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Title	LDPC Code Proposal – Technology Overview	
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Re:	IEEE 802.20 Call for Proposal	
Abstract	This document proposes an LDPC coding scheme for Mobile Broadband Wireless Access Systems.	
Purpose	For consideration and adoption as a feature supported by 802.20 standard	
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802.20 LDPC Code Proposal

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March 11-16, 2007

Outline

- Introduction
- A Proposed LDPC Code Structure
  - Structured LDPC Code
  - Multi-Edge-Type LDPC code
  - Code Length Flexibility
  - LDPC Design for HARQ transmission
  - Efficient Encoding Algorithm
- Packet Formats
- Performance Comparison
- Conclusions

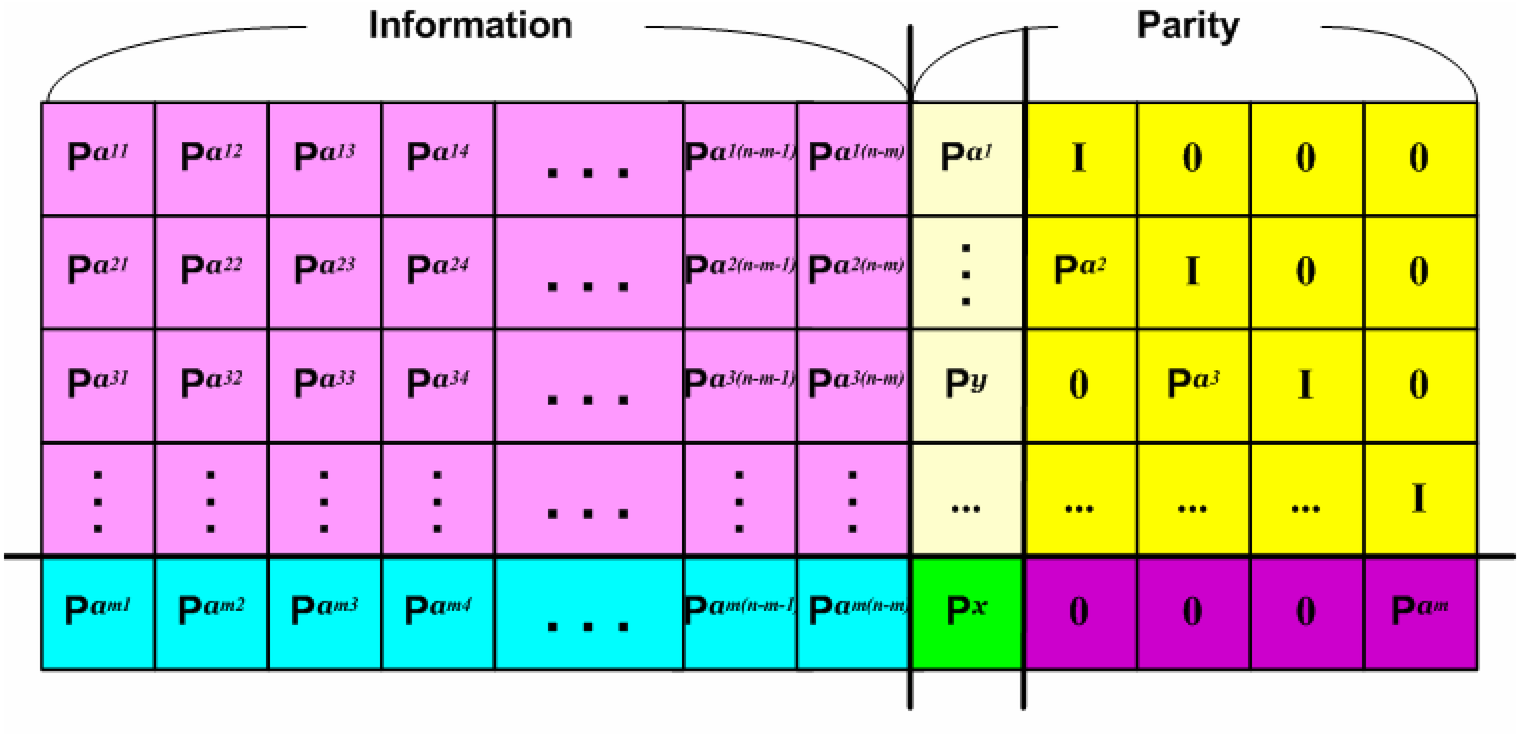
Introduction (1/2)

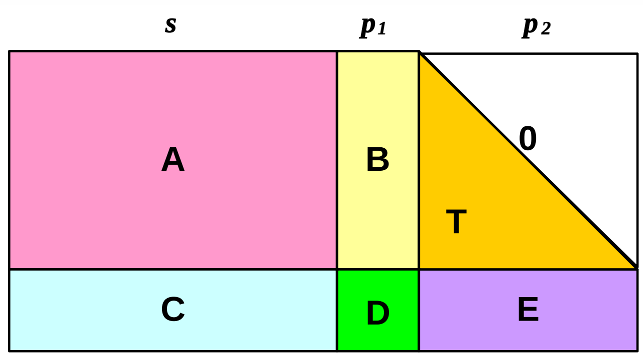
- Current 802.20 Air Interface [1] supports two channel coding schemes
  - Rate 1/5 Turbo code (PCCC) for large packet size (k > 128 bits)
  - Rate 1/3 Convolutional code for small packet size (k ≤ 128 bits)
- Low-Density Parity-Check (LDPC) code is proposed as an optional coding scheme for high data rates (large packet size)
  - Efficient support of Type II HARQ (Incremental Redundancy)
  - Similar or better performance than Turbo codes through all HARQ retransmissions
  - Highly parallelizable encoder/decoder architectures, thus resulting in high-throughput encoder/decoder implementations

Introduction (2/2)

- LDPC codes are fully defined by a sparse parity-check matrix
  - Can also be represented by bipartite graph (Tanner graph)
  - Two types of nodes (variable and check nodes) and edges
- LDPC codes can be decoded by Message-Passing algorithms
  - Pearl's Belief-Propagation (BP) algorithm which passes beliefs in the form of Log-Likelihood Ratios (LLRs) along the edges of the bipartite graph.
  - Optimal only for cycle free tree structure graph codes, but sub-optimal on the graph with cycles
  - The complexity of BP algorithm is proportional to the number of edges in the bipartite graph
  - Due to the sparseness of the parity-check matrix, and thus of the corresponding bipartite graph, the resulting decoding complexity is quite affordable

Structured LDPC Code





$$P_{L\times L} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

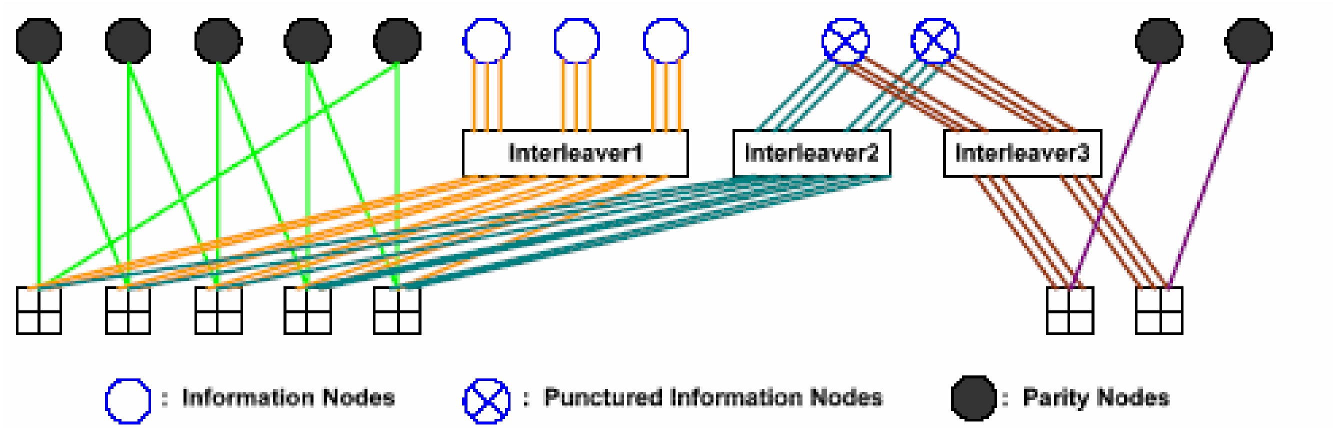
Permutation matrix should be cyclic permutation matrix

Parallelizing Structured LDPC Code

- Full Parallel Implementation [2] – High Throughput, High Complexity
- Semi-Parallel Implementation of Structured LDPC code of size mL x nL
  - Edge Parallel Decoder
    - Basic Parallelization Factor: L
    - $L/2$, $L/4$, *etc* are also possible (implementation issue)
  - Node Parallel Decoder
    - Basic Parallelization Factor: (m, n)
    - $2(m, n)$, $4(m, n)$, *etc* are also possible (implementation issue)
- Structured LDPC code well suited for both edge parallel and node parallel approach
- Node Parallel Decoder is matched with the proposed scheme for code length flexibility

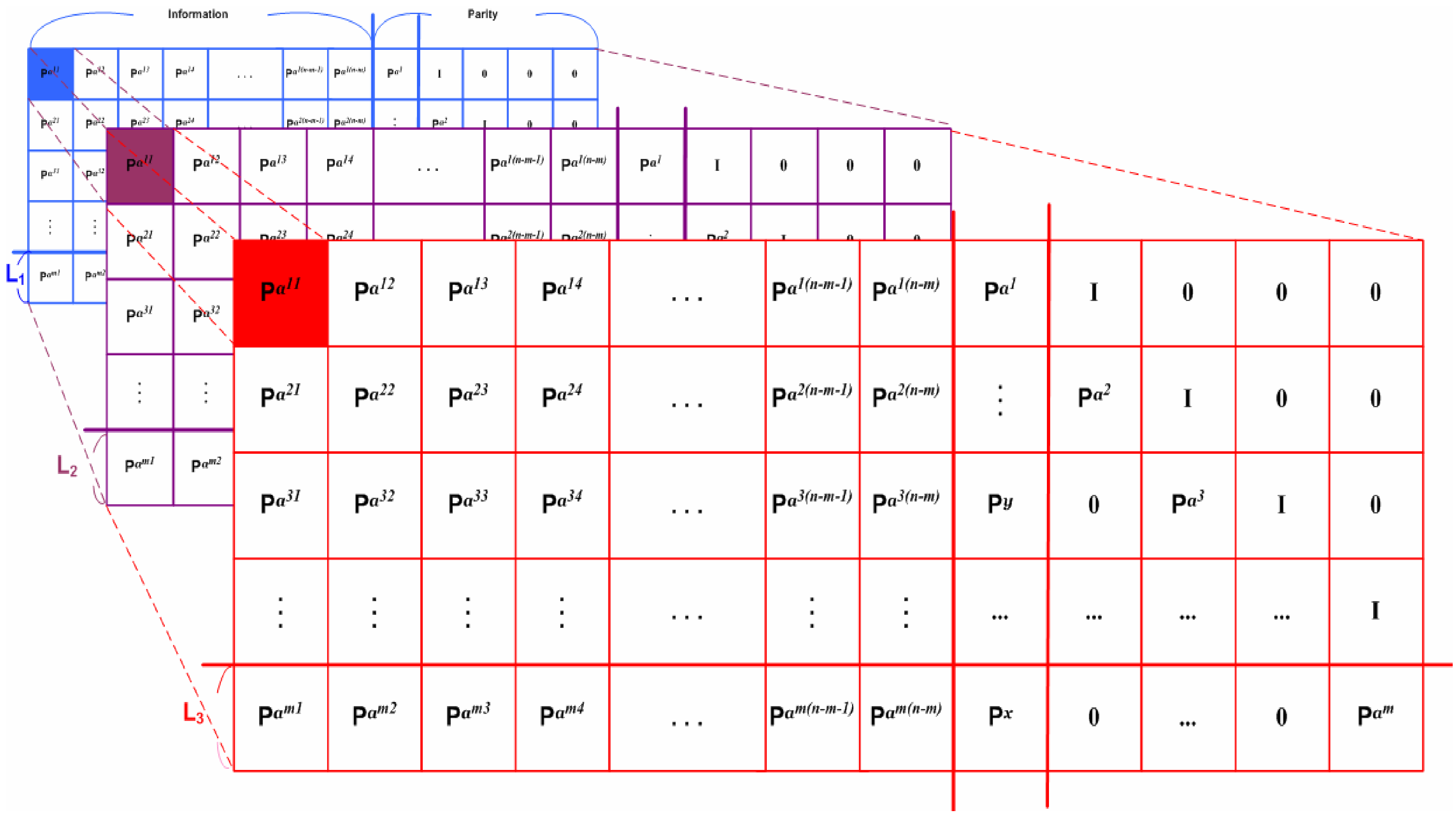
Multi-Edge-Type LDPC Code

- Multi-Edge-Type (MET) LDPC codes are generalizations of regular and irregular LDPC codes
- Perform better with lower error floor than standard irregular LDPC codes, while requiring lower complexity



- Type of the edges: need a detailed degree distribution
- Degree-one variable nodes: may not sacrifice the threshold
- Punctured variable nodes : increase thresholds and lower error floors

Code Length Flexibility (1/2)



Code length flexibility is obtained by increasing or decreasing the size of cyclic permutation matrix P

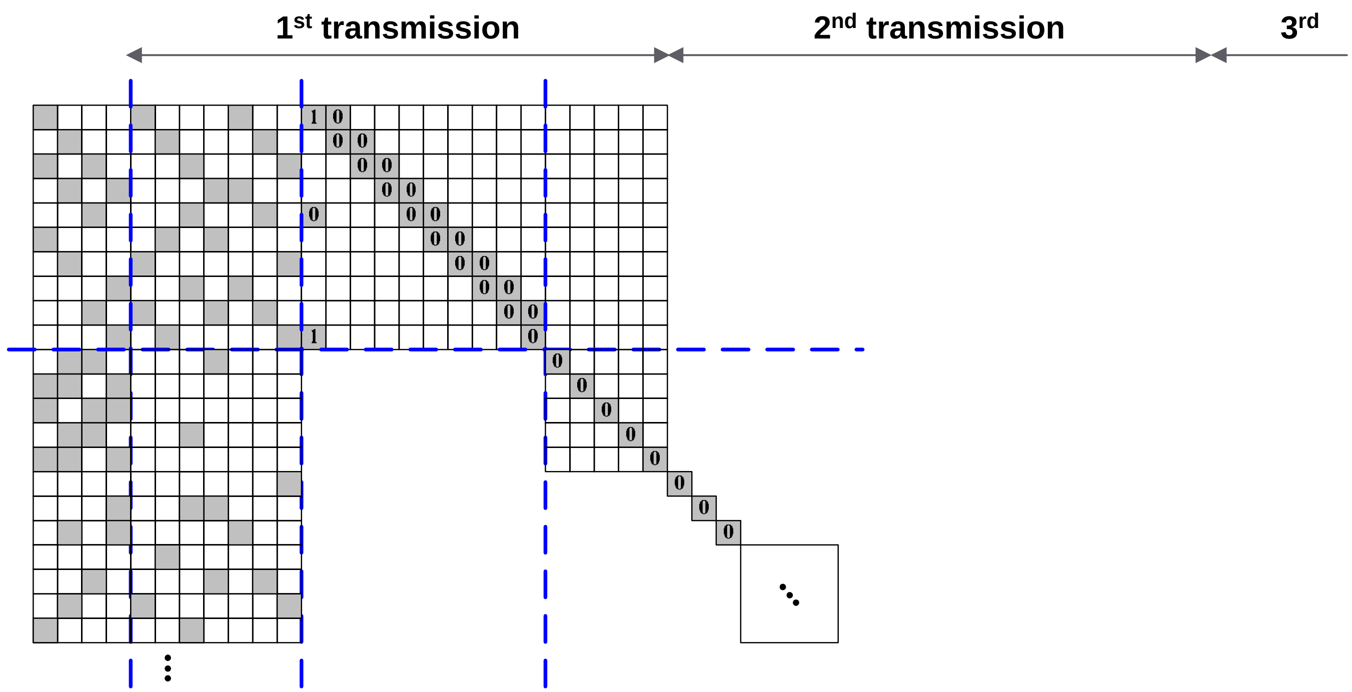
Code Length Flexibility (2/2)

- LDPC codes of variable length need to be expressed by only one parity check matrix (called base matrix), thus reducing the memory storage requirements
- The flexibility with respect to code length is achieved by adopting modulo function on the expansion factor of the non-zero sub-matrices in the parity-check matrix
- Assuming that $(i, j)^{th}$ element in base matrix is non-zero. Then shift factor $p(f, i, j)$ corresponding to the expansion factor L_f is derived from the original expansion factor $p(i, j)$ by following:

$$p(f, i, j) = \mathrm{mod}\left(p(i, j), L_f \right)$$

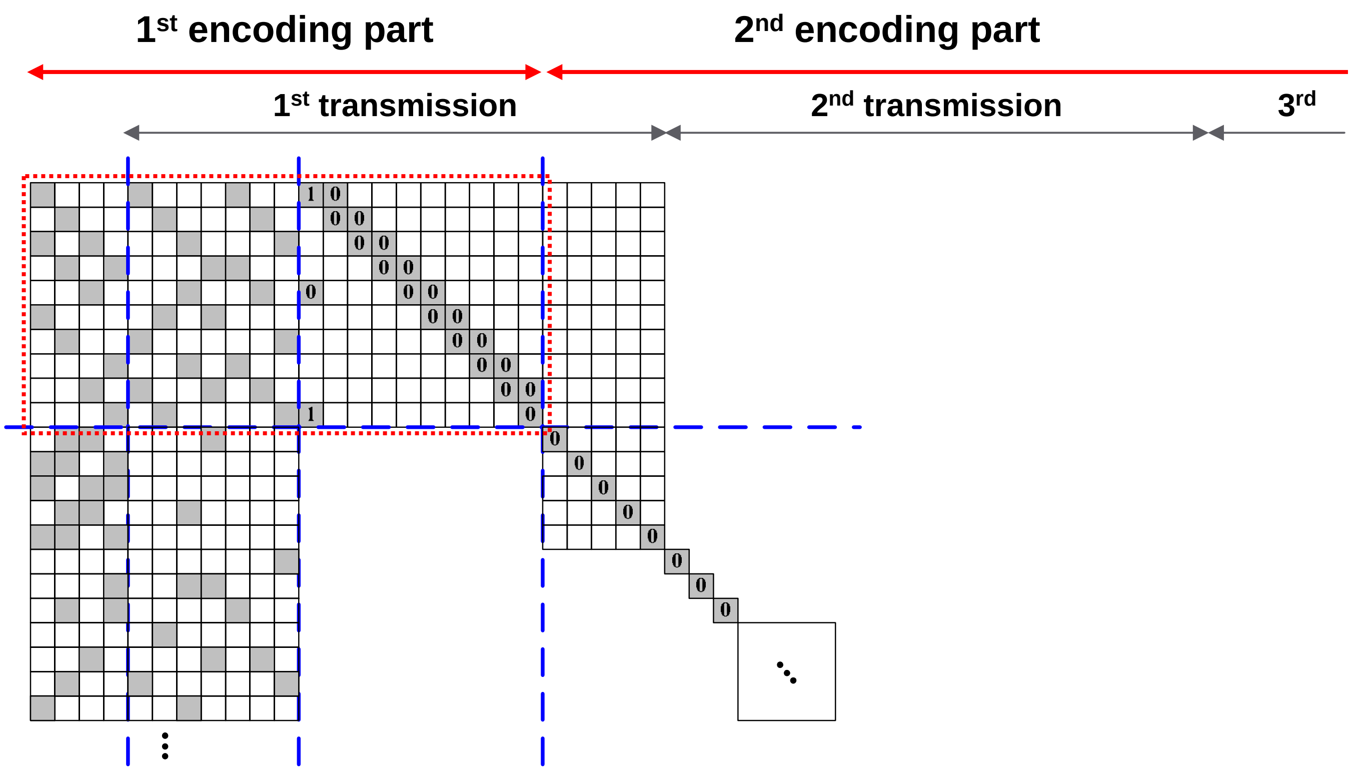
LDPC Design for HARQ

- Construct H matrix of lowest code-rate
- Only a part of codeword is transmitted during each HARQ transmission



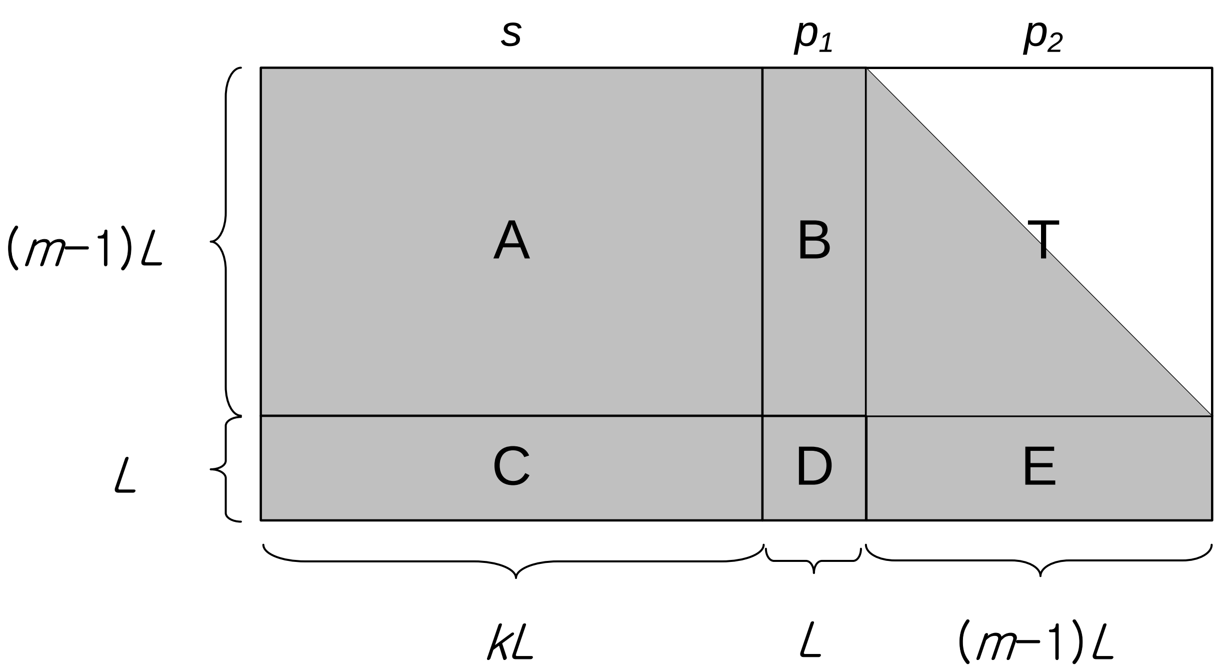
Encoding Algorithm (1/5)

- Encoding of proposed LDPC code is accomplished by following two steps:
  - 1st part: Richardson & Urbanke's encoding algorithm [3]
  - 2nd part: Single parity-check coding



Encoding Algorithm (2/5)

- Richardson & Urbanke's encoding algorithm



$$\begin{cases} As^T + Bp_1^T + Tp_2^T = 0 \\ Cs^T + Dp_1^T + Ep_2^T = 0 \end{cases}$$

$$(ET^{-1}A + C)s^T + (ET^{-1}B + D)p_1^T = 0$$

$$p_1^T = \phi^{-1}(ET^{-1}A + C)s^T, \ \phi := ET^{-1}B + D$$

Encoding Algorithm (3/5)

- Encoding Procedure
  - Step 1) Compute As^T and Cs^T
  - Step 2) Compute $T^{-1}As^T$
  - Step 3) Compute $E(T^{-1}As^T)$ and $E(T^{-1}As^T) + Cs^T$
  - Step 4) Compute $\phi = ET^{-1}B + D$ and ϕ^{-1}
  - Step 5) Compute $p_1^T = \phi^{-1}(ET^{-1}As^T + Cs^T)$
  - Step 6) Compute p_2^T using $As^T + Bp_1^T + Tp_2^T = 0$
    by back-substitution

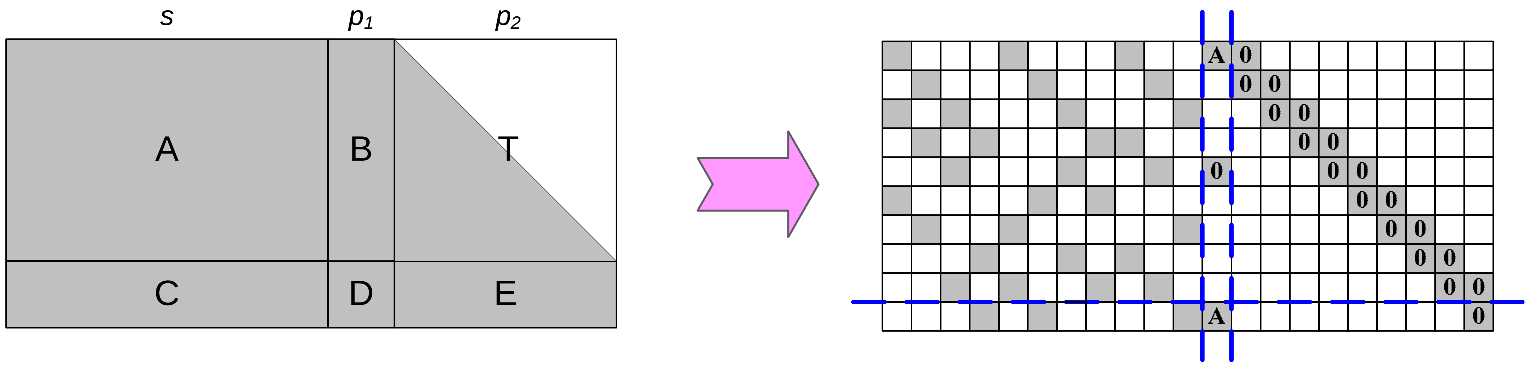
  - Computational complexity of encoding procedure is $O(N) + O(L^2)$.
    The second term comes from multiplying by ϕ^{-1} in Step 5)

Encoding Algorithm (4/5)

- The matrix ϕ^{-1} is NOT a sparse matrix
- The multiplication by ϕ^{-1} is a main source to increase the complexity of encoding procedure
- If we can make ϕ an identity matrix, we can skip the multiplication by ϕ^{-1} in the procedure, and can reduce the encoding complexity

Encoding Algorithm (5/5)

- The simple solution to make ϕ an identity matrix [4]:



- B: two non-zero element.
  - Position: 1st and arbitrary.
  - Shift Parameter: A (arbitrary number) and zero
- T: dual diagonal structure (accumulate chain)
  - Shift Parameters: all zero
- D: 1x1
  - Shift Parameter: A (same as 1st non-zero element in B)
- E: one non-zero element.
  - Position: right most.
  - Shift Parameter: zero

Packet Formats (1/2)

- FL Packet Formats [1]

Packet Format Index	Spectral efficiency on 1st trans-mission	Spectral efficiency on 2nd trans-mission	Max number of trans-missions	Modulation order for each transmission					
				1	2	3	4	5	6
0	0.2	--	6	2	2	2	2	2	2
1	0.5	--	6	2	2	2	2	2	2
2	1.0	--	6	2	2	2	2	2	2
3	1.5	--	6	3	2	2	2	2	2
4	2.0	--	6	4	3	3	3	3	3
5	2.5	--	6	6	4	4	4	4	4
6	3.0	--	6	6	4	4	4	4	4
7	4.0	--	6	6	6	4	4	4	4
8	5.0	--	6	6	6	4	4	4	4
9	6.0	3.0	6	6	6	4	4	4	4
10	non-decodable	3.5	6	6	6	4	4	4	4
11	non-decodable	4.0	6	6	6	6	4	4	4
12	non-decodable	4.5	6	6	6	6	4	4	4
13	non-decodable	5.0	6	6	6	6	6	4	4
14	non-decodable	5.5	6	6	6	6	6	4	4
15	NULL	NULL							

Packet Formats (2/2)

- RL Packet Formats [1]

Packet format index	Spectral efficiency on 1st transmission	Spectral efficiency on 2nd transmission	Max number of transmissions	Modulation order for each transmission					
				1	2	3	4	5	6
0	0.25	--	6	2	2	2	2	2	2
1	0.50	--	6	2	2	2	2	2	2
2	1.0	--	6	2	2	2	2	2	2
3	1.5	--	6	3	2	2	2	2	2
4	2.0	--	6	3	3	2	2	2	2
5	2.67	--	6	4	4	3	3	3	3
6	4.0	--	6	4	4	3	3	3	3
7	6.0	3.0	6	4	4	4	3	3	3
8	non-decodable	4.0	6	4	4	4	4	4	3
9	4.0	--	6	6	6	4	4	4	4
10	5.0	--	6	6	6	4	4	4	4
11	6.0	3.0	6	6	6	4	4	4	4
12	non-decodable	3.5	6	6	6	4	4	4	4
13	non-decodable	4.0	6	6	6	6	4	4	4
14	non-decodable	4.5	6	6	6	6	4	4	4

Simulation Assumptions

• **System Parameters**

Number of resource channels	4 (110 data symbols per resource channel)
Modulation	Modulation order step-down
Channel	AWGN
Packet Format (PF) index	FL 2, 4, 8, 14

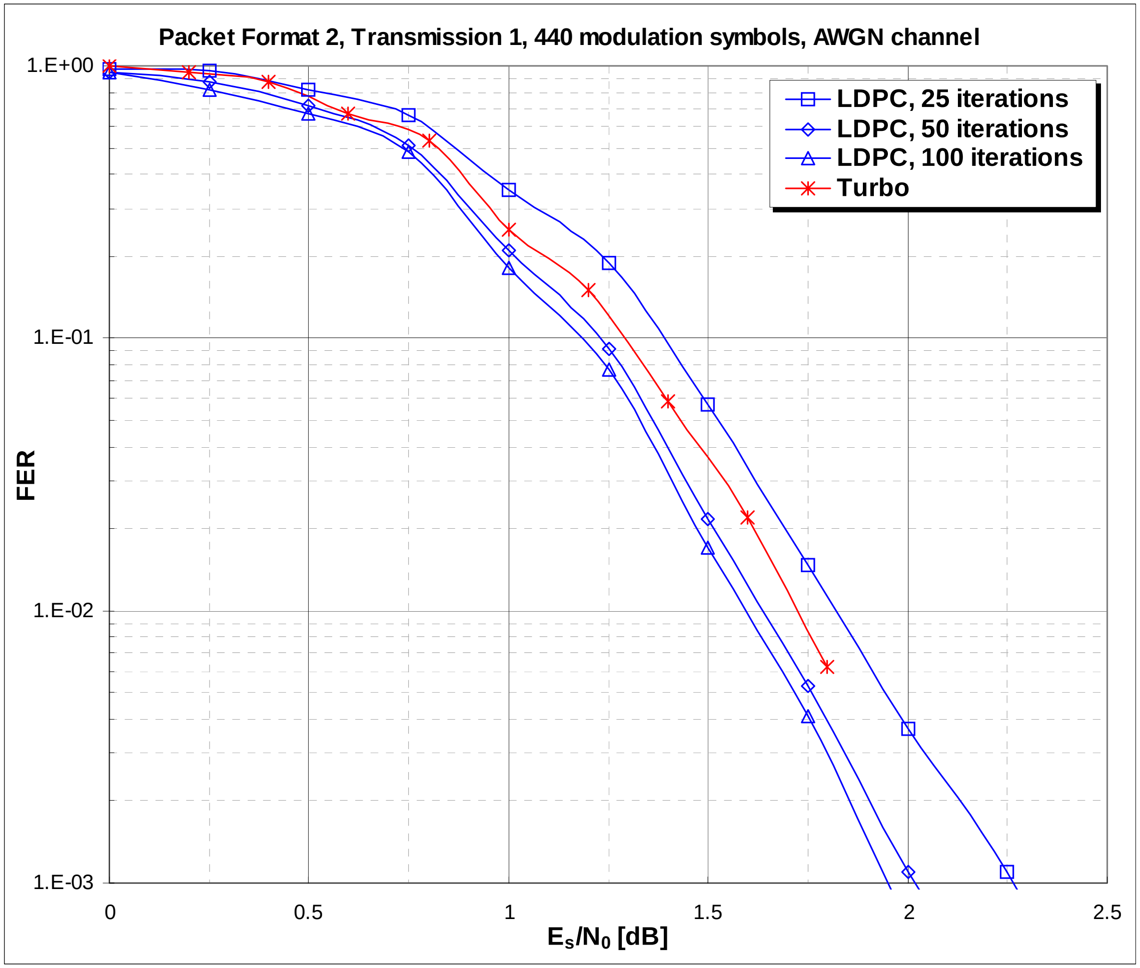
• **LDPC Code Parameters**

Code length	440 modulation symbols for 1st HARQ transmission
Code rate	Defined in PFI
Operation Point	1% FER
Decoding Algorithm	Standard Belief Propagation, floating-point
Scheduling Algorithm	Flooding
Number of Iterations	25, 50, 100

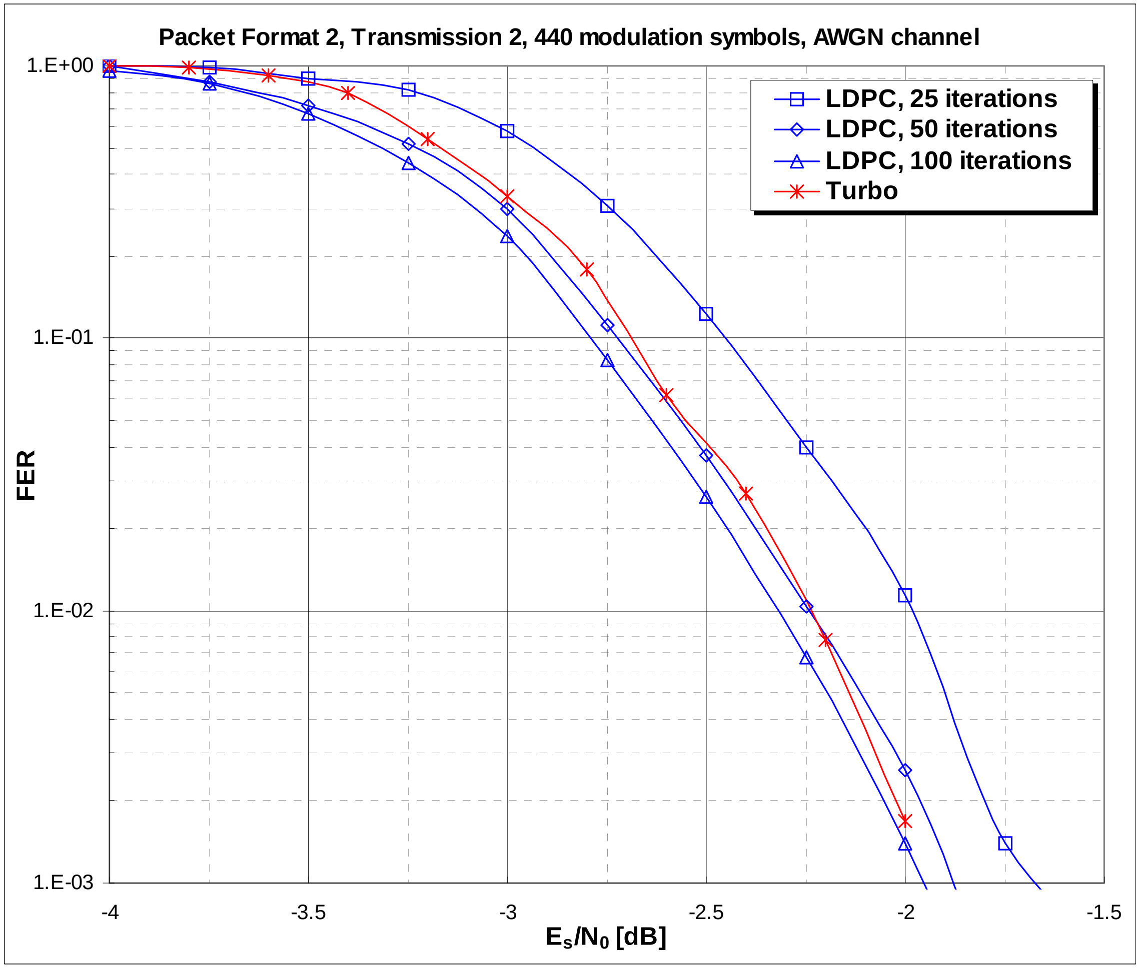
• **Turbo Code Parameters**

Decoding Algorithm	Log MAP, floating-point
Number of Iterations	12

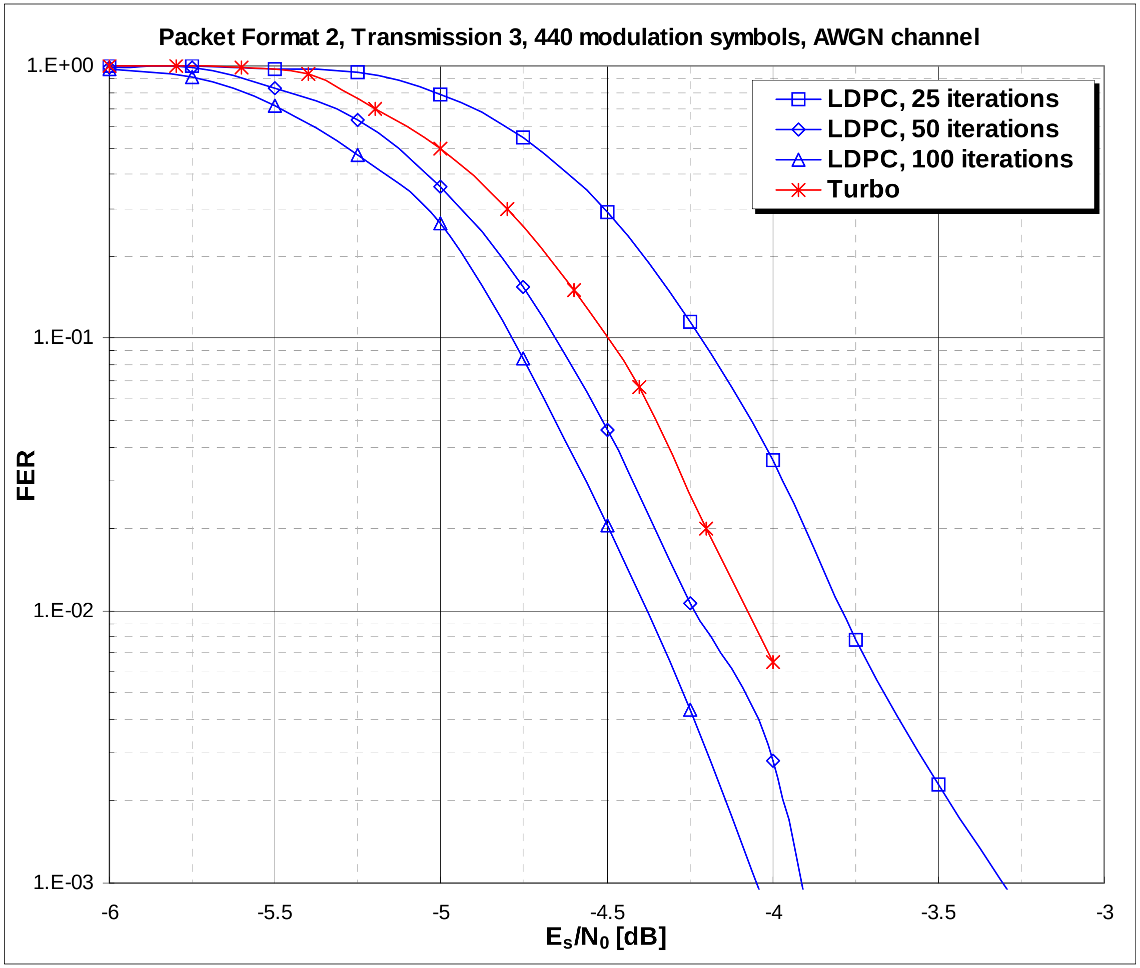
FL PF 2, Transmission 1



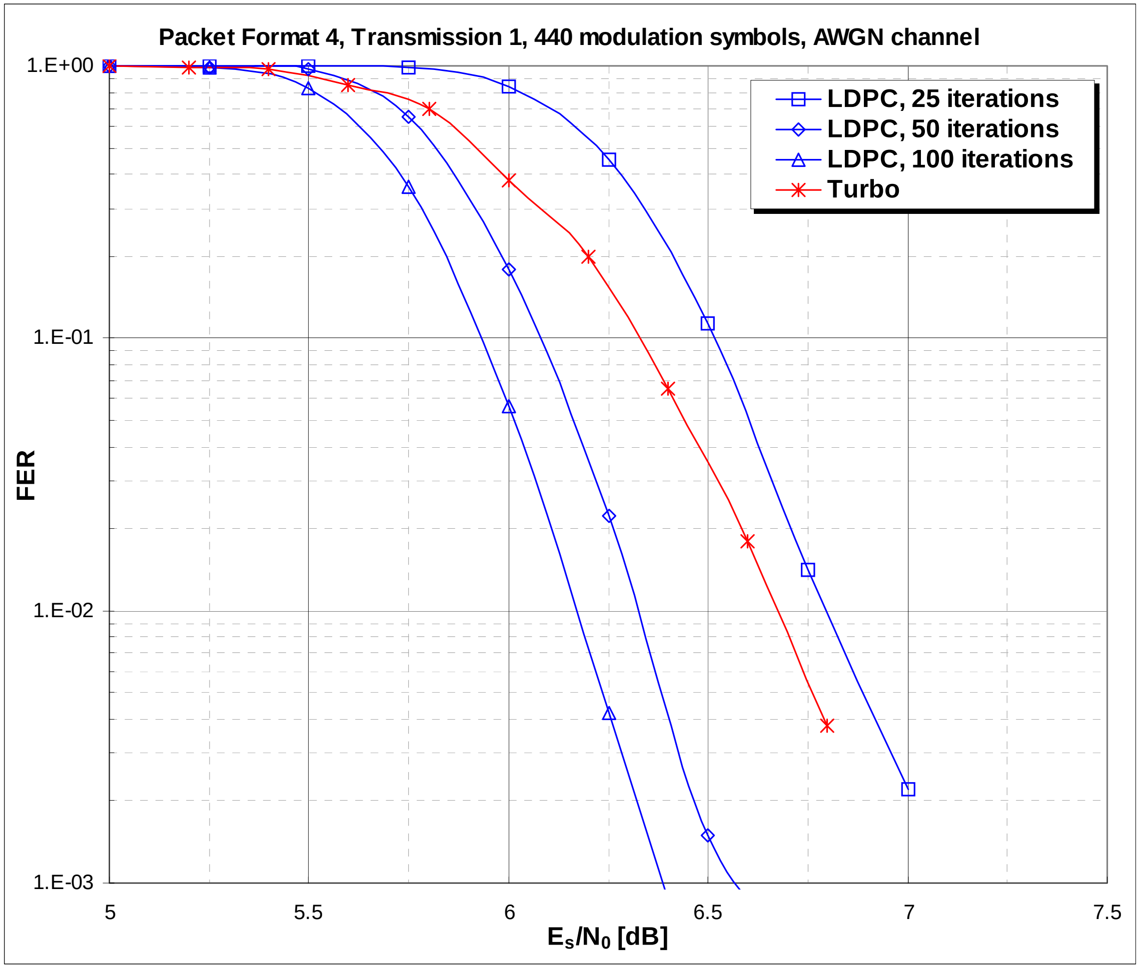
FL PF 2, Transmission 2



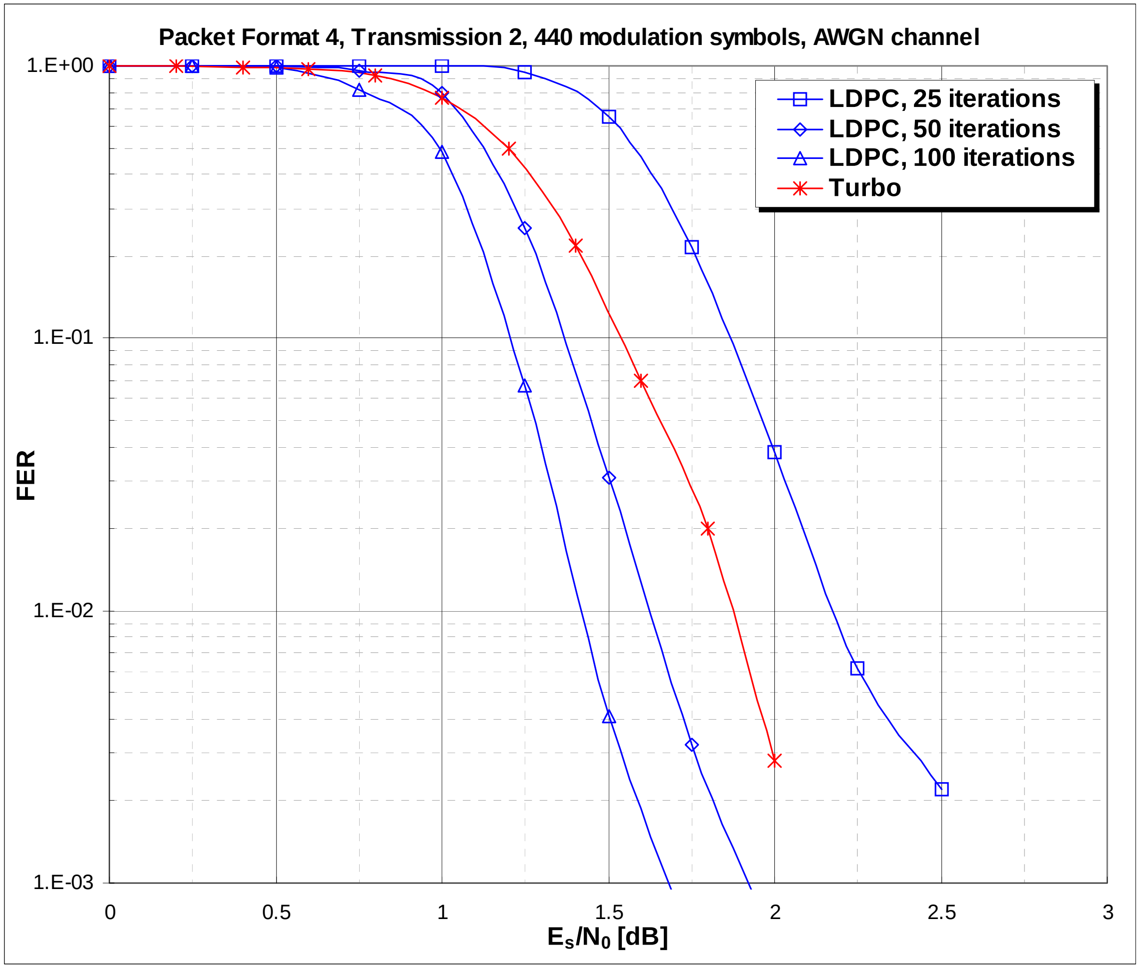
FL PF 2, Transmission 3



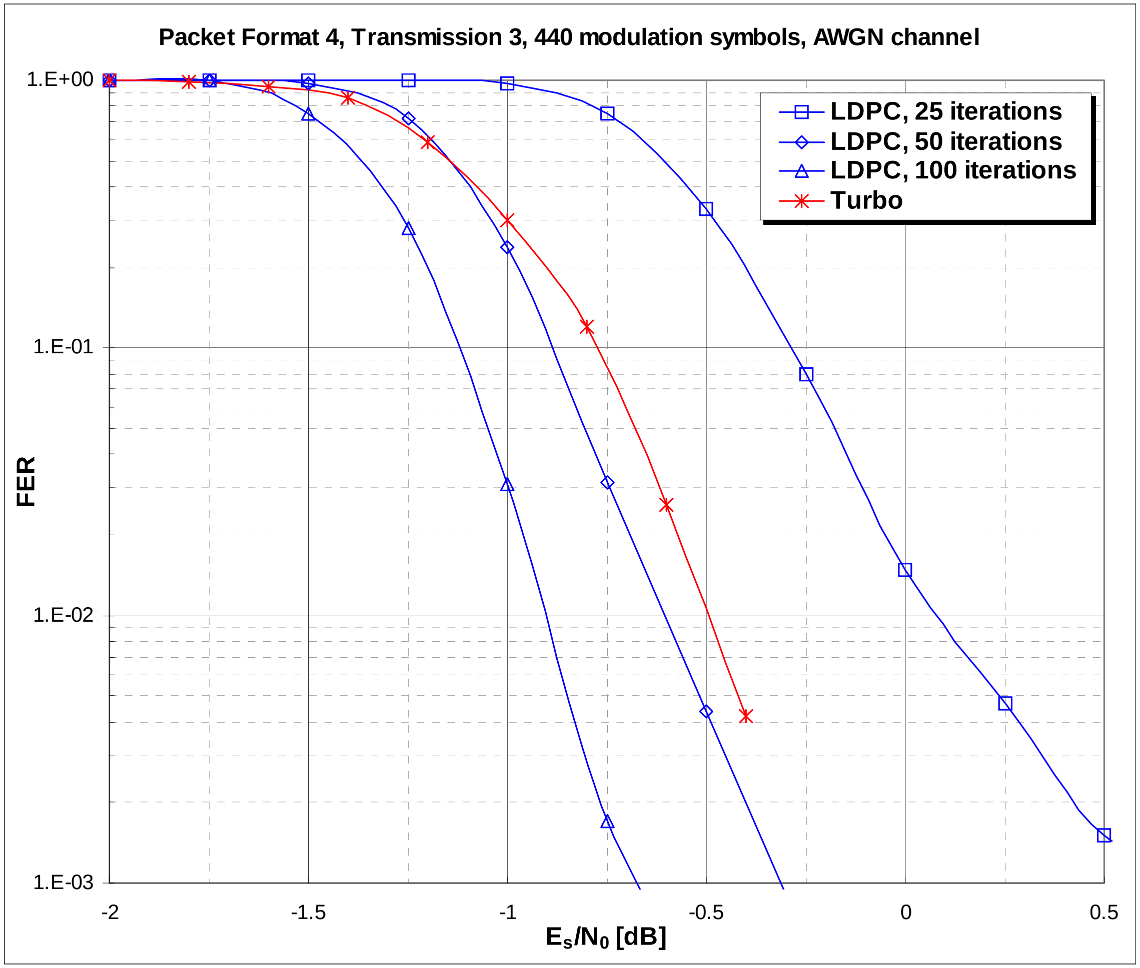
FL PF 4, Transmission 1



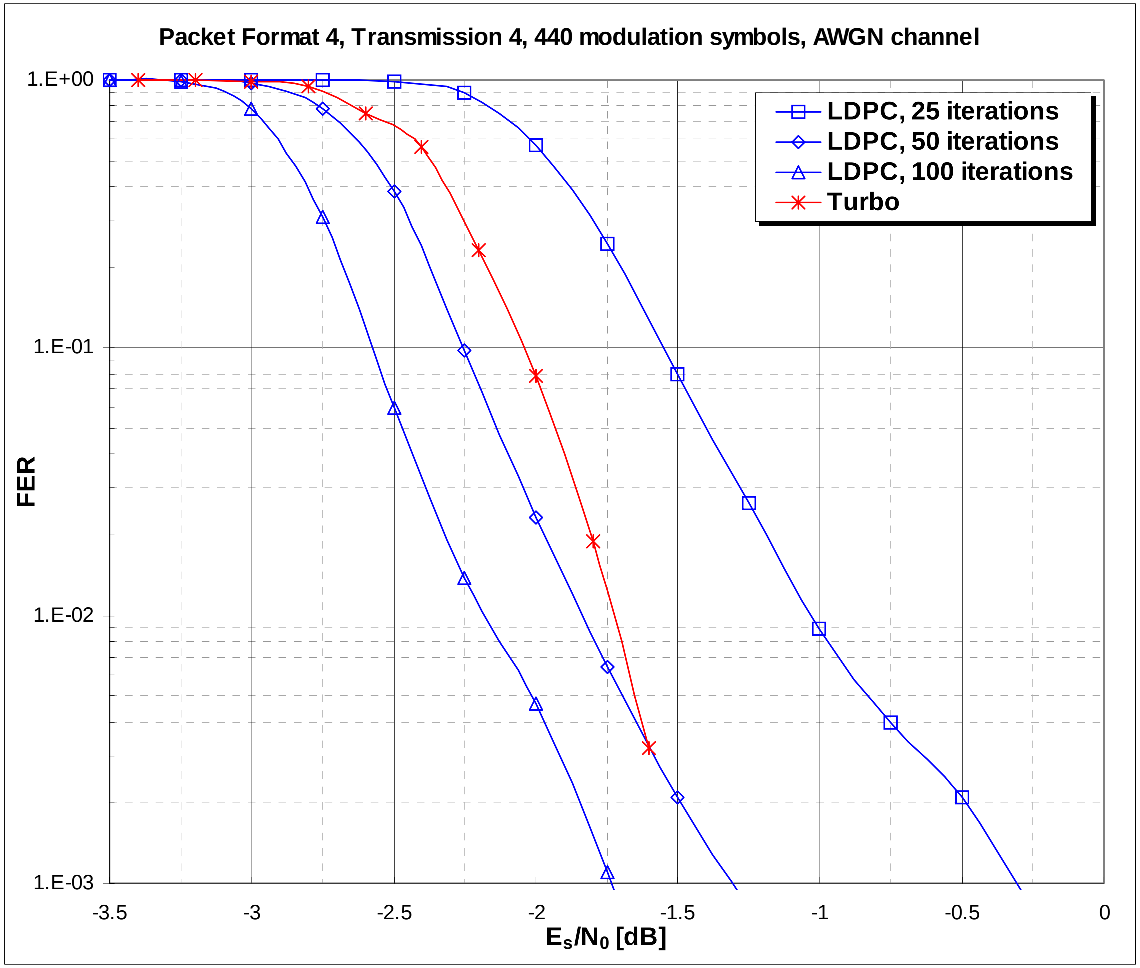
FL PF 4, Transmission 2



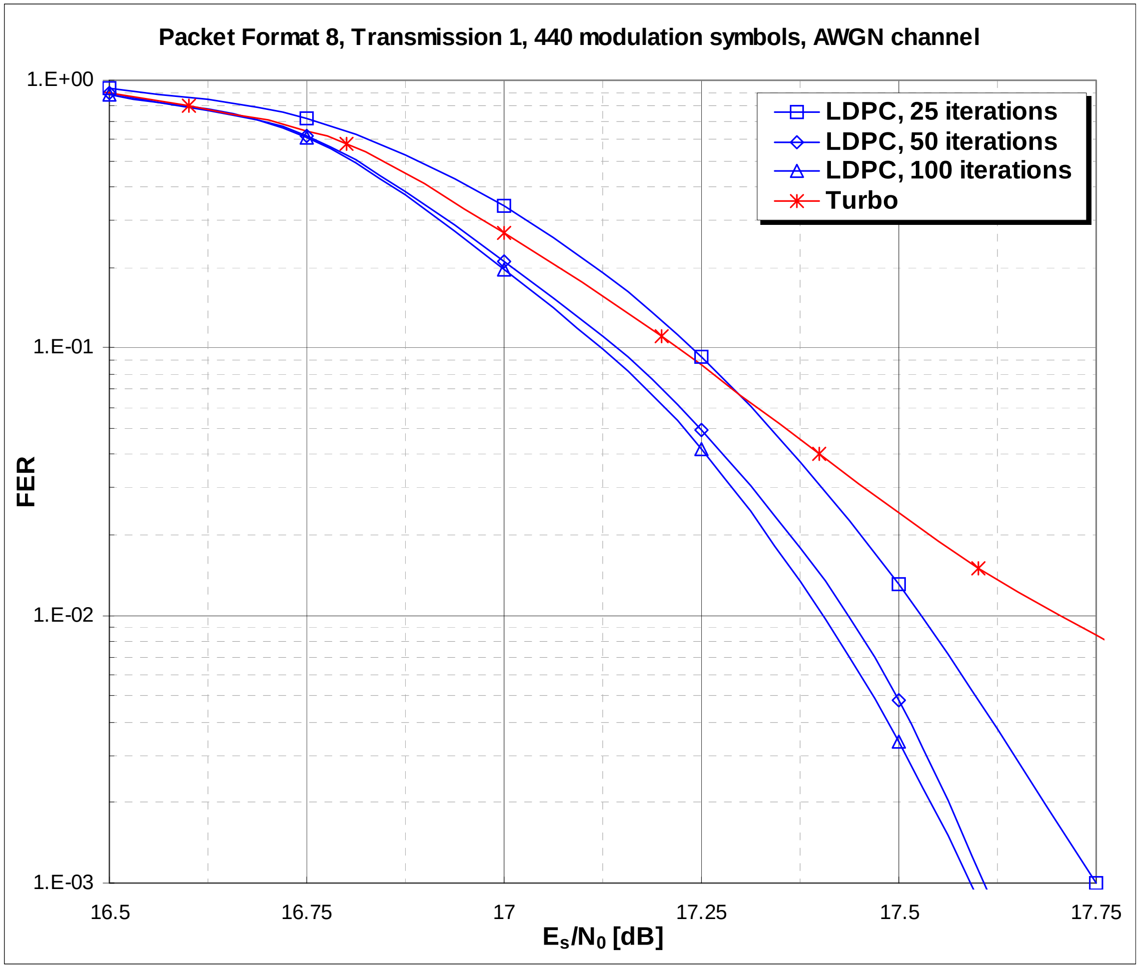
FL PF 4, Transmission 3



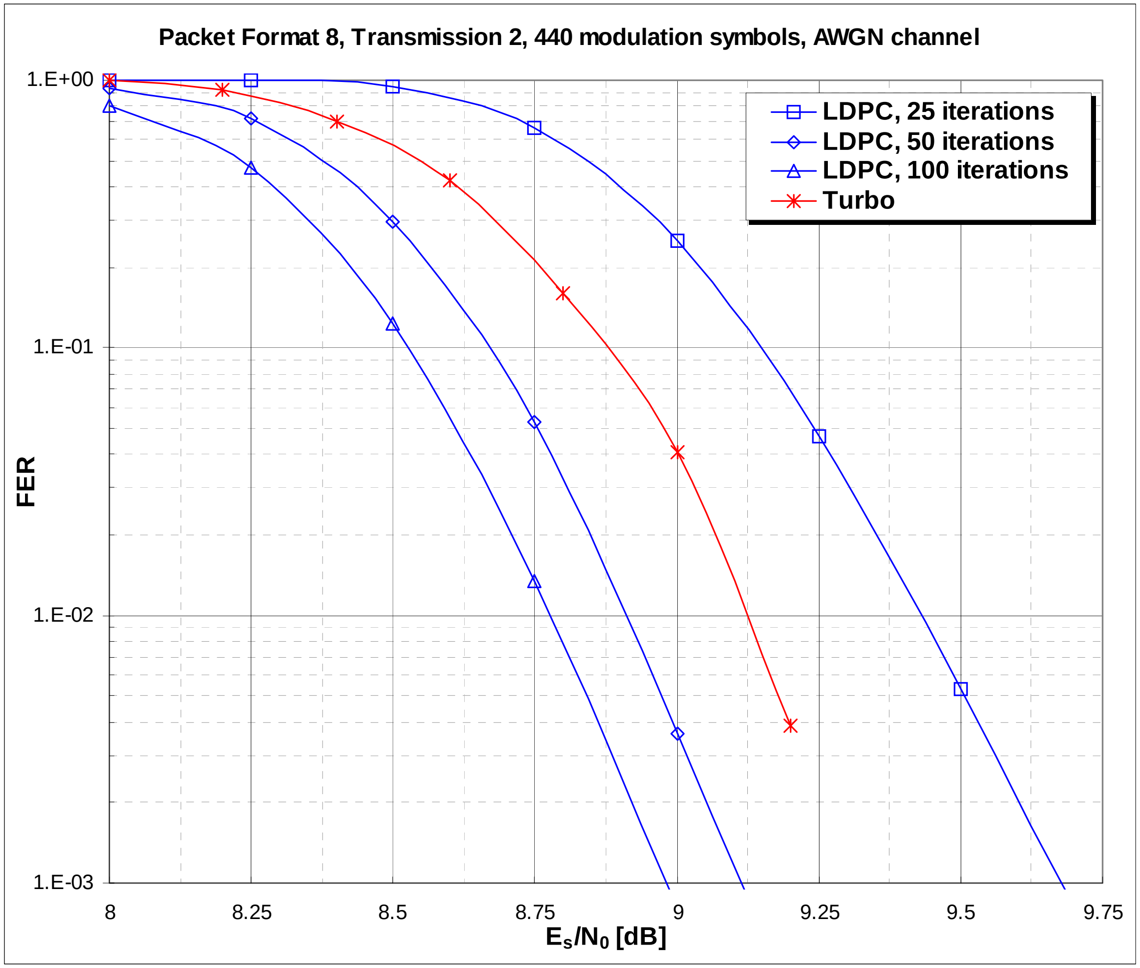
FL PF 4, Transmission 4



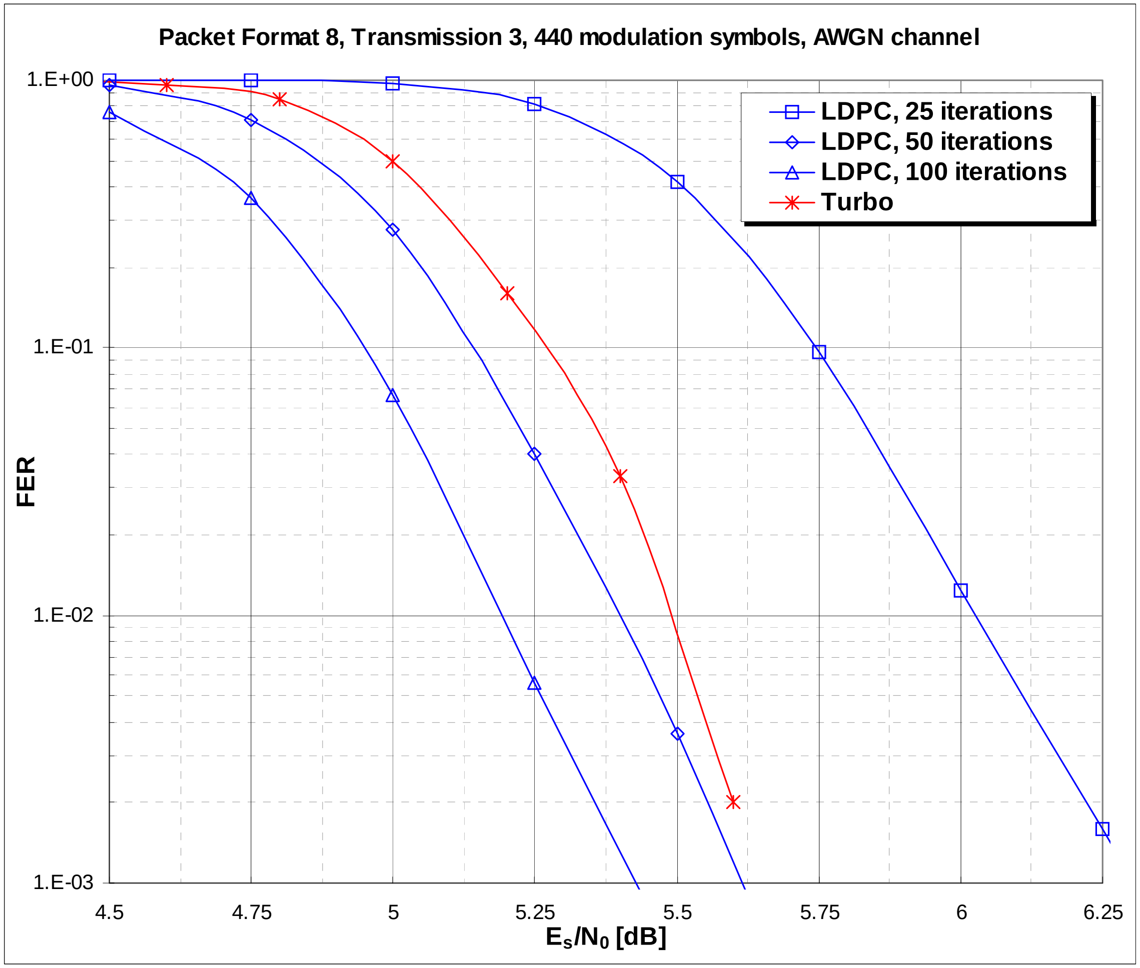
FL PF 8, Transmission 1



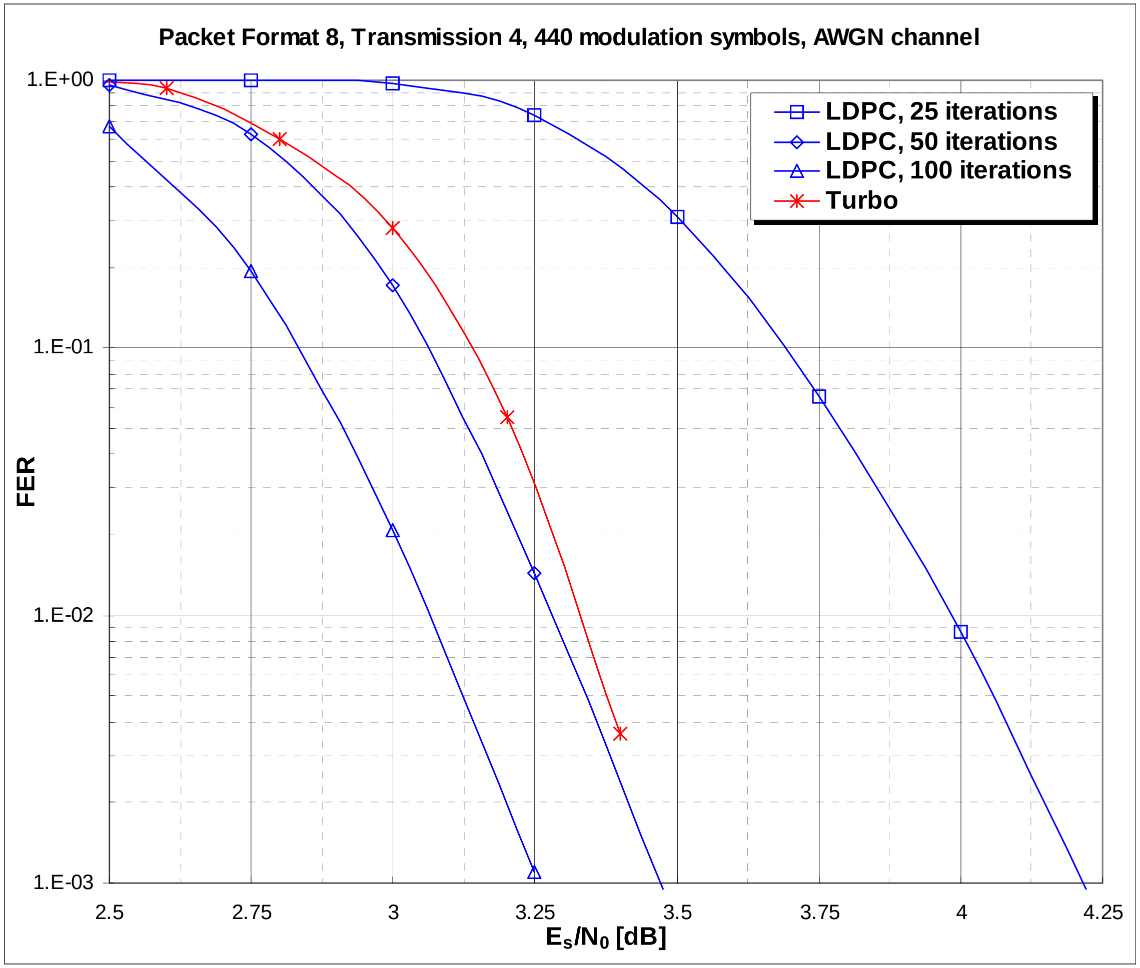
FL PF 8, Transmission 2



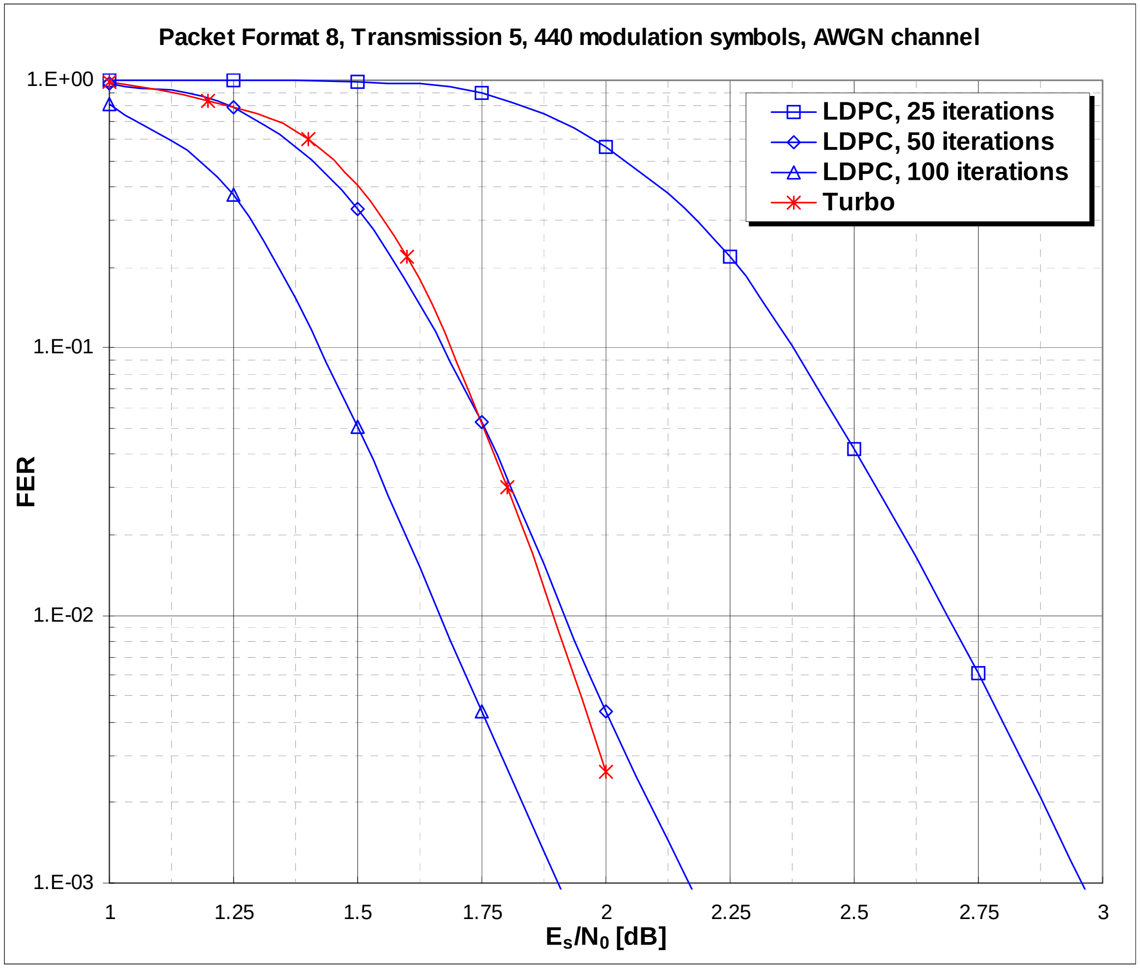
FL PF 8, Transmission 3



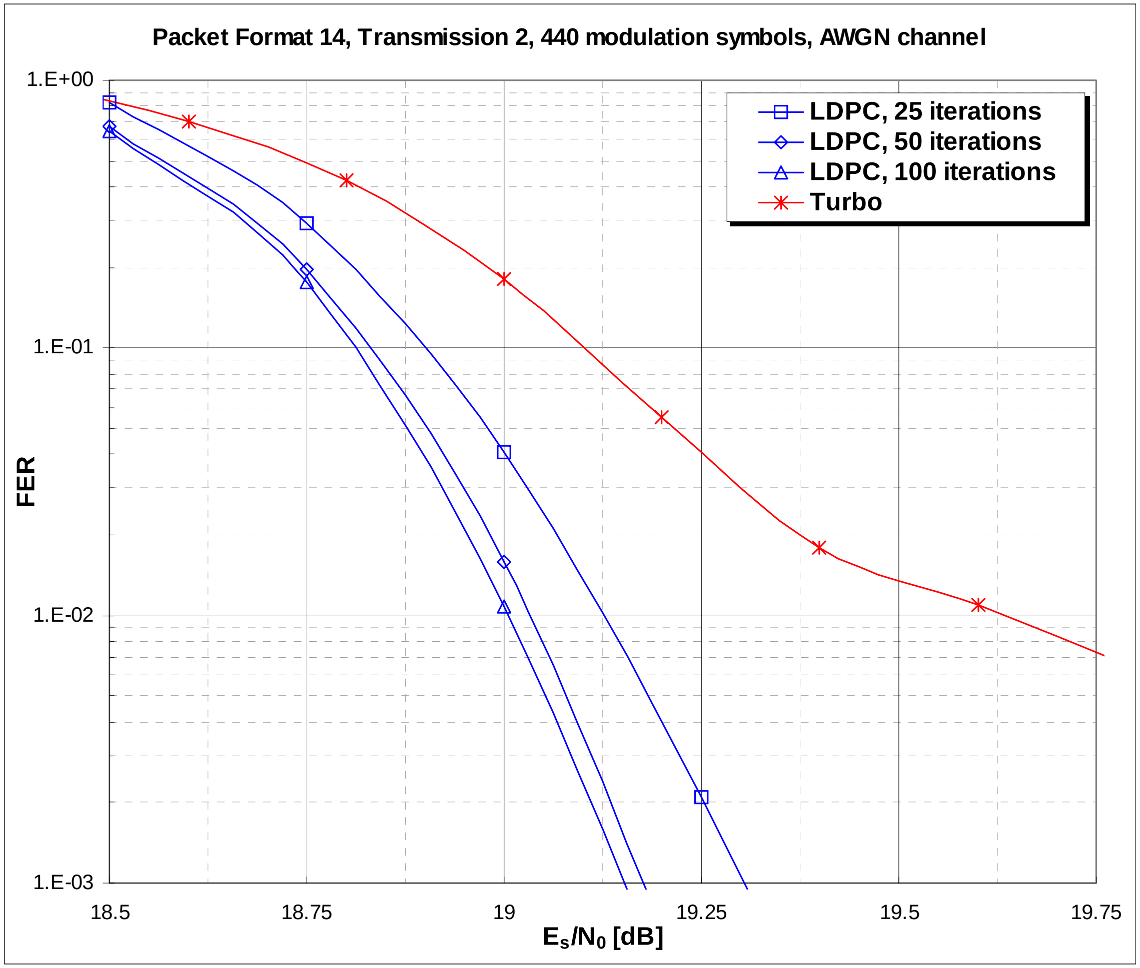
FL PF 8, Transmission 4



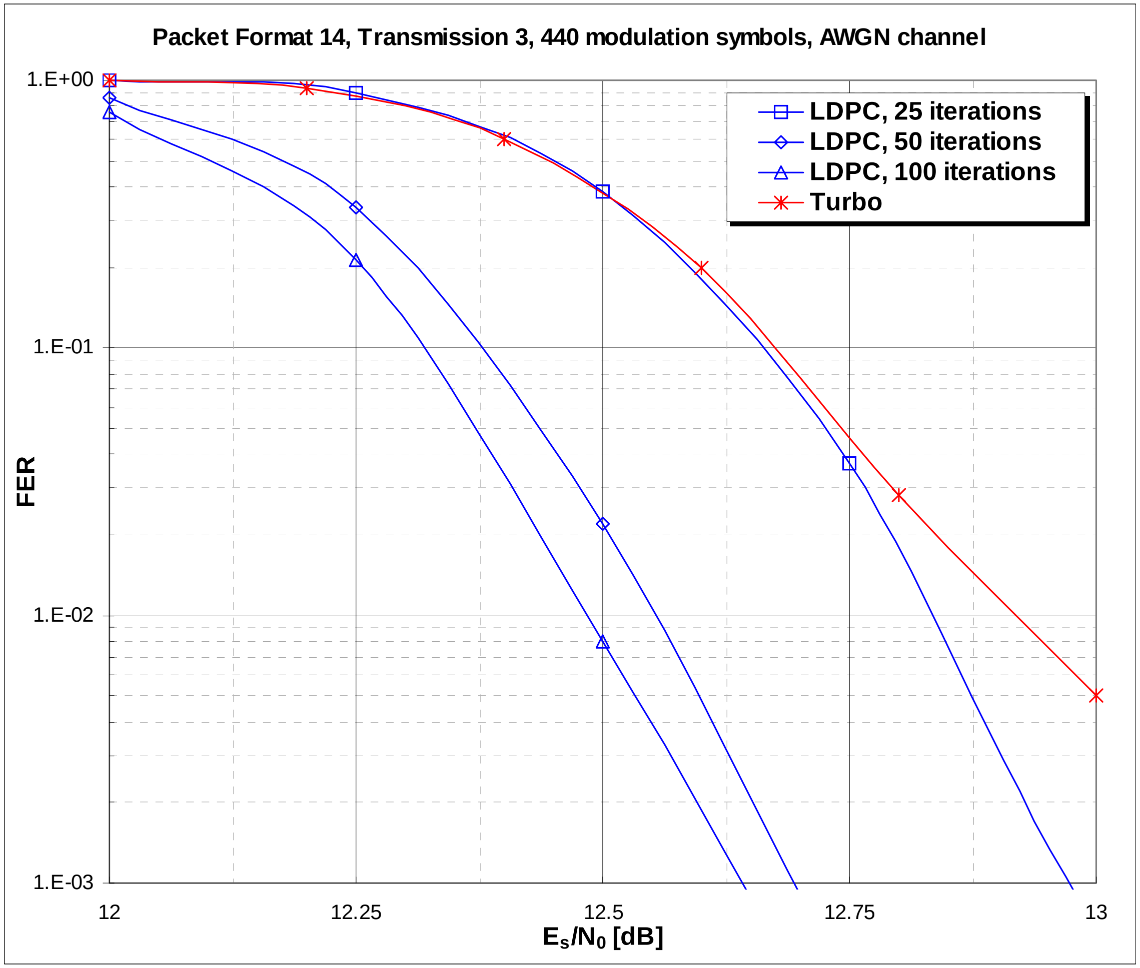
FL PF 8, Transmission 5



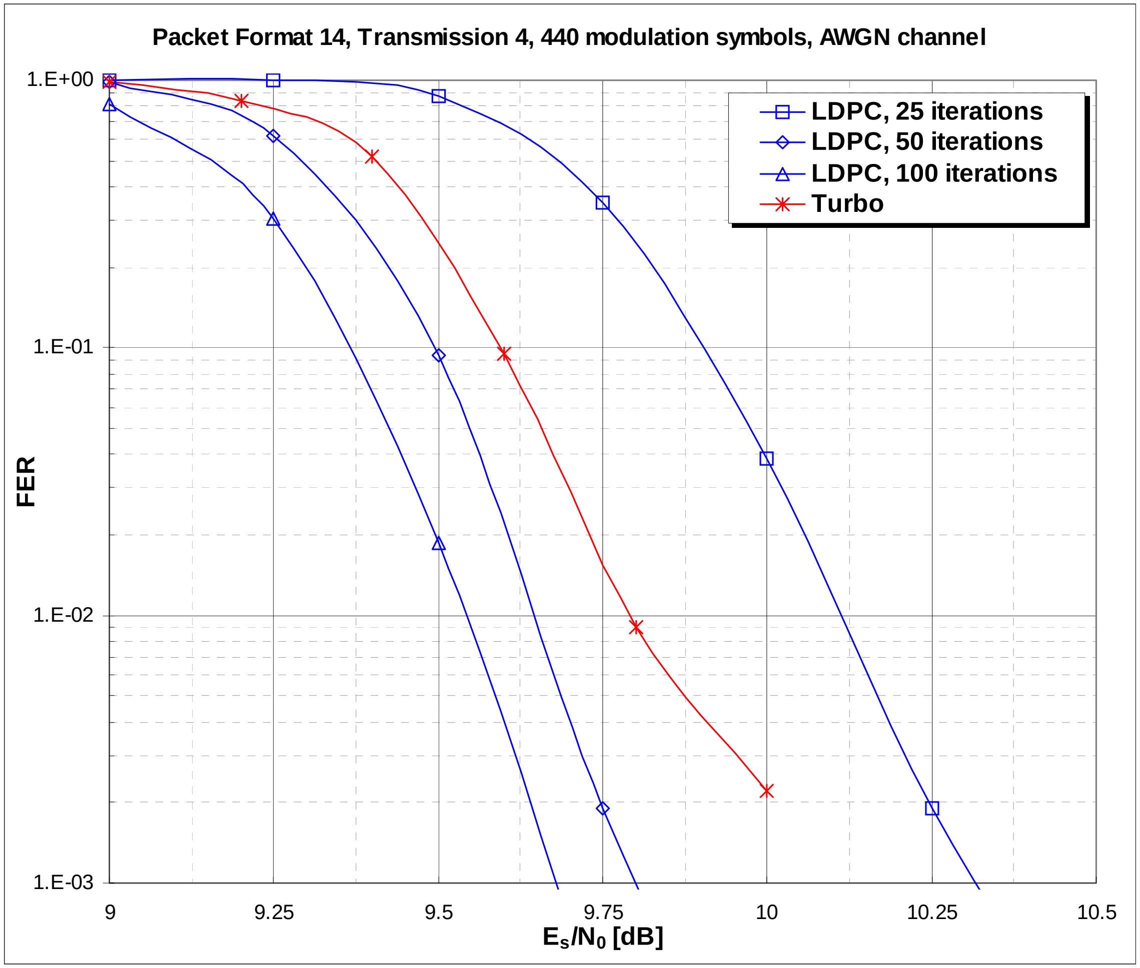
FL PF 14, Transmission 2



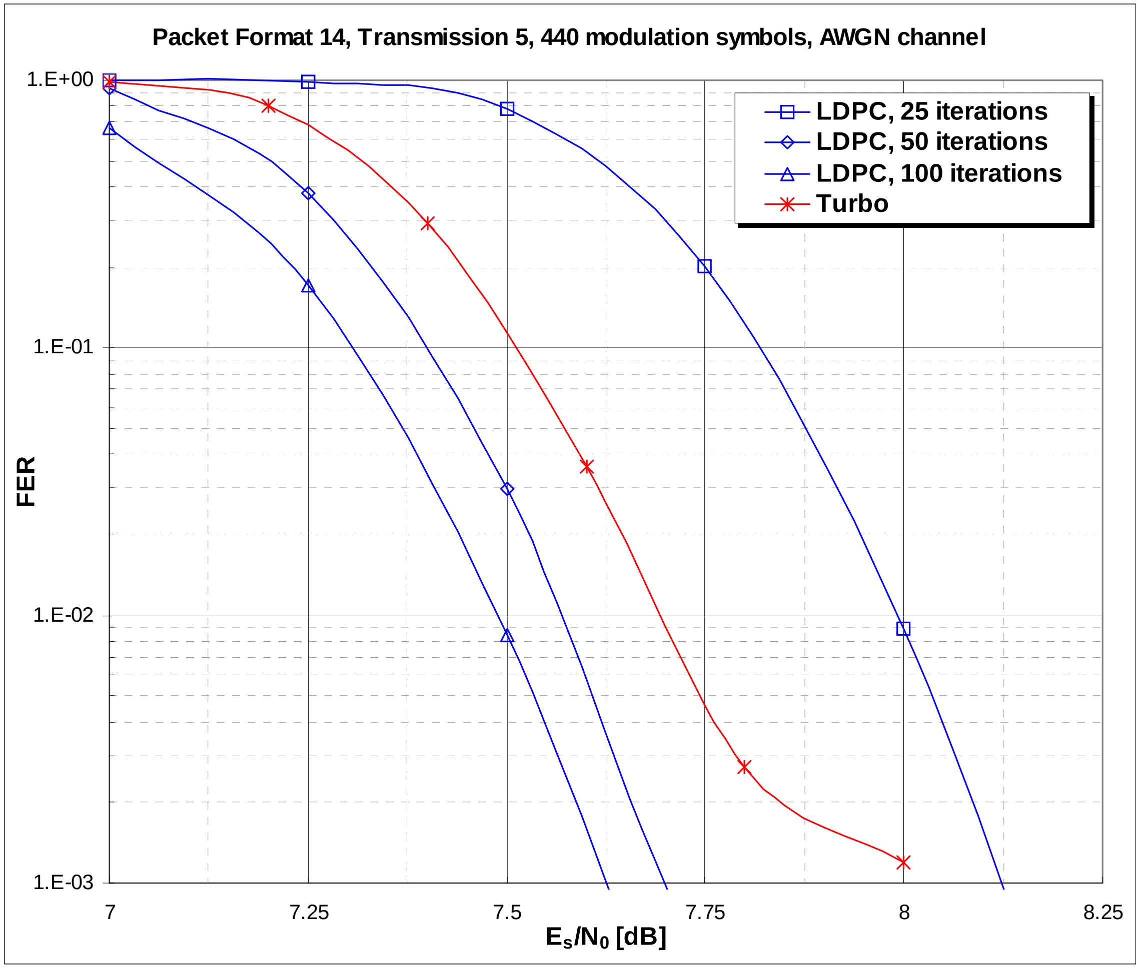
FL PF 14, Transmission 3



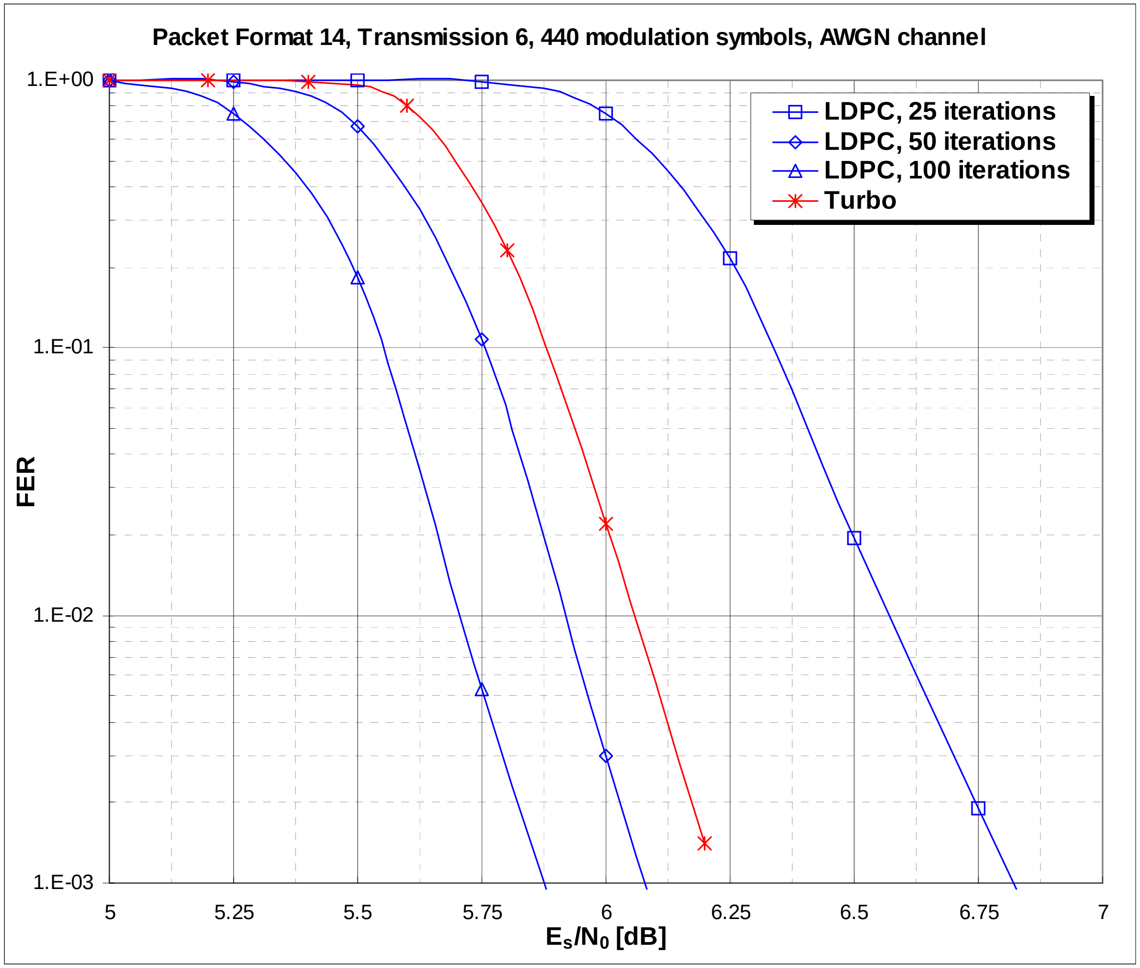
FL PF 14, Transmission 4



FL PF 14, Transmission 5



FL PF 14, Transmission 6



Conclusions

- Proposed LDPC codes offer both efficient support of Type II HARQ (Incremental Redundancy) together with similar or better performance than Turbo codes through all HARQ retransmissions

- Proposed code structure enables highly parallelizable decoder architectures, thus resulting in high-throughput decoder implementations.

References

[1] "Draft Standard for Local and Metropolitan Area Networks - Standard Air Interface for Mobile Broadband Wireless Access Systems Supporting Vehicular Mobility - Physical and Media Access Control Layer Specification", IEEE P802.20/D2.1, May 2006.

[2] A. J. Blanksbyand C. J. Howland, "A 690-mW 1-Gb/s 1024-b, Rate-1/2 Low-Density Parity-Check Code Decoder, IEEE Journal of solid-state circuits, vol. 37, no. 3, March 2002.

[3] T. J. Richardson and R. Urbanke, "Efficient encoding of low-density parity-check codes," IEEE Transactions on Information Theory, vol. 47, no. 2, pp.638-656, Feb. 2001.

[4] S. Myung, K. Yang and J. Kim, "Quasi-Cyclic LDPC Codes for Fast Encoding", IEEE Trans. on Info. Theory, Vol.51, N.8, Aug. 2005