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Source(s)	M. A. Khojastepour, NEC, amir@nec-labs.com Ron Porat, Nextwave, rporat@nextwave.com
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Abstract	This document proposes efficient demodulators for the 4 TX rate-2 DSTTD Code
Purpose	Discussion and Decision
Notice	
Release	
Patent Policy	

Efficient Demodulators for the DSTTD Code

N. Prasad, M. Jiang, M. A. Khojastepour, X. Wang, S. Rangarajan

I. Introduction

In this contribution we obtain a fast (hard decision) maximum-likelihood (ML) decoder for the rate-2 4×2 DSTTD code (or the DSFBC code), that has a complexity of $O(M^2)$, where M is the constellation size. Moreover, an efficient soft-output version of $O(M^2)$ complexity is also possible. Further, it is shown that the structure of the DSTTD code can be exploited to obtain substantial complexity reduction when computing the filters for the sub-optimal MMSE demodulation.

II. MIMO System Model

The complex baseband model for a MIMO link employing the DSTTD code is given by

$$\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2] = \mathbf{H}\mathbf{X} + \mathbf{V}, \tag{1}$$

where $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3, \mathbf{h}_4]$ denotes the $N_r \times 4$ complex-valued channel matrix representing the fading, with $N_r \geq 2$. $\mathbf{X}$ denotes the 4×2 DSTTD code which transmits 4 quadrature amplitude modulation (QAM) symbols every two channel uses and hence has a symbol rate of 2. In particular

$$\mathbf{X} = \begin{bmatrix} x_1 & -x_2^\dagger \\ x_2 & x_1^\dagger \\ x_3 & -x_4^\dagger \\ x_4 & x_3^\dagger \end{bmatrix} \tag{2}$$

where $\{x_i\}_{i=1}^4$ are the QAM symbols and $(.)^\dagger$ denotes the conjugate transpose operation.

We can express (1) in an equivalent representation as

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{x}} + \tilde{\mathbf{v}} \tag{3}$$

where $\tilde{\mathbf{v}}$ is the noise vector with i.i.d. complex proper normal, zero-mean and unit variance elements, $\tilde{\mathbf{x}} = [x_1, x_2, x_3, x_4]^T$, $\tilde{\mathbf{y}} = [\mathbf{y}_1^T, \mathbf{y}_2^\dagger]^T$ and

$$\tilde{\mathbf{H}} = \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 & \mathbf{h}_4 \\ \bar{\mathbf{h}}_2 & -\bar{\mathbf{h}}_1 & \bar{\mathbf{h}}_4 & -\bar{\mathbf{h}}_3 \end{bmatrix} \tag{4}$$

where $\bar{(.)}$ denotes the conjugate operation.

Assuming that $\mathbf{H}$ is perfectly known to the receiver, the optimal detector minimizing the error probability is the maximum likelihood (ML) detector, which solves the following least squares minimization problem.

$$\tilde{\mathbf{x}}^* = \arg\min_{\tilde{\mathbf{x}} \in \mathcal{O}^4} \left\| \tilde{\mathbf{y}} - \tilde{\mathbf{H}}\tilde{\mathbf{x}} \right\|^2 \tag{5}$$

where $\mathcal{O}$ stands for the set of underlying constellation points in M-QAM. It is clear that an exhaustive search for the ML solution has the complexity $O(M^4)$.

Applying the QR decomposition to the equivalent MIMO channel matrix $\tilde{\mathbf{H}}$ in (4) , we have

$$\tilde{\mathbf{H}} = \tilde{\mathbf{\Phi}}\tilde{\mathbf{L}} \tag{6}$$

where $\tilde{\mathbf{L}} = [\tilde{\mathbf{l}}_1, ..., \tilde{\mathbf{l}}_4]$ is a 4×4 lower triangular matrix with positive diagonal elements. $\tilde{\mathbf{\Phi}}$ is a semi-unitary matrix with $\tilde{\mathbf{\Phi}}^\dagger\tilde{\mathbf{\Phi}} = \mathbf{I}$.

We shall first optimize over symbols $\{x_3, x_4\}$ by fixing the choice of $\{x_1, x_2\}$. For convenience, we define the vectors $\tilde{\mathbf{z}}_{12} = [x_1, x_2]^T$ for symbols $\{x_1, x_2\}$ and $\tilde{\mathbf{z}}_{34} = [x_3, x_4]^T$ for symbols $\{x_3, x_4\}$.

By left multiplying $\tilde{\mathbf{y}}$ in (3) by $\tilde{\mathbf{\Phi}}^\dagger$, we obtain

$$\tilde{\mathbf{\Xi}} = \tilde{\mathbf{\Phi}}^\dagger\tilde{\mathbf{y}} = \tilde{\mathbf{L}}\tilde{\mathbf{x}} + \tilde{\tilde{\mathbf{v}}} \tag{7}$$

The maximum likelihood detector chooses $\tilde{\mathbf{x}}$ using the rule

$$\tilde{\mathbf{x}}^* = \arg\min_{\tilde{\mathbf{x}} \in \mathcal{O}^4} \left\| \tilde{\mathbf{\Xi}} - \tilde{\mathbf{L}}\tilde{\mathbf{x}} \right\|^2 \tag{8}$$

For any fixed choice of $\tilde{\mathbf{z}}_{12}$, i.e. $\tilde{\mathbf{z}}_{12}^0$, defining $\tilde{\mathbf{\Omega}} = \tilde{\mathbf{\Xi}} - [\tilde{\mathbf{l}}_1, \tilde{\mathbf{l}}_2]\tilde{\mathbf{z}}_{12}^0$, the minimization in (8) reduces to

$$\min_{\tilde{\mathbf{z}}_{34}\in\mathcal{O}^2} \left\| \tilde{\mathbf{\Omega}} - [\tilde{\mathbf{l}}_3, \tilde{\mathbf{l}}_4]\tilde{\mathbf{z}}_{34} \right\|^2. \tag{9}$$

The particular structure of the DSTTD code defined in (2) ensures a nice structure of $\tilde{\mathbf{L}}$ and dramatically reduces the detection complexity. It is shown in Appendix 1 that the overall complexity of determining the optimal $\tilde{\mathbf{z}}_{34}^*$ in (9) for any given $\tilde{\mathbf{z}}_{12}^0$ (i.e., any fixed $\{x_1^0, x_2^0\}$) is $\mathbf{O}(1)$. Thus, for all combinations of symbols x_1 and x_2, we can determine the M^2 soft metrics denoted by $\lambda(x_{1i}, x_{2j}), 1 \le i, j \le M$, where x_{nm} refers to choosing m-th constellation point in the M-ary constellation set for symbol x_n, with $\mathbf{O}(M^2)$ complexity.

$$\lambda(x_{1i}, x_{2j}) = \left\| \tilde{\mathbf{\Omega}} - [\tilde{\mathbf{l}}_3, \tilde{\mathbf{l}}_4]\tilde{\mathbf{z}}_{34}^* \right\|^2 = \min_{\tilde{\mathbf{z}}_{34}\in\mathcal{O}^2} \left\| \tilde{\mathbf{\Omega}} - [\tilde{\mathbf{l}}_3, \tilde{\mathbf{l}}_4]\tilde{\mathbf{z}}_{34} \right\|^2 \tag{10}$$

where note that $\tilde{\mathbf{\Omega}}$ depends on (x_{1i}, x_{2j}).

Thus, the ML metric which corresponds to the optimal decision for four transmitted symbols $\{x_1^*, x_2^*, x_3^*, x_4^*\}$ (or $\tilde{\mathbf{x}}^* = [\tilde{\mathbf{z}}_{12}^{*T} \quad \tilde{\mathbf{z}}_{34}^{*T}]^T$ in the vector form) is given by

$$\lambda^{ML} = \min_{1\le i,j\le M} \lambda(x_{1i}, x_{2j}). \tag{11}$$

The metrics $\{\lambda(x_{1i}, x_{2j}), 1 \le i, j \le M\}$ also allow us to compute log likelihood ratios (LLRs) for the coded bits associated with symbols x_1, x_2. A similar approach can be applied to compute the metrics and LLRs for the coded bits associated with symbols x_3, x_4. Thus, the structure of the DSTTD code enables an efficient soft-output demodulator with $\mathbf{O}(M^2)$ complexity.

Note that the efficient demodulator derived above readily applies to any row-permuted version of the DSTTD. In particular let $\mathbf{P}$ denote a 4×4 permutation matrix and let $\mathbf{S} = \mathbf{PX}$ denote a row-permuted version of the DSTTD code matrix $\mathbf{X}$ in (2). The resulting model is now given by

$$\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2] = \mathbf{HS} + \mathbf{V} = \mathbf{HPX} + \mathbf{V}. \tag{12}$$

Assuming $\mathbf{H}' = \mathbf{HP}$ to be the channel matrix, we can proceed as before.

Finally, the efficient demodulators also readily apply to the following case. Let $\mathbf{X}_1$ denote the

DSTTD code in (2) and let $\mathbf{S}_2 = \mathbf{P}\mathbf{X}_2$ denote a row-permuted DSTTD, where

$$\mathbf{X}_2 = \begin{bmatrix} x_5 & -x_6^\dagger \\ x_6 & x_5^\dagger \\ x_7 & -x_8^\dagger \\ x_8 & x_7^\dagger \end{bmatrix}, \quad (13)$$

is a DSTTD with QAM symbols $\{x_i\}_{i=5}^{8}$. Consider the received model over four channel uses

$$\mathbf{Y} = [\mathbf{Y}_1, \mathbf{Y}_2] = [\mathbf{H}_1\mathbf{X}_1, \mathbf{H}_2\mathbf{S}_2] + \mathbf{V}. \quad (14)$$

Assuming the additive noise in (14) to be temporally uncorrelated, without loss of optimality, we can consider the demodulation of $\{x_i\}_{i=1}^{4}$ and $\{x_i\}_{i=5}^{8}$ separately, using $\mathbf{Y}_1$ and $\mathbf{Y}_2$, respectively. Each such demodulation can be efficiently performed using the demodulator proposed above.

III. MMSE DEMODULATOR

In this section we show that the structure of the DSTTD code also simplifies the sub-optimal MMSE demodulation. In particular, the computation of the MMSE filters can be greatly simplified by exploiting the structure of the effective channel matrix in (4). For convenience we assume two receive antennas and consider the equivalent representation in (3). Note that to compute the MMSE filters, we need to determine $\tilde{\mathbf{R}}^{-1}$, where $\tilde{\mathbf{R}} = E[\tilde{\mathbf{y}}\tilde{\mathbf{y}}^\dagger]$ is the 4×4 covariance matrix. It can be verified that the covariance matrix $\tilde{\mathbf{R}}$ has the following structure

$$\tilde{\mathbf{R}} = \begin{bmatrix} a & c & 0 & d \\ \bar{c} & b & -d & 0 \\ 0 & -\bar{d} & a & \bar{c} \\ \bar{d} & 0 & c & b \end{bmatrix}, \quad (15)$$

where a, b are positive-valued scalars and c, d are complex-valued scalars such that $\tilde{\mathbf{R}} \succ \mathbf{0}$. The set of all such matrices forms a matrix group $\underline{\mathcal{G}}$ under matrix multiplication so that if $\tilde{\mathbf{R}} \in \underline{\mathcal{G}}$

then $\tilde{\mathbf{R}}^{-1} \in \underline{\mathcal{G}}$. Using these results we can obtain $\tilde{\mathbf{R}}^{-1}$ very easily as:

$$\tilde{\mathbf{R}}^{-1} = q \begin{bmatrix} b & -c & 0 & d \\ -\bar{c} & a & -d & 0 \\ 0 & -\bar{d} & b & -\bar{c} \\ \bar{d} & 0 & -c & a \end{bmatrix}, \quad q = \frac{1}{|c|^2 + |d|^2 - ab}. \tag{16}$$

IV. CONCLUSIONS

The 4 TX rate-2 DSTTD (or DSFBC) code allows for efficient demodulation. This code was shown to have superior performance in both correlated and un-correlated scenarios in [1]. Thus, it should be selected as the 4 TX rate-2 open loop SU-MIMO scheme.

APPENDIX 1: FINDING THE OPTIMAL $\tilde{\mathbf{z}}_{34}^{*}$ FOR A GIVEN $\tilde{\mathbf{z}}_{12} = \tilde{\mathbf{z}}_{12}^{0}$

For the DSTTD code, the QR decomposition of $\tilde{\mathbf{H}}$ defined in (6) produces a special lower-triangular matrix $\tilde{\mathbf{L}}$ in which $[\tilde{\mathbf{l}}_3, \tilde{\mathbf{l}}_4]$ (a 4×2 matrix) has the the following structure, which can be utilized to substantially reduce the detection complexity.

$$[\tilde{\mathbf{l}}_3, \tilde{\mathbf{l}}_4] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ a & 0 \\ 0 & a \end{bmatrix} \tag{17}$$

where a is a positive scalar.

Letting $\tilde{\mathbf{b}} = [b_1, b_2, b_3, b_4]^T = \tilde{\mathbf{\Xi}} - [\tilde{\mathbf{l}}_1, \tilde{\mathbf{l}}_2]\tilde{\mathbf{z}}_{12}^{0}$, for any given $\tilde{\mathbf{z}}_{12} = \tilde{\mathbf{z}}_{12}^{0}$, the optimal $\tilde{\mathbf{z}}_{34}^{*}$ can be obtained as

$$\begin{aligned} \tilde{\mathbf{z}}_{34}^{*} &= \arg \min_{\tilde{\mathbf{z}}_{34} \in \mathcal{O}^2} \left\| \tilde{\mathbf{b}} - [\tilde{\mathbf{l}}_3, \tilde{\mathbf{l}}_4]\tilde{\mathbf{z}}_{34} \right\|^2 \\ &= \begin{bmatrix} \arg\min_{\{x_3\} \in \mathcal{O}} \|b_3 - a x_3\|^2 \\ \arg\min_{\{x_4\} \in \mathcal{O}} \|b_4 - a x_4\|^2 \end{bmatrix} \end{aligned} \tag{18}$$

where $\mathcal{O}$ stands for the set of underlying constellation points in M-QAM. Each minimization in the RHS of (18) can be done using simple quantization in $\mathbf{O}(1)$ complexity. Thus, for each choice of $\tilde{\mathbf{z}}_{12}$ (i.e. symbols $\{x_1, x_2\}$), we can determine the best $\tilde{\mathbf{z}}_{34}$ (i.e. symbols $\{x_3, x_4\}$) with a complexity that is $\mathbf{O}(1)$.

REFERENCES

[1] Yinggang Du, Hongjie Si, Yinwei Zhao, Xin Chang, Yang Tang and Ying Du, "Performance evaluation of DL OL SU-MIMO Transmit Scheme," *IEEE C802.16m-08/1306*, Nov. 2008.

Proposed Text

[Section 11.8.2.1.1 “Open-loop SU-MIMO”]

Accept the 4Tx rate-2 Double SFBC + Antenna hopping as an open-loop SU-MIMO scheme in the downlink.

[Section 11.12.2.1.1 “Open-loop SU-MIMO”]

Accept the 4Tx rate-2 Double SFBC + Antenna hopping as an open-loop SU-MIMO scheme in the uplink.