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Re:	P802.16m/D2 LB30a
Abstract	The contribution proposes the text for best-M subband indexing in P802.16m/D2
Purpose	Discussion and Decision
Notice	
Release	
Patent Policy	

Efficient Sub-band Indexing and Reverse Indexing Methods

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1 Introduction

The subband indexing section in the current standard draft P802.16m/D2 is missing [1]. This contribution proposes the text for sub-band indexing to be included in P802.16m/D2.

2 Indexing the permutations

Let subband positions be denoted by a binary sequence of length N where only M positions corresponding to the selected subbands are 1 and the rest are zero. We call such sequences SPS (Subband Positions Sequence). There will be only $\binom{N}{M}$ possibilities for SPS out of all 2^N possible binary sequences. In this contribution, we present a simple and efficient way to index such sequences. Our indexing has the following properties:

- the index are efficient in a sense that they are exactly between 0 and $\binom{N}{M} - 1$
- If you treat such sequences as the binary number and sort them, the proposed index shows the order of the sequence in the sorted list
- For any SPS, there is a closed form expression to calculate the corresponding index

- The computational complexities of calculating the index for any SPS and the reverse procedure of calculating SPS from its index are very low.

2.1 Calculating index from SPS

Let P_k denote the k^{th} nonzero position in the SPS where $P_M > \ldots > P_{k+1} > P_k > P_{k-1} > \ldots > P_1$. We assume the positions are indexed from zero for the least significant position to $N-1$ for the most significant position. The index I for this SPS is calculated as

$I(\text{SPS}) = \sum_{k=1}^{M} \binom{P_k}{k}$

Please note that $\binom{P_k}{k}$ is defined to be zero if and only if $P_k < k$ which signifies that it is not possible to choose k elements out of P_k elements.

2.2 Calculating SPS from its index

To find the SPS from its index I, we use the following procedure

$I_M = I$

foreach $k = M, M-1, \ldots, 2$ **do**

  Find the largest value for x such that $I_k - \binom{x}{k} \geq 0$

  $I_{k-1} = I_k - \binom{x}{k}$

  $P_k = x$

end

$P_1 = I_1$

Algorithm 1: Reverse indexing procedure

The above algorithm can be further optimized by considering the range of the possible values for x when searching for a given index k. The first (most significant) nonzero position would be somewhere in the position $M-1$ up to $N-1$. We can perform binary search which requires at most $\log_2(M-N)$ operation to find the first nonzero position P_M. We update the residual index I_{M-1}. Now, the second nonzero position would be somewhere in the range of $M-2$ to P_M. We gain to the binary search and update the residual index and continue the procedure till we find the second nonzero position. For the last nonzero position, the residual index I_1 directly points to the position of the nonzero bit and there is no additional computation necessary. a rough upper bound on the total number of the comparison and the binomial (choose) function calculation would be $\log_2(N-M) + \sum_{k=M-1}^{1} \log_2(P_{k+1}-k) \leq M\log_2(N-M)$. In Table **??** and **??** summarizes the actual average value of the average binomial function calls by simulation and generating all the possible subband position patterns.

Here is the detail of the procedure

```
P = zeros(M + 1);
P(M + 1) = N;
foreach k = M : −1 : 2 do
    up = P(k + 1) − 1;
    lo = k − 1;
    while up ≠ lo do
        j = lo+ceil((up − lo)/2); if I ≥ (j choose k) then
            lo = j;
            else
            up = j − 1;
        end
    end
    P(k) = lo;
    SPS(N − P(k)) = k;
    I = I − (P(k) choose k);
end
P(1) = I;
SPS(N − P(1)) = 1;
```

Algorithm 2: Procedure with minimized binomial operation

3 Discussion and Conclusion

We believe the proposed indexing method is the most natural way because of its simplicity of representation, finding the index and reverse operation of finding the subband positions from its index.

The proposed reverse indexing also has the least computational complexity among the candidate subband indexing proposals and methods [2,3,4,5]. The complexity of the algorithm has been compared in terms of the number of binomial (choose) operations in [4]. Since the number of multiplications in different binomial computations is different this is not the best measure to do the comparison. Here, we follow the same comparison to show the gain of our proposed algorithm. It is also possible to show the same gains by using a more appropriate method of comparing the number of multiplications. Table 1 and 2 summarizes the comparison for the reverse indexing procedure.

M	4	3	2
Reference Method[2]	23.8095	23.8636	23.9130
Intel Method[3]	19.8095	18.6136	16.5797
Samsung Method[5]	19.8095	18.6136	16.5797
Proposed Method	12.6137	8.7870	4.7018
Gain	57.05%	111.83%	252.62%

Table1: Average binomial (choose) operation for reverse indexing procedure , $N = 24$

M	4	3	2
Reference Method[2]	11.5556	11.7	11.8182
Intel Method[3]	9.9556	9.45	8.4848
Samsung Method[5]	9.9556	9.45	8.4848
Proposed Method	9.1842	6.6301	3.7385
Gain	8.4%	42.53%	126.95%

Table2: Average binomial (choose) operation for reverse indexing , $N = 12$

4 References

[1] IEEE P802.16m/D2, “DRAFT Amendment to IEEE Standard for Local and metropolitan area networks”, Oct. 2009.

[2] T. M. Cover, “Enumerative source encoding”, IEEE Trans. Inform. Theory, vol. IT-19, pp. 73-77, January 1973

[3] Intel, “Proposed Text for sub-band indexing (15.3.9.3.1.4) for the IEEE 802.16m/D1”, IEEE C80216m-09/1720r3

[4] Intel, “Supporting document for C80216m-09/1720r3”, IEEE S80216m-09/1720r2

[5] Samsung, “Proposed Text for sub-band indexing (15.3.9.3.1.3)”, IEEE C802.16m-09/2132r5

5 text proposal

Insert the following text at section 15.3.9.3.1.4 (reference to page 679 in IEEE 802.16m/D2)

------------ Beginning of the text proposal ------------

The AMS shall report the subband selection by an index

$I(\mathtt{SPS}) = \sum_{k=1}^{M} \binom{P_k}{k}$

where N is the total number of subands, M is the number of selected subbands and $P_M > P_{M-1} > ... > P_1$ are the positions of the M selected subbands.

------------ End of the text proposal ------------