Project	**IEEE 802 Executive Committee Study Group on Mobile Broadband Wireless Access** **<http://grouper.ieee.org/groups/802/mbwa>**
Title	**Error-Control Coding for Mobile Wireless Communications**
Date Submitted	**2002-11-08**
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Re:	MBWA ECSG Presentation
Abstract	The design parameters for ECCs in a mobile wireless system are discussed.
Purpose	For informative use only
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Error-Control Coding for Mobile Wireless Communications

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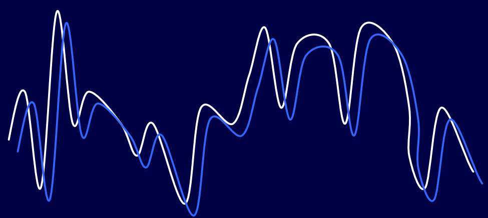
802 MBWA ECSG

November 14, 2002

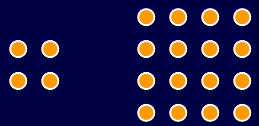
Mobile Wireless

Mobile wireless channel



- Rapid variation in signal strength
- Frequency-selective fading

Mobile wireless system



- Variable code rates and constellations
- Traffic channels – long blocks for coding gain
- Control channels – short messages, “fine granularity”

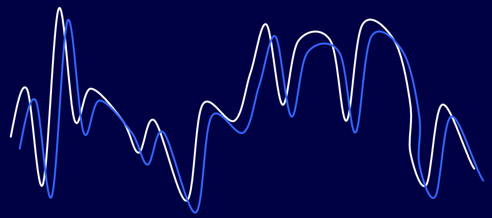
Mobile wireless devices

- Low cost
- Low power

Desired characteristics for ECC

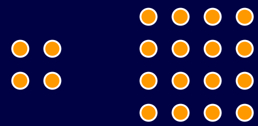
Fading in wireless channel

- High coding gain
- Fast ARQ (low latency)



Multiple varied code design

- Flexibility
- Programmability



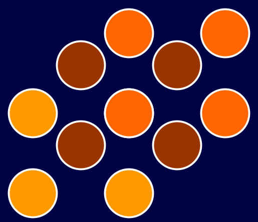
Mobile wireless devices

- Low complexity to reduce hardware costs
- Low power implementation

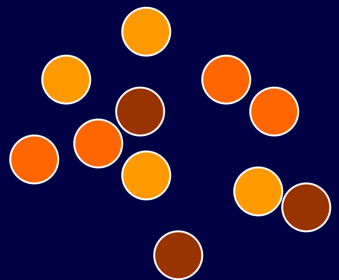
Art of Error-Control Coding

- Introduce redundancy into a data sequence
- Allows for correction against noise and fading introduced by the wireless channel

- Structure:
  - Enable implementation of encoding, decoding
  - E.g., linear block codes, convolutional codes



- Randomness:
  - Random codes are good
  - Shannon's random coding argument



A Brief History of ECC

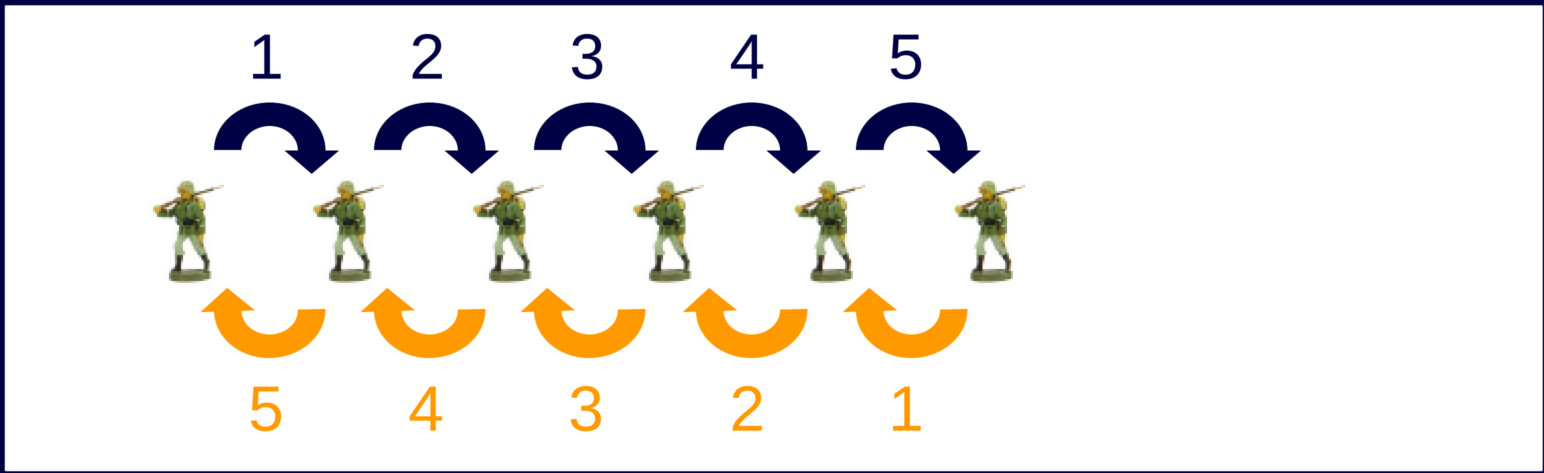
- Hamming code
- Convolutional Codes
- BCH / Reed-Solomon codes

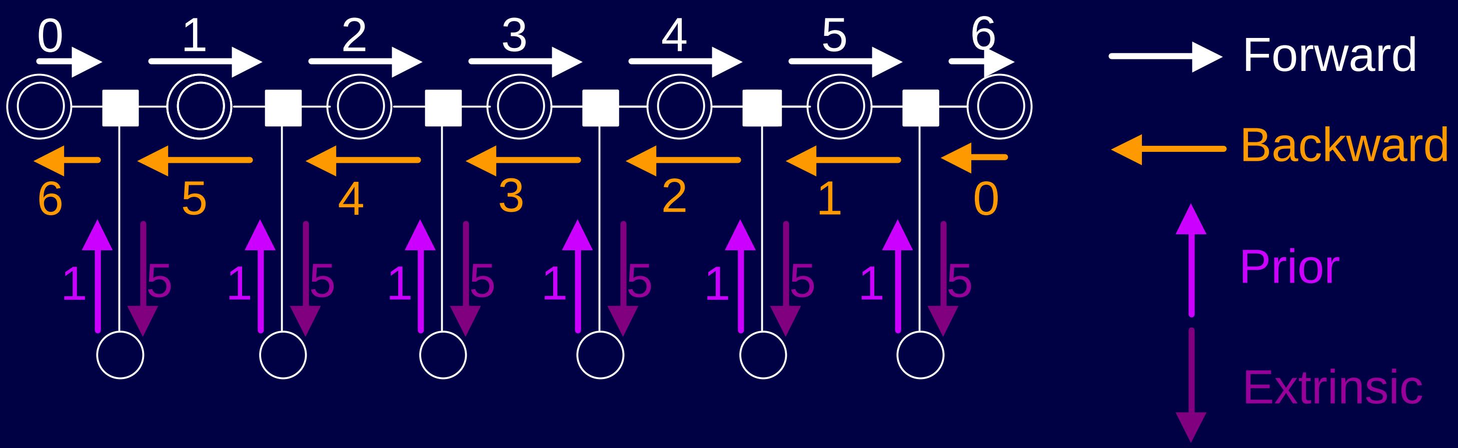
- Low-density parity-check (LDPC) codes
- Turbo decoding
  - concatenated convolutional codes
  - product codes

Soft iterative decoding paradigm

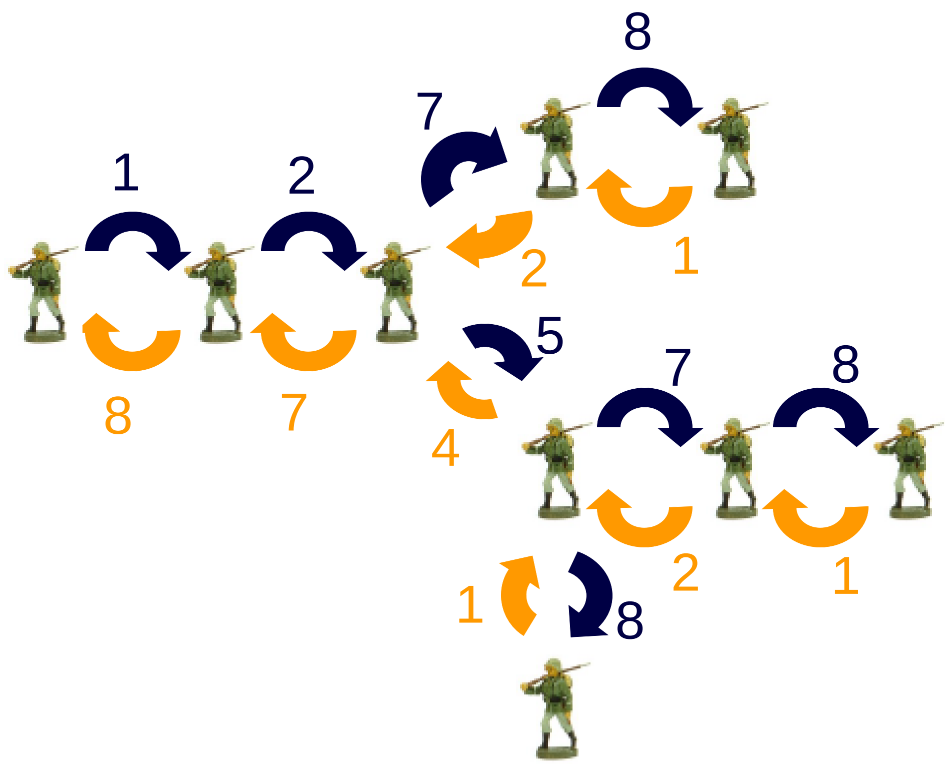
- Random-like structure
  - Random: for good performance
  - Structure: to enable encoding and decoding
- Soft decoding (probabilistic, soft-in, soft-out)
- Local decoding
- Iteration of messages (“turbo,” message-passing)
- *Near-capacity performance for reasonable complexity*

Example: Counting soldiers using message-passing

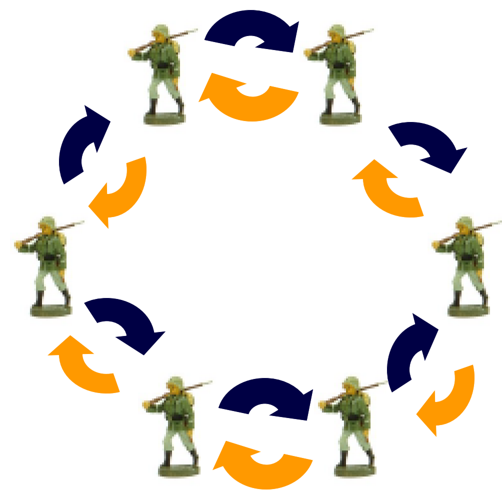




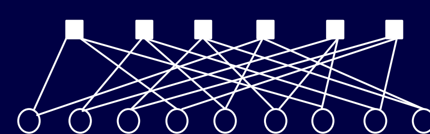
Message-passing gives the correct answer for graphs without cycles.



In general, message-passing does not work for graphs with cycles…

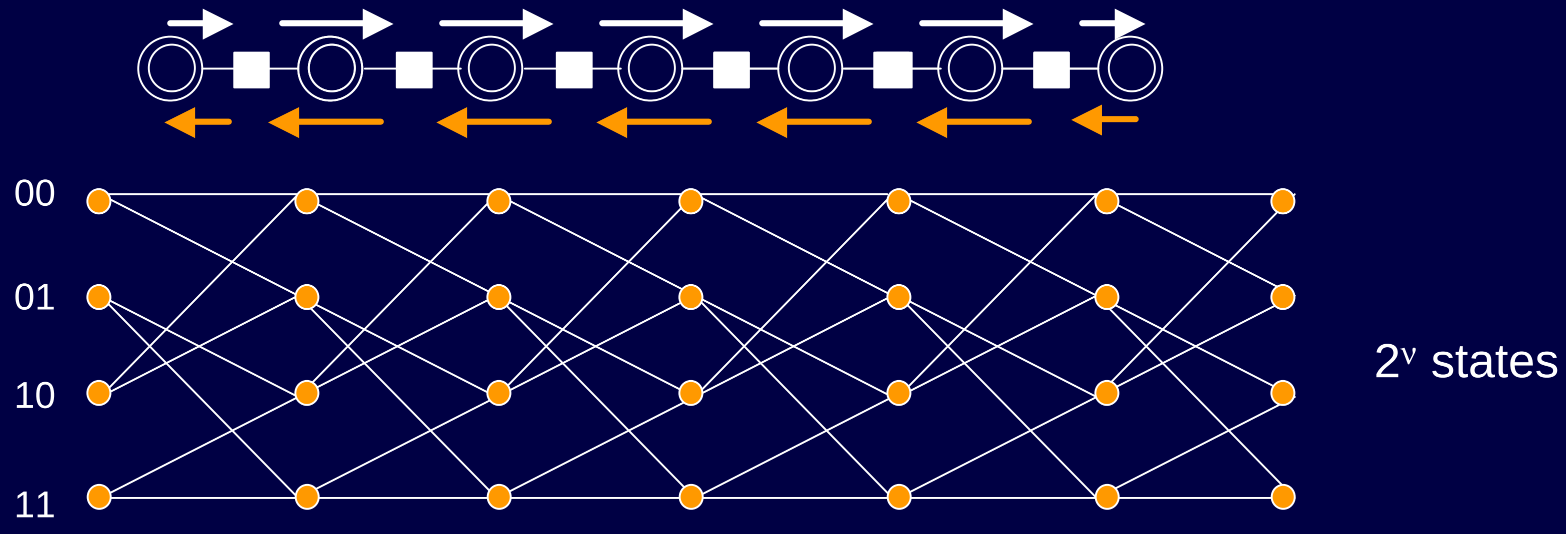


In practice, however, message-passing works very well for turbo and LDPC codes, despite the presence of cycles.

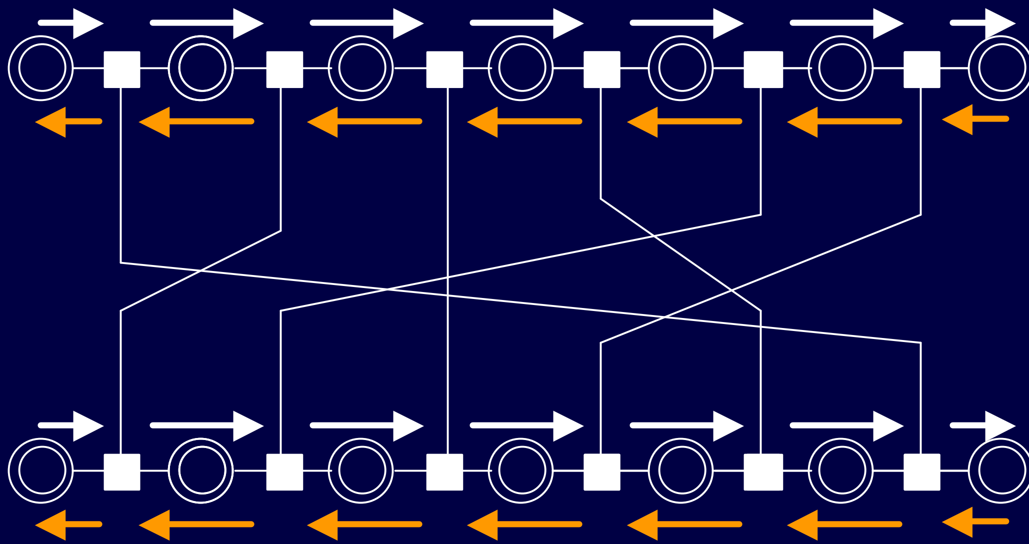


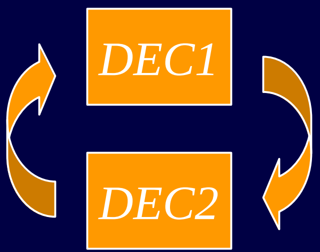
BCJR algorithm

- Soft-in, soft-out decoder
- "APP decoder", "MAP decoder"
- Used for convolutional codes and ISI channels



Turbo coding:
Parallel concatenated convolutional codes





Low Density Parity Check codes

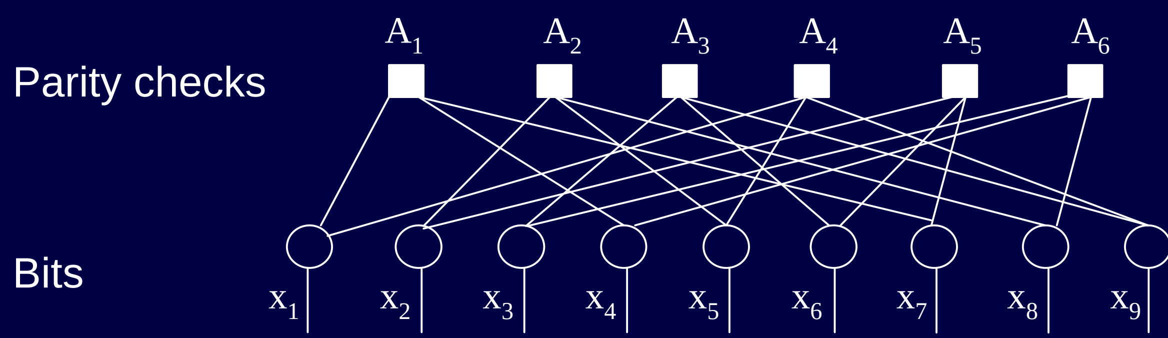
$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- LDPC codes are binary, linear error-correcting codes, defined by sparse parity check matrices.
  - Gallager (1962), Tanner, MacKay
- Encoded using a generator matrix
- Decoded using the message-passing algorithm

Tanner graph for a LDPC code

For a M by N parity check matrix, we can set up a graph where the edges correspond to 1's in the matrix.

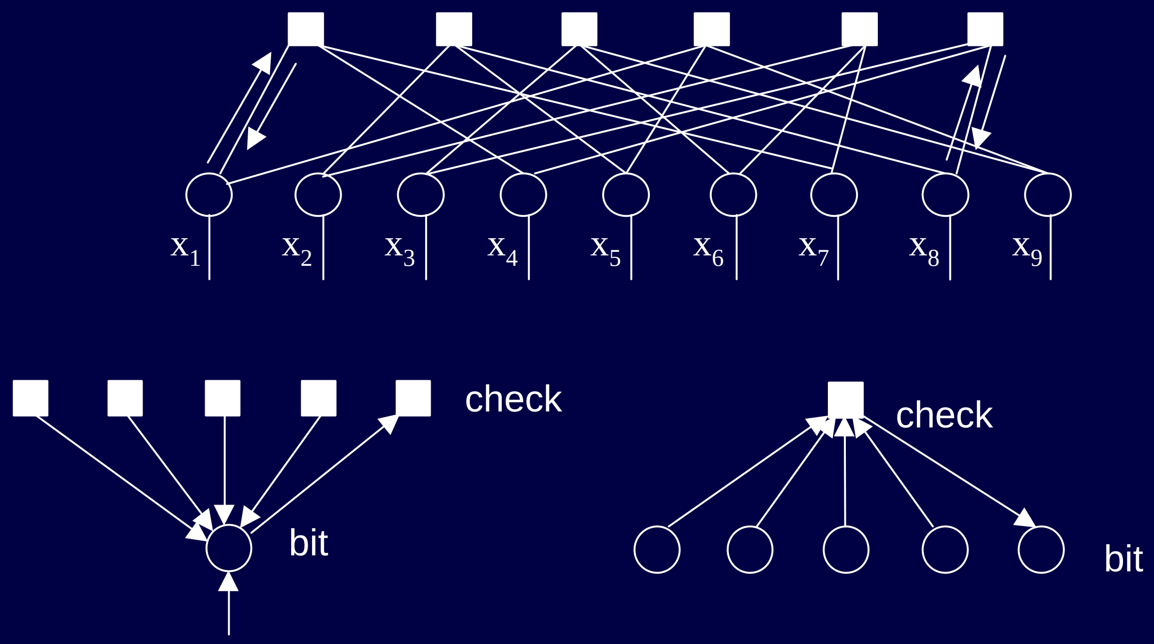
$$
\begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_9 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$



Message-passing algorithm

An efficient distributed method of solving probabilistic problems by passing messages on a graph.

Related to artificial intelligence (belief propagation, inference on Bayesian networks)



Soft information

For a binary variable x, we have

- a probability p=P(x=1)
- a log-likelihood ratio LLR(x)=log(p/(1-p))

Given evidence y, there are 3 types of soft information:

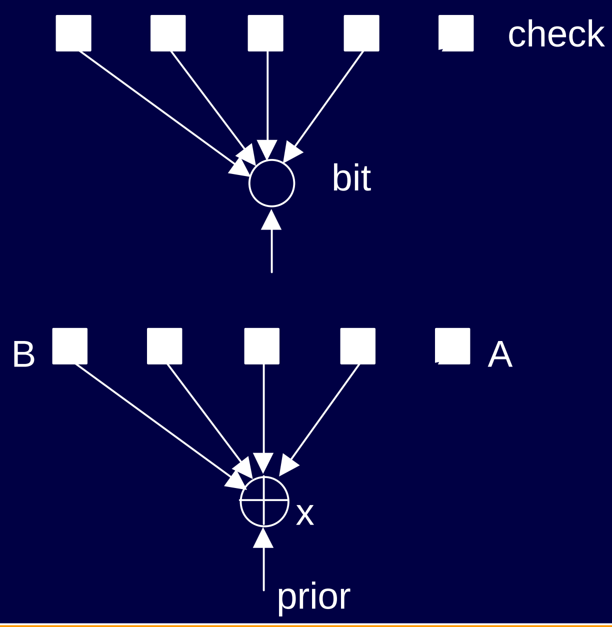
- *Prior* information - previously known, P(x=1)
- Posterior information - updated estimate, P(x=1|y)
- Extrinsic information - “new” knowledge acquired from y.

Simple relationship of log-likelihood ratios:

LLR(prior) + LLR(extrinsic) = LLR(posterior)

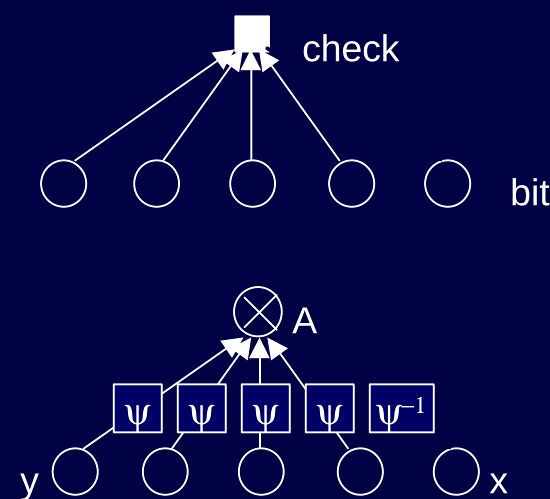
Message-passing for LDPC codes

message from bit to check



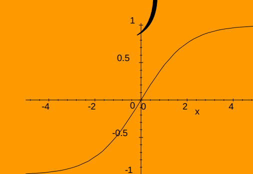
$$\mathrm{LLR}_{x\to A} = \mathrm{LLR}_x^{\mathrm{prior}} + \sum_{\substack{B\in \mathrm{Nbr}(x)\\ B\neq A}} \mathrm{LLR}_{B\to x}$$

message from check to bit

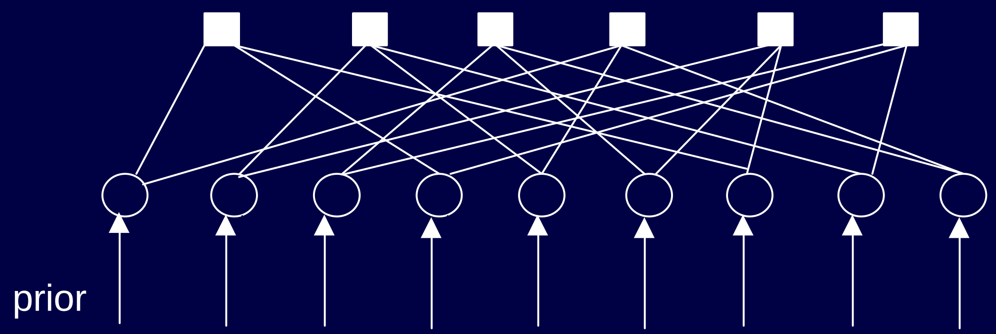


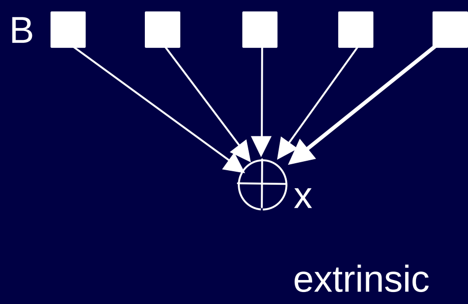
$$\mathrm{LLR}_{A\to x} = y^{-1}\left(\prod_{\substack{y\in Nbr(A)\\ y\neq x}} y\left(\mathrm{LLR}_{y\to A}\right) \right)(-1)^{|\mathrm{Nbr}(A)|}$$

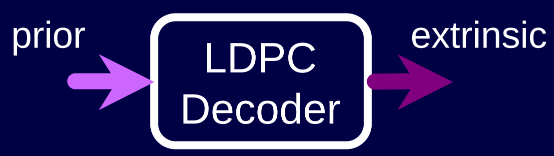
$$\text{where } y(z) = \tanh\left(\frac{1}{2}z\right)$$



Summing out







$$\mathrm{LLR}_x^{\mathrm{extrinsic}} = \sum_{B \in \mathrm{Nbr}(x)} \mathrm{LLR}_{B \to x}$$

$$\mathrm{LLR}_x^{\mathrm{posterior}} = \mathrm{LLR}_x^{\mathrm{prior}} + \mathrm{LLR}_x^{\mathrm{extrinsic}}$$

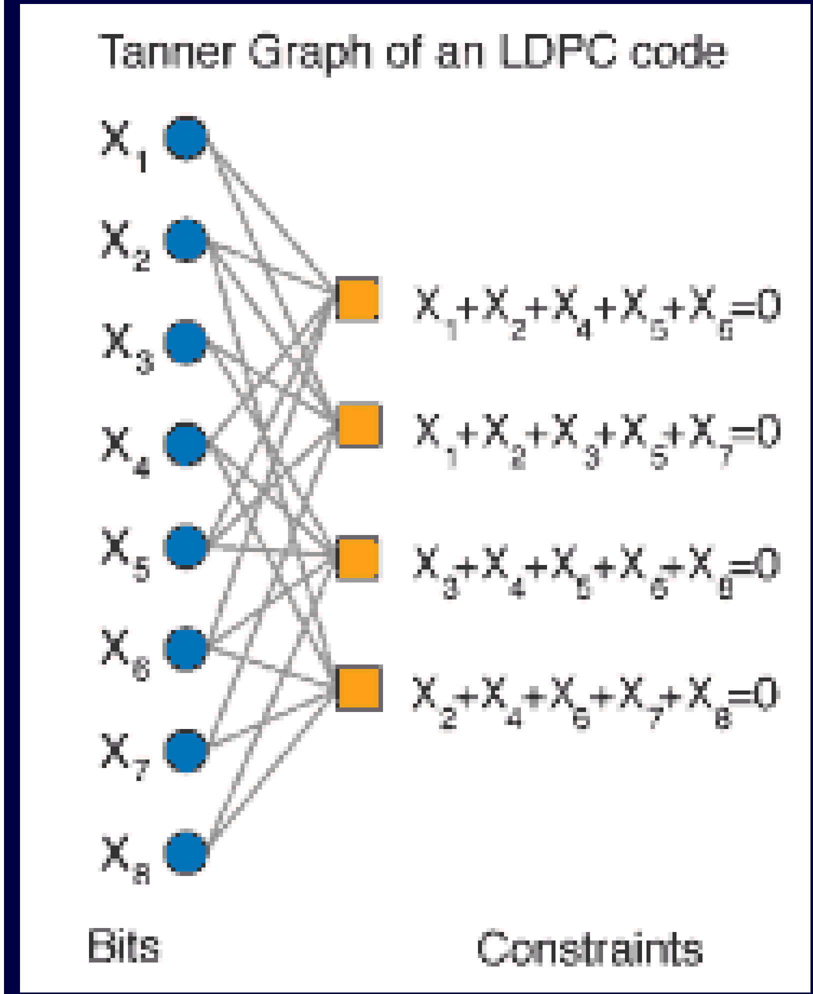


Figure 1

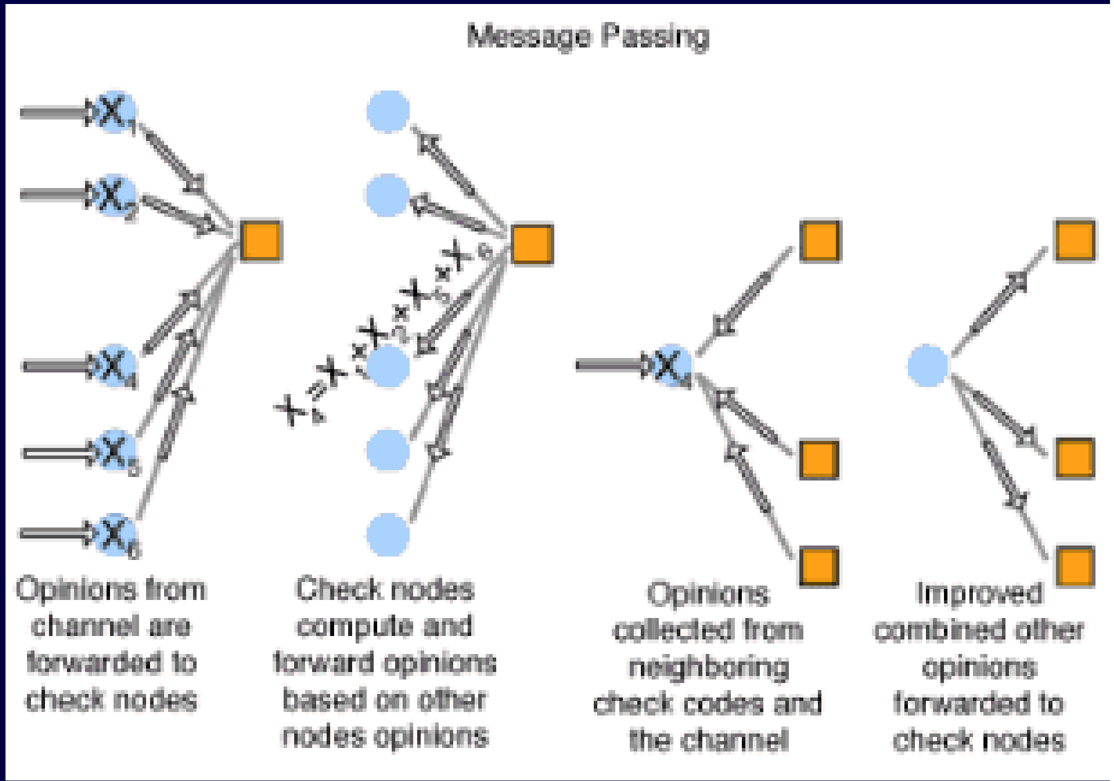
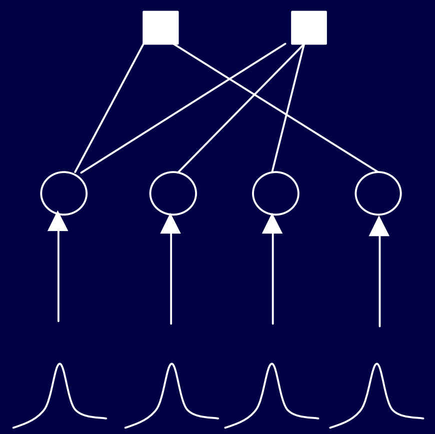


Figure 2

Density Evolution and LDPC Code Design

- A key tool for the analysis of LDPC codes
- Consider an infinite graph specified by its distribution of edges

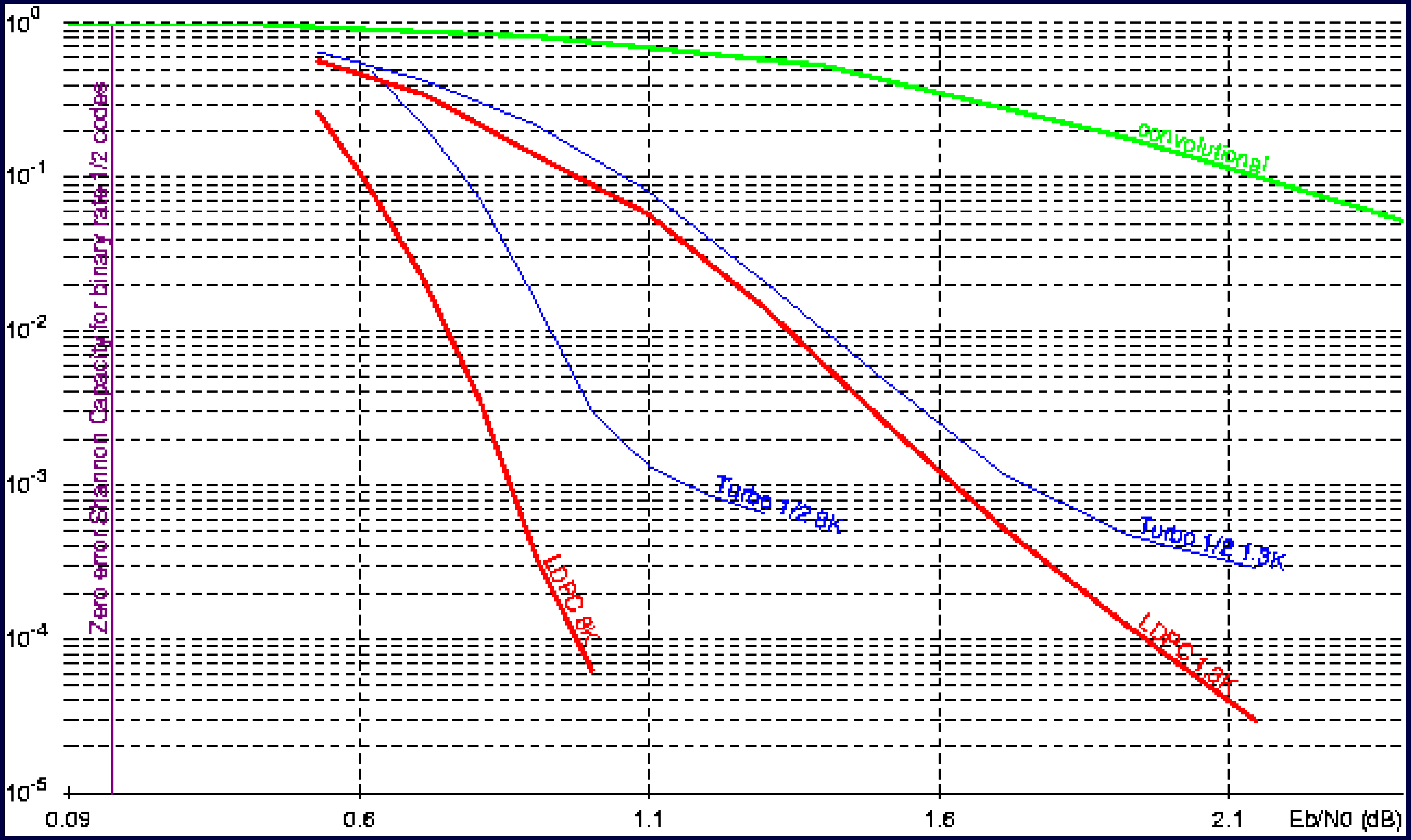


Results:

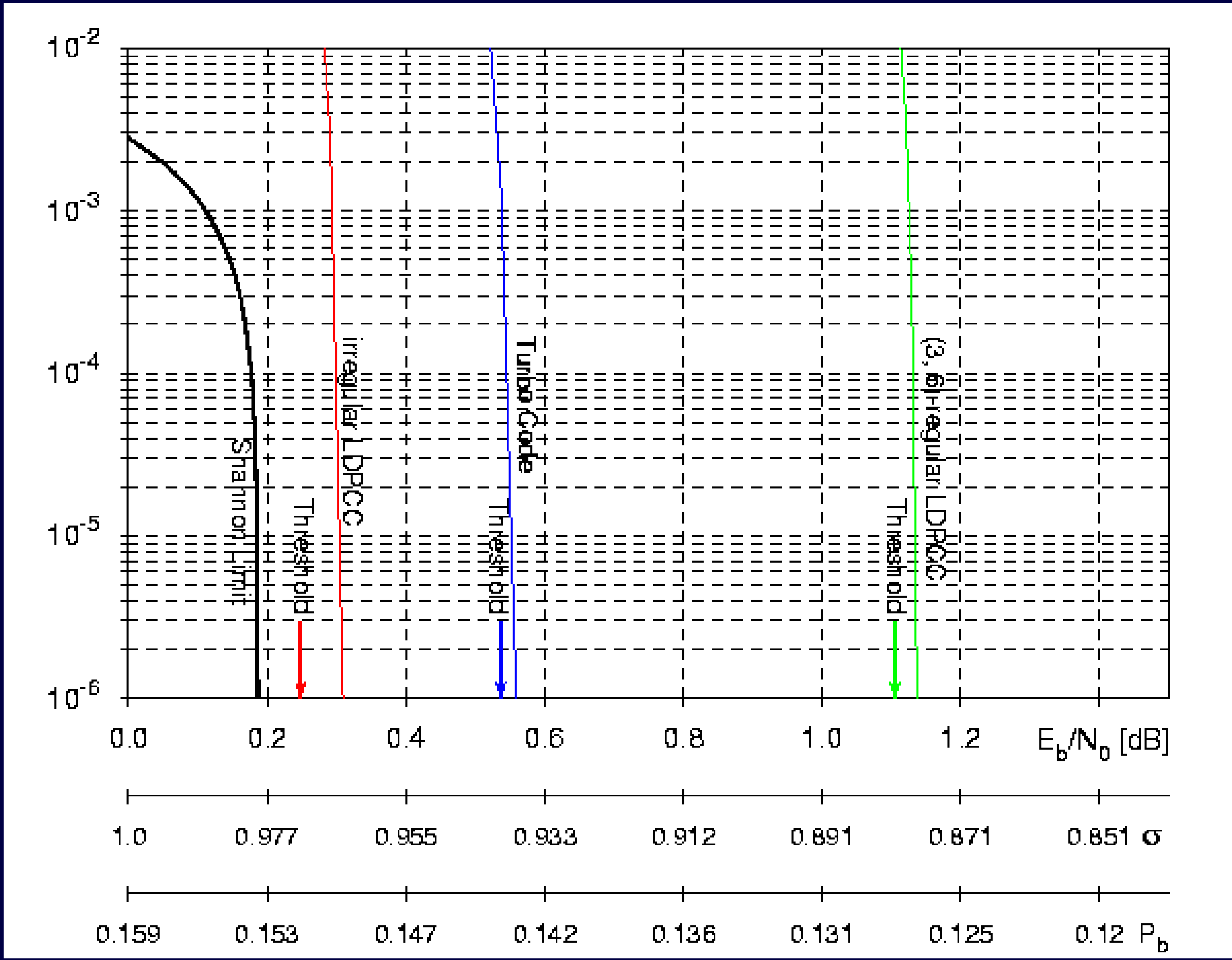
- It is possible to evaluate the channel threshold for the LDPC code
- The performance of finite codes approaches the infinite case reasonably fast.

- This tool enables LDPC graph design to improve the threshold and optimize for the channel

Performance curves



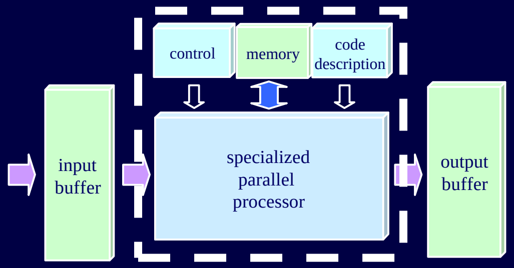
Comparison of LDPC codes, Turbo codes and IS-95 convolutional code of rate 1/2



10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
Shannon Limit
Threshold
irregular LDPCC
Turbo Code
Threshold
Threshold
(3, 6)-regular LDPCC
0.0
0.2
0.4
0.6
0.8
1.0
1.2
E_b/N_0 [dB]
1.0
0.977
0.955
0.933
0.912
0.891
0.871
0.851 σ
0.159
0.153
0.147
0.142
0.136
0.131
0.125
0.12 P_b

Areas of innovation for LDPC codes

- Improved code designs
  - using density evolution and other tools
- Parallelized hardware implementation
  - for high-speed throughput
- Programmable architecture
  - supports multiple codes
- Error floor analysis
- Turbo equalization



Conclusions

- Mobile wireless influences the design parameters for error-control schemes
- As an example, LDPC codes can be used…
  - soft iterative decoding for coding gain
  - can be implemented by programmable architecture
  - relatively low complexity, small hardware

- In the design of a MBWA system, one should be free to use ECCs optimized for mobile wireless.
- *The best mobile system is one designed for mobility.*