Evaluation of Differential Codebooks for IEEE 802.16m Amendment Working Document

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Re: Category: AWD comments / Area: Chapter 15.3.7 (DL-MIMO)

"Comments on AWD 15.3.7 DL-MIMO"

Base Contribution:

IEEE C80216m-09_0790

Purpose:

Discussion and approval

Notice:

Release:

Patent Policy:

Background

- This contribution presents the performance evaluation of differential codebooks
- SDD supports a differential feedback mode for codebook based precoding in DL SU and MU-MIMO
- In San Diego meeting, SDD supports “rotation based schemes”.

2 kinds of rotation based schemes

[C80216m-09_0058r4.doc]

- Rotation Scheme 1: right quantization
  - Differentiation at SS: $\mathbf{D} = \mathbf{Q}^H(t-1)\mathbf{V}(t)$
  - Quantization at SS: $\hat{\mathbf{D}} = \arg\max_{\mathbf{D}_i \in C_d} \left\| \mathbf{D}^H \mathbf{D}_i \right\|_F$
  - Beamforming matrix reconstruction at BS:

$$\hat{\mathbf{V}}(t) = \mathbf{Q}(t-1)\hat{\mathbf{D}}$$

  - Beamforming at BS: $\mathbf{y} = \mathbf{H}\hat{\mathbf{V}}(t)\mathbf{s} + \mathbf{n}$

Our Proposal

- Rotation Scheme 2: left quantization
  - Differentiation at SS: $\mathbf{D} = \mathbf{V}(t)\mathbf{Q}^H(t-1)$
  - Quantization at SS: $\hat{\mathbf{D}} = \arg\max_{\mathbf{D}_i \in C_d} \left\| \mathbf{D}^H \mathbf{D}_i \right\|_F$
  - Beamforming matrix reconstruction at BS:

$$\hat{\mathbf{V}}(t) = \hat{\mathbf{D}}\mathbf{Q}(t-1) \qquad \mathbf{Q}(t-1) = \hat{\mathbf{V}}(t-1)$$

  - Beamforming at BS: $\mathbf{y} = \mathbf{H}\hat{\mathbf{V}}(t)\mathbf{s} + \mathbf{n}$

Properties

	Rotation Scheme 1	Rotation Scheme 2
Design Principle	Quantize the right side combining weight space	Quantize a rotation matrix space
Codeword size	Nt x Ns	Nt x Nt
# of codebooks	•One codebook per rank and scenarios (spatially uncorrelated and correlated), i.e. 4 x 2 = 8 codebooks •difficult to select in practice the appropriate codebook for a given rank	•The same (=1) codebook for all ranks and scenarios •Robust design for various spatial correlation
Adaptation to time and spatial correlation	no adaptation to time correlation, but adaptation to spatial correlation -> redundant with adaptive mode	Adaptation to time correlation (i.e. primary objective of a differential codebook) and spatial correlation through a single parameter ρ
Codebook sizes	3 and 4 bits	4 bits
CQI, PMI calculation and testing complexity	Higher	Lower
Quantization	Nt x Ns space to quantize sensitivity to quantization error difficult to control and assess	Nt-dimensional unitary space Compactly packed rotation codebook
Rank adaptation	Rank feedback is typically 20 ms. A typical reset period for differential codebook is 20-30 ms. Hence no need to adapt the rank during the differential transmissions	
Design for 8x8	Optimization for each rank	1 optimization for all ranks

Complexity comparisons

<table>
<tr><th>Rotation Scheme 1</th><th>Rotation Scheme 2</th></tr>
<tr><td colspan="2">• Searching complexity directly proportional to the codebook size and is implementation dependent
• Quantization at MS: $\hat{\mathbf{D}} = \arg\max_{\mathbf{D}_i \in C_d} \left\| \mathbf{D}^H \mathbf{D}_i \right\|_F$
For 4bits, complexity is pretty much the same for rotation schemes 1 and 2</td></tr>
<tr><td>• Q(t-1) updated at each feedback period and subband based on complex operations (multiple Householder transformation, Gram-Schmidt orthogonalization and/or QR decomposition of $\hat{\mathbf{V}}(t-1)$)</td><td>• calculation of Q(t-1) is straightforward
$\mathbf{Q}(t-1) = \hat{\mathbf{V}}(t-1)$</td></tr>
<tr><td>• Q(t-1) update is rank dependent, therefore requiring a different implementation of Q(t-1) for each rank</td><td>• same straightforward operation for all ranks</td></tr>
<tr><td>• no adaptation to time correlation</td><td>• possible adaptation to time correlation through parameter ρ and SVD
– SVD only necessary if we change the codebook. A predefined codebook can be stored otherwise.</td></tr>
<tr><td>Large testing time required
• Householder, Gram-Schmidt and QR
• multiple codebooks (CB per rank and scenarios)</td><td>Small testing time required
• only one codebook</td></tr>
</table>

Quantization properties

Rotation Scheme 1: Nt x Ns space to quantize	Rotation Scheme 2: Nt-dimensional unitary space
• no equivalence relation of the codebook – No distance measure related to system performance – No guarantee that the CB does not overquantize the space	•Proof of the equivalence relation which decreases the volume of the codebook space • The base rotation codebook is in Riemannian manifold -> distance measure can be defined
•The quantization error not only depends on the rank of **D** but also on the quant. error induced in **Q** $\hat{\mathbf{V}}(t) = \hat{\mathbf{Q}}(t-1)\hat{\mathbf{D}}(t)$ Build based on $\hat{\mathbf{V}}(t-1) = \hat{\mathbf{Q}}(t-2)\hat{\mathbf{D}}(t-1)$ • Quantization error in D appears at 2 levels • Householder operation in Q boosts and spreads the quantization error over 4x4 space	•Rotation schemes 2 never have to quantize a full NtxNt matrix. The density/magnitude of off-diagonal elements of D codebook is much lower than diagonal elements
•Some ambiguity when applying transformation for generating **Q** (for columns from Nt-Ns to Nt) • rank 3 and rank 4 transformation gives very weak performance	
•sensitivity to quantization error difficult to control and assess	**•Compactly packed rotation codebook**

Differential 4Tx codebooks

	rank	label	Codebook size	reference
Rotation scheme 1	Rank 1	'Rot1 Uncorr CB rank1' 'Rot1 Corr CB rank1'	3 bit 3 bit	C80216m-09_0528.ppt (Qinghua Li et al.)
	Rank 2	'Rot1 Uncorr CB rank2' 'Rot1 Corr CB rank2'	3 bit 3 bit	
	Rank 3	'Rot1 Uncorr CB rank3' 'Rot1 Corr CB rank3'	3 bit 3 bit	
	Rank 4	'Rot1 Uncorr CB rank4' 'Rot1 Corr CB rank4'	3 bit 3 bit	
	Rank 1	'Rot1 Uncorr CB rank1'	4 bit	
	Rank 2	'Rot1 Uncorr CB rank2'	4 bit	
	Rank 3	'Rot1 Uncorr CB rank3'	4 bit	
	Rank 4	'Rot1 Uncorr CB rank4'	4 bit	
Rotation scheme 2	For all Ranks	'Rot2 1' (ρ = 0.9, 0.95)	4 bit	C80216m-09_0677.doc (David Mazzarese et al.) S80216m-09_0790.pdf (Bruno Clerckx et al.)
	For all Ranks	'Rot2 2'	4 bit	C80216m-09_038r1_LGE_r1.doc (WookBong Lee et al.)

Simulated codebooks

SU-MIMO performance

	Uncorrelated
Rot 1 {Uncorr CB for rank1 to 4} 3bits	moderate refinement in 4x2 Big Loss in 4x4
Rot 1 {Uncorr CB for rank1 to 4} 4bits	The best refinement in 4x2 Big Loss in 4x4
'Rot2 1' 4bits ($\rho = 0.9$)	The best overall: excellent refinement in 4x2 and 4x4
'Rot2 2'	Loss
Adaptive mode	No gain for rank 1 Loss for rank>1

SU-MIMO Performance comparisons

4x2 SU-MIMO Uncor-related	**SNR**	**0dB**	**5dB**	**10dB**	**15dB**	**20dB**
	Gain of 'Rot 1' 4bit {Uncorr CB for rank1 to 2} over 4bit AWD standard mode	1.99%	6.87%	10.73%	6.84%	0.00%
	Gain of 'Rot2 1' 4bit (**ρ**=0.9) over 4bit AWD standard mode	1.70%	5.26%	9.77%	5.30%	0.00%
	Gain of 'Rot 1' 4bit {Uncorr CB for rank1 to 2} over 'Rot2 1' 4bit (**ρ**=0.9)	0.29%	1.53%	0.88%	1.46%	0.00%
4x4 SU-MIMO Uncor-related	**SNR**	**0dB**	**5dB**	**10dB**	**15dB**	**20dB**
	Gain of 'Rot 1' 4bit {Uncorr CB for rank1 to 4} over 4bit AWD standard mode	3.17%	-2.87%	-3.91%	-4.47%	-1.82%
	Gain of 'Rot2 1' 4bit (**ρ**=0.9) over 4bit AWD standard mode	1.50%	4.13%	7.21%	1.30%	-0.68%
	Gain of 'Rot 1' 4bit {Uncorr CB for rank1 to 4} over 'Rot2 1' 4bit (**ρ**=0.9)	1.64%	-6.72%	-10.37%	-5.69%	-1.15%

* Gain averaged over 30ms (i.e. reset period=30ms)

MU-MIMO performance

	Uncorrelated	Correlated
'Rot1 Uncorr CB rank1' 3 bits	good refinement	No refinement
'Rot1 Corr CB rank1' 3bits	Not robust enough	The best among diff. CB at high SNR (excellent refinement)
'Rot1 Uncorr CB rank1' 4 bits	The best refinement at high SNR (20 dB)	small refinement
'Rot2 1' 4bits	(ρ=0.95, ρ=0.9) excellent refinement at all SNRs Enables tracking of mobility (fct. of ρ)	(ρ=0.95) The best among diff. CB at low SNR. Very Robust. (ρ=0.9) small refinement
'Rot2 2'	Loss	Loss
Adaptive mode	No gain	The best

MU-MIMO Performance comparisons

4x2 MU-MIMO Uncor-related	SNR	0dB	5dB	10dB	15dB	20dB
	Gain of ‘Rot 1 Uncorr CB for rank1 ’ 4bit over 4bit AWD standard mode	17.69%	16.65%	17.99%	17.35%	25.47%
	Gain of ‘Rot2 1’ 4bit (ρ=0.95) over 4bit AWD standard mode	18.82%	16.07%	17.30%	19.67%	21.83%
	Gain of ‘Rot2 1’ 4bit (ρ=0.9) over 4bit AWD standard mode	17.01%	15.05%	16.91%	15.04%	20.82%

4x2 MU-MIMO Correl-ated	SNR	0dB	5dB	10dB	15dB	20dB
	Gain of ‘Rot 1 Uncorr CB for rank1’ 4bit over 4bit AWD standard mode	2.17%	3.20%	3.12%	3.71%	1.67%
	Gain of ‘Rot 1 Corr CB for rank1’ 3bit over 4bit AWD standard mode	2.36%	4.74%	7.39%	10.21%	5.55%
	Gain of ‘Rot2 1’ 4bit (ρ=0.95) over 4bit AWD standard mode	5.29%	7.10%	6.63%	8.10%	3.94%
	Gain of ‘Rot2 1’ 4bit (ρ=0.9) over 4bit AWD standard mode	4.41%	3.21%	2.04%	4.54%	1.42%

* Gain averaged over 30ms (i.e. reset period=30ms)

Conclusions

- Differential mode should mainly target spatially uncorrelated channels
  - Significantly outperforms the standard and adaptive modes in spatially uncorrelated channels
  - Is outperformed by the adaptive mode in spatially correlated channels

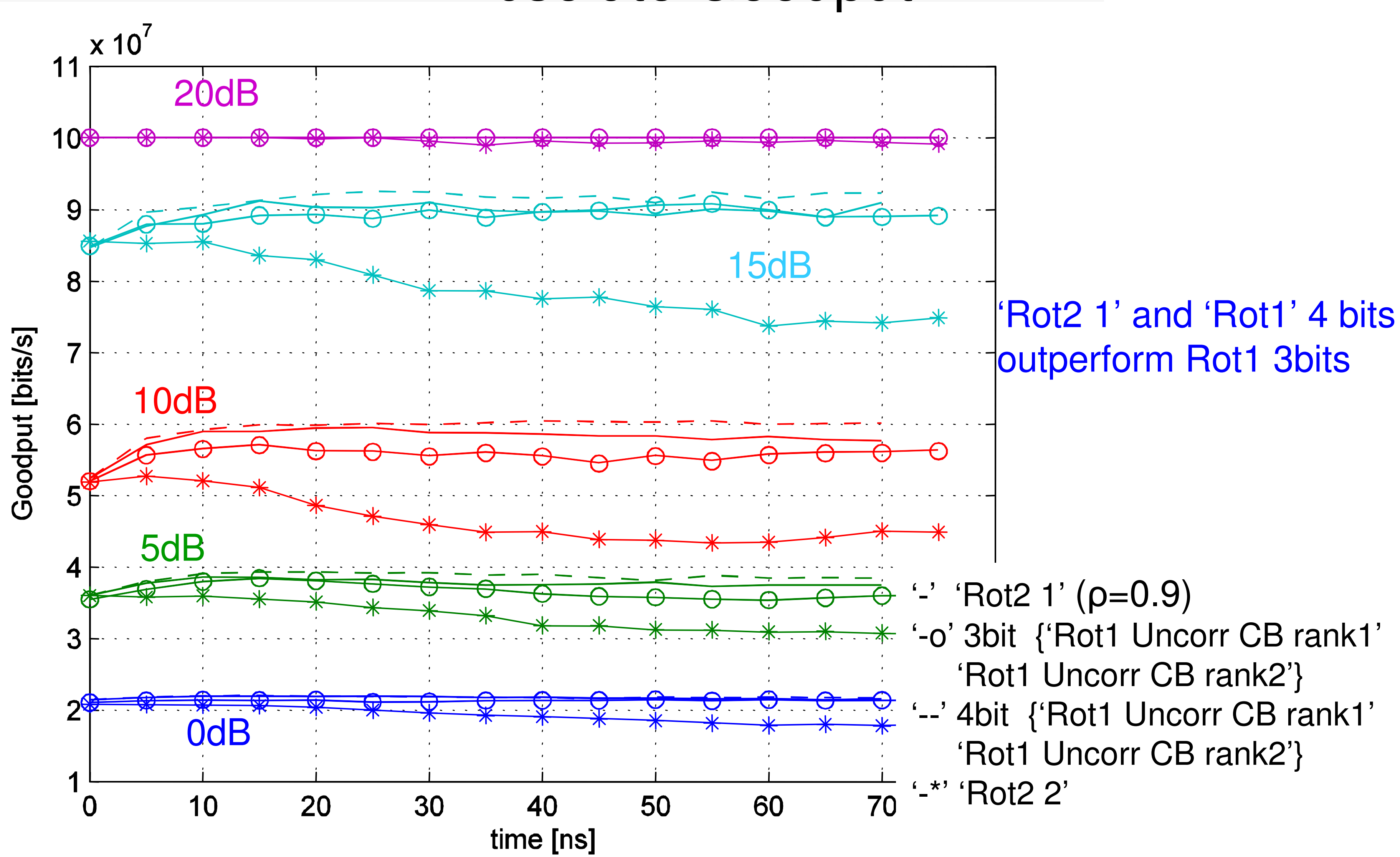
- We propose to adopt ‘Rot2 1’ as the differential feedback mode for codebook based feedback
  - The best overall performance and robustness in 4x2 and 4x4 SU MIMO
  - Excellent performance in MU MIMO uncorrelated channels
  - Very robust in correlated channels
  - For the same codebook size, it has lower complexity compared to rotation schemes 1
  - A single codebook for all scenarios and ranks
  - Easily adaptable to various environment and mobile speed
    - Recommended value $\rho = 0.9$

Appendix
simulation results

CL SU MIMO

4x2 CL SU MIMO: uncorrelated (4 λ,15°AS), 3km/h

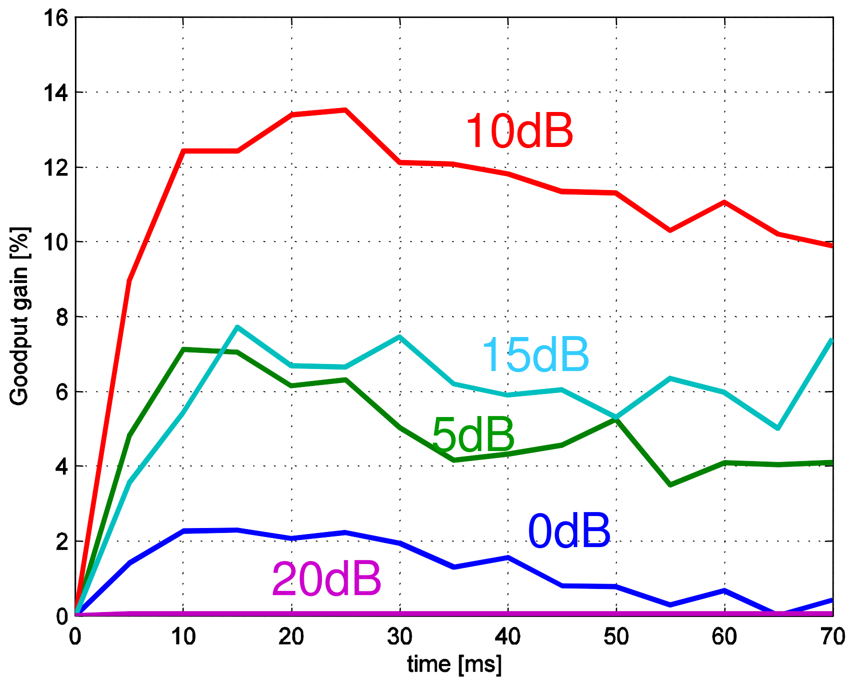
Absolute Goodput



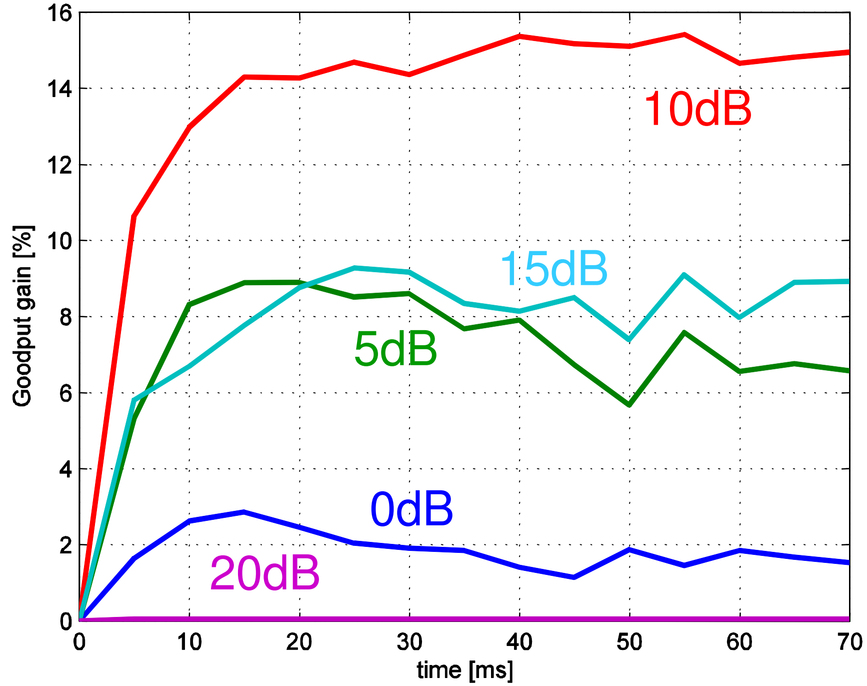
4x2 CL SU MIMO: uncorrelated (4 λ, 15°AS), 3km/h

Relative Goodput Gain [%] **vs. standard** mode

- Relative Goodput Gain [%] of **'Rot2 1' (ρ=0.9) 4bits** over **standard** mode



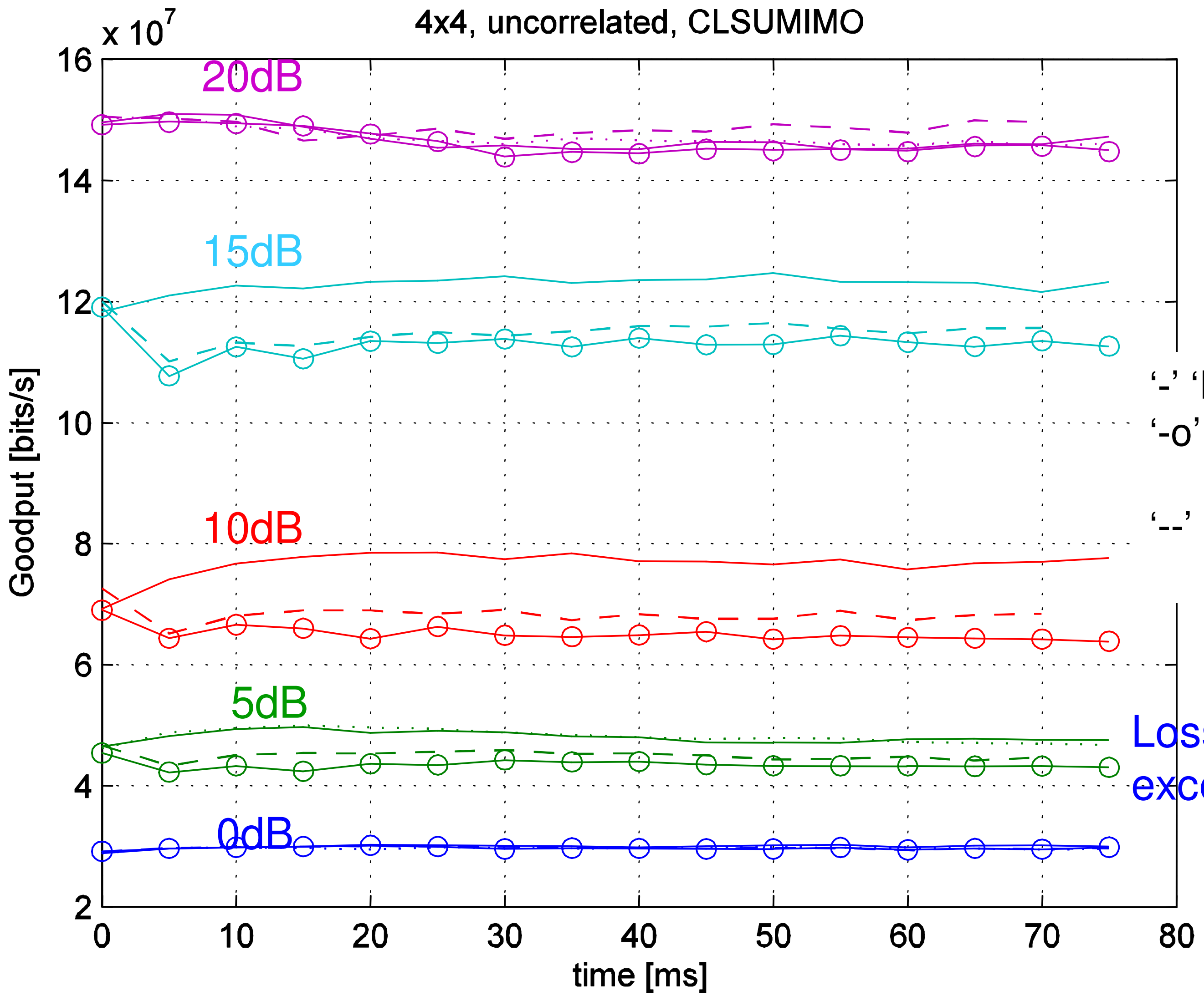
- Relative Goodput Gain [%] of **'Rot 1' 4bit {Uncorr CB for rank1 to 2}** over **standard** mode



Slight performance gain for 'Rot1' 4bits over 'Rot2 1' 4bits

4x4 CL SU MIMO: uncorrelated (4 λ, 15°AS), 3km/h

Absolute Goodput

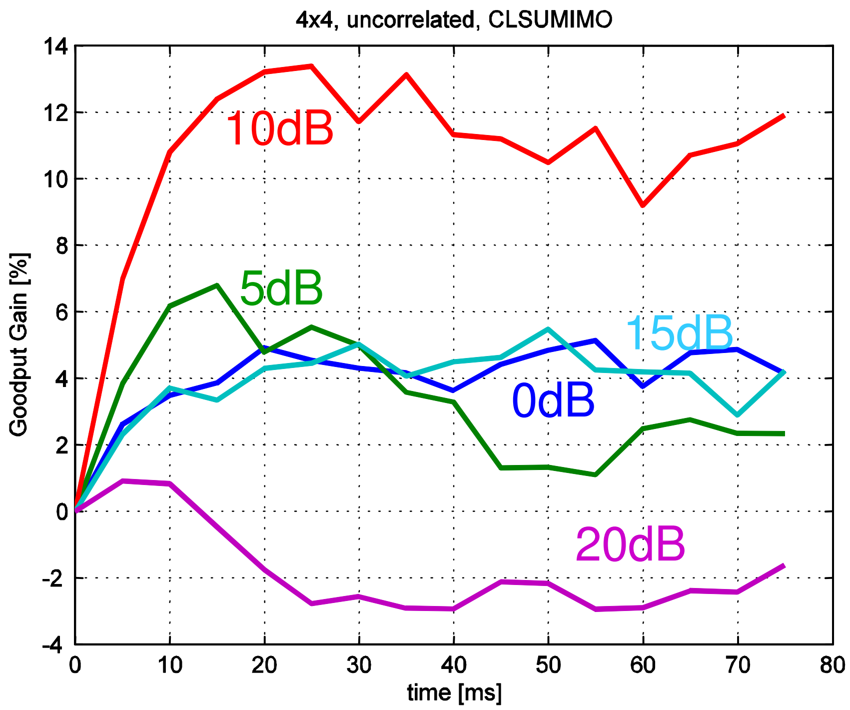


'-' 'Rot2 1' (ρ=0.9)
'-o' Rot 1 3bit {Uncorr CB for rank1 to 4}
'--' Rot 1 4bit {Uncorr CB for rank1 to 4}

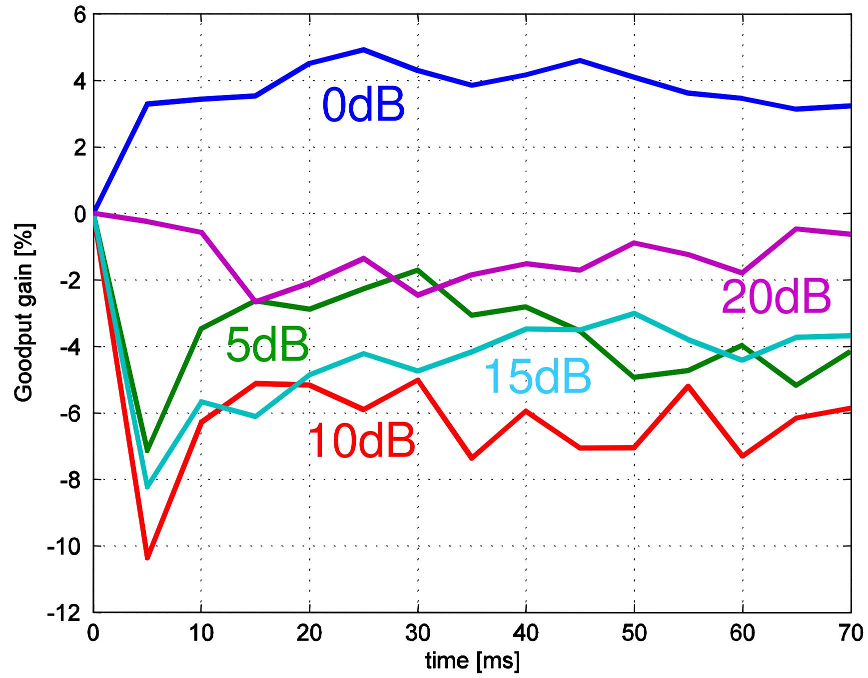
Loss for 'Rot 1' 3 and 4bits, except at very low SNR

4x4 CL SU MIMO: uncorrelated (4 λ, 15°AS), 3km/h
Relative Goodput Gain [%] **vs. standard** mode

- Relative Goodput Gain [%] of **'Rot2 1' (ρ=0.9) 4bits** over **standard** mode



- Relative Goodput Gain [%] of **'Rot 1' 4bit {Uncorr CB for rank1 to 4}** over **standard** mode



'Rot2 1' 4bits significantly outperforms 'Rot1' 4bits
'Rot1' 4bits is worse than the standard mode for SNR>=5dB

Performance comparisons

4x2 SU-MIMO Uncor-related	SNR	0dB	5dB	10dB	15dB	20dB
	Gain of 'Rot 1' 4bit {Uncorr CB for rank1 to 2} over 4bit AWD standard mode	1.99%	6.87%	10.73%	6.84%	0.00%
	Gain of 'Rot2 1' 4bit (ρ=0.9) over 4bit AWD standard mode	1.70%	5.26%	9.77%	5.30%	0.00%
	Gain of 'Rot 1' 4bit {Uncorr CB for rank1 to 2} over 'Rot2 1' 4bit (ρ=0.9)	0.29%	1.53%	0.88%	1.46%	0.00%
4x4 SU-MIMO Uncor-related	**SNR**	**0dB**	**5dB**	**10dB**	**15dB**	**20dB**
	Gain of 'Rot 1' 4bit {Uncorr CB for rank1 to 4} over 4bit AWD standard mode	3.17%	-2.87%	-3.91%	-4.47%	-1.82%
	Gain of 'Rot2 1' 4bit (ρ=0.9) over 4bit AWD standard mode	1.50%	4.13%	7.21%	1.30%	-0.68%
	Gain of 'Rot 1' 4bit {Uncorr CB for rank1 to 4} over 'Rot2 1' 4bit (ρ=0.9)	1.64%	-6.72%	-10.37%	-5.69%	-1.15%

* Gain averaged over 30ms (i.e. reset period=30ms)

observations

4x2 MIMO

- 4bits outperform 3bits
- Good refinement for ‘Rot2 1’ and ‘Rot1 Uncorr CB rank1’ 4bits with a slight advantage for ‘Rot1 Uncorr CB rank1’ 4bits
- ‘Rot2 2’ shows significant loss due to small distance on the Riemannian manifold

	Distance on Riemannian manifold	properties
‘Rot2 1’	0.9822	**Equally spaced** codebook
‘Rot2 2’	0.0266	**Not equally spaced** codebook

4x4 MIMO

- Good refinement for ‘Rot2 1’
- Significant loss for ‘Rot1 Uncorr CB rank1’ 3bits and 4bits

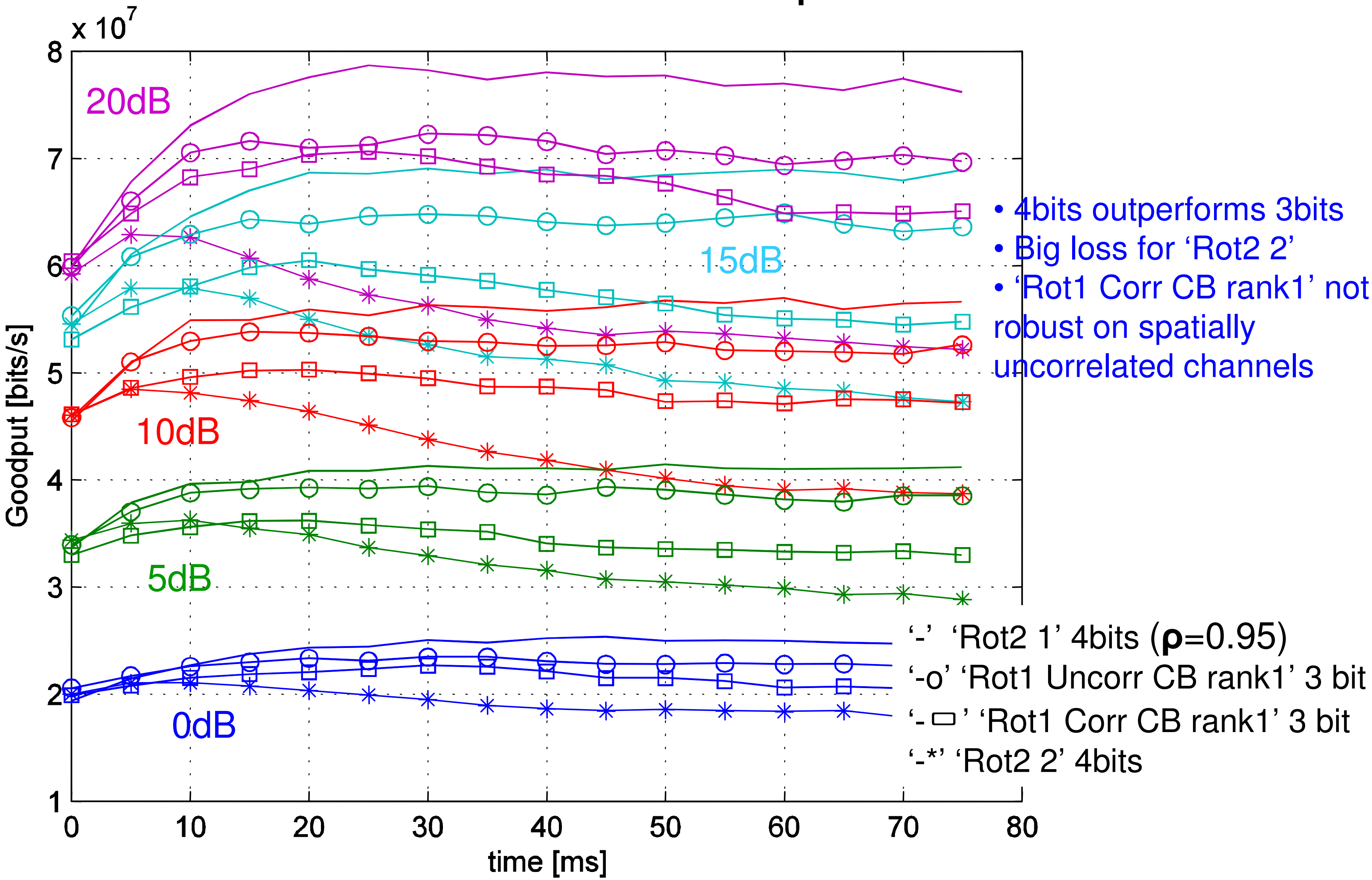
Overall

- **Significant gain of differential codebooks in uncorrelated channels over the standard mode**
- **‘Rot2 1’ 4bits (0.9) shows the best performance overall**

CL MU MIMO

4x2 MU MIMO: uncorrelated (4 λ, 15° AS), 3km/h

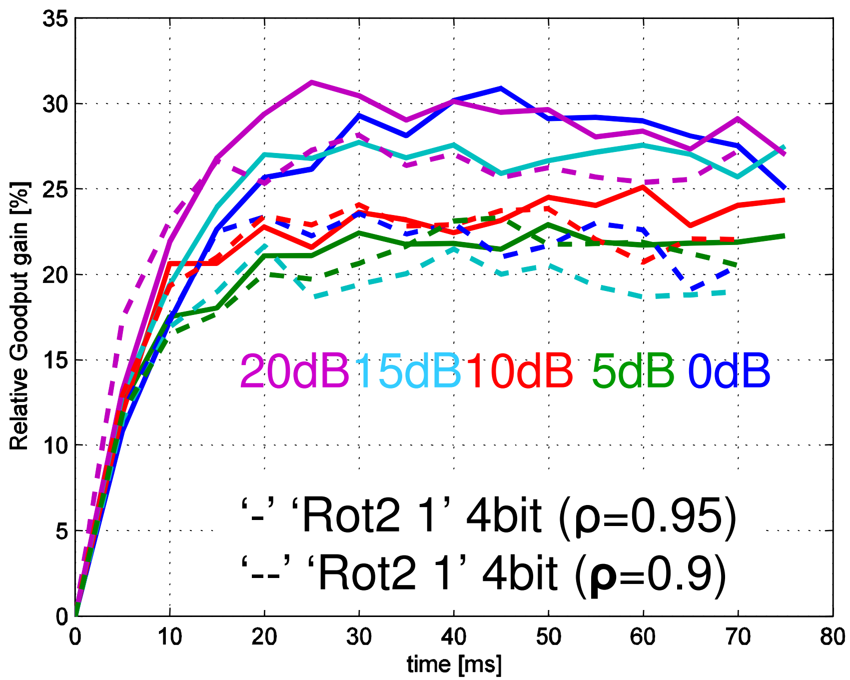
Absolute Goodput



- 4bits outperforms 3bits
- Big loss for 'Rot2 2'
- 'Rot1 Corr CB rank1' not robust on spatially uncorrelated channels

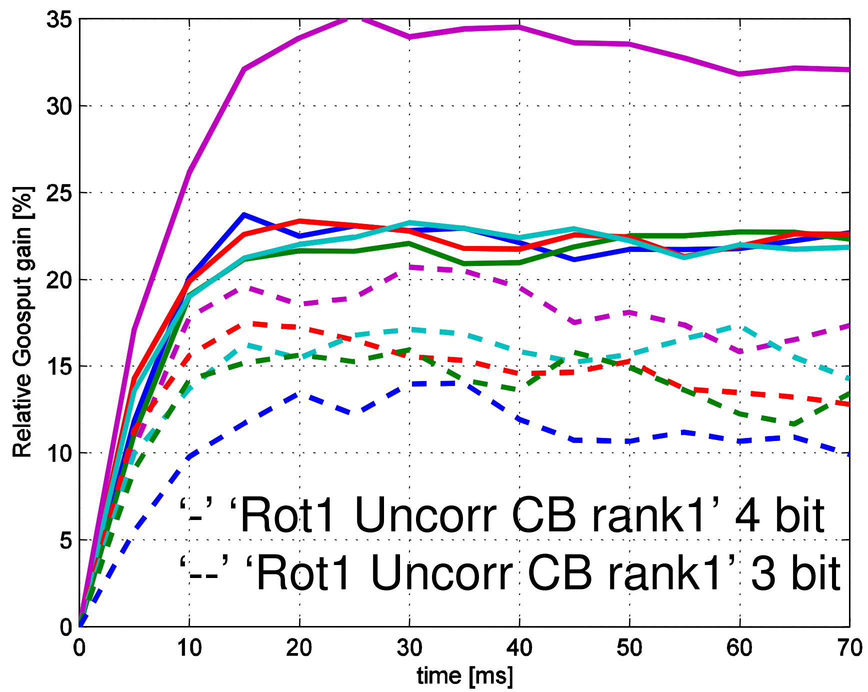
4x2 MU MIMO: uncorrelated (4 λ, 15° AS), 3km/h
Rot 1 vs. Rot 2

- Relative Goodput Gain [%] of **'Rot2 1' (ρ=0.95 and ρ=0.9) 4bits** over **standard** mode



- Average gain between 20-30%
- **ρ**=0.95 slightly outperforms **ρ**=0.9

- Relative Goodput Gain [%] of **'Rot1 Uncorr CB rank1' 3 bits and 4bits** over **standard** mode



- Average gain between 20-35% for 4bits
- 4bits outperforms 3bits
- 'Rot1 Uncorr CB rank1' 4 bit the best at 20dB

Performance comparisons

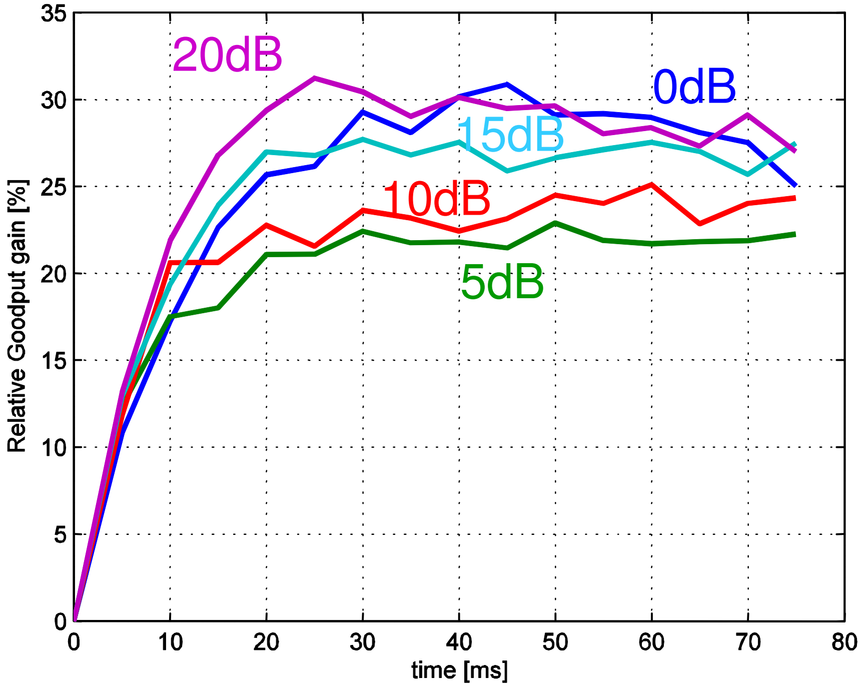
4x2 MU-MIMO Uncor-related	SNR	0dB	5dB	10dB	15dB	20dB
	Gain of 'Rot 1 Uncorr CB for rank1 ' 4bit over 4bit AWD standard mode	17.69%	16.65%	17.99%	17.35%	25.47%
	Gain of 'Rot2 1' 4bit (**ρ**=0.95) over 4bit AWD standard mode	18.82%	16.07%	17.30%	19.67%	21.83%
	Gain of 'Rot2 1' 4bit (**ρ**=0.9) over 4bit AWD standard mode	17.01%	15.05%	16.91%	15.04%	20.82%

Similar performance at realistic SNR
'Rot2 1' 4bit (**ρ**=0.9) slightly lower performance
'Rot 1 Uncorr CB for rank1 ' 4bit the best at very high SNR (20 dB)

* Gain averaged over 30ms (i.e. reset period=30ms)

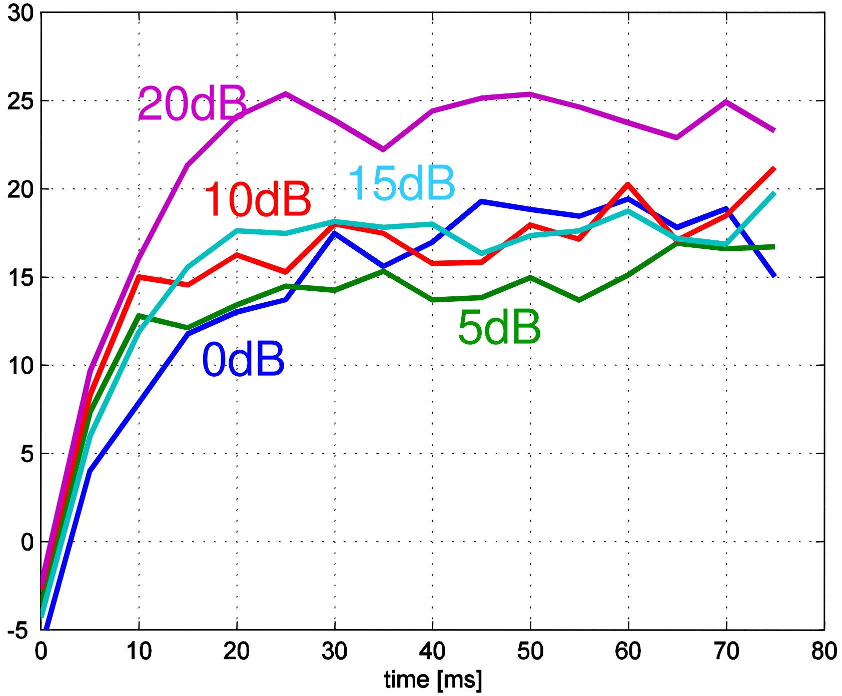
4x2 MU MIMO: uncorrelated (4 λ, 15° AS), 3km/h
Gain over standard and adaptive modes

- Relative Goodput Gain [%] of **'Rot2 1' (ρ=0.95) 4bits** over **standard** mode



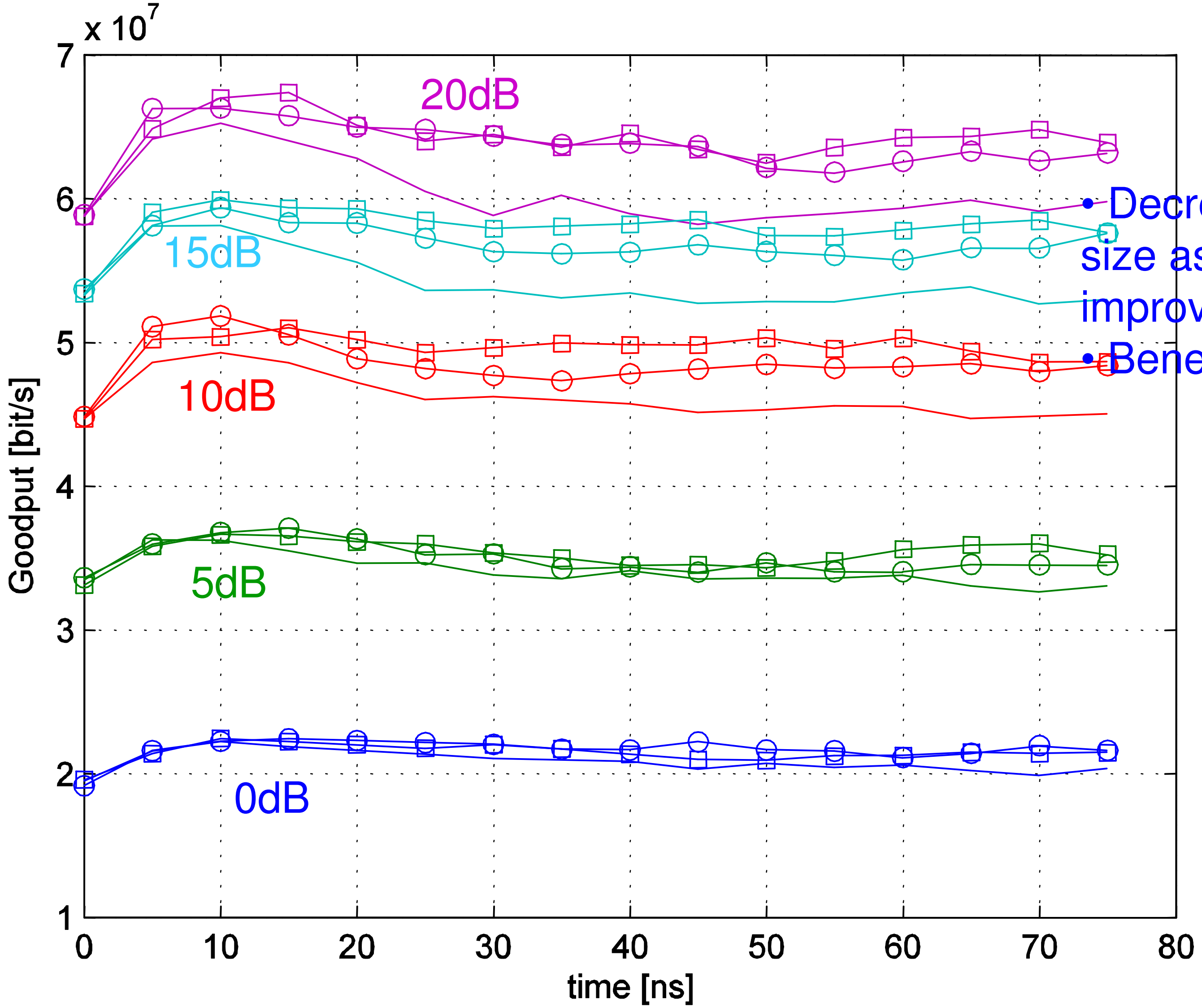
Significant gain (20-30%) over standard mode

- Relative Goodput Gain [%] of **'Rot2 1' (ρ=0.95) 4bits** over **adaptive** mode



Significant gain (15-25%) over adaptive mode

4x2 MU MIMO: uncorrelated (4 λ, 15° AS), 6km/h
Adaptation to time correlation



- Decreasing the polar cap size as speed increases improves the performance
- Beneficial feature of 'Rot 2'

'Rot2 1'

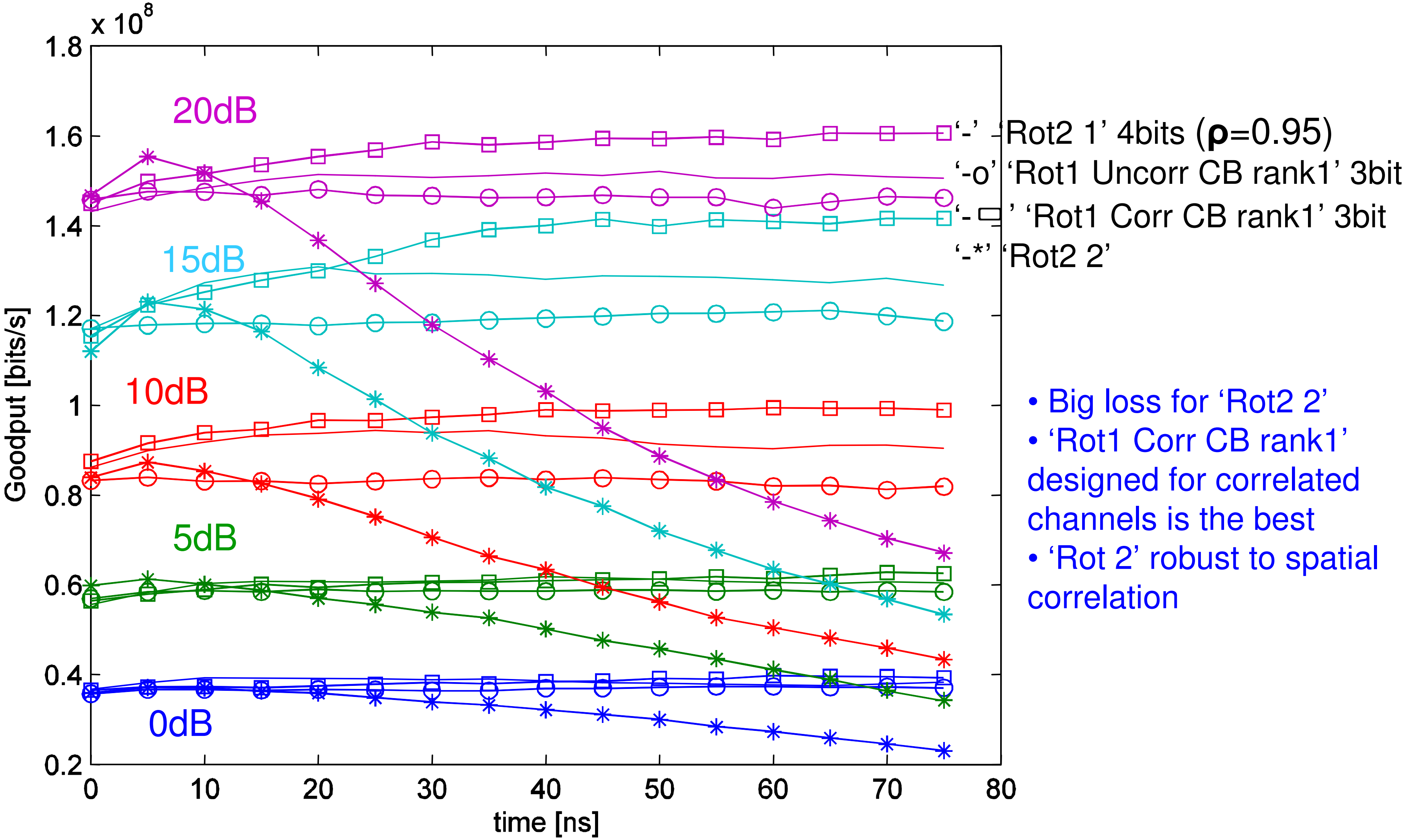
'-' ρ=0.95
'-o' ρ=0.9
'-▭' ρ=0.85

observations

- Good refinement for ‘Rot2 1’ and ‘Rot1 Uncorr CB rank1’ 3 and 4bits
- 4bits outperform 3bits
- ‘Rot1 Corr CB rank1’ 3bits optimized for small spacing shows loss or weak refinement in uncorrelated channels
- ‘Rot2 2’ shows significant loss due to small distance on the Riemannian manifold
- ‘Rot2 1’ easily adapts to mobile speed (i.e. parameter ρ)
- **Overall**
  - **Significant gain of differential codebooks in uncorrelated channels over the standard and adaptive modes**
  - **‘Rot2 1’ (0.95 and 0.9) and ‘Rot1 Uncorr CB rank1’ 4bits show the best performance with some additional performance gain for ‘Rot1 Uncorr CB rank1’ 4bits at high SNR (20dB)**
  - **‘Rot2 1’ benefits from better flexibility**

4x2 MU MIMO: correlated (0.5 λ, 3° AS), 3km/h

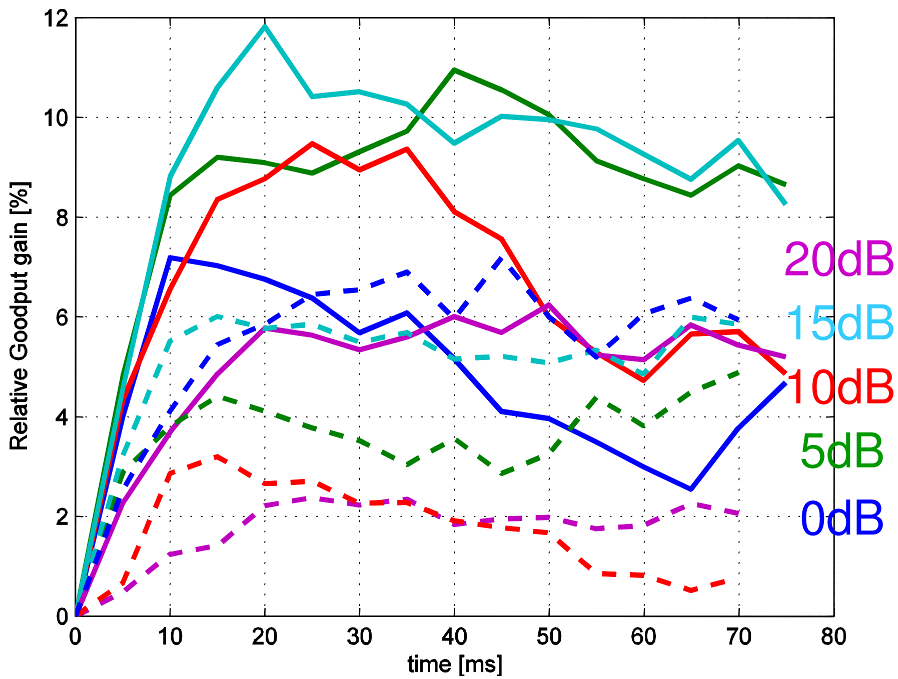
Absolute Goodput



- Big loss for 'Rot2 2'
- 'Rot1 Corr CB rank1' designed for correlated channels is the best
- 'Rot 2' robust to spatial correlation

4x2 MU MIMO: correlated (0.5 λ, 3° AS), 3km/h
Rot 1 vs. Rot 2

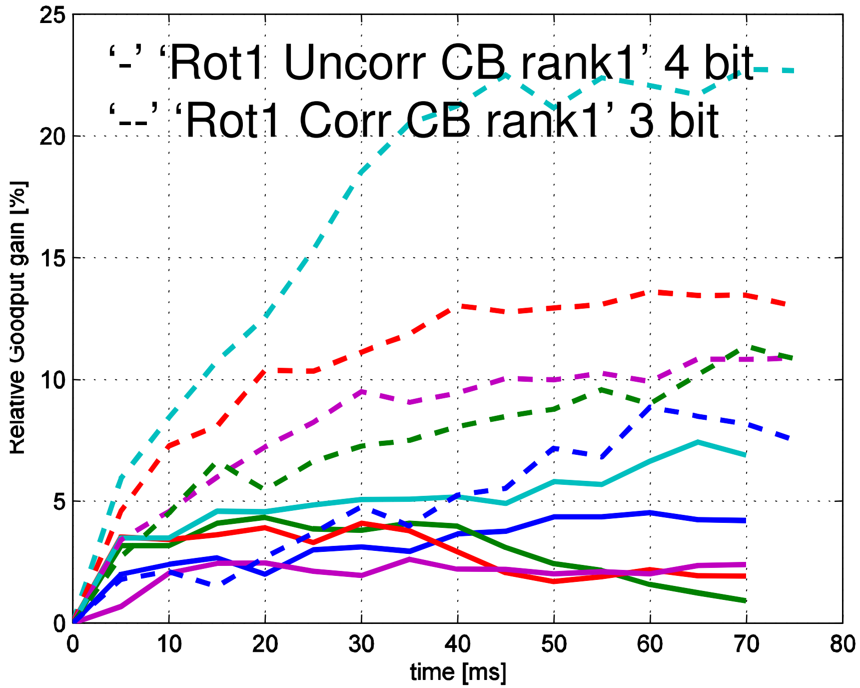
- Relative Goodput Gain [%] of **'Rot2 1' (ρ=0.95 and ρ=0.9) 4bits** over **standard** mode



'-' 'Rot2 1' 4bit (ρ=0.95)
'--' 'Rot2 1' 4bit (**ρ**=0.9)

- **ρ**=0.95 outperforms **ρ**=0.9

- Relative Goodput Gain [%] of **'Rot1 Corr CB rank1' 3 bit and 'Rot1 Unorr CB rank1' 4bits** over **standard** mode



- 3bits designed for spatial correlation outperforms 4bits designed for uncorrelated
- 'Rot1 corr CB rank1' 3 bit the best at high SNR

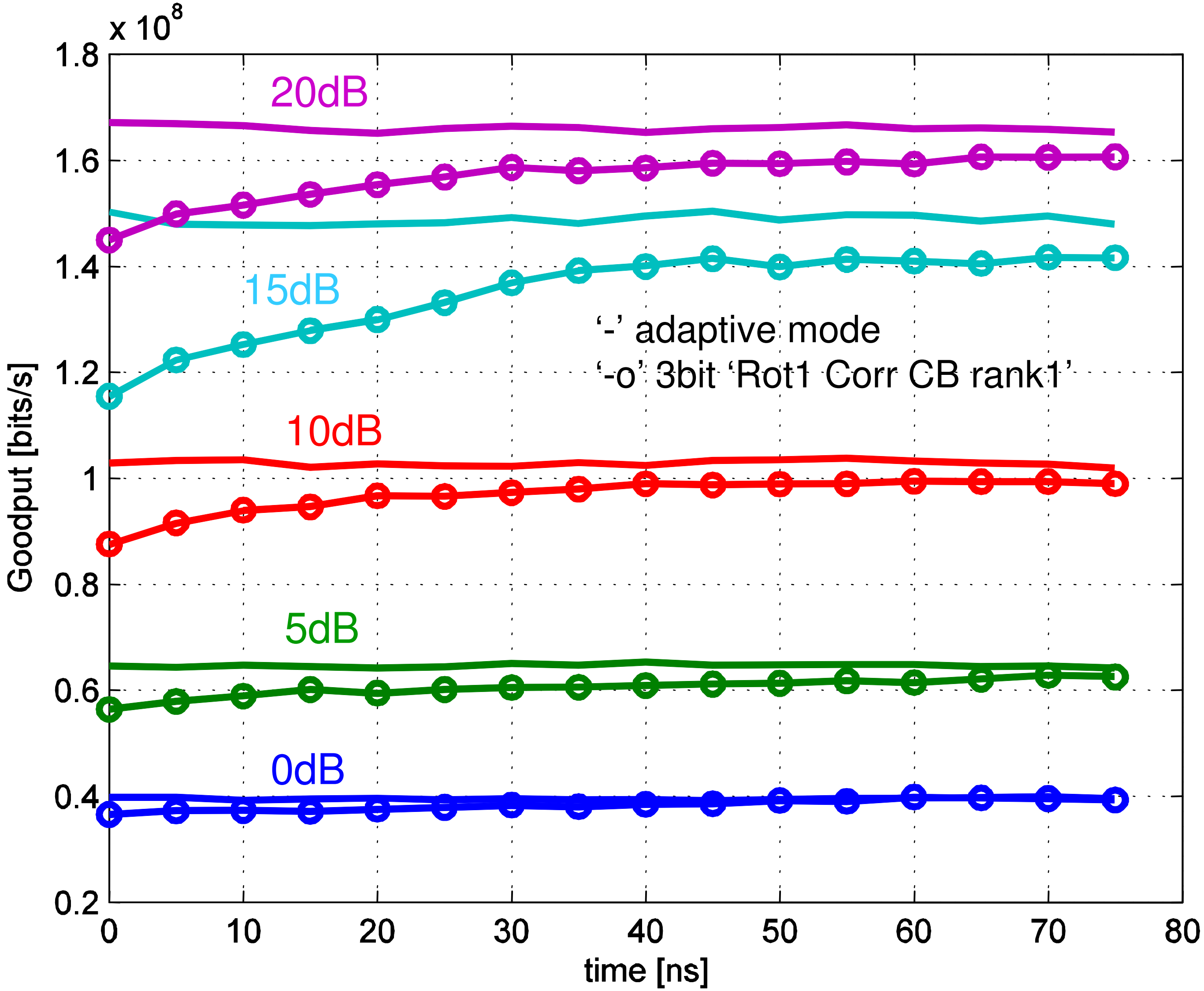
Performance comparisons

4x2 MU-MIMO Correl-ated	SNR	0dB	5dB	10dB	15dB	20dB
	Gain of 'Rot 1 Uncorr CB for rank1' 4bit over 4bit AWD standard mode	2.17%	3.20%	3.12%	3.71%	1.67%
	Gain of 'Rot 1 Corr CB for rank1' 3bit over 4bit AWD standard mode	2.36%	4.74%	7.39%	10.21%	5.55%
	Gain of 'Rot2 1' 4bit (**ρ**=0.95) over 4bit AWD standard mode	5.29%	7.10%	6.63%	8.10%	3.94%
	Gain of 'Rot2 1' 4bit (**ρ**=0.9) over 4bit AWD standard mode	4.41%	3.21%	2.04%	4.54%	1.42%

'Rot2 1' 4bit (**ρ**=0.95) the best at low SNR
'Rot 1 Uncorr CB for rank1' 3bit the best at high SNR

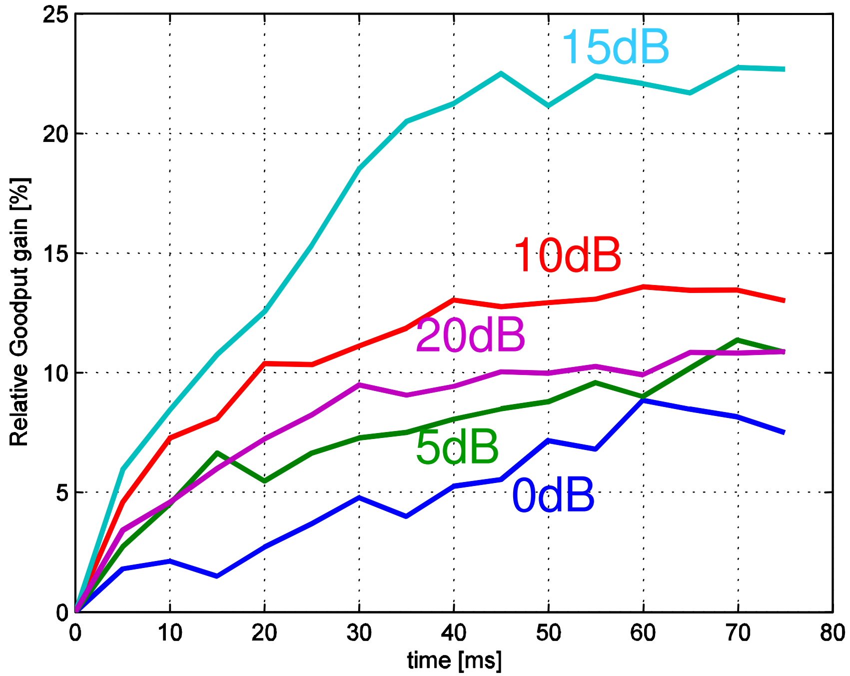
* Gain averaged over 30ms (i.e. reset period=30ms)

4x2 MU MIMO: correlated (0.5 λ, 3° AS), 3km/h
differential vs. adaptive mode



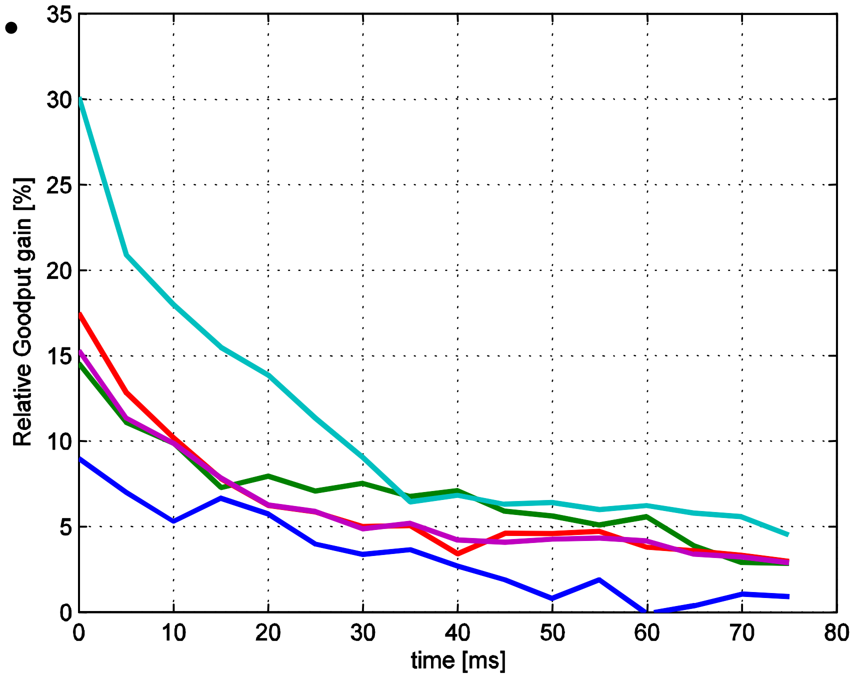
4x2 MU MIMO: correlated (0.5 λ, 3° AS), 3km/h Gain over standard and adaptive modes

- Relative Goodput Gain [%] of **'Rot1 Corr CB rank1' 3bits** over **standard** mode



Gain much lower than in uncorrelated scenarios

- Relative Goodput Gain [%] **adaptive** mode over **'Rot 1 Corr CB rank 1' 3 bits**
-



Significantly outperformed by the adaptive mode

observations

- Differential codebook less beneficial in correlated channels than in uncorrelated channels
- Good refinement for ‘Rot1 Corr CB rank1’ 3bits
- Good Robustness and refinement for ‘Rot2 1’ (0.95) 4bits
- ‘Rot2 1’ (0.9) and ‘Rot1 Uncorr CB rank1’ 4bits show some small refinement
- ‘Rot1 Uncorr CB rank1’ 3bits shows no throughput improvement compared to base codebook
- **Overall:**
  - **Moderate gain (smaller than in uncorrelated) of differential codebooks in correlated channels over the standard mode**
  - **Adaptive mode outperforms the differential mode in correlated channels**
  - **‘Rot1 Corr CB rank1’ 3bits shows the best performance at high SNR**
  - **‘Rot2 1’ (0.95) 4bits shows the best performance at low SNR**

Simulation Assumptions

- Channel model: Pedestrian B channel model, 3km/h, linear array
  - Uncorrelated: AS= 15, d/λ=4
  - Correlated: AS= 3, d/λ=0.5
- 10 MHz
- HARQ (Chase Combining, non-adaptive) with 3 retransmissions
  - Delay first transmission: 8 subframes
  - Delay between re-transmissions: 1 frame (8 subframes)
- CQI, PMI feedback period: every frame (5 ms)
- Link Adaptation (PHY abstraction): QPSK 1/2 with repetition 1/2/4/6, QPSK 3/4, 16QAM 1/2, 16QAM 3/4, 64QAM 1/2, 64QAM 2/3, 64QAM 3/4, 64QAM 5/6
- Ideal channel estimation
- MMSE receiver, MMSE CQI and PMI selection
- No CQI transmission errors
- ZFBF and SCW CL SU MIMO with rank adaptation
- LLRU (4 PRUs)
- Base codebook: 4bit subset AWD C80216m-09_0513r2.doc
- Ideal antenna calibration
- No constraint on PAPR
- adaptive mode: correlation matrix feedback every 100ms and unquantized

Text proposal

15.3.7.2.6.6.4. Differential codebook-based feedback mode

The procedure of differential feedback scheme is described as follows.

At time instant $\tau = 0$, the MS chooses the appropriate codeword and rank from the CL SU MIMO base codebook (in the CL SU MIMO mode) or the appropriate codeword from the CL MU MIMO base codebook (in CL MU MIMO mode) presented in previous sections 15.3.7.2.6.6.2. Denote this codeword as $\mathbf{F}_0$. The index of this codeword in the base codebook is fed back to the BS.

For time instant $\tau = 1, 2, \cdots T_{\max}$, the MS differentially rotates the previous codeword as: $\mathbf{F}_\tau = \tilde{\mathbf{\Theta}}_i \mathbf{F}_{\tau-1}$. The index of the preferred rotation matrix $\tilde{\mathbf{\Theta}}_i$ is fed back to the BS. At time $T_{\max}$+1, the process is reset and τ is fixed to 0 again.

The rotation and differential codebook are constructed according to the following 2-step procedure.

Firstly, we construct an equally spaced finite set differential codebook, $\{\theta\} = \{\mathbf{\Theta}_1, \ldots, \mathbf{\Theta}_{2^B}\}$, where each codeword is a $N_t \times N_t$ unitary matrix.

Secondly, the rotation codebook is then build using the following operations:

Step 1: Generate the matrix for $i = 1, 2, \ldots, 2^B$

$$\mathbf{\Psi}_i(\rho, \mathbf{\Theta}_i) = \rho \mathbf{I} + \sqrt{1-\rho^2}\,\mathbf{\Theta}_i$$

where ρ is an effective time correlation coefficient.

Step 2: Then, take $\tilde{\mathbf{\Theta}}_i$ such that

$$\tilde{\mathbf{\Theta}}_i = \arg\min_{\tilde{\mathbf{\Theta}}_i} \left\| \mathbf{\Psi}_i(\rho, \mathbf{\Theta}_i) - \tilde{\mathbf{\Theta}}_i \right\|_F \quad \text{for } i = 1, 2, \ldots, 2^B$$

Let the SVD of $\mathbf{\Psi}_i(\rho, \mathbf{\Theta}_i)$ be $\mathbf{\Phi}_i \mathbf{\Lambda}_i \mathbf{B}_i^*$, then the solution of the above problem is given as $\tilde{\mathbf{\Theta}}_i = \mathbf{\Phi}_i \mathbf{B}_i^*$

Step 3: Construct the rotation codebook (adapted to current ρ value)

$$\{\tilde{\theta}\} = \{\tilde{\mathbf{\Theta}}_1, \ldots, \tilde{\mathbf{\Theta}}_{2^B}\}$$

When differential codebook feedback mode is enabled, the MS shall report the PMI corresponding to the index *i* of its preferred rotated codebook entry $\tilde{\mathbf{\Theta}}_i$.

The unitary matrices of the equally spaced finite differential 3-bit codebook for two transmit antennas at BS are given in Table 1.

Table 1 – 3-bit Differential Codebook Unitary Matrices $\left\{\theta\right\}=\left\{\mathbf{\Theta}_1,\ldots,\mathbf{\Theta}_{2^B}\right\}$ for 2 Antennas at BS

Index i of $\mathbf{\Theta}_i$	Column 1	Column 2
1	1.0000	0.0000 + 0.0000i
	0.0000 - 0.0000i	1.0000
2	0.5732 + 0.1150i	0.5343 + 0.6105i
	-0.7161 + 0.3814i	0.5767 + 0.0958i
3	-0.3396 + 0.1940i	0.6153 - 0.6844i
	0.0883 - 0.9161i	-0.0867 - 0.3814i
4	-0.0685 + 0.7437i	-0.4689 + 0.4715i
	-0.3493 + 0.5658i	0.7073 - 0.2399i
5	-0.3065 - 0.4181i	0.5613 + 0.6452i
	-0.7872 + 0.3341i	-0.4587 + 0.2415i
6	0.2983 - 0.2900i	0.2784 - 0.8657i
	-0.9078 - 0.0541i	0.3783 - 0.1730i
7	-0.6555 - 0.2242i	0.4919 + 0.5274i
	-0.3710 - 0.6184i	-0.0344 - 0.6919i
8	0.7811 - 0.1004i	0.1436 - 0.5993i
	-0.1963 + 0.5842i	-0.6088 - 0.4996i

Table 2 – 4-bit Differential Codebook Unitary Matrices $\{\theta\}=\{\mathbf{\Theta}_1,\ldots,\mathbf{\Theta}_{2^B}\}$ for 4 Antennas at BS

Index i of $\mathbf{\Theta}_i$	Column 1	Column 2	Column 3	Column 4
1	1.0000	0.0000 - 0.0000i	-0.0000 + 0.0000i	-0.0000 + 0.0000i
	0.0000 + 0.0000i	1.0000 + 0.0000i	-0.0000 + 0.0000i	0.0000 + 0.0001i
	-0.0000 - 0.0000i	-0.0000 - 0.0000i	1.0000	-0.0001 + 0.0000i
	-0.0000 - 0.0000i	0.0000 - 0.0001i	-0.0001 - 0.0000i	1.0000
2	0.2345 + 0.5093i	0.0710 - 0.1613i	-0.5599 - 0.3206i	-0.4835 - 0.0669i
	0.1248 - 0.3603i	0.1195 - 0.0899i	0.3867 - 0.4896i	-0.4611 + 0.4799i
	0.3584 - 0.0083i	-0.7321 + 0.4322i	0.0416 - 0.3476i	0.1121 - 0.1166i
	-0.5598 - 0.3137i	-0.4676 - 0.0714i	-0.1958 + 0.1833i	-0.5217 - 0.1432i
3	-0.2840 + 0.2036i	-0.0482 - 0.2558i	-0.3197 + 0.6799i	-0.4323 + 0.2426i
	0.3537 - 0.4401i	0.2661 - 0.0081i	0.0165 + 0.6220i	0.2938 - 0.3699i
	-0.5263 - 0.3729i	0.1497 - 0.1631i	-0.1030 - 0.0172i	0.5579 + 0.4612i
	-0.1222 - 0.3580i	-0.8114 + 0.3923i	-0.1563 + 0.1146i	0.0200 - 0.0815i
4	0.0979 + 0.4428i	-0.3465 + 0.1727i	-0.1084 + 0.7806i	0.0942 - 0.1206i
	0.6779 + 0.2144i	-0.0967 - 0.3529i	-0.2463 - 0.2200i	-0.3461 - 0.3629i
	0.2421 - 0.3300i	-0.0275 + 0.4333i	-0.3076 - 0.1315i	0.5891 - 0.4301i
	0.2546 - 0.2375i	0.4284 - 0.5866i	0.0177 + 0.3971i	0.4157 + 0.1425i
5	0.3515 + 0.0167i	-0.1554 + 0.1117i	0.0386 + 0.2929i	-0.6705 + 0.5501i
	-0.4836 + 0.1506i	-0.2138 - 0.6278i	0.2294 + 0.4721i	0.0464 + 0.1608i
	0.7730 + 0.0887i	0.0390 - 0.4240i	-0.0937 + 0.2664i	0.3482 - 0.1104i
	0.0523 - 0.1062i	-0.1601 - 0.5627i	-0.1443 - 0.7326i	-0.1560 + 0.2489i
6	0.0157 + 0.0106i	-0.3825 + 0.1709i	0.4178 + 0.0436i	-0.6451 - 0.4812i
	-0.3924 + 0.6839i	0.2337 - 0.0749i	0.1400 - 0.4785i	0.0870 - 0.2489i
	-0.4873 - 0.1883i	-0.5361 - 0.0966i	-0.5783 - 0.2922i	-0.1002 - 0.0222i
	0.2439 + 0.2134i	-0.6684 + 0.1438i	0.3365 - 0.2051i	0.4463 + 0.2704i
7	-0.1484 - 0.6909i	-0.5976 - 0.1255i	-0.2197 - 0.1921i	0.1260 + 0.1630i
	0.4980 - 0.1609i	0.0538 + 0.6937i	0.1318 - 0.0756i	0.2942 + 0.3638i
	0.2783 + 0.2876i	-0.2132 - 0.2287i	-0.3228 + 0.2323i	0.7335 - 0.2141i
	-0.2306 + 0.1152i	-0.1937 - 0.0875i	0.7945 - 0.3202i	0.3806 - 0.0997i

8	0.0167 - 0.5145i	0.3110 - 0.4278i	-0.1853 + 0.0053i	-0.5452 + 0.3516i
	-0.3835 - 0.5844i	-0.1035 + 0.1825i	-0.4601 - 0.0298i	0.1926 - 0.4666i
	0.3207 + 0.0490i	-0.7369 - 0.2643i	-0.1650 + 0.2279i	-0.3629 - 0.2666i
	0.3435 - 0.1519i	0.2496 - 0.0338i	-0.0637 + 0.8185i	0.3485 - 0.0061i
9	-0.5688 - 0.0120i	-0.3898 + 0.2975i	-0.3493 + 0.0604i	-0.1820 + 0.5264i
	-0.3039 - 0.0476i	0.1928 - 0.1487i	0.5434 - 0.5795i	0.2284 + 0.4034i
	-0.3365 + 0.3560i	0.6043 - 0.3477i	-0.1999 + 0.0494i	-0.4809 - 0.0159i
	-0.3121 - 0.4942i	0.1682 - 0.4311i	-0.1826 + 0.4092i	0.4929 + 0.0219i
10	-0.3930 + 0.1272i	0.1383 - 0.5915i	0.6426 + 0.0499i	-0.1626 + 0.1355i
	0.4393 + 0.3725i	0.4381 - 0.1760i	-0.0303 + 0.1562i	0.5067 + 0.4041i
	0.2293 + 0.5314i	0.1544 + 0.3313i	0.4033 + 0.0137i	-0.1122 - 0.5968i
	0.3684 + 0.1638i	-0.5052 - 0.1390i	-0.0042 + 0.6297i	-0.3747 + 0.1619i
11	-0.2373 + 0.0490i	-0.6173 + 0.5692i	0.4226 + 0.0197i	-0.1270 - 0.2027i
	-0.3278 - 0.2836i	-0.2696 + 0.1643i	-0.4266 - 0.1075i	-0.0345 + 0.7195i
	0.4137 - 0.1395i	-0.0639 - 0.0419i	0.5764 - 0.2381i	0.4628 + 0.4476i
	0.6550 - 0.3655i	-0.0578 + 0.4314i	-0.3151 + 0.3732i	0.0127 - 0.0963i
12	0.2417 + 0.1763i	0.0450 + 0.2953i	0.2078 - 0.4861i	0.5801 + 0.4530i
	-0.0820 - 0.4701i	0.5723 - 0.3346i	0.0930 + 0.3083i	0.1824 + 0.4424i
	-0.3244 - 0.0925i	-0.3863 + 0.0475i	-0.7244 + 0.0567i	0.1373 + 0.4336i
	0.6554 + 0.3734i	-0.1363 - 0.5489i	-0.1929 + 0.2273i	0.0541 + 0.1393i
13	0.3348 + 0.2001i	-0.5934 + 0.2450i	-0.5398 + 0.0970i	-0.3558 - 0.0914i
	0.3233 + 0.2147i	-0.5011 - 0.1514i	0.4454 + 0.2845i	0.5079 + 0.1952i
	-0.5250 - 0.2586i	-0.4139 - 0.0765i	0.3692 + 0.3400i	-0.4036 - 0.2559i
	0.3373 + 0.4909i	0.3681 - 0.0348i	0.3008 + 0.2783i	-0.2955 - 0.5033i
14	-0.2795 - 0.2884i	0.0154 + 0.6932i	0.0028 + 0.3217i	-0.1104 - 0.4920i
	0.2550 - 0.0730i	-0.0510 - 0.2703i	0.2359 + 0.8828i	-0.0799 + 0.1121i
	0.2400 + 0.5271i	-0.4736 + 0.4282i	0.0362 - 0.0025i	-0.4443 + 0.2416i
	-0.6298 + 0.1907i	-0.0845 + 0.1692i	-0.0729 + 0.2342i	0.4251 + 0.5390i
15	0.4995 + 0.4688i	-0.3954 - 0.3825i	0.1135 + 0.2966i	0.2189 - 0.2817i
	-0.2746 + 0.0992i	-0.2530 + 0.6514i	0.0345 + 0.3733i	0.5337 - 0.0295i
	0.2411 + 0.5303i	-0.0982 + 0.4310i	-0.5151 - 0.3534i	-0.2623 + 0.0790i
	-0.0352 - 0.3240i	-0.0016 - 0.1165i	-0.5692 - 0.2106i	0.3326 - 0.6335i
16	-0.0246 - 0.1239i	-0.1148 - 0.4967i	0.7805 + 0.0955i	-0.0974 - 0.3103i
	0.3017 - 0.2799i	0.0232 - 0.3740i	-0.0315 - 0.3722i	0.6925 + 0.2666i
	-0.2235 - 0.4672i	-0.4954 + 0.0779i	0.0580 + 0.4264i	-0.0101 + 0.5430i
	-0.5613 - 0.4811i	0.5716 - 0.1464i	-0.0689 - 0.2284i	-0.2050 + 0.0803i

Recommended values for the effective time correlation coefficient are between ρ=0.85 and ρ=0.95 for 4Tx. ρ=0.9 gives overall good performance tradeoff for 4Tx. For 2Tx, ρ=0.95 is a good choice.

The unitary matrices for 2Tx antennas at BS with ρ=0.95 are specified as follows.

$\tilde{\mathbf{\Theta}}_1$ =

1.0000	-0.0000 - 0.0000i
0.0000 - 0.0000i	1.0000

$\tilde{\mathbf{\Theta}}_2$ =

0.9755 + 0.0380i	0.1647 + 0.1407i
-0.1744 + 0.1283i	0.9757 + 0.0328i

$\tilde{\mathbf{\Theta}}_3$ =

0.9525 + 0.1066i	0.0918 - 0.2701i
-0.0866 - 0.2718i	0.9544 - 0.0882i

$\tilde{\mathbf{\Theta}}_4$ =

0.9524 + 0.2575i	-0.0360 + 0.1589i
0.0011 + 0.1630i	0.9855 - 0.0477i

$\tilde{\mathbf{\Theta}}_5$ =

0.9471 - 0.1233i	0.2410 + 0.1727i
-0.2387 + 0.1757i	0.9485 + 0.1113i

$\tilde{\mathbf{\Theta}}_6$ =

0.9667 - 0.1139i	0.1663 - 0.1576i
-0.1941 - 0.1219i	0.9702 - 0.0783i

$\tilde{\mathbf{\Theta}}_7$ =

0.9899 - 0.0591i	0.1252 - 0.0299i
-0.1287 + 0.0032i	0.9720 - 0.1967i

$\tilde{\mathbf{\Theta}}_8$ =

0.9963 - 0.0376i	0.0762 - 0.0134i
-0.0771 + 0.0065i	0.9729 - 0.2177i

The unitary matrices for 4Tx antennas at BS with ρ=0.9 are specified as follows

$\tilde{\boldsymbol{\Theta}}_1$ =

1.0000 - 0.0000i	-0.0000 - 0.0000i	-0.0000 - 0.0000i	-0.0000 - 0.0000i
0.0000 - 0.0000i	1.0000 - 0.0000i	0.0000 - 0.0000i	-0.0000 + 0.0000i
0.0000 - 0.0000i	-0.0000 + 0.0000i	1.0000 + 0.0000i	-0.0000 + 0.0000i
-0.0000 - 0.0000i	0.0000 + 0.0000i	-0.0000 + 0.0000i	1.0000 - 0.0000i

$\tilde{\boldsymbol{\Theta}}_2$ =

0.9458 + 0.1964i	0.0206 - 0.0913i	-0.2010 - 0.0858i	0.0168 - 0.1001i
0.0553 - 0.1154i	0.9501 - 0.0706i	0.2576 - 0.0507i	0.0096 + 0.0838i
0.1939 - 0.0664i	-0.2757 + 0.0127i	0.9144 - 0.1960i	0.0824 - 0.0210i
-0.0138 - 0.0911i	0.0190 + 0.0841i	-0.0794 + 0.0494i	0.9825 - 0.1000i

$\tilde{\boldsymbol{\Theta}}_3$ =

0.9548 + 0.1582i	-0.1024 - 0.1342i	0.0840 + 0.1483i	-0.0515 - 0.0566i
0.1186 - 0.1565i	0.9391 - 0.0538i	-0.0762 + 0.1081i	0.2372 - 0.0540i
-0.0736 + 0.1268i	0.0693 + 0.0481i	0.9503 + 0.0250i	0.1911 + 0.1764i
0.0085 - 0.0574i	-0.2819 - 0.0105i	-0.1441 + 0.1701i	0.9266 - 0.0934i

$\tilde{\boldsymbol{\Theta}}_4$ =

0.9401 + 0.1598i	-0.2353 + 0.0952i	-0.1021 + 0.0985i	0.0101 - 0.0771i
0.2079 + 0.1076i	0.9192 - 0.1702i	-0.0131 + 0.0593i	-0.1800 - 0.1878i
0.1013 + 0.1000i	0.0140 + 0.0174i	0.9718 - 0.1296i	0.1336 - 0.0147i
0.0553 - 0.1119i	0.1563 - 0.1919i	-0.1208 + 0.0194i	0.9523 + 0.0376i

$\tilde{\boldsymbol{\Theta}}_5$ =

0.9566 - 0.0183i	0.0577 - 0.0173i	-0.1634 + 0.0562i	-0.1846 + 0.1308i
-0.0559 - 0.0081i	0.9355 - 0.3304i	-0.0042 + 0.0524i	-0.0149 - 0.0978i
0.2160 + 0.0598i	0.0029 + 0.0295i	0.9242 + 0.1339i	0.1948 - 0.1972i
0.1449 + 0.1011i	0.0156 - 0.1048i	-0.1631 - 0.2622i	0.9182 + 0.1383i

$\tilde{\boldsymbol{\Theta}}_6$ =

0.9201 + 0.0719i	-0.0359 + 0.1568i	0.2672 - 0.0209i	-0.2168 - 0.0599i
-0.0134 + 0.1918i	0.9381 - 0.0418i	0.2007 - 0.1776i	0.0929 - 0.0270i
-0.2718 - 0.0429i	-0.2227 - 0.1586i	0.8879 - 0.2018i	-0.1050 - 0.0966i
0.1882 - 0.0138i	-0.1323 + 0.0043i	0.1255 - 0.1068i	0.9518 + 0.1175i

$\tilde{\boldsymbol{\Theta}}_7 =$

0.8972 - 0.3030i	-0.2504 - 0.0586i	-0.1497 + 0.0157i	0.1073 + 0.0543i
0.2446 - 0.0662i	0.9065 + 0.3021i	0.0637 - 0.0438i	0.1061 + 0.0751i
0.1438 + 0.0089i	-0.0587 - 0.0720i	0.9463 + 0.2156i	0.0198 - 0.1678i
-0.1071 + 0.0827i	-0.1036 + 0.0391i	0.0336 - 0.1679i	0.9694 + 0.0202i

$\tilde{\boldsymbol{\Theta}}_8 =$

0.9064 - 0.2375i	0.1330 - 0.2260i	-0.1066 - 0.0118i	-0.1965 + 0.0556i
-0.1651 - 0.2268i	0.9496 + 0.0983i	0.0671 - 0.0168i	-0.0133 - 0.0703i
0.0871 + 0.0132i	-0.0373 - 0.0406i	0.9738 + 0.1405i	-0.0995 + 0.1062i
0.1864 + 0.0270i	0.0629 - 0.1134i	0.0617 + 0.1090i	0.9653 + 0.0166i

$\tilde{\boldsymbol{\Theta}}_9 =$

0.9924 + 0.0369i	-0.0064 + 0.0377i	-0.0003 + 0.1063i	0.0255 + 0.0206i
0.0293 + 0.0404i	0.9674 + 0.0094i	0.0077 - 0.2405i	0.0558 - 0.0242i
-0.0084 + 0.0956i	0.0347 - 0.2420i	0.9431 + 0.1406i	-0.1041 + 0.1052i
-0.0394 + 0.0241i	-0.0234 - 0.0484i	0.0700 + 0.1295i	0.9855 + 0.0466i

$\tilde{\boldsymbol{\Theta}}_{10} =$

0.9434 + 0.0321i	-0.0524 - 0.0921i	0.1122 + 0.1597i	-0.2209 + 0.1037i
0.0891 - 0.0504i	0.9573 - 0.0253i	-0.0055 + 0.0607i	0.2434 + 0.0978i
-0.1064 + 0.1614i	0.0295 + 0.0571i	0.9782 - 0.0333i	0.0012 - 0.0210i
0.1883 + 0.1600i	-0.2553 + 0.0493i	0.0005 + 0.0120i	0.9271 + 0.1081i

$\tilde{\boldsymbol{\Theta}}_{11} =$

0.9724 + 0.0436i	-0.0423 + 0.0413i	0.0326 - 0.0292i	-0.1707 - 0.1340i
0.0661 + 0.0921i	0.9569 + 0.0902i	-0.0966 - 0.0559i	-0.0016 + 0.2257i
0.0253 - 0.0167i	0.0511 - 0.0135i	0.9611 - 0.0872i	0.1813 + 0.1793i
0.1596 - 0.1152i	-0.0208 + 0.2637i	-0.1520 + 0.1769i	0.9143 - 0.0316i

$\tilde{\boldsymbol{\Theta}}_{12} =$

0.9492 + 0.0182i	0.0142 - 0.0212i	0.1551 - 0.2117i	-0.0001 + 0.1707i
-0.0167 - 0.0625i	0.9719 - 0.0979i	0.1554 + 0.1151i	0.0643 + 0.0085i
-0.1702 - 0.1851i	-0.1664 + 0.1084i	0.9035 + 0.0532i	0.1458 + 0.2386i
0.0644 + 0.1646i	-0.0614 - 0.0437i	-0.1373 + 0.2362i	0.9416 + 0.0422i

$\tilde{\mathbf{\Theta}}_{13}$ =

0.9447 + 0.1012i	-0.2465 + 0.0950i	-0.0260 + 0.0307i	-0.1341 + 0.0886i
0.2290 + 0.1183i	0.9205 - 0.1061i	0.2256 + 0.0693i	-0.0008 + 0.1390i
-0.0620 - 0.0018i	-0.2253 + 0.0261i	0.9455 + 0.1655i	-0.1527 - 0.0021i
0.1276 + 0.1033i	-0.0261 + 0.1397i	0.1445 + 0.0207i	0.9300 - 0.2582i

$\tilde{\mathbf{\Theta}}_{14}$ =

0.9176 - 0.2298i	-0.1133 + 0.1737i	-0.0163 + 0.2101i	0.0991 - 0.0894i
0.1187 + 0.1484i	0.9128 - 0.1253i	0.1170 + 0.2986i	-0.0062 + 0.1099i
0.0191 + 0.2108i	-0.1391 + 0.2755i	0.9107 + 0.0247i	-0.1235 + 0.1210i
-0.1482 - 0.0482i	-0.0271 + 0.1098i	0.0762 + 0.1300i	0.9498 + 0.1949i

$\tilde{\mathbf{\Theta}}_{15}$ =

0.9550 + 0.1665i	-0.0740 - 0.0516i	-0.0201 + 0.2007i	0.0651 - 0.0843i
0.0267 - 0.0147i	0.8964 + 0.3294i	0.0483 + 0.2503i	0.1476 - 0.0173i
0.0303 + 0.2155i	-0.0753 + 0.2450i	0.9034 - 0.2476i	0.0399 - 0.0891i
-0.0609 - 0.0904i	-0.1153 - 0.0298i	-0.0741 - 0.1067i	0.9442 - 0.2559i

$\tilde{\mathbf{\Theta}}_{16}$ =

0.9233 - 0.0334i	-0.1047 - 0.1296i	0.2562 - 0.0511i	0.1185 - 0.1905i
0.0550 - 0.1409i	0.9601 - 0.1892i	0.0883 - 0.1007i	0.0028 + 0.0403i
-0.2390 - 0.1106i	-0.0831 - 0.0593i	0.9269 + 0.2085i	-0.0092 + 0.1320i
-0.1024 - 0.2088i	0.0000 + 0.0653i	-0.0568 + 0.0888i	0.9646 - 0.0047i