\\1\\-0.5000+j0.8660\\-0.5000-j0.8660\end{bmatrix},\frac{1}{\sqrt{3}}\begin{bmatrix}0\\1\\-0.5000-j0.8660\\-0.5000+j0.8660\end{bmatrix},$$

$$\frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\0\\1\end{bmatrix},\frac{1}{\sqrt{3}}\begin{bmatrix}1\\-0.5000+j0.8660\\0\\-0.5000-j0.8660\end{bmatrix},\frac{1}{\sqrt{3}}\begin{bmatrix}1\\-0.5000-j0.8660\\0\\-0.5000+j0.8660\end{bmatrix},$$

$$\frac{1}{\sqrt{3}}\begin{bmatrix}1\\0\\1\\1\end{bmatrix},\frac{1}{\sqrt{3}}\begin{bmatrix}1\\0\\-0.5000+j0.8660\\-0.5000-j0.8660\end{bmatrix},\frac{1}{\sqrt{3}}\begin{bmatrix}1\\0\\-0.5000-j0.8660\\-0.5000+j0.8660\end{bmatrix},$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\\0\\0\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}0\\0\\1\\1\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\\0\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\0\\1\end{bmatrix},$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\\0\\0\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}0\\0\\1\\-1\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\-1\\0\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\-1\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\0\\-1\end{bmatrix},$$

$$\begin{bmatrix}1\\0\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\\0\end{bmatrix},\begin{bmatrix}0\\0\\1\\0\end{bmatrix},\begin{bmatrix}0\\0\\0\\1\end{bmatrix} \qquad (4)$$

Extension of codebook size can be accomplished by the application of three different kinds of phase shift matrix like equation (5)-(7).

$$\mathbf{P}(\phi_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{j\phi_i} & 0 & 0 \\ 0 & 0 & e^{j2\phi_i} & 0 \\ 0 & 0 & 0 & e^{j3\phi_i} \end{bmatrix} \quad (5)$$

$$\mathbf{P}(\phi_i) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{j\phi_i} & 0 \\ 0 & 0 & e^{j2\phi_i} \end{bmatrix} \quad (6)$$

$$\mathbf{P}(\phi_i) = \begin{bmatrix} 1 & 0 \\ 0 & e^{j\phi_i} \end{bmatrix} \quad (7)$$

Reference

[1] IEEE 802.16m-07/002r4, “IEEE 802.16m System Requirements”

[2] IEEE 802.16m-08/004, “Project 802.16m Evaluation Methodology Document (EMD)”

[3] P80216Rev2/D4, Section 8.4.5.4.10.15 MIMO feedback for Tx beamforming

[4] R1-071799, “Way Forward on 4-Tx Antenna Codebook for SU-MIMO,” Texas Instruments and others