Project	**IEEE 802.16 Broadband Wireless Access Working Group** <http://ieee802.org/16>
Title	**Hybrid Analog/Digital Feedback for CL-MIMO**
Date Submitted	**2008-09-05**
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Re:	PHY: MIMO; in response to the TGm Call for Contributions and Comments 802.16m-08/033 for Session 57
Abstract	This document describes a proposal for CL-MIMO feedback
Purpose	To be discussed and adopted by 802.16m SDD
Notice	
Release	
Patent Policy	

Hybrid Analog/Digital Feedback for CL-MIMO

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Nextwave Wireless

1. Introduction

In a previous contribution C802.16m-08/529r1 we analyzed the performance of analog feedback and compared it to digital codebook-based feedback.

The current trend of digital communications is based on Shannon's source channel separation theorem. Applying it to the CL-MIMO problem means that the source (e.g. V from SVD of the channel) is first quantized by using a codebook and the quantization bits are coded using a channel code and transmitted.

However, in some cases, and in particular for CL-MIMO, this approach is suboptimal since:

- The communication system is delay constrained and operating in time selective Rayleigh fading (where strong FEC codes and HARQ can't be used)
- The SNR is unknown at the transmitter.

As is known, digital communication suffer a cliff effect more noticeable than analog communication. When SNR is below the design point, performance deteriorates rapidly whereas above the design point there is no throughput improvement as the system throughput is limited by the choice of input MCS.

This was clearly seen in -08/529r1 by looking at the slopes of the capacity ratio of digital vs analog. At low SNR digital deteriorates at a higher rate than analog. At high SNR the digital approach doesn't improve due to the quantization effect of V whereas the analog approach continues to improve.

However, analog transmission of the source is only optimal for transmitting a Gaussian source over a Gaussian channel where the source bandwidth is the same as the channel bandwidth. In some cases the channel bandwidth may be greater than the source bandwidth and hence a pure analog scheme may be suboptimal as well. For example, transmitting the analog V of a 2x4 channel (8 complex values) over 16 subcarriers means that the channel bandwidth is twice the source bandwidth.

As a result of these short-comings of both analog and digital transmission we propose to use a joint source-channel code which consists of a hybrid of digital and analog transmission to feed back the channel information.

It has been shown in e.g. [3] that joint source-channel codes are more robust than separate source and channel codes over a channel with unknown SNR and have better performance for a given delay. Hence these codes are more suitable for our application. Here, we will construct a suitable hybrid digital-analog code for the channel feedback application. Our scheme consists of first quantizing the feedback information using few bits, sending those using a short algebraic code and then sending the quantization error using analog transmission.

Before explaining the details we provide some useful information on the various analog feedback options, an improved digital feedback mechanism and sensitivity to interference results.

2. What Type of Analog Information?

The following plots compare the performance of several analog feedback options for CL-MIMO using the tile structure of 802.16e (a tile contains 8 data subcarriers and 4 pilots in a 4x3 structure).

The simulations assume uncorrelated 2x4 channel with real UL channel estimation at the BS. The UL channel is assumed Ped-B 3kmph. Due to power concentration at the MS the DL SNR was assumed equal to UL SNR. For more information on the simulation assumptions please refer to the description in -08/529r1. One, three or six tiles per channel feedback were used.

1. V- denotes feedback of 8 complex values of the columns of V(from SVD of the channel) whereas if rank adaptation was done at the MS and rank-1 was chosen, only the first column (4 complex values) was transmitted.
2. R- denotes feedback of the 8 complex values of the 4x4 channel covariance matrix
3. H- denotes feedback of the 8 complex values of the channel H
4. SV – denotes feedback of the product SV' which constitutes the minimum sufficient feedback information for any MU-MIMO algorithm
5. QR - denotes feedback of R from the QR decomposition of the channel H.

Conclusions:

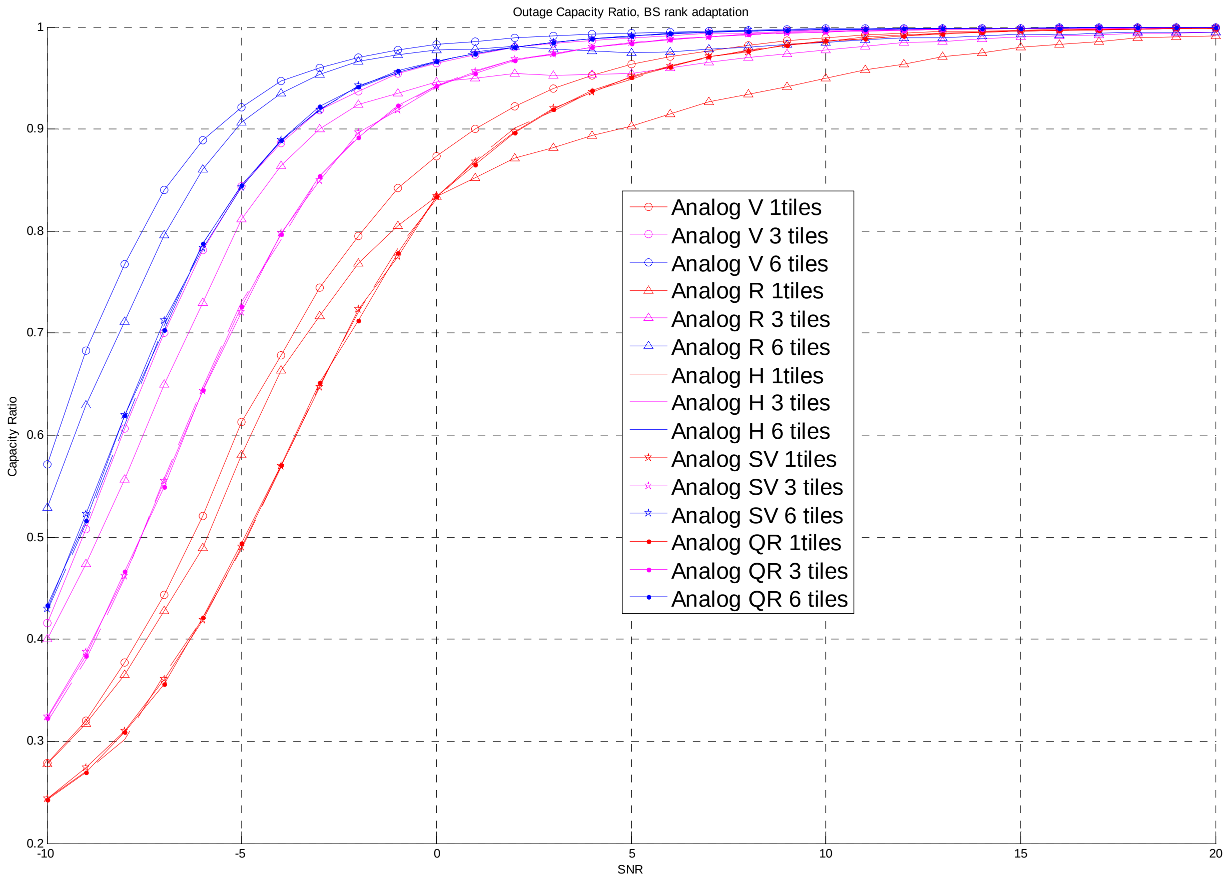
1. Feeding back rank adapted V is optimal at any SNR for SU-MIMO and is a good pragmatic approach for MU-MIMO and we therefore use it for our hybrid scheme.

    a. Note that we previously proposed feeding back V for a pure analog based feedback and provided two related algorithms in -08/522r1 and -08/526r1.

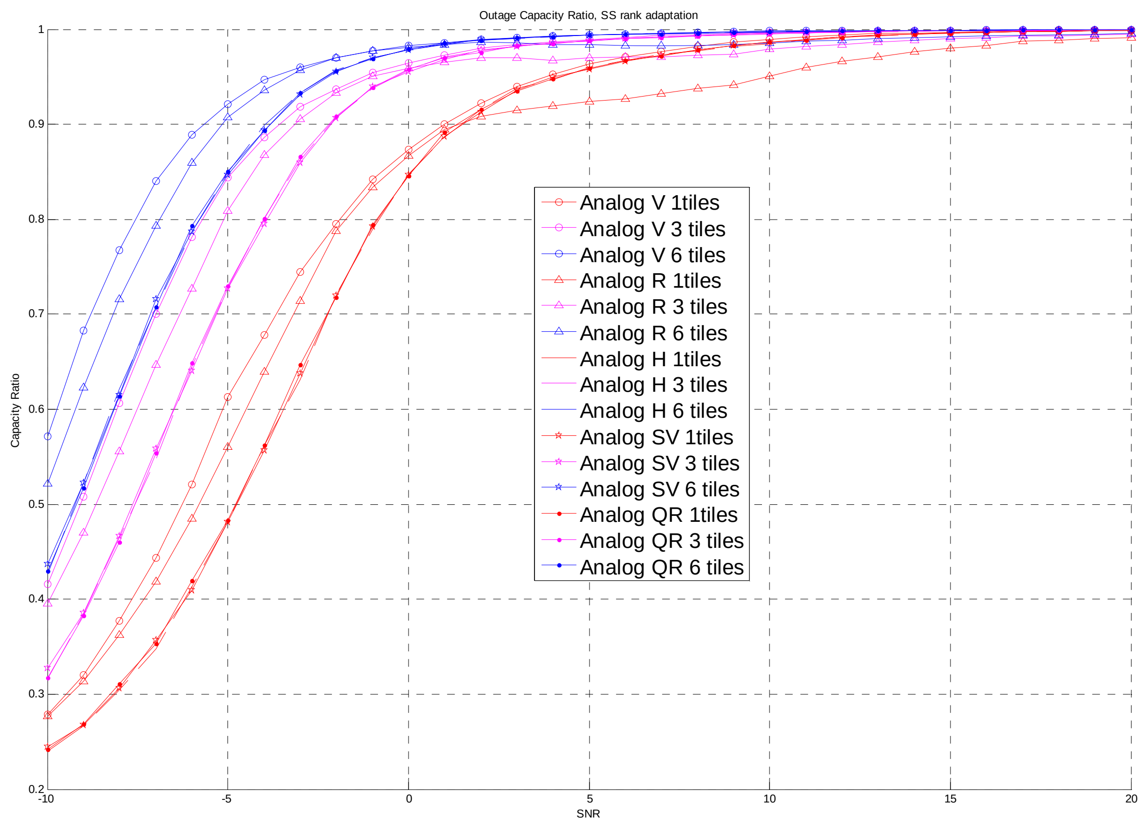
2. Feeding back the channel covariance R is second best below 0dB and the worst scheme above 0dB. The reason is that all schemes besides rank adaptation V, feed back a 'weighted' sum of the two singular vectors. At low SNR where rank-1 is more likely, R which has stronger relative weight on the larger singular vector (due to squaring of the singular values in R) provides closer to optimal ratio whereas at high SNR the second singular vector suffers from weaker weight for the same reason. This causes poor estimation of the second singular vector at high SNR.

3. The plot comparing rank adaptation at the BS and MS shows that while feeding back R gives full channel knowledge to the BS, as a result of noise in the UL there is reduced performance in the medium SNR range due to wrong rank decisions.

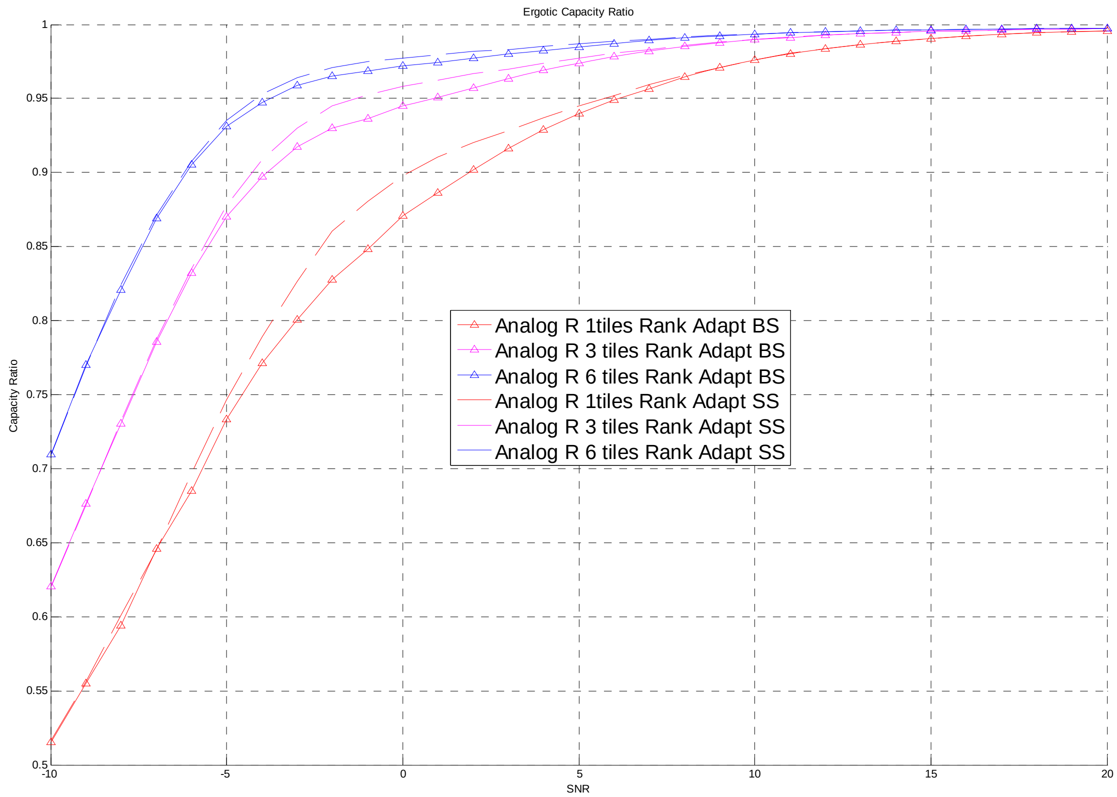
Outage Capacity ratio with Rank adaptation done at the BS



Outage Capacity ratio with Rank adaptation done at the MS



Rank adaptation at the BS vs MS



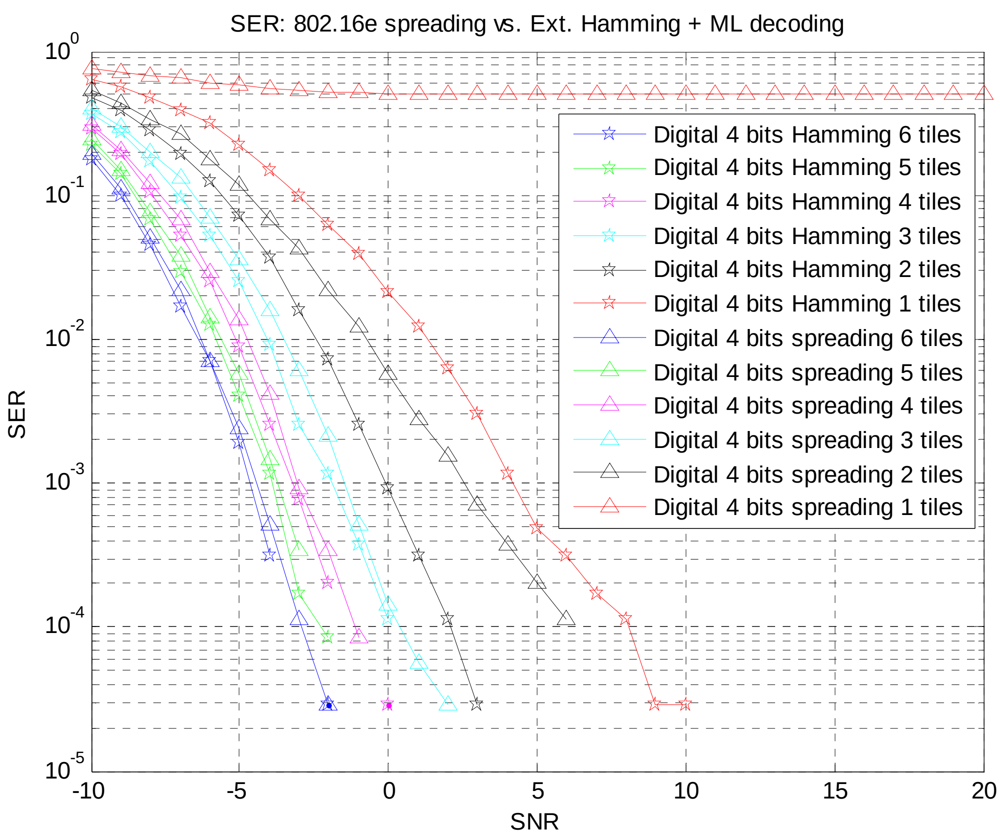
3. UL Digital Control Channel Design

In previous simulations we used the 802.16e CQICH mapping for one, two, three and six tiles (note the mapping for one tile is not part of 802.16e Rev2).

We have improved the performance of the digital control channel by using algebraic codes. Specifically, Reed-Muller codes (of which Hamming codes are a special case) are good candidates as they are the optimal codes for short block lengths.

Specifically, the following plot shows the performance of a new digital control channel for feeding back multiples of 4 bits of information using the extended Hamming code (8,4,4). The 8 coded bits are mapped to 4 QPSK subcarriers and are repeated until all tiles are full.

We can observe that regardless of the number of tiles and especially for low spreading (one or two tiles) the performance of this new UL digital control channel is improved.

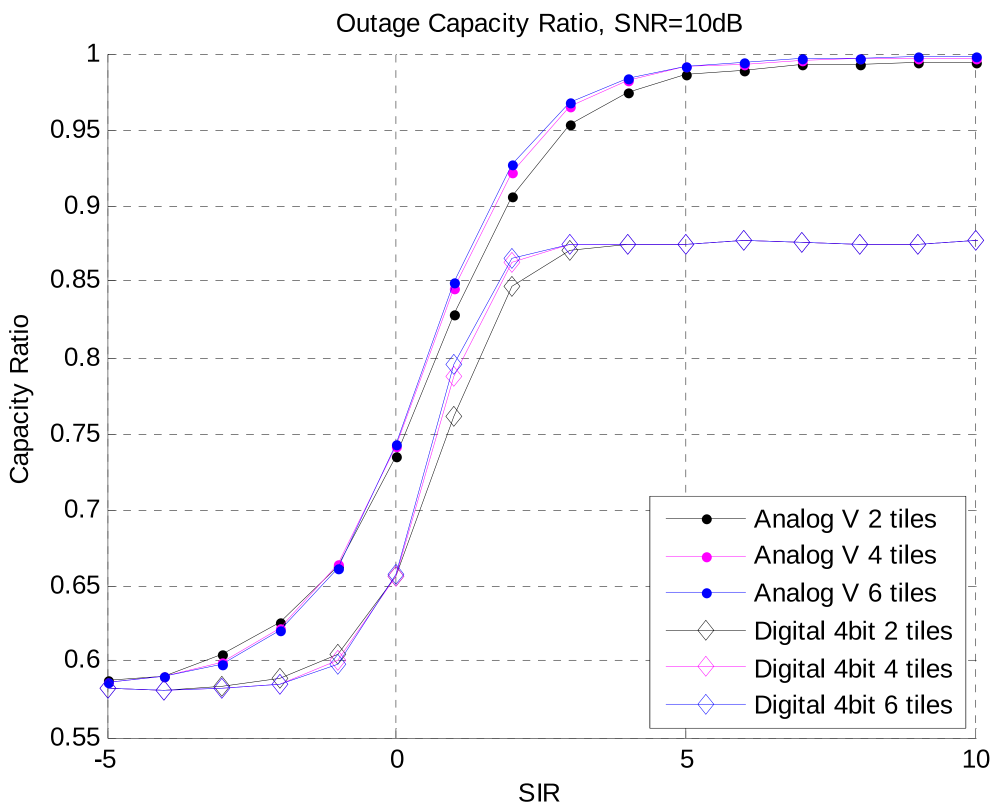


4. Sensitivity to Interference - Analog vs Digital

Contrary to some belief, it was shown in -08/529r1 that analog feedback provides superior performance in AWGN in all SNR ranges when both feedback schemes use the same amount of UL resources.

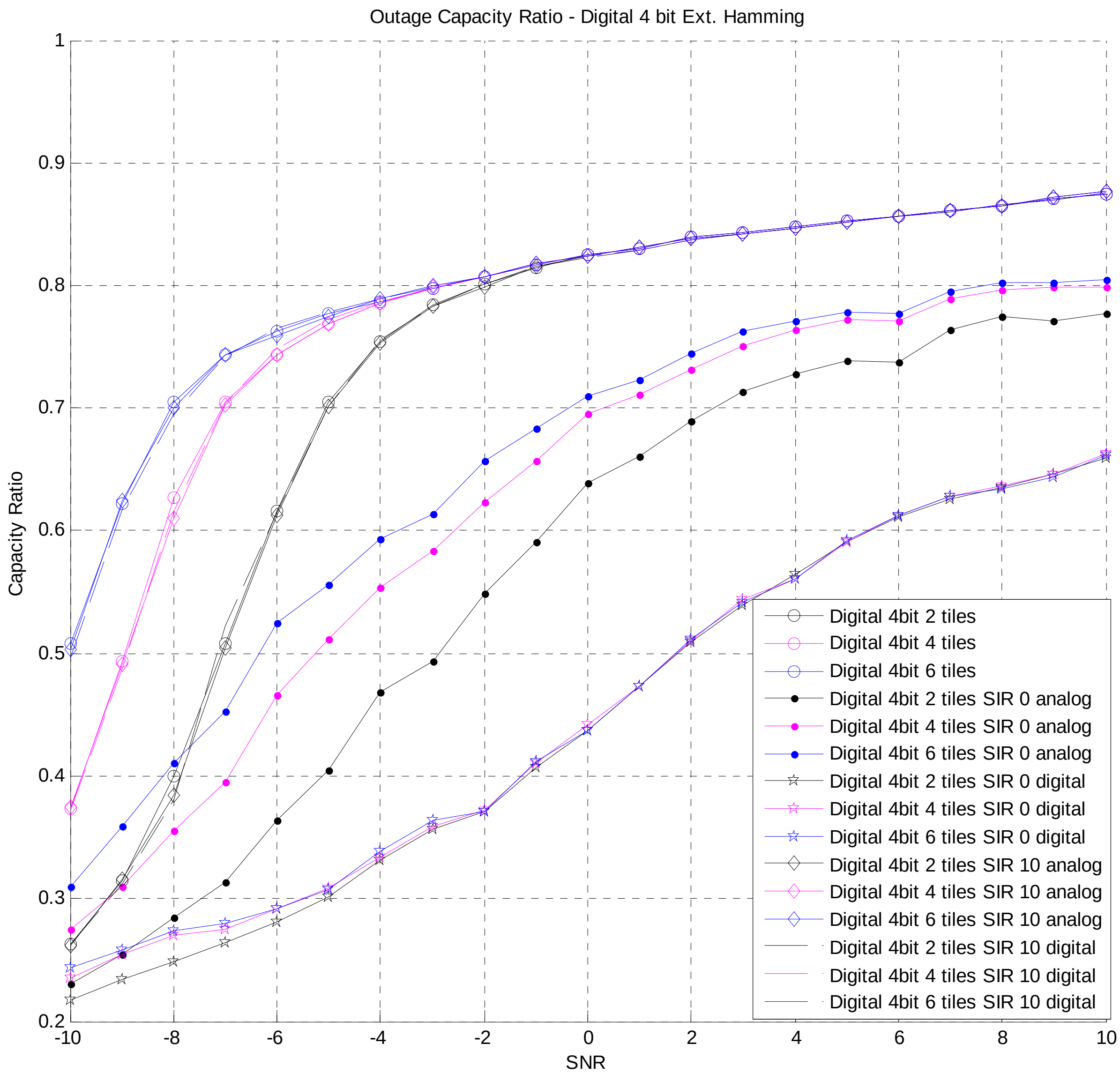
Here we expand this result for the colored interference case. The following result assumes one interferer of the same type where SNR is fixed at 10dB and SIR is varied.
We used 4 bit codebook using the improved UL control channel from section 3.

Again, analog feedback is superior. Note that with proper UL control channel design we can expect positive SIR values.

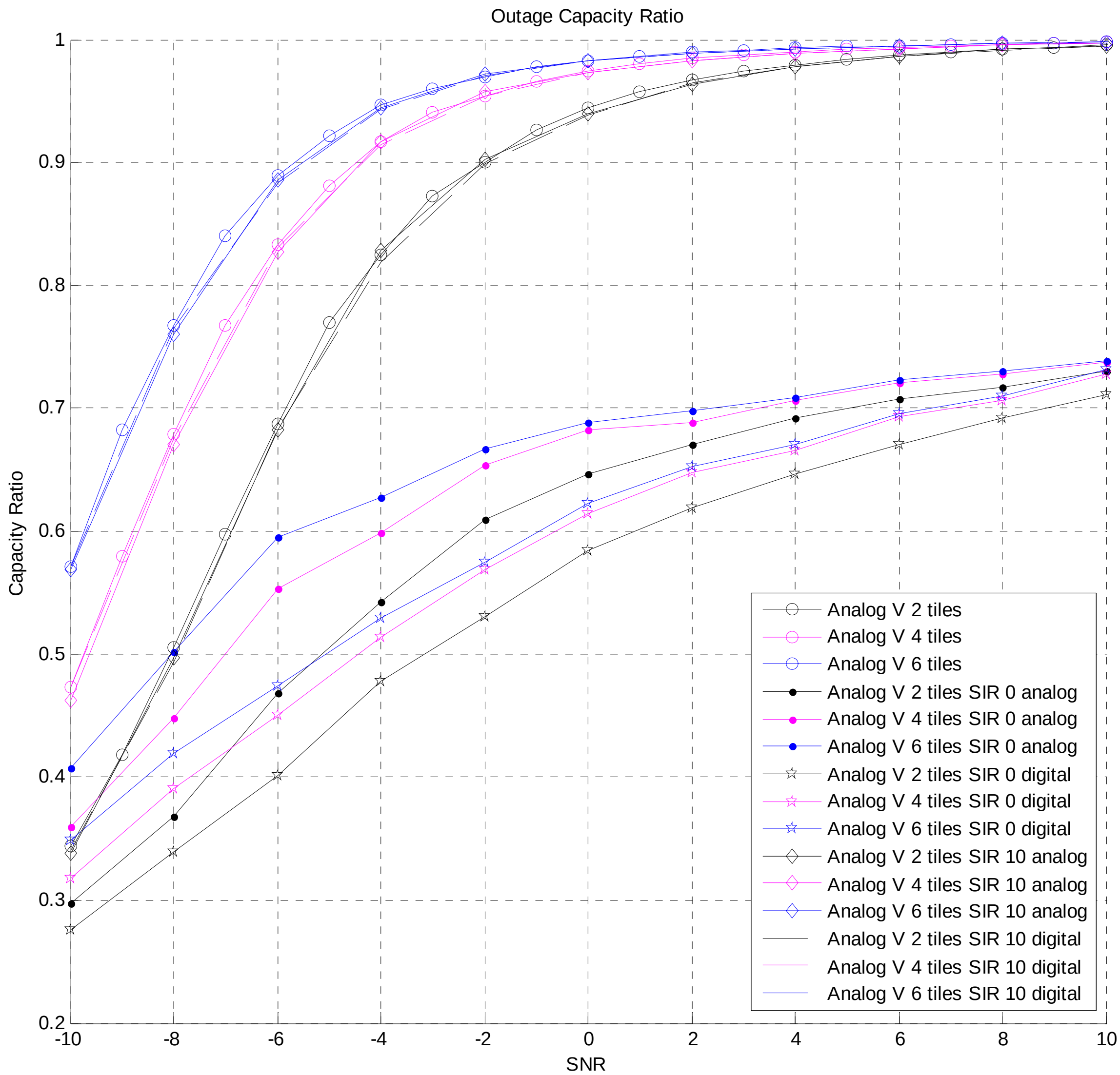


We also provide for reference some plots with fixed SIR for mixed types of interferences.

Digital V (4bit codebook) using (8,4,4) code with Interference



Analog V (4bit codebook) with Interference


Outage Capacity Ratio
Capacity Ratio
SNR
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
-10
-8
-6
-4
-2
0
2
4
6
8
10
Analog V 2 tiles
Analog V 4 tiles
Analog V 6 tiles
Analog V 2 tiles SIR 0 analog
Analog V 4 tiles SIR 0 analog
Analog V 6 tiles SIR 0 analog
Analog V 2 tiles SIR 0 digital
Analog V 4 tiles SIR 0 digital
Analog V 6 tiles SIR 0 digital
Analog V 2 tiles SIR 10 analog
Analog V 4 tiles SIR 10 analog
Analog V 6 tiles SIR 10 analog
Analog V 2 tiles SIR 10 digital
Analog V 4 tiles SIR 10 digital
Analog V 6 tiles SIR 10 digital

5. Hybrid Channel Feedback Algorithm

Figure 1 shows the schematics of the feedback algorithm. The algorithm is as follows:

1. Perform a singular value decomposition (SVD) of H.
2. Decide on the transmission rank, r_{opt}, based on a desired criterion. For example we can pick the optimal rank that maximizes capacity:
$$r_{opt} = \arg\max_{r} C_r = \arg\max_{r} \log\det(I + \frac{SNR}{r} H V_{1:r} V_{1:r}^{*} H^{*})$$
3. Quantize the selected V using a k bit unitary codebook with the mapping criterion of interest such as maximum capacity criterion. Denote the resulting quantization bits by $(b_1, \ldots, b_k)$ and the quantized V by $\hat{V}$.
4. Align V to $\hat{V}$ by performing a unitary transformation on V. Since the precoder V is invariant to unitary transformations on the right, the goal is to find the unitary transformation matrix Q_{opt} such that
$$Q_{opt} = \arg\min_{Q} \| \hat{V} - VQ \|_F^2$$
   It is shown in Appendix A that Q_{opt} is given by $Q_{opt} = V_{corr} U_{corr}^{*}$ where V_{corr} and U_{corr} are the right and left singular vectors of the correlation matrix, $\hat{V}^{*} V$, respectively, i.e., $\hat{V}^{*} V = U_{corr} \Sigma V_{corr}^{*}$. We denote the aligned V by $V_a = VQ_{opt}$.
   a. Note that for rank-1 precoding, the unitary transformation becomes a simple phase rotation: $e^{j\phi} = \frac{V^{*}\hat{V}}{|V^{*}\hat{V}|}$.
   b. Alternatively for rank-2 or higher, we can use a diagonal unitary rotation matrix whereby each column of V is independently phase aligned.
5. Construct the analog error signal: $E = \hat{V} - V_a$.
6. Use a short (n, k, d) algebraic code such as Reed Muller codes to encode the k quantization bits into a codeword $(c_1, \ldots, c_n)$.
7. Send E and $(c_1, \ldots, c_n)$ over the channel. This should be done by splitting the resources in the channel, i.e., power and bandwidth, between E and the digital code $(c_1, \ldots, c_n)$ depending on the application. Specifically for an OFDM system we can superpose the analog and digital signal on the same set of subcarriers (by adding them) or split the subcarriers into two groups of subcarriers each carrying one type of signal.

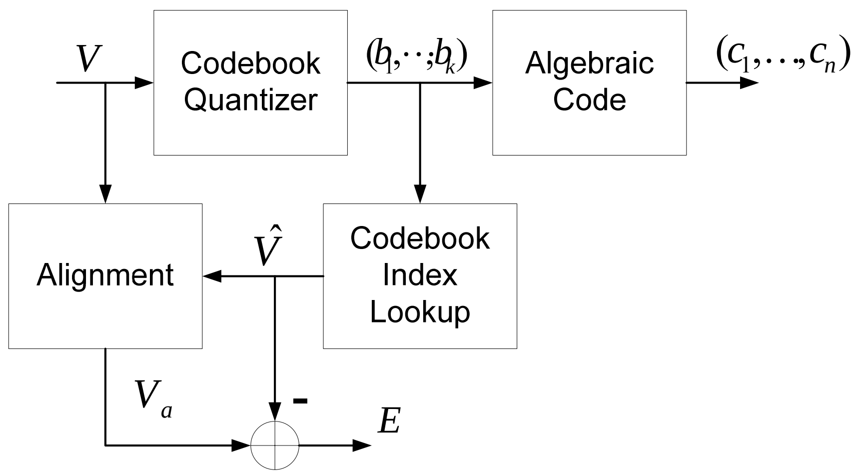


Figure 1: Hybrid Feedback System

This scheme can be used for multi-user MIMO algorithms that benefit from full channel knowledge as the rank adapted V provides good approximation for SV which by itself is sufficient information for all multiuser MIMO algorithms (including DPC).

The same algorithm could be used to send back the channel matrix H or the channel covariance by properly designing vector or scalar quantizers for these quantities.

For example, a scalar quantizer for the elements of H uses 1 bit for the real and 1 bit for the imaginary of each element. Since the elements of H, i.e., h_{ij}'s, are Gaussian, we use the following scalar quantizer for the Gaussian variables

$$q(x) = \begin{cases} \sqrt{\frac{2}{\Pi}}\sigma & \text{if } x \geq 0 \\ -\sqrt{\frac{2}{\Pi}}\sigma & \text{if } x < 0 \end{cases}$$

where σ is the standard deviation of the real and imaginary parts of h_{ij}'s and in the case of Raleigh fading channel is equal to $\sqrt{0.5}$.

6. Implementation Considerations

In the following simulations we use the tile structure of the UL PUSC tile in 802.16e but other structures are possible. Also, for simplicity we split the OFDM subcarriers into two groups – one carrying the digital information (PMI) and one the analog error. Superposition of the information on the same subcarriers is possible as well.

The digital data is constructed by using 4 bit codebook described in -08/372r3.

(8,4,4) extended Hamming code (which is RM(3,1) code) is used to encode the 4 bit into 8 coded bits that are mapped to 4 QPSK subcarriers.

Two tiles are assumed the basic uplink transmission unit. The analog error occupies 4/8 subcarriers for rank 1/2 respectively.
In rank-1 transmission, the digital and analog data are mapped into the first tile as shown in Figure 2 and then repeated over the second tile.
In rank-2 transmission, the digital data is repeated and sent over the 2 tiles just as in rank 1 transmission while the analog data is split between the two tiles.

In order to improve diversity, we can apply a unitary transformation on the 8 analog data using an 8 dimensional rotational unitary matrix, Φ_8, constructed according to

$$\Phi_n = \begin{bmatrix} \Phi_{n/2}\cos\theta & \Phi_{n/2}\sin\theta \\ -\Phi_{n/2}\sin\theta & \Phi_{n/2}\cos\theta \end{bmatrix}$$

where Φ_2 is the standard 2 dimensional rotation matrix

$$\Phi_2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

and θ is a properly chosen angle. Note that with $\theta = \frac{\Pi}{4}$, the rotational matrices reduce to the well-known Walsh-Hadamard matrices.

The mapping to 4 or 6 tiles uses the 2 tile mapping with repetition.

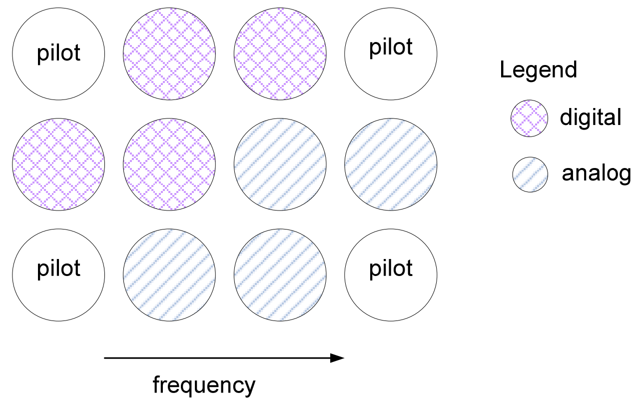


Figure 2: Hybrid data mapping on a single tile.

In order to satisfy the power constraint, the average power per subcarrier needs to be normalized to 1 (or equivalently per tile to 8).

We introduce a design parameter β that is used to control the average power allocated to the digital portion versus the analog portion.

A reasonable choice selects β to make the resulting analog average power the same as the digital power as it provides good balance between the detection of the digital and analog portions.

Hence, writing the power equation for the basic transmission unit of 2 tiles we have:

$$16 = \alpha^2 (P_{digital} + \beta^2 \| E \|_F^2) = \alpha^2 (8 + \beta^2 \| E \|_F^2)$$
$$\alpha = \sqrt{\frac{16}{8 + \beta^2 \| E \|_F^2}}$$

β can be fixed or a parameter controlled by the BS.
α is the normalization factor of the digital and normalized analog information.

If different number of subcarriers are allocated to the digital and analog portions, or a superposition on the same subcarriers is used the formula can be easily derived.

At the receiver α can be estimated by finding the ratio of the average power of the pilot signals to the average power of the subcarriers carrying the digital part of the data.

At the receiver, the digital part of the feedback data is first combined using MRC or MMSE linear combiner and then a Maximum Likelihood (ML) decoder is used to decode the Hamming code. Assuming that the i th coded bit sees an equivalent vector channel $\vec{h}_i$ (resulting from both the SIMO channel and repetition), i.e., $\vec{y}_i = \vec{h}_i c_i + \vec{n}$, then the corresponding MRC performs the following operation on the i th received vector:

$$\hat{y}_i = \frac{\vec{h}_i^*}{\|\vec{h}_i\|} \vec{y}_i$$

An ML decoder is then used on the combined symbols to decode the digital code as follows:

$$(c_1^{opt}, \ldots, c_n^{opt}) = \underset{(c_1, \ldots, c_n) \in RM(3,1)}{\arg\max} \sum_{i=1}^{n} | \hat{y}_i - \|\vec{h}_i\| c_i |^2$$

The analog portion of the data is reconstructed using a linear estimator. Assuming the equivalent vector channel on the i th analog symbol, e_i, is given by $\vec{h}_i$, i.e., $\vec{y}_i = \vec{h}_i e_i + \vec{n}$, the estimator is given by

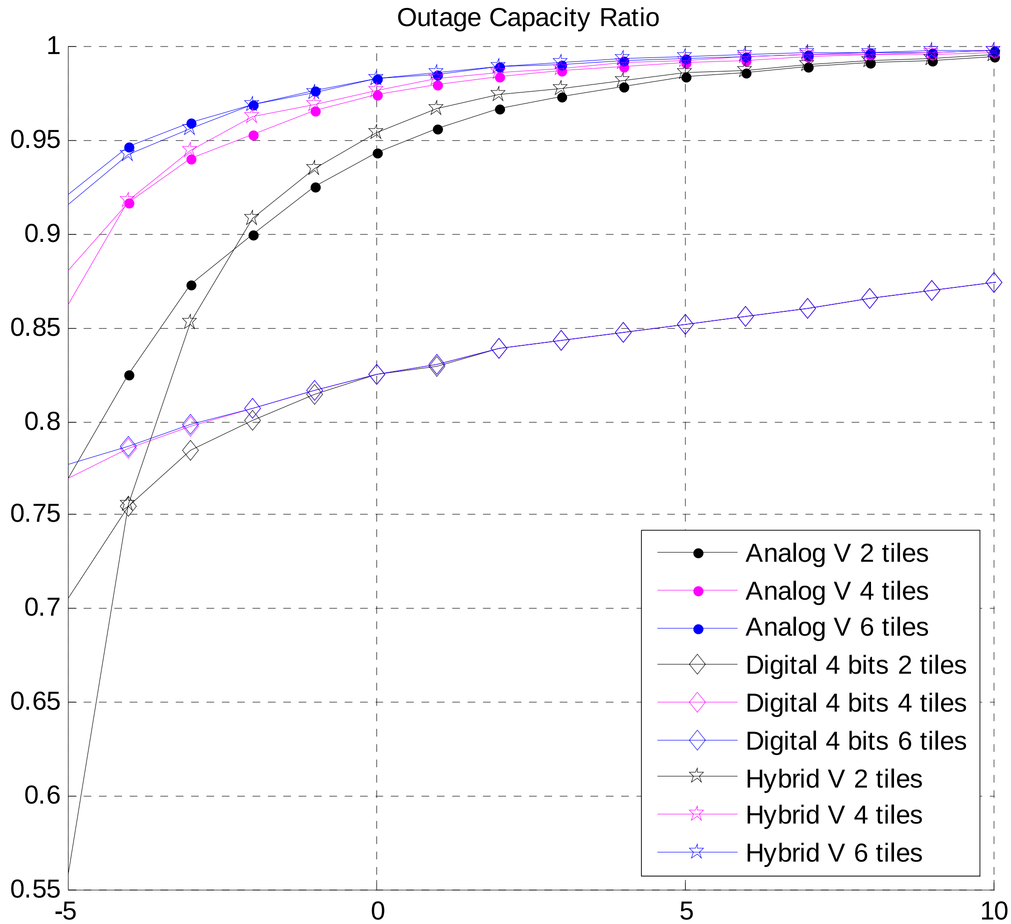
$$\hat{e}_i = \frac{\vec{h}_i^*}{\|\vec{h}_i\|^2 + 1/SNR} \vec{y}_i$$

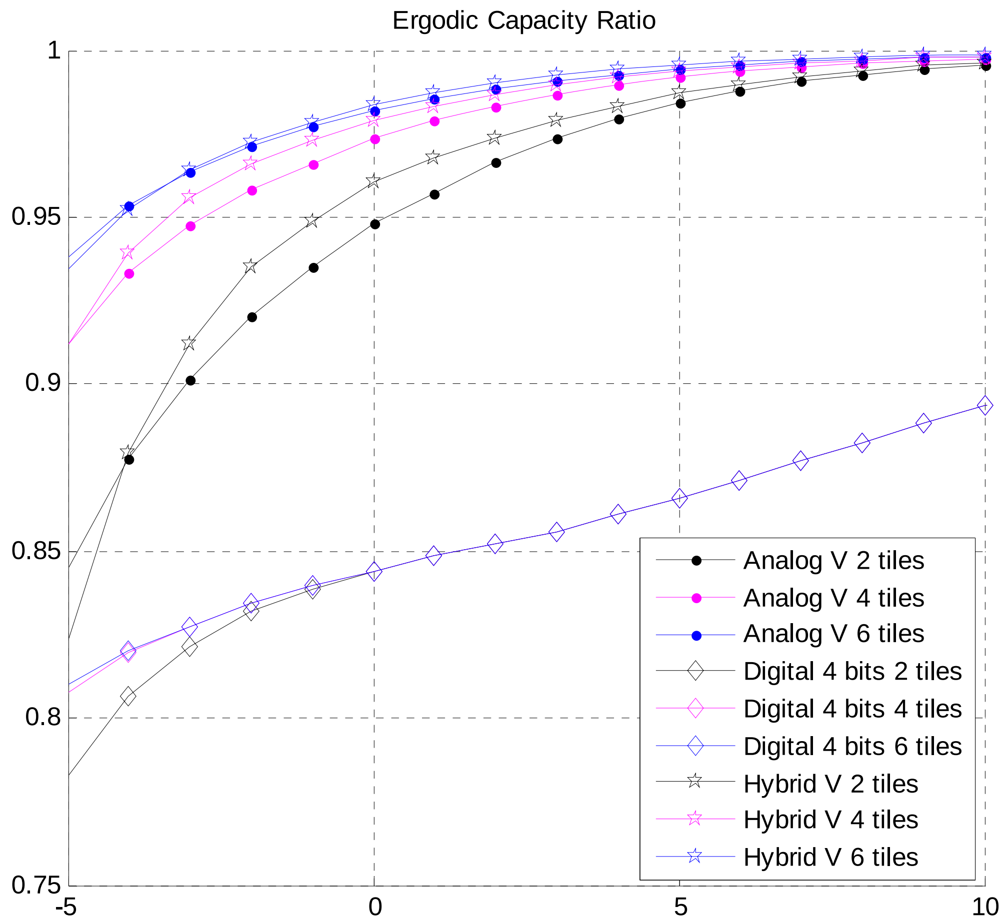
The precoder V is then reconstructed by adding the reconstructed analog error to the decoded quantized precoder.

7. SU-MIMO Simulation Results

The following plots show the ratio of the average achievable Ergodic and 10% Outage capacities (as compared to perfect feedback) using the 3 approaches on 2, 4, and 6 tiles. SNR range is -5 to 10dB.

We can see that the hybrid scheme performs better than both analog and digital schemes in the entire SNR range in terms of the Ergodic capacity and after -3dB in terms of the outage capacity.




Ergodic Capacity Ratio
1
0.95
0.9
0.85
0.8
0.75
-5
0
5
10
Analog V 2 tiles
Analog V 4 tiles
Analog V 6 tiles
Digital 4 bits 2 tiles
Digital 4 bits 4 tiles
Digital 4 bits 6 tiles
Hybrid V 2 tiles
Hybrid V 4 tiles
Hybrid V 6 tiles

8. MU-MIMO Simulation results

It has been shown that to achieve the full multiplexing gain for a MU-MIMO system with a zero-forcing (ZF) precoder, the number of feedback bits per mobile station should increase linearly with SNR in dB [9].
A hybrid feedback algorithm is well suited for MU-MIMO applications where a single codebook of few bits could be used and the additional accuracy required to achieve the full multiplexing gain at higher SNR is obtained by sending the quantization error in an analog fashion.

We use the same simulation setup as before and implement the multi-user eigenmode transmission (MET) technique of [10]. In this algorithm at most one spatial stream is sent to each user. The mobile stations feed back their strongest eigen-vector.

Note that as shown in [10], limiting the number of spatial streams per user to one, and the feedback to the strongest singular vector only, reduces the achievable MU rate by a non-negligible amount.

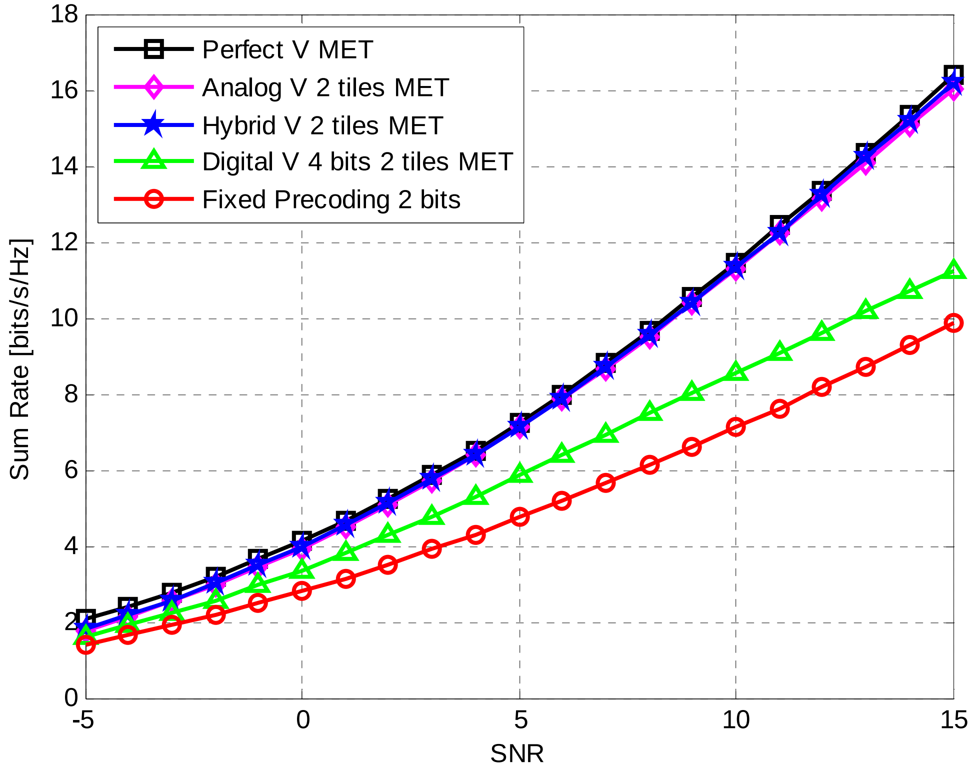
We assume that the base station has information on 4 users and performs an exhaustive search to find the number and combination of users that will maximize the MU rate.

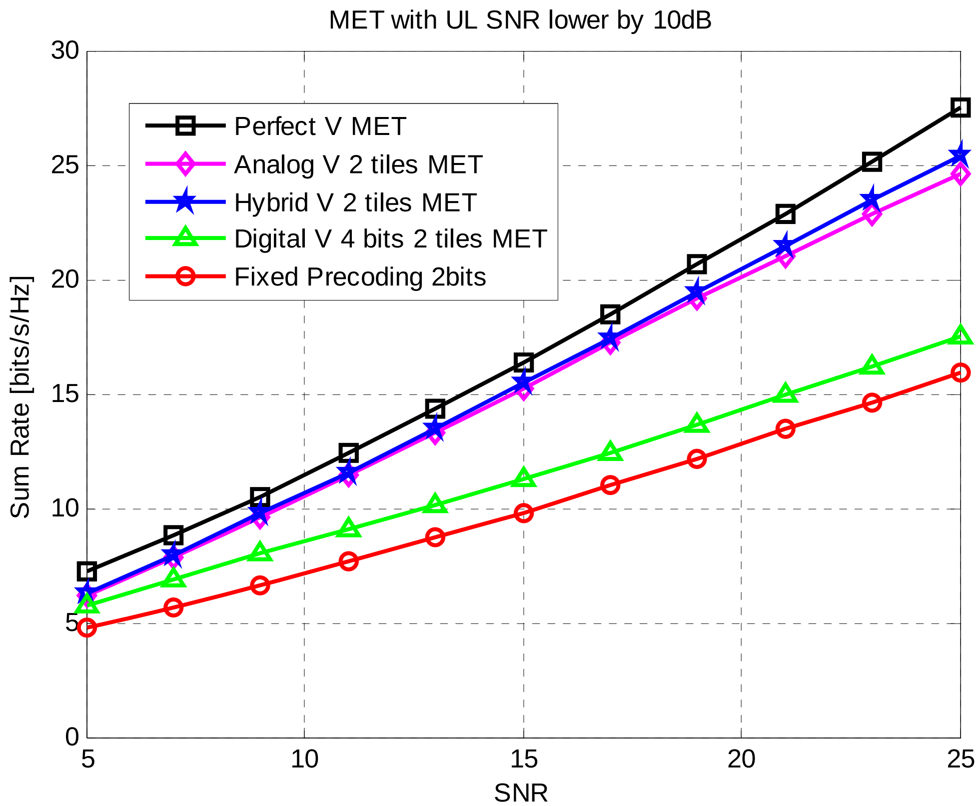
We also simulate the algorithm of [11] (fixed precoding) assuming a 2 bit MU codebook that are fed back error free.

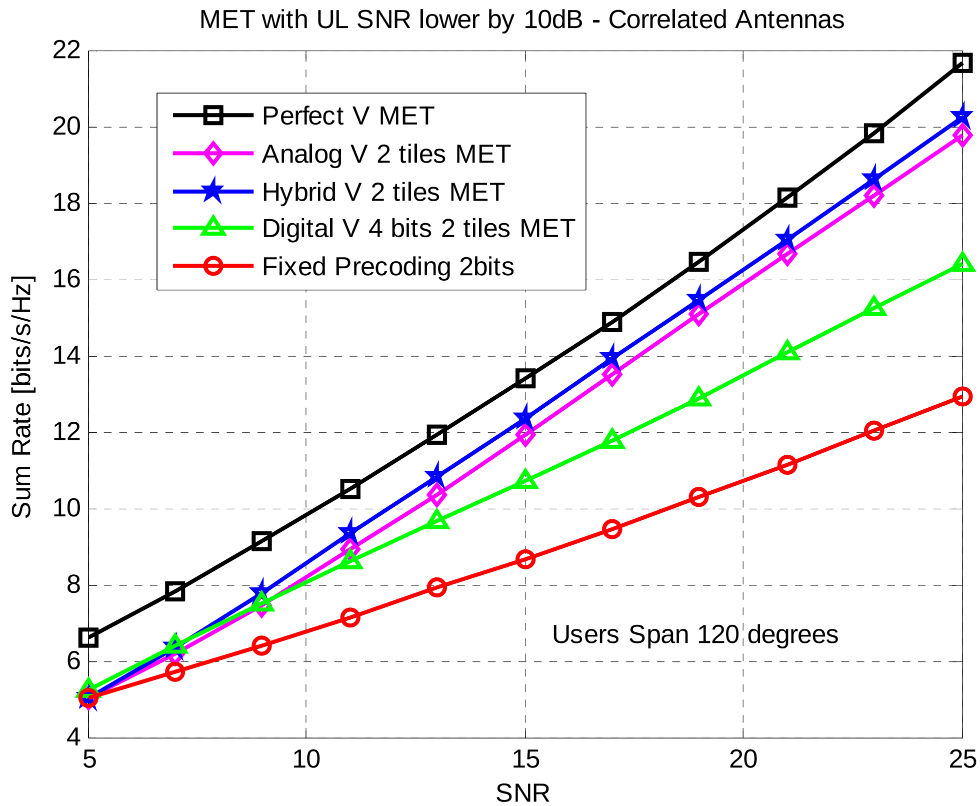
The following plots compare the sum rate as a function of DL SNR achieved for the different algorithms. We simulated correlated and uncorrelated antenna configurations as well as equal UL/DL SNR and DL SNR higher than UL SNR by 10dB. We further looked at the effect of angle spread of users on the MU performance for correlated antennas since in this case spatial signatures differ mostly by AoA.

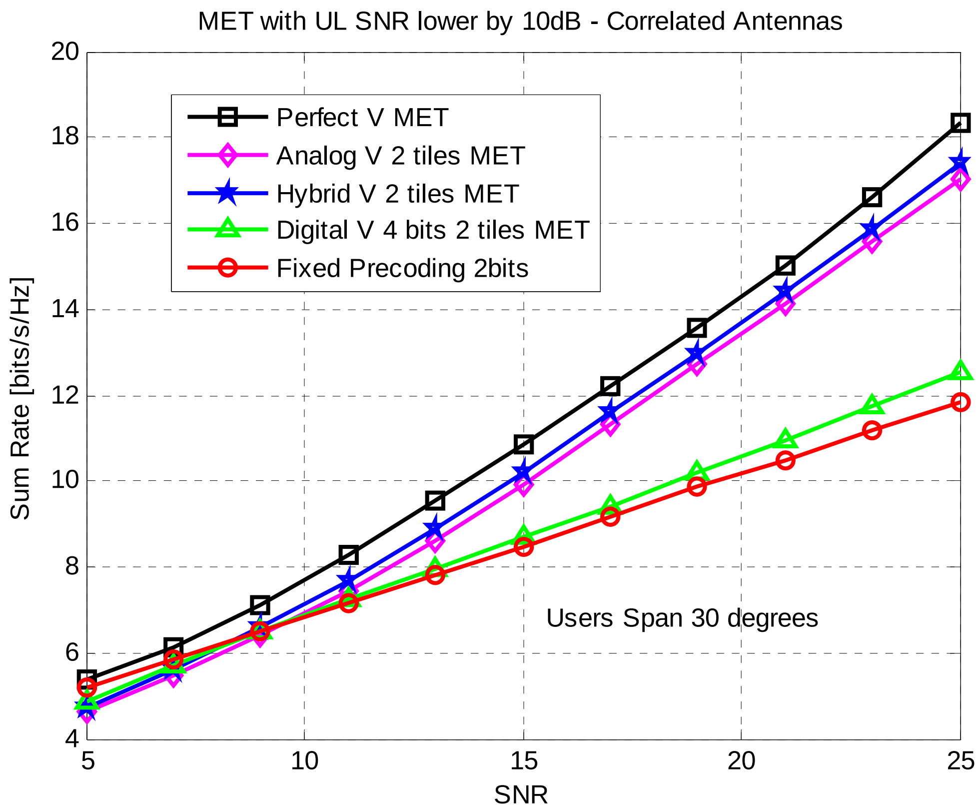
We can see that the MET with hybrid feedback of the eigenvectors performs very close to the MET with perfect knowledge of these vectors. On the other hand, using the digital approach with 4 bit codebooks, the sum rate is significantly reduced compared to the hybrid feedback algorithm especially at high SNR. With 4 users, the baseline method of 11 with 2 bits, even assuming perfect feedback of these bits, does worse than MET even with digital feedback as it relies heavily on the existence of many users.

These results confirm the need for very large number of feedback bits at higher SNR (for adaptive precoding) and show that our algorithm can overcome this problem by using a combination of a single small codebook and an analog error.


Perfect V MET
Analog V 2 tiles MET
Hybrid V 2 tiles MET
Digital V 4 bits 2 tiles MET
Fixed Precoding 2 bits
Sum Rate [bits/s/Hz]
SNR
0
2
4
6
8
10
12
14
16
18
-5
0
5
10
15


MET with UL SNR lower by 10dB
Perfect V MET
Analog V 2 tiles MET
Hybrid V 2 tiles MET
Digital V 4 bits 2 tiles MET
Fixed Precoding 2bits
Sum Rate [bits/s/Hz]
SNR
0
5
10
15
20
25
30
5
10
15
20
25


MET with UL SNR lower by 10dB - Correlated Antennas
Perfect V MET
Analog V 2 tiles MET
Hybrid V 2 tiles MET
Digital V 4 bits 2 tiles MET
Fixed Precoding 2bits
Users Span 120 degrees
Sum Rate [bits/s/Hz]
SNR
4
6
8
10
12
14
16
18
20
22
5
10
15
20
25


MET with UL SNR lower by 10dB - Correlated Antennas
Perfect V MET
Analog V 2 tiles MET
Hybrid V 2 tiles MET
Digital V 4 bits 2 tiles MET
Fixed Precoding 2bits
Users Span 30 degrees
Sum Rate [bits/s/Hz]
SNR
4
6
8
10
12
14
16
18
20
5
10
15
20
25

9. Conclusions

Hybrid Analog/Feedback provides very good performance for SU and MU-MIMO, reduces the need for optimized or large size codebooks, provides harmonized feedback structure between pure codebooks and pure analog and is therefore proposed for inclusion in 802.16m.

If codebooks for a given antenna size are defined in the standard, we can use the feedback as explained here where the MS feeds back the rank and PMI in a digital format and the error in an analog format.
If codebooks are not defined for a particular antenna size we propose to feed back pure analog rank adapted singular vectors.

10. References

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12. Appendix A

The objective is to minimize

$$\begin{aligned}\|\hat{V}-VQ\|_F^2 &= \mathrm{Trace}((\hat{V}-VQ)^*(\hat{V}-VQ))\\ &= \mathrm{Trace}(\hat{V}^*\hat{V}-\hat{V}^*VQ-Q^*V^*\hat{V}+Q^*V^*VQ)\\ &= \mathrm{Trace}(\hat{V}^*\hat{V}-\hat{V}^*VQ-Q^*V^*\hat{V}+QQ^*V^*V)\\ &= \mathrm{Trace}(2I-\hat{V}^*VQ-Q^*V^*\hat{V})\end{aligned}$$

Defining the correlation matrix $C=\hat{V}^*V$, and writing its SVD as $C=U_{corr}\Sigma V_{corr}^*$, the optimum unitary transformation Q_{opt} is given by

$$\begin{aligned}Q_{opt} &= \arg\min_Q \|\hat{V}-VQ\|_F^2\\ &= \arg\max_Q \mathrm{Trace}(\hat{V}^*VQ+Q^*V^*\hat{V})\\ &= \arg\max_Q \mathrm{Trace}(CQ+Q^*C^*)\\ &= \arg\max_Q \mathrm{Trace}(\mathrm{Re}(CQ))\\ &= \arg\max_Q \mathrm{Trace}(\mathrm{Re}(U_{corr}\Sigma V_{corr}^*Q))\\ &= \arg\max_Q \mathrm{Trace}(\mathrm{Re}(\Sigma V_{corr}^*QU_{corr}))\end{aligned}$$

Now since $V_{corr}^*QU_{corr}$ is unitary, the trace is maximized if $\Sigma V_{corr}^*QU_{corr}$ is diagonal which requires $V_{corr}^*QU_{corr}=I$ and hence $Q_{opt}=V_{corr}U_{corr}^*$.