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Re:	Change request for document 802.16m-08/004r2
Abstract	This document describes a problem encountered in the 16m EVM Link-to-System Mapping (RBIR, Section 4) and a remedy to solve the problem
Purpose	Discuss and adopt
Notice	
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Correction to RBIR Link-to-System Mapping in 802.16m Evaluation Methodology

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1. Purpose

The purpose of this contribution is to document a change request necessary to correct an error in the mandatory Link-to-System Mapping method (RBIR), section 4 of [1].

2. Introduction

Received Bit Information Rate (RBIR) has been chosen as the mandatory link-to-system mapping method in the 16m EVM. A fair amount of effort has been devoted to streamline the application of this method and to make it computationally efficient. Unfortunately, some errors have been found and in this contribution we document the extent of the problem, its solution and a remedy to make a change in the EVM (see Section 5 below).

3. RBIR Mapping for the Maximum-Likelihood MIMO Receiver

Fix an OFDM symbol and a subcarrier at the output of the FFT, and let Y denote the received vector of dimension N_r.

Then $Y=HX+U$, where H is the channel matrix, X is the transmitted symbol vector of dimension N_t, and U is a zero mean complex Gaussian noise vector of dimension N_r with covariance $\sigma^2 I$.

Let M_i denote the constellation size of the ith stream (i.e., M_i = 4, 16, and 64 for QPSK, QAM16, and QAM64, respectively) and x_i denote the ith coordinate of X. In the SIMO case, we simply use M to denote the constellation size.

Given random variables z, w, and u, let $I(z,w)$ denote the mutual information of z and w and $I(z,w|u)$ denote the conditional mutual information of z and w given u.

Define

$$RBIR_1 = \frac{I(Y,x_1)}{\log_2(M_1)}, \qquad RBIR_2 = \frac{I(Y,x_2)}{\log_2(M_2)}.$$

According to the EVM document [1], the RBIR for MIMO Matrix B system is calculated from the quantities $RBIR_1$ and $RBIR_2$ as follows: For a 2x2 system using MIMO Matrix B and horizontal encoding, the RBIR metric is computed individually for each stream by computing $RBIR_1$ and $RBIR_2$ for streams 1 and 2, respectively. For a 2x2 system using MIMO Matrix B and vertical encoding, RBIR is computed as a weighted sum of the individual RBIRs, i.e., $RBIR = p_1 RBIR_1 + p_2 RBIR_2$, where p_1 and p_2 are given in Table 27 in [1].

It can be verified that

$$I(Y,x_1) = I(Y,X) - I(Y,x_2 | x_1),$$

$$I(Y,x_2) = I(Y,X) - I(Y,x_1 | x_2). \qquad (3.1)$$

Let us examine the term $I(Y,X)$ for arbitrary N_r and N_t. Let $\overline{H} = \frac{H}{|H|}$ and $\varphi = \left(\frac{|H|}{\sigma}\right)^2$, where $|H|$ denotes the Frobenius norm of H. If $N_t > 1$, X is taken from a super-constellation of $L = M_1 M_2 \cdots M_{N_t}$ symbols $X_1, X_2, \ldots, X_L$. Then assuming equiprobable symbols,

$$I(Y,X) = E\left(\log_2\left(\frac{p(Y,X)}{p(Y)p(X)}\right)\right) = E\left(\log_2\left(\frac{p(Y|X)}{\frac{1}{L}\sum_i p(Y|X_i)}\right)\right) = \log_2 L - \frac{1}{L}\psi(\varphi,\overline{H}),$$

(3.2)

where

$$\psi(\varphi,\overline{H}) = E\left(\sum_j \log_2\left(\sum_i \exp\left(-\frac{|H(X_j - X_i)|^2 + 2\operatorname{Re}((X_j - X_i)^H H^H U)}{\sigma^2}\right)\right)\right) =$$

$$\int_{C^{Nr}} \sum_j \log_2\left(\sum_i \exp\left(-\frac{|H(X_j - X_i)|^2 - 2\operatorname{Re}(X_i^H H^H U)}{\sigma^2}\right)\right)\frac{1}{(\pi\sigma^2)^{N_r}}\exp\left(-\frac{|U|^2}{\sigma^2}\right)dU =$$

$$\frac{1}{\pi^{N_r}}\int_{C^{N_r}} \sum_j \log_2\left(\sum_i \exp\left(-\varphi|\overline{H}(X_j - X_i)|^2 + 2\sqrt{\varphi}\operatorname{Re}(X_i^H \overline{H}^H U)\right)\right)\exp\left(-|U|^2\right)dU$$

(3.3)

and C^{N_r} denotes the N_r-dimensional complex space. Thus the integral in the above equation is a $2N_r$ -dimensional integral.

In the SIMO case ($N_t = 1$), it can be shown that

$$\psi(\varphi) = \frac{1}{\pi}\int_C \sum_j \log_2\left(\sum_i \exp\left(-\varphi|(X_j - X_i)|^2 + 2\sqrt{\varphi}\operatorname{Re}(X_i^H U)\right)\right)\exp\left(-|U|^2\right)dU.$$

(3.4)

It follows that ψ depends only on the SINR φ and does not depend on H except through its norm.

Thus using numerical integration, a table of RBIR vs. SINR can be calculated as done in table 24 in the EVM [1].

Unfortunately, in the case $N_t > 1$, ψ generally depends on $\overline{H}^H\overline{H}$. It is impossible to calculate the RBIR using only the SINR φ, since the structure of the matrix $\overline{H}^H\overline{H}$ may have a significant effect on the term $I(Y,X)$.

However, in the MIMO case ($N_t = 2$, $N_r = 2$), $\psi(\varphi,\overline{H})$ can still be calculated using numerical integration.

Consider the integral $\int_{-\infty}^{\infty} g(z)\exp(-z^2)dz$. Using the Gauss Quadratures method with Gauss-Hermite polynomials, n-point numerical integration is precise if g is a polynomial of degree $2n-1$. Let $w_1, w_2, \ldots, w_n$ and $a_1, a_2, \cdots, a_n$ denote the weights and abscissas, respectively, for n-point integration. For the values of these abscissas and weights, the reader is referred to [3]. Then if g is smooth enough so that it can be approximated by a polynomial of degree *2n-1*,

$$\int_{-\infty}^{\infty} g(z)\exp(-z^2)\,dz \approx \sum_{i=1}^{n} g(a_i)w_i. \tag{3.5}$$

Note that the weights and abscissas obey the following relation:

$$w_i = w_{n-i+1},$$
$$a_i = -a_{n-i+1}. \tag{3.6}$$

For an integral over the complex plane, we have the trivial extension:

$$\int_C g(u)\exp(-|u|^2)du \approx \sum_{j=1}^{n}\sum_{i=1}^{n} g(a_i + ia_j)w_i w_j. \tag{3.7}$$

Similarly, for an integral over C^2,

$$\int_C\int_C g(u_1,u_2)\exp(-|u_1|^2 - |u_2|^2)\,du_1\,du_2 \approx \sum_{l=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n} g(a_i + ia_j, a_k + ia_l)w_i w_j w_k w_l. \tag{3.8}$$

Let us define the real valued function ρ over C^2 as follows:

$$\rho(U) = \sum_j \log_2\left(\sum_i \exp\left(-\varphi\,|\overline{H}(X_j - X_i)|^2 + 2\sqrt{\varphi}\,\mathrm{Re}(X_i^{\ H}\overline{H}^{H}U)\right)\right). \tag{3.9}$$

Then

$$\psi = \frac{1}{\pi^2}\int_{C^2}\rho(U)\exp\left(-|U|^2\right)dU \approx \frac{1}{\pi^2}\sum_{l=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\rho(U_{ijkl})w_i w_j w_k w_l, \tag{3.10}$$

where $U_{ijkl} = [a_i + ia_j, a_k + ia_l]^T$.

Since ρ is symmetric[1], i.e., $\rho(U) = \rho(-U)$, the computation of $\psi(\varphi,\overline{H})$ can be cut by half, if n is chosen to be even, as follows:

$$\psi \approx \frac{2}{\pi^2}\sum_{l=1}^{n/2}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\rho(U_{ijkl})w_i w_j w_k w_l, \tag{3.11}$$

[1] Although not readily apparent from 3.9, it can be shown that symmetry results from the summation over the *i* and *j* indices

Therefore $I(Y,X)$ can be calculated using (3.2) and the numerical integration in (3.11).

Our simulation results suggest that *n*-point integration is very accurate for *n* as low as 10 since the function ρ is very smooth. Larger values of *n* yield only a small improvement. Thus for *n=10* we would need on the order of $\frac{n^4}{2} = 5000$ multiplications and evaluations of ρ. Certainly it is the evaluations of ρ that are expensive and not the multiplications with the weights. An efficient way to evaluate or approximate ρ can significantly speed up the integration.

Let H_i denote the ith column of H and $\varphi_i = \frac{|H_i|^2}{\sigma^2} = \varphi \frac{|H_i|^2}{|H|^2}$. It can be shown that $I(Y, x_2 | x_1)$ (or in entirely analogous way $I(Y, x_1 | x_2)$) can be calculated as follows:

$$I(Y, x_2 | x_1) = \log_2 M_2 - \frac{1}{M_2} f(\varphi_2), \tag{3.12}$$

where

$$f(z) = \frac{1}{\pi} \int_C \sum_{j=1}^{M_2} \log_2 \left(\sum_{i=1}^{M_2} \exp\left(-z\,|(b_j - b_i)|^2 + 2\sqrt{z}\,\mathrm{Re}(b_i^H v) \right) \right) \exp\left(-|v|^2 \right) dv \tag{3.13}$$

and $b_1, b_2, \ldots, b_{M_2}$ are the symbols of the M_2-size constellation.

Note that $f(z)$ is equivalent to $\psi(z)$ of the SIMO case. Thus $I(Y, x_2 | x_1)$ can be calculated using Table 24 in EVM [1] by replacing the SINR parameter in that table with φ_2.

To conclude, $I(Y, x_1)$ in the MIMO case ($N_t = 2$, $N_r = 2$) can be calculated using (3.1), (3.2), and (3.11)-(3.13).

The main difficulty in calculating $I(Y, x_1)$ is in calculating $I(Y, X)$. As mentioned earlier, the latter can be done using numerical integration, but it may be helpful to find more efficient methods for the sake of the reduced complexity of the system simulation. Until this is done, the method proposed here can be used in order to gauge the performance of other approximations or derivations of the RBIR for the MIMO case.

4. Previously Proposed methods for the Maximum-Likelihood MIMO Receiver

We would like to compare our approach for MIMO Matrix B system with the approach adopted in a previous contribution [2]. We would need to digress briefly to the SISO case in order to make our arguments in the sequel clearer.

In the SISO case, it is certainly true that eqn. (1.3) in [2] holds:

$$I(Y, X) = \frac{1}{M} \sum_{i=1}^{M} E\left\{ \log_2 \left(\frac{M}{1 + \exp(-LLR_i)} \right) \right\}, \tag{4.1}$$

where LLR_i is the Symbol Level LLR of the ith symbol (see eqn. (1.2) in [2]) .

Eqn. (4.1) requires one dimensional integration. However, $p(LLR_i)$ is generally cumbersome.

It is proposed in [2] to approximate $p(LLR_i)$ as Gaussian with mean *AVE* and variance *VAR*.

In the SISO case, since the LLR depends only on the SINR, a lookup table for AVE and VAR can be calculated and used. An attempt to generate such a table was done in Table 25 in [1]. It is important to note that currently there is an error in that table. Indeed, if the RBIR is plotted based on this table, by performing the integration in (4.1) numerically, the RBIR is not even monotonic (see Fig. 1 below), which means that in a certain region of SINR, we may decrease the transmit power and have better performance! Observing Appendix Q in [1], the error follows from lines 12 and 19 in pp. 161, which are the expressions for $E(K_1)$ and $E(K_1^{\ 2})$.

The error is, in fact, in eqn. (1.13) in [2]. In order to calculate these expectations, it is necessary to perform double integration, but there is an error in the derivation and a one dimensional integral is performed instead.

The equations should be corrected as described below. Certainly

$$E(K)=E\left(\log_e\left(e^{-\frac{2d(h_rn_r+h_in_i)}{\sigma^2}}+e^{-\frac{2d(h_rn_i-h_in_r)}{\sigma^2}}+e^{-\frac{d^2|h|^2}{\sigma^2}}e^{-\frac{2d(h_rn_r+h_in_i)}{\sigma^2}}e^{-\frac{2d(h_rn_i-h_in_r)}{\sigma^2}}\right)\right) \tag{4.2}$$

holds for QPSK as mentioned in [2], where n_i and n_r are independent zero mean Gaussian random variables with variance $\frac{\sigma^2}{2}$, *d* is the distance between neighboring symbols, *h* is the channel, and h_r and h_i are the real and imaginary parts of *h*, respectively. Let $u_1=\frac{h_rn_r+h_in_i}{\sigma^2}$ and $u_2=\frac{h_rn_i-h_in_r}{\sigma^2}$. Then u_1 and u_2 are independent zero mean Gaussian random variables with variance $\frac{|h|^2}{2\sigma^2}$. Thus

$$E(K)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\log_e\left(e^{-2du_1}+e^{-2du_2}+e^{-\frac{d^2|h|^2}{\sigma^2}}e^{-2du_1}e^{-2du_2}\right)\frac{\sigma^2}{\pi|h|^2}e^{-\frac{\sigma^2u_1^2}{|h|^2}}e^{-\frac{\sigma^2u_2^2}{|h|^2}}du_1du_2 \tag{4.3}$$

Similarly,

$$E(K^2)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\log_e\left(e^{-2du_1}+e^{-2du_2}+e^{-\frac{d^2|h|^2}{\sigma^2}}e^{-2du_1}e^{-2du_2}\right)\right)^2\frac{\sigma^2}{\pi|h|^2}e^{-\frac{\sigma^2u_1^2}{|h|^2}}e^{-\frac{\sigma^2u_2^2}{|h|^2}}du_1du_2. \tag{4.4}$$

Now, with the corrected expression a new table for AVE and VAR can be generated.

Using the corrected table (see Table 1 below), we can compare the RBIR calculated using Table 24 in [1], which calculates RBIR directly from the definition, not using the mean and variance of the LLR, the RBIR using Table 25 in [1], and the RBIR using Table 1 below. Certainly, the RBIR vs. SINR plot should be similar whether the calculation is done by the direct method of Table 24 in [1], or using the Gaussian approximation for the LLR. It can be seen that this is indeed the case once the corrected table (Table 1) is used.

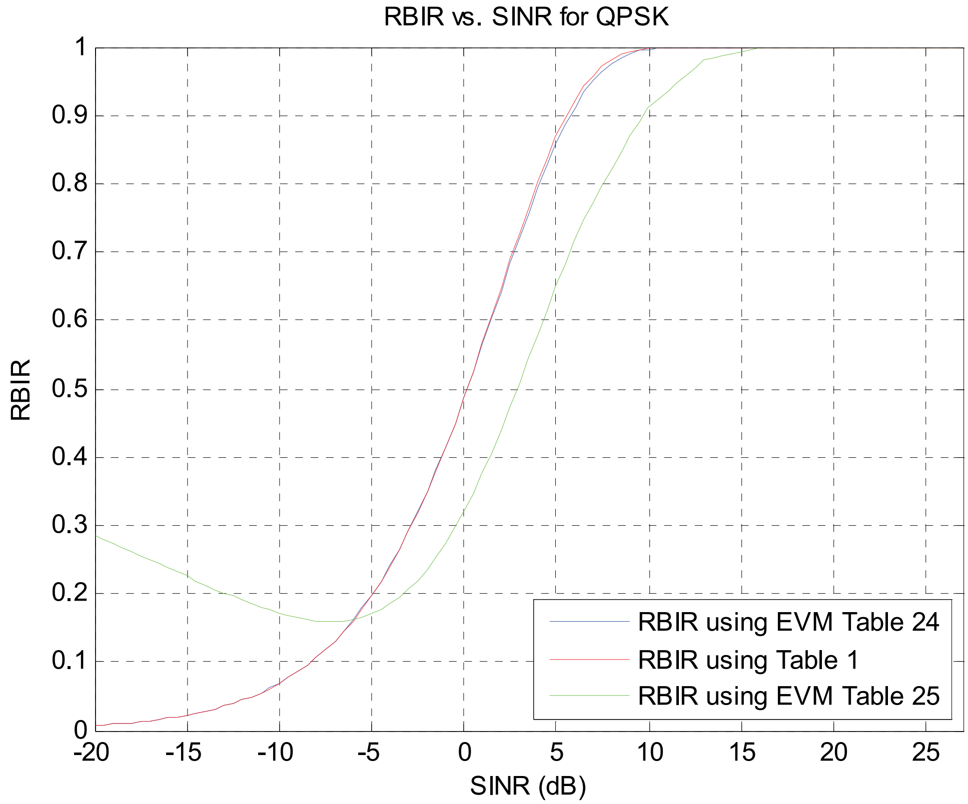

Figure 1. RBIR vs. SINR for QPSK in the SISO case

Let $\gamma = d^2\varphi = \dfrac{d^2 |h|^2}{\sigma^2}$. The corrected table is given below.

γ_{dB} (dB)	[-20:0.5:30]
AVE	[-1.0897 -1.0886 -1.0874 -1.0861 -1.0845 -1.0828 -1.0809 -1.0787 -1.0763 -1.0736 -1.0706 -1.0672 -1.0633 -1.0590 -1.0542 -1.0488 -1.0428 -1.0360 -1.0284 -1.0199 -1.0104 -0.9997 -0.9878 -0.9744 -0.9594 -0.9426 -0.9237 -0.9027 -0.8791 -0.8528 -0.8233 -0.7903 -0.7534 -0.7121 -0.6660 -0.6144 -0.5568 -0.4923 -0.4202 -0.3396 -0.2494 -0.1485 -0.0356 0.0908 0.2324 0.3910 0.5690 0.7687 0.9930 1.2451 1.5287 1.8481 2.2081 2.6139 3.0720 3.5892 4.1734 4.8336 5.5796 6.4229 7.3758 8.4526 9.6691 11.0431 12.5943 14.3453 16.3210 18.5497 21.0631 23.8966 27.0902 30.6887 34.7425 39.3081 44.4491 50.2368 56.7512 64.0822 72.3308 81.6103 92.0481 103.7868 116.9869 131.8285 148.5137 167.2693 188.3500 212.0417 238.6653 268.5809 302.1929 339.9550 382.3764 430.0288 483.5536 543.6709 611.1887 687.0139 772.1641 867.7816 975.1480]
VAR	[1.7724e-002 1.9879e-002 2.2296e-002 2.5005e-002 2.8041e-002 3.1445e-002 3.5259e-002 3.9533e-002 4.4321e-002 4.9684e-002 5.5691e-002 6.2416e-002 6.9944e-002 7.8370e-002 8.7796e-002 9.8339e-002 1.1013e-001 1.2330e-001 1.3802e-001 1.5446e-001 1.7280e-001 1.9327e-001 2.1609e-001 2.4151e-001 2.6983e-001 3.0134e-001 3.3638e-001 3.7533e-001 4.1858e-001 4.6657e-001 5.1979e-001 5.7876e-001 6.4407e-001 7.1634e-001 7.9627e-001 8.8465e-001 9.8233e-001 1.0903e+000 1.2096e+000 1.3415e+000 1.4874e+000 1.6488e+000 1.8278e+000 2.0264e+000 2.2472e+000 2.4930e+000 2.7674e+000 3.0743e+000 3.4183e+000 3.8048e+000 4.2399e+000 4.7305e+000 5.2845e+000 5.9107e+000 6.6189e+000 7.4200e+000 8.3258e+000 9.3493e+000 1.0505e+001 1.1808e+001 1.3277e+001 1.4930e+001 1.6790e+001 1.8880e+001 2.1229e+001 2.3867e+001 2.6830e+001 3.0158e+001 3.3895e+001 3.8091e+001 4.2803e+001 4.8093e+001 5.4033e+001 6.0702e+001 6.8190e+001 7.6596e+001 8.6033e+001 9.6628e+001 1.0852e+002 1.2187e+002 1.3686e+002 1.5368e+002 1.7257e+002 1.9377e+002 2.1756e+002 2.4426e+002 2.7424e+002 3.0788e+002 3.4564e+002 3.8801e+002 4.3557e+002 4.8895e+002 5.4885e+002 6.1608e+002 6.9153e+002 7.7619e+002 8.7121e+002 9.7784e+002 1.0975e+003 1.2318e+003 1.3825e+003]

Table 1: Mean and Variance for Symbol Level LLR

It is important to note that the approach of using the mean and the variance of the Symbol Level LLR does not provide any gain for the SISO case since Table 24 in [1] can be used instead.

It only makes sense to use this approach in the MIMO case if it provides any computation advantage over direct numerical integration.

Let us return to the MIMO case for which $N_r = N_t = 2$. In eqn. (1.18) in [2] the LLR of the first stream is approximated and a rather simple expression is given. In that expression the LLR of the first stream depends only on H_1, the first column of *H*, and does not depend at all on H_2, the second column of *H*. This expression is in fact the LLR of the first stream when the second stream is given (i.e., second stream is perfectly known). The exact expression for the LLR is much more complicated and depends, in general, on H_2. If the RBIR is calculated based on the approximation (1.18) in [2], it is not calculated based on $I(Y, x_1)$, but based on $I(Y, x_1 | x_2)$. Hence the approximation (1.18) is accurate when $I(Y, x_1) \approx I(Y, x_1 | x_2)$.

It can be shown that this is the case when $\frac{\alpha}{\sigma^2} << 1$, where $\alpha = | H_2^H H_1 |$. The case $\frac{\alpha}{\sigma^2} << 1$ can certainly occur, but it is not guaranteed to be the only case of interest. In general, the mean and variance of the LLR of the first stream can depend on H_2. It can be shown that the mean and variance of the LLR depend, in general, on 4 real parameters: $\varphi, \frac{\alpha}{|H|^2}, \frac{|H_1|}{|H|}$, and $ARG(H_2^H H_1)$. Hence lookup tables can be prohibitively large. Calculating the mean and variance requires two 4-dimensional integrations, with roughly the same complexity as in (3.11).

Moreover, each symbol in the super-constellation has a different LLR and even if an approximation based on the dominant constellation points is used, as proposed in [2], still the LLRs are different because of the channel *H*.

In [2] a fudge factor '*a'* was introduced and carried over into the EVM text in [1]. In a sense the factor '*a'* introduces back some dependency on the structure of the matrix *H* through its singular values.

The danger in introducing this fudge factor is that while it may be optimized for certain values of *H*, it may fail for other values of *H*. As can be seen in Table 26 in [1], the values of '*a*' for the two different streams are not always the same, which does not make sense because of the following symmetry argument: we should certainly obtain the same BLER results for each stream if we interchange the two streams and exchange the columns of the matrix *H*. However, replacing the columns of the matrix *H* does not change its singular values, but Table 26 in [1] implies that we would get a different RBIR mapping for both streams in these two cases (i.e., before and after we interchange the two streams) whenever the parameter '*a*' for those streams is not identical and the BLER results for each stream will be different.

Also, the table has different values for different coding rates, which also does not make sense since RBIR is calculated based on the constellation and not based on coding rate.

To illustrate the points above, we randomly picked the following normalized channel matrix $\overline{H} = \begin{bmatrix} 0.1673 + 0.6764i & -0.1036 - 0.0401i \\ 0.6964 - 0.1239i & -0.0434 - 0.0026i \end{bmatrix}$. We then calculated the RBIR using numerical integration and using Tables 25 and 26 in [1]. The results are presented in Fig. 2.

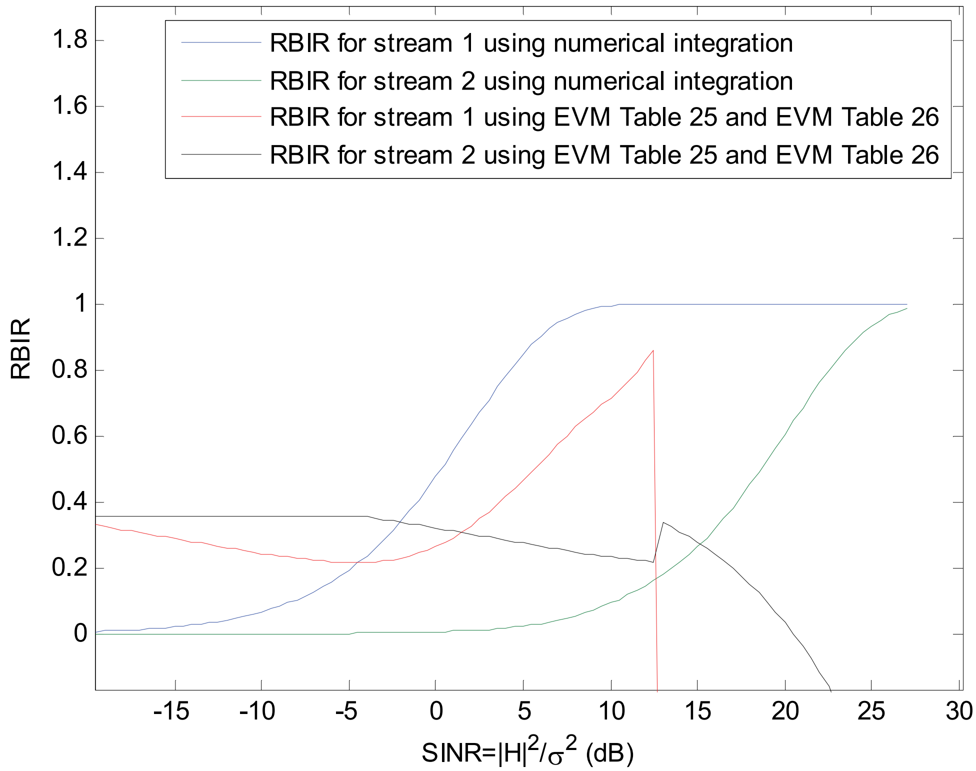

Figure 2. RBIR vs. SINR for QPSK in the MIMO case: numerical integration vs. tables in the EVM

It can be seen that the RBIR calculated using Tables 25 and 26 in [1] does not match the correct one using numerical integration as described in Section 3. In fact, the behavior of the RBIR calculated from Tables 25 and 26 is anomalous since it is not monotonically increasing and suffers from discontinuities because of the use of the factor '*a*'. Moreover, in some regions the curve is not even positive!

Since we already know that Table 25 in [1] is erroneous, we use Table 1 instead of Table 25 in [1], but still use the factor '*a*' as described in Table 26 in [1] to get Fig. 3. It can be seen that RBIR calculated using these tables is still significantly different than the one using numerical integration and that some anomalous behavior is still there. Note that in the MIMO case γ used in Table 1 is $\gamma = d^2\varphi_i = \frac{d^2 |H_i|^2}{\sigma^2}$.

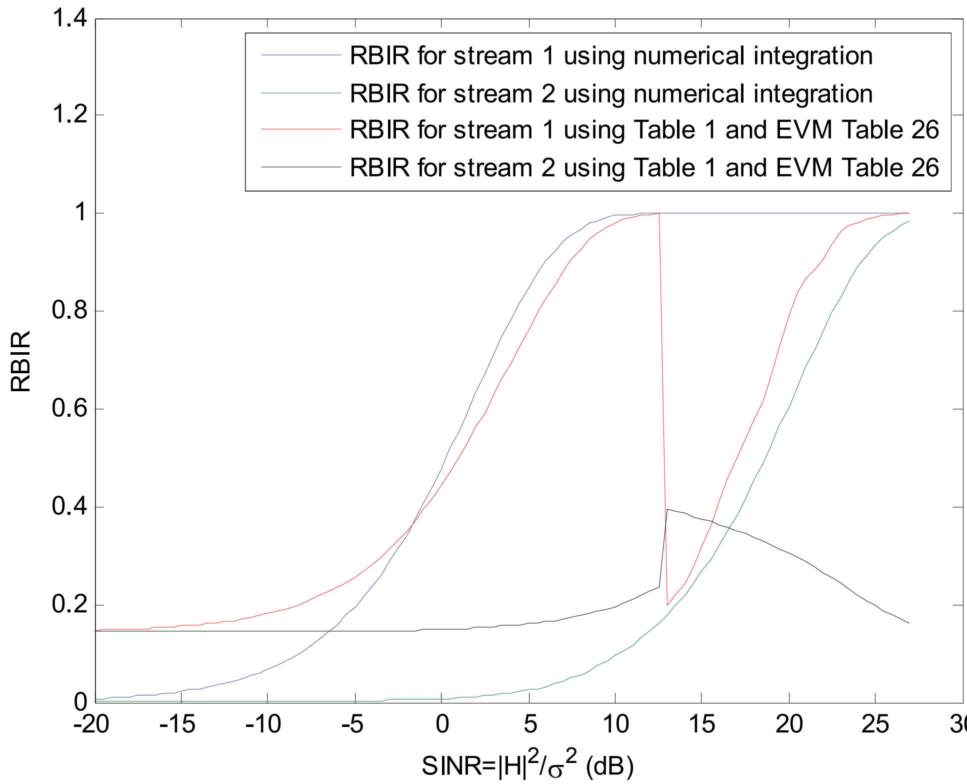

Fig. 3. RBIR vs. SINR for QPSK in the MIMO case: numerical integration vs. Table 1 and EVM Table 26

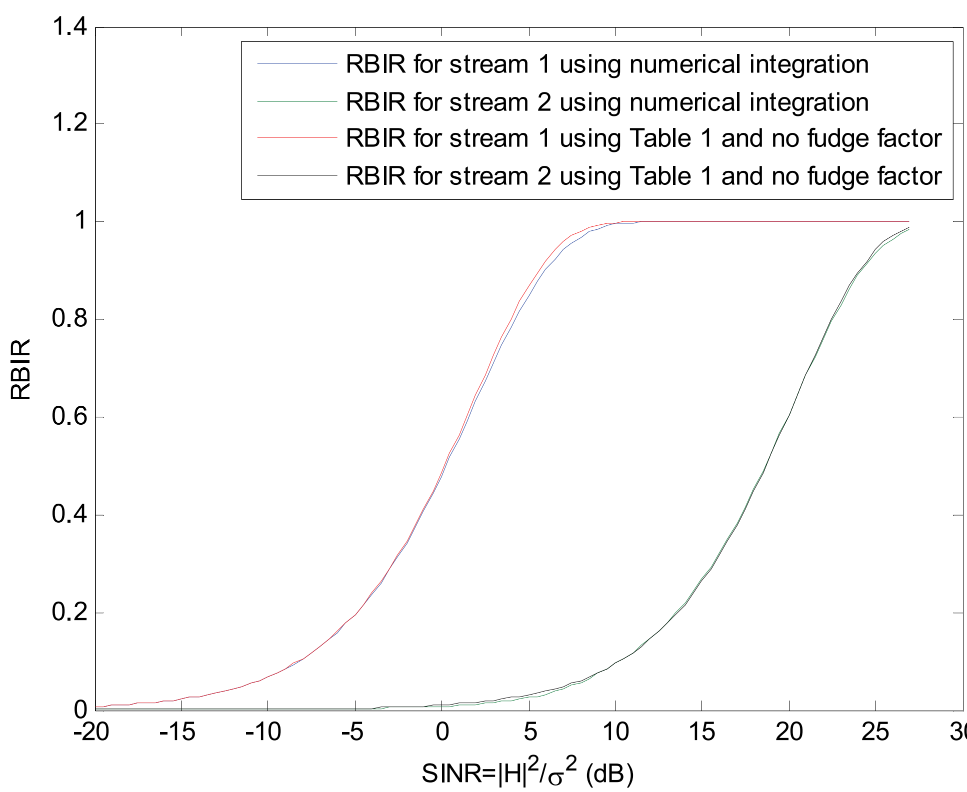

Figure 4. RBIR vs. SINR for QPSK in the MIMO case: numerical integration vs. Table 1 (small α)

Finally, we use just Table 1 with no fudge factor 'a' to get Fig. 4. Note that for the chosen channel $\frac{\alpha}{\sigma^2}$ is small for low and moderate SINR φ since $\frac{|H_2^H H_1|}{|H|^2} = 0.09$. Hence we expect to have a good approximation of the RBIR when Table 1 is used. It can be seen in Fig. 4, that for both streams, the RBIR curves generated using Table 1 are very similar to the ones calculated using numerical integration.

It is important to note that although Table 1 provides good results when α is small, there is no reason to use the approach of calculating the RBIR using the mean and variance of the LLR. Indeed, Table 24 in the EVM [1] can still be used if the 'SINR' parameter in this table is replaced by $SINR_1$ and $SINR_2$ for the first and second streams, respectively, where $SINR_1$ and $SINR_2$ are defined as $\varphi_1 = \frac{|H_1|^2}{\sigma^2}$ and $\varphi_2 = \frac{|H_2|^2}{\sigma^2}$, respectively.

In Fig. 5, we show that if we calculate the RBIR for the first and second streams using Table 24 in [1] as described above, we get a very similar figure to Fig. 4.

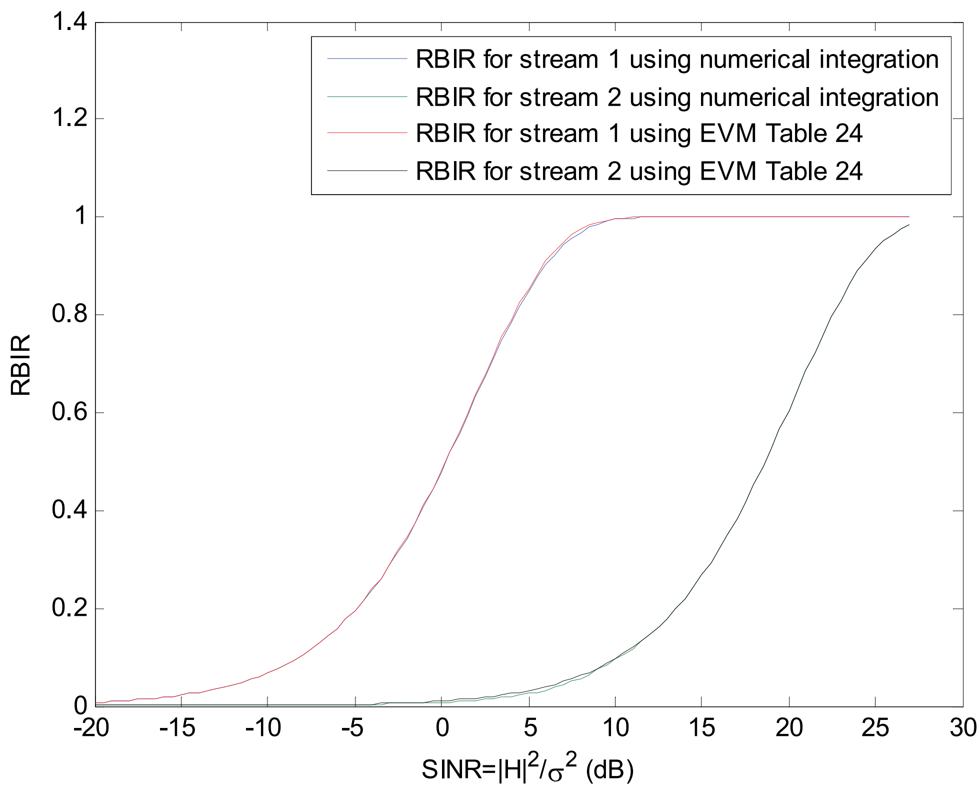

Figure 5. RBIR vs. SINR for QPSK in the MIMO case: numerical integration vs. EVM Table 24 (small α)

We emphasize that when α is not small, the approximation of $I(Y, x_1)$ by $I(Y, x_1 | x_2)$ may not be accurate anymore and hence using Table 24 in [1] or Table 1 as described above may not give us accurate results.

Consider the following channel as an example: $\overline{H} = \begin{bmatrix} 0.1078 + 0.4358i & -0.6675 - 0.2584i \\ 0.4487 - 0.0798i & -0.2796 - 0.0168i \end{bmatrix}$. In this normalized channel matrix the columns of the channel from the previous example are scaled (i.e., columns H_1 and H_2 are weighted by α_1 and α_2, respectively). In this case $\frac{|H_2{}^H H_1|}{|H|^2} = 0.38$ and α is larger than in the previous example. As can be seen in Fig. 6, the RBIR curves calculated using Table 24 in [1] (i.e., assuming the small α approximation) differ from the correct RBIR curves, calculated using numerical integration.

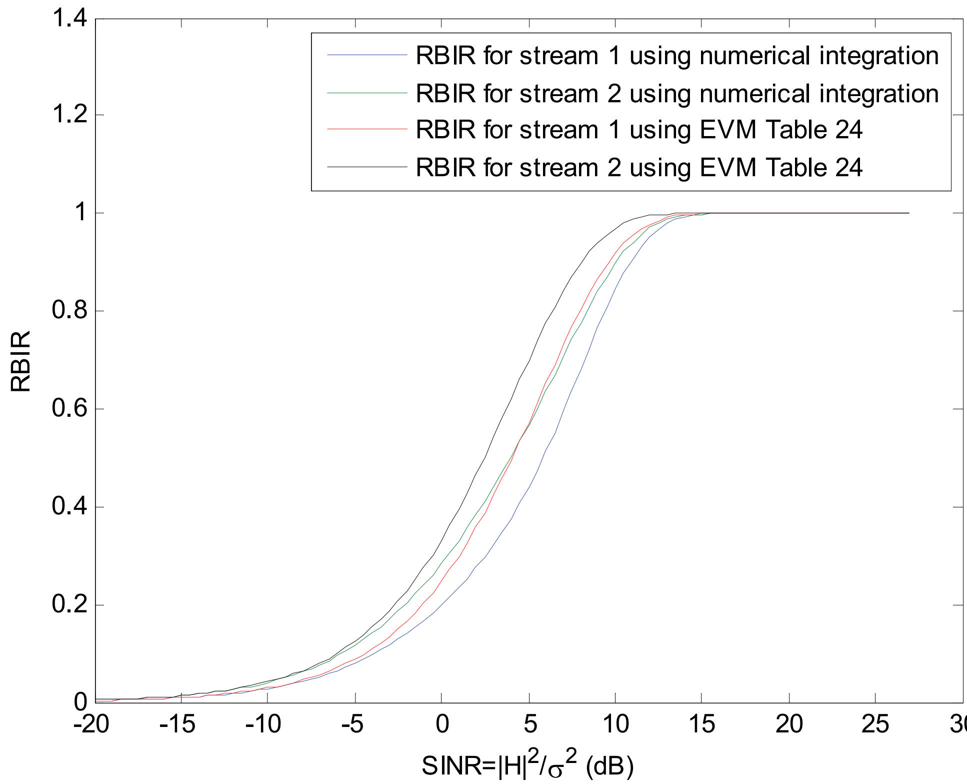

Figure 6. RBIR vs. SINR for QPSK in the MIMO case: numerical integration vs. EVM Table 24 (moderate α)

Thus, in general, except for the case of small α , RBIR in the MIMO case should be preferably calculated using the numerical integration method of Section 3. Of course, it would be very useful to derive an approximation that would enable us to use a simple lookup table, but this approximation should hold in all cases of interest and not only for the case of small α .

5. Suggested Remedy

Due to the discussion in the previous sections, we propose to make the following changes in the EVM document [1]. We propose to remove from section 4.3.1.3 lines 13-23 in page 63, pages 64, 65, and lines 1-2 on page 66 (including deletion of Table 26), and replace them with the following results from Section 3 above:

{

Consider the case of channel input $X = [x_1, x_2]^T$, channel output $Y = [y_1, y_2]^T$ and channel matrix $H = [H_1, H_2]$, where H_1 and H_2 are each two-element column vectors. The symbols x_1, and x_2 are both taken from a constellations of size M_1 and M_2, respectively, and hence X is taken from a super-constellation with symbols $X_1, X_2, \ldots, X_{M_1 M_2}$. Then

$Y = HX + U$, where U is a zero mean complex Gaussian noise vector of dimension 2 with covariance $\sigma^2 I$.

Define

$$RBIR_1 = \frac{I(Y, x_1)}{\log_2(M_1)}, \qquad RBIR_2 = \frac{I(Y, x_2)}{\log_2(M_2)},$$

where $I(Y, x_i)$ denotes the mutual information of Y and x_i. It can be verified that [Insert reference number for (C80216m-08_543)]

$$I(Y, x_1) = I(Y, X) - I(Y, x_2 \mid x_1),$$

$$I(Y, x_2) = I(Y, X) - I(Y, x_1 \mid x_2),$$

where $I(Y, X)$ denotes the mutual information of Y and X, and $I(Y, x_2 \mid x_1)$ and $I(Y, x_1 \mid x_2)$ denote the conditional mutual information of Y and x_2 given x_1 and Y and x_1 given x_2, respectively.

It can be shown that

$$I(Y, X) = \log_2(M_1 M_2) - \frac{1}{M_1 M_2} \psi(\varphi, \overline{H}),$$

where

$$\psi(\varphi, \overline{H}) = \frac{1}{\pi^2} \int_{C^2} \sum_j \log_2\left(\sum_i \exp\left(-\varphi \,|\, \overline{H}(X_j - X_i) \,|^2 + 2\sqrt{\varphi}\, \mathrm{Re}(X_i^H \overline{H}^H U) \right) \right) \exp\left(-|U|^2 \right) dU,$$

C^2 is a 2-D complex space (i.e., the above is a four-dimensional integral), $\overline{H} = \frac{H}{|H|}$, $\varphi = \left(\frac{|H|}{\sigma} \right)^2$, and $|H|$ denotes the Frobenius norm of H. The conditional mutual information $I(Y, x_2 \mid x_1)$ (and in entirely analogous way $I(Y, x_1 \mid x_2)$) is given by

$$I(Y, x_2 \mid x_1) = \log_2 M_2 - \frac{1}{M_2} f(\varphi_2), \text{ where } \varphi_i = \frac{|H_i|^2}{\sigma^2} = \varphi \frac{|H_i|^2}{|H|^2},$$

$$f(z) = \frac{1}{\pi} \int_C \sum_{j=1}^{M_2} \log_2\left(\sum_{i=1}^{M_2} \exp\left(-z \,|\, (b_j - b_i) \,|^2 + 2\sqrt{z}\, \mathrm{Re}(b_i^H v) \right) \right) \exp\left(-|v|^2 \right) dv,$$

and $b_1, b_2, \ldots, b_{M_2}$ are the symbols of the M_2-size constellation.

Note that $I(Y, x_2 \mid x_1)$ can be evaluated using Table 24 by replacing the SINR parameter in that table with φ_2.

Also note that $I(Y, X)$ above can be calculated using Gauss-Hermite quadrature integration. Let $w_1, w_2, \ldots, w_n$ and $a_1, a_2, \cdots, a_n$ denote the weights and abscissas, respectively, for n-point integration [3]. Let us define the real valued function ρ over C^2 as follows:

$$\rho(U) = \sum_{j=1}^{M_1 M_2} \log_2\left(\sum_{i=1}^{M_1 M_2} \exp\left(-\varphi \,|\, \overline{H}(X_j - X_i)\,|^2 + 2\sqrt{\varphi}\,\mathrm{Re}(X_i^{\,H} \overline{H}^{\,H} U)\right)\right).$$

Then

$$\psi = \frac{1}{\pi^2} \int_{C^2} \rho(U) \exp\left(-\,|\,U\,|^2\right) dU \approx \frac{1}{\pi^2} \sum_{l=1}^{n} \sum_{k=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \rho(U_{ijkl}) w_i w_j w_k w_l ,$$

where $U_{ijkl} = [a_i + ia_j , a_k + ia_l]^T$.

Since ρ is symmetric[2], i.e., $\rho(U) = \rho(-U)$, the computation of $\psi(\varphi, \overline{H})$ can be cut in half, by choosing *n* even, as follows:

$$\psi \approx \frac{2}{\pi^2} \sum_{l=1}^{n/2} \sum_{k=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \rho(U_{ijkl}) w_i w_j w_k w_l .$$

}

We propose to replace lines 5-11 in page 66 with the following:

{

For a 2x2 system using MIMO Matrix B and horizontal encoding, the RBIR metric is computed individually for each stream by computing $RBIR_1$ and $RBIR_2$ for streams 1 and 2, respectively.

}

We propose to replace lines 15-18 on page 66 and lines 1-5 on page 67 with the following:

{

For a 2x2 system using MIMO Matrix B and vertical encoding, RBIR can be computed as a weighted sum of the individual RBIRs, i.e.,

$$RBIR = p_1 RBIR_1 + p_2 RBIR_2 ,$$

where p_1 and p_2 are given in Table 27.

(Note to the editor: Equation 44 from C80216m-08/004r2 needs to be saved in order to use Table 27)

}

In addition, we propose to replace the text in appendix Q with the following

{

Appendix Q

Search for the values of p1 and p2

The procedure used to obtain the parameters p_1 and p_2 [78] can be described as follows:

---

[2] It can be shown that symmetry results from the summation over the *i* and *j* indices

Step 1: From the AWGN SINR-to-BLER curve, calculate the $SINR_{AWGN}(BLER)$ from the measured BLER.
Step 2: Calculate the corresponding RBIR metric over the two streams for a given channel matrix 'H' and SINR.
Step 3: Calculate the average RBIR metric as a weighted sum of '*p1*' and '*p2*' and then calculate the effective $SINR_{eff}$ value using the averaged RBIR from the SINR to RBIR mapping in Table 24.
Step 4: Find the parameters *p1* and *p2* which result in the smallest gap over all values of BLER between the interpolated SINR (step 1) and effective SNR (step 3).

$$\Delta = \min_{(p_1, p_2; p_1 + p_2 = 1)} | SINR_{AWGN}(BLER) - SINR_{eff}(BLER) |^2$$

$\forall BLER$ and $\forall H$ which belongs to a particular range of k and λ_{min}dB.
}

6. Conclusions

We have shown that there are some errors in section 4.3.1.3 of the EVM document [1] that cause the RBIR calculation for MIMO to be erroneous. We propose to perform numerical integration using the Gauss Quadratures method with Gauss-Hermite polynomials to fix this problem. While from the theoretical view point, our solution is complete, it may be too computationally complex if RBIR mappings are done intensively.

We showed that in some cases a simple approximation (which is equivalent to an approximation in [2]) can be done and a lookup table may be used, but that in the general case, this approximation may fail. Hence it would be desirable to develop accurate approximations for the RBIR in the MIMO case in order to replace the numerical integration by a more computationally-efficient method. The numerical integration method proposed here can then be used as an evaluation tool to gauge the accuracy of future approximations.

7. References

[1] IEEE 802.16m Evaluation Methodology Document (EMD), IEEE802.16m-08/004r2, 3 July 2008.
[2] Hongming Zheng et al., "Link Performance Abstraction for ML Receivers based on RBIR Metrics," IEEE C802.16m-08/119, 4 March 2008. (Note: This contribution is reference 78 in the 16m EVM)
[3] M. Abramowitz and I. Stegun, Handbook of Mathematical Functions, (see Gauss-Hermite integration, Table 25.10, p. 924), Dover Publications, 1970.