Undetected Error Analysis
HEC-16 & HEC-32 Polynomials

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Outline

Problem Statement

- HEC-16 vs HEC-32

Coding Preliminaries

- Weight Distribution of Codes
- Undetected Error Probability Formula

Undetected Error Probability Results

- Without Scrambling Artifacts
- With Scrambling Artifacts

Conclusions

Problem Statement

Does HEC-16 Provide Adequate Error Coverage ?

- 16 Byte (128 bits) RPR Header
- Realistic Fiber Bit Error Rates ($10^{-9} \sim 10^{-13}$)
- Generator Polynomial

  $X^{16}+X^{12}+X^{5}+1$

Do We Need HEC-32 ?

- Generator Polynomial

  $X^{32}+X^{26}+X^{23}+X^{22}+X^{16}+X^{12}+X^{11}+X^{10}+X^{8}+X^{7}+X^{5}+X^{4}+X^{2}+X+1$

Approach– Determine Undetected Error Probability

- HEC-16, Code Length L = 144 (128+16), Check Bits r = 16
- HEC-32, Code Length L = 160 (144+32), Check Bits r = 32

Coding Preliminaries

Bit Error Rate– ε

Weight Distribution Enumerator N[L,w]

- For A Given Generator Polynomial

N[L,w] – Number of Length *L* Code Words with *w* ones

Probability of Undetected Error

$$\sum_{w=1}^{w=L} N(L,w)(1-\varepsilon)^{L-w}\varepsilon^{w}$$

Coding Preliminaries

Undetected Error Probability

- Computationally More Efficient to Use Dual Codes
- Every (L,r) Code Has a (L,L-r) Dual Code
  See Reference [1] for Definition of Dual Codes
- Dual Code Weight Enumerator M[L,w]
  See Reference [2] for Algorithms to Compute M[L,w]

Probability of Undetected Error

$$\sum_{w=1}^{w=L} N(L,w)(1-\varepsilon)^{L-w}\varepsilon^{w}$$

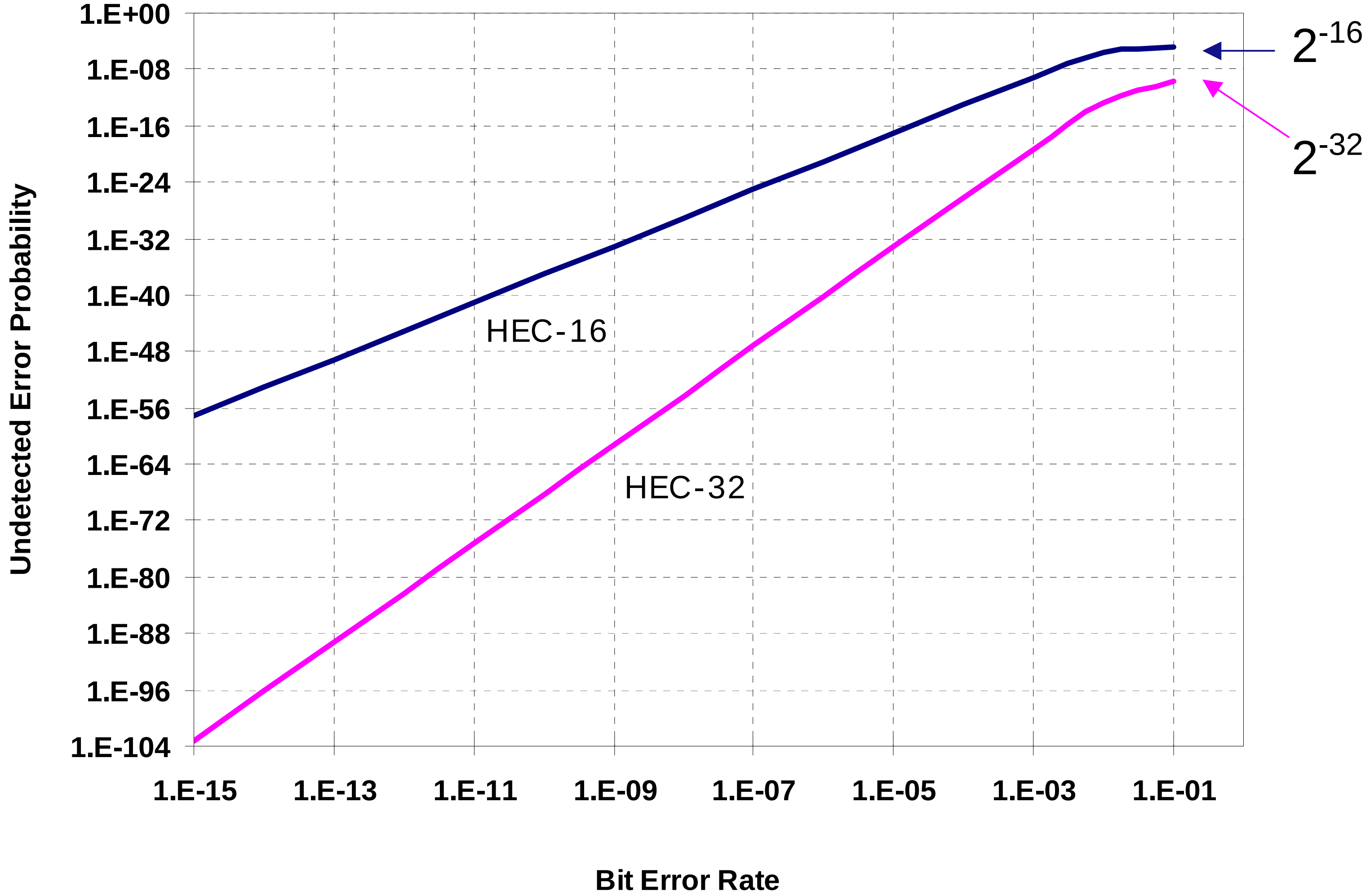
$$= 2^{-r}\sum_{w=0}^{w=L} M(L,w)(1-2\varepsilon)^{w} - (1-\varepsilon)^{L}$$

Results without Scrambling Consideration

Undetected Error Probability

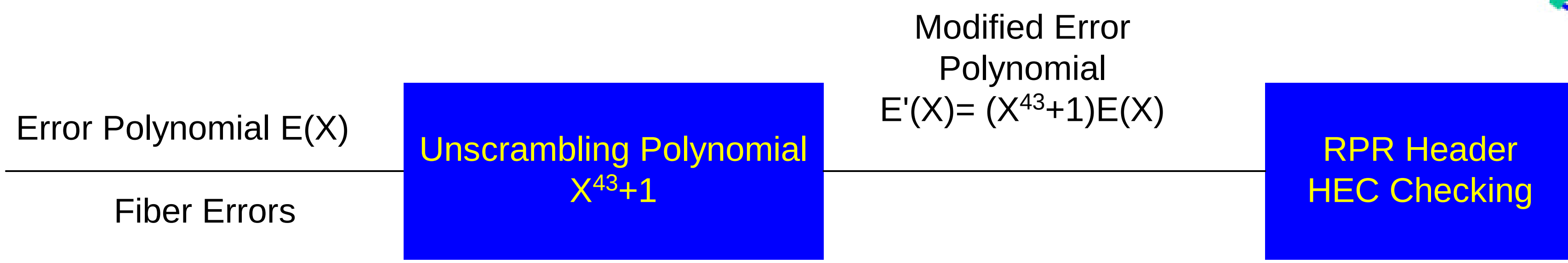
Bit Error Rate	HEC-16	HEC-32
10^{-15}	8.31×10^{-58}	6.00×10^{-104}
10^{-14}	8.31×10^{-54}	6.00×10^{-97}
10^{-13}	8.31×10^{-50}	6.00×10^{-90}
10^{-12}	8.31×10^{-46}	6.00×10^{-83}
10^{-11}	8.31×10^{-42}	6.00×10^{-76}
10^{-10}	8.31×10^{-38}	6.00×10^{-69}
10^{-09}	8.31×10^{-34}	6.00×10^{-62}
10^{-08}	8.31×10^{-30}	5.99×10^{-55}
10^{-07}	8.31×10^{-26}	5.99×10^{-48}
10^{-06}	8.31×10^{-22}	5.99×10^{-41}
10^{-05}	8.29×10^{-18}	5.99×10^{-34}
10^{-04}	8.19×10^{-14}	5.93×10^{-27}
10^{-03}	7.22×10^{-10}	5.36×10^{-20}
10^{-02}	2.12×10^{-06}	1.91×10^{-13}
10^{-01}	1.53×10^{-05}	2.32×10^{-10}

HEC-16 and HEC-32 Comparison



Results with Scrambling Artifacts

Scrambling Artifacts

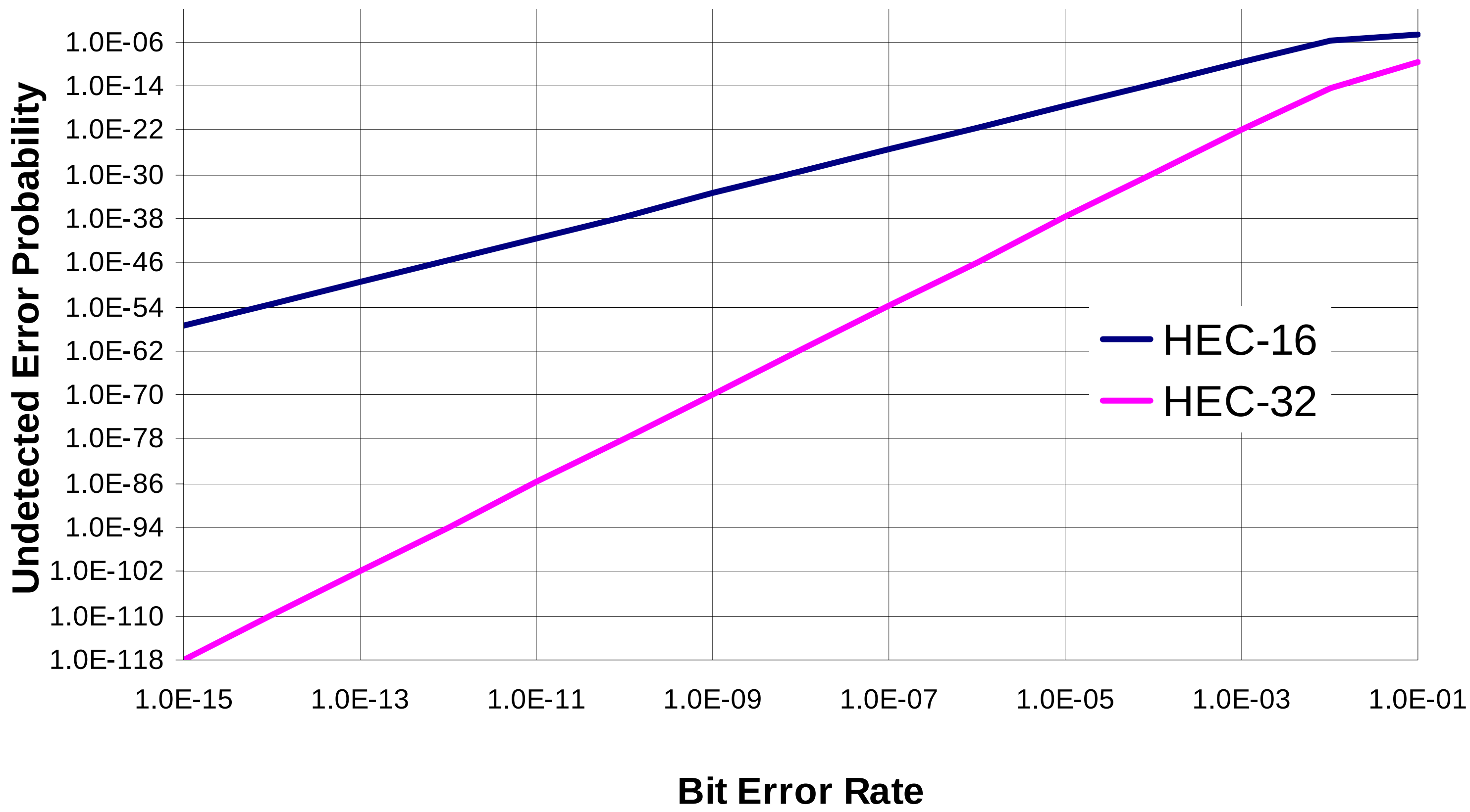


- Bit Errors on Fiber Can Be Represented By Error Polynomial E(X)
  - Unscrambling Modifies the Error Polynomial By A Multiplication Factor
- Undetected Error Probability Analysis Approach
  - Need Only Consider Error Polynomials E(X) of Degree Up To L-43
  - For E(X) with Degree Greater Than L-43
    - Errors Spill into Payload of RPR Packet
    - Detected By FCS-32 of RPR Packet (Up To Detection Probability)
- HEC-16 & HEC-32 Properties
  - HEC-16 shares a Common Factor (1+X) with $(1+X^{43})$
  - HEC-32 is Primitive Polynomial and Relatively Prime to $(1+X^{43})$
  - Undetected Error Probability Calculation Includes These Properties

Undetected Error Probability

Bit Error Rate	HEC-16	HEC-32
10^{-15}	2.94×10^{-58}	1.45×10^{-118}
10^{-14}	2.94×10^{-54}	1.45×10^{-110}
10^{-13}	2.94×10^{-50}	1.45×10^{-102}
10^{-12}	2.94×10^{-46}	1.45×10^{-94}
10^{-11}	2.94×10^{-42}	1.45×10^{-86}
10^{-10}	2.94×10^{-38}	1.45×10^{-78}
10^{-09}	2.94×10^{-34}	1.45×10^{-70}
10^{-08}	2.94×10^{-30}	1.45×10^{-62}
10^{-07}	2.94×10^{-26}	1.45×10^{-54}
10^{-06}	2.94×10^{-22}	1.45×10^{-46}
10^{-05}	2.94×10^{-18}	1.45×10^{-38}
10^{-04}	2.91×10^{-14}	1.44×10^{-30}
10^{-03}	2.68×10^{-10}	1.32×10^{-22}
10^{-02}	1.21×10^{-06}	5.61×10^{-15}
10^{-01}	3.08×10^{-05}	2.10×10^{-10}

HEC-16 and HEC-32 Comparison



Conclusions

- HEC-16 Provides More than Adequate Protection
- For Realistic Fiber Error Rates ($10^{-9} \sim 10^{-13}$)
  - With or Without Scrambling Artifacts
  - Undetected Error Probability is of Order 10^{-34}
- Even for Signal Degrade Conditions (BER ~ 10^{-6})
  - Undetected Error Probability for HEC-16 is of Order 10^{-22}

References

1. F. J. MacWilliams and N.J.A Sloane– *The Theory of Error-Correcting Codes*, North Holland, Elsevier Science Publishers, 1986.
2. N. R. Saxena, P. Franco, and E.J. McCluskey, "Simple Bounds on Serial Signature Analysis Aliasing for Random Testing," IEEE Transactions on Computers, Vol. 41, No. 5, pp. 638-645, May 1992.