Signaling Terminology; PAM-M and Partial Response Precoders

IEEE 802.3
100Gb/s Backplane and Copper Cable Study Group

March 7, 2011

Will Bliss

Motivation and Process

- Looking at the ‘backplane’ literature of the last 7 years, I see significant differences in usage and meaning on the topics of
  - PAM-M
  - Partial Response names, polynomials, and precoders
- The Partial Response (PR) literature is now at least 48 years old
  - The context of that time is hard for us to recreate now
- I’m confused!
  - I’ve probably got company
- We need a clear common language to make progress as a group
- I’ll start by defining these signaling terms
  - Hopefully consistent with the wider communications literature
- Everyone please review, and
  - Offer suggestions for change, clarity, improvement, etc., and
  - Offer your own exact definitions for other items of interest that aren’t covered here

Things You've Heard About Partial Response & PAM-M
Might be Misleading?

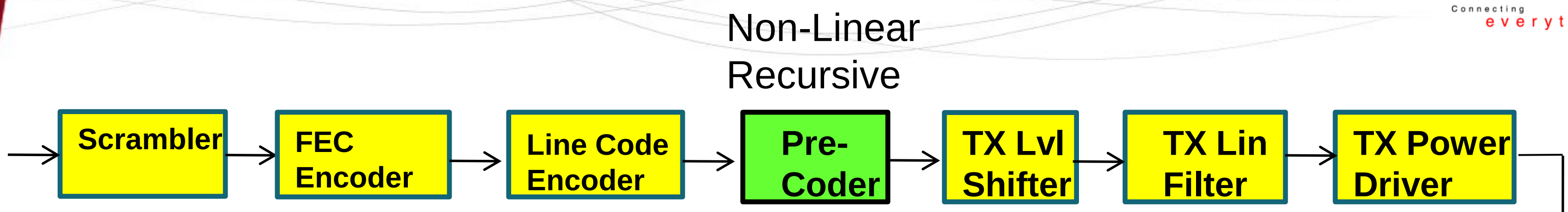
- Partial Response has lower Bandwidth than NRZ (or PAM-M)?
  - Action: Define meaning of 'compare Bandwidth'
- Partial Response is like (or is) Multi-level modulation PAM-M?
  - Action: Define PAM-M
- Partial Response 'Class X' is the polynomial … ?
  - Action: Review historical Partial Response polynomials
- Partial Response precoders 'Invert the channel'?
  - Action: Define Partial Response precoders

What is a MODERN Partial Response System?

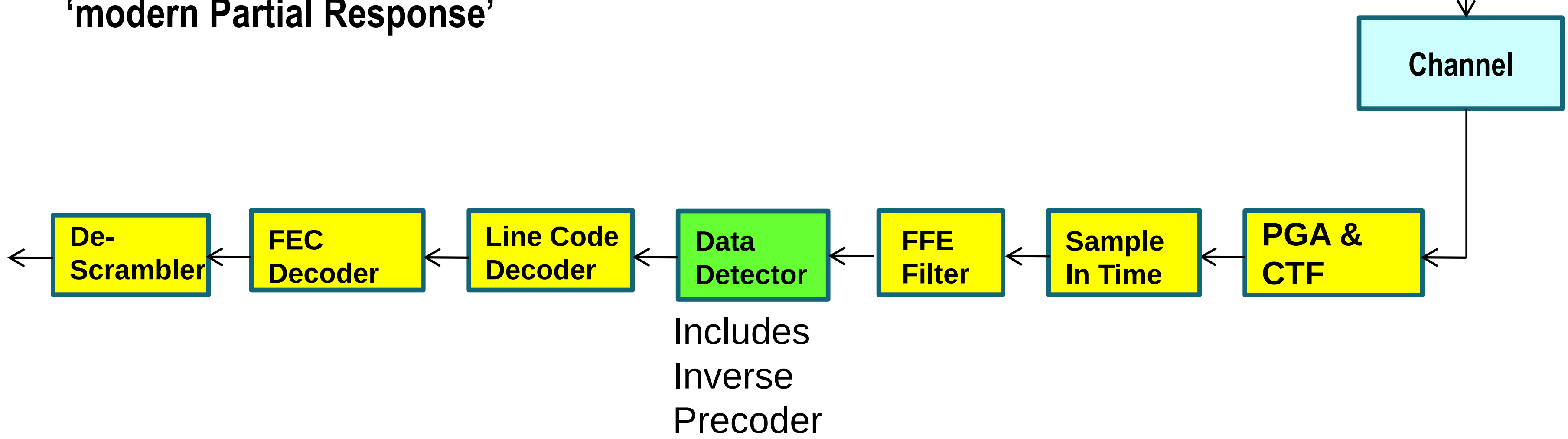
- On the TX side, always a certain type of NON-LINEAR and RECURSIVE (feedback) filter called a ‘PRECODER’
  - ‘Non-Linear’ here is over the REAL number algebra of ‘impulse responses’, etc.
    - In general the precoder will be ‘linear over the ring of integers {0,1,…M-1}’
  - If there is no non-linear precoder, then its not Modern Partial Response, just another system with another linear TX filter
  - Some of the old PR literature did detail systems w/o any TX precoder
    - These are only alternate RX implementations that seem of no interest for modern implementations

- On the RX side
  - Standards don’t really define what is done in the RX, but
  - There are several interesting RX structures that take advantage of the non-linear recursive precoders of modern Partial Response
    - One will be described later

A block diagram of a TX & RX with 'PR Precoder'

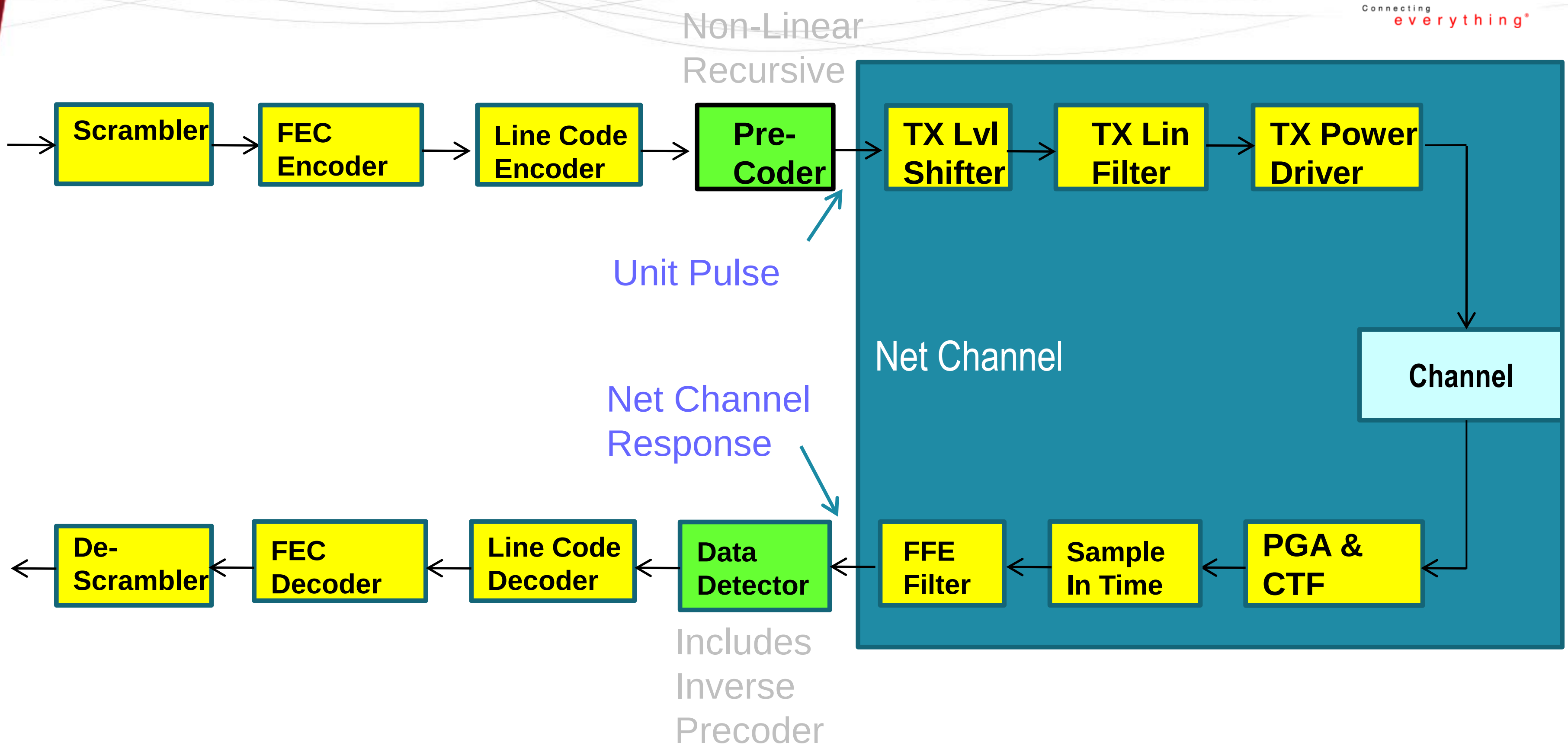


- The non-linear and recursive Precoder is what makes the system 'modern Partial Response'



- The Data Detector block here must also 'undo' the non-linear pre-coder
- One example of such a Data Detector for modern Partial Response will follow

‘Net Channel’ includes the TX and RX

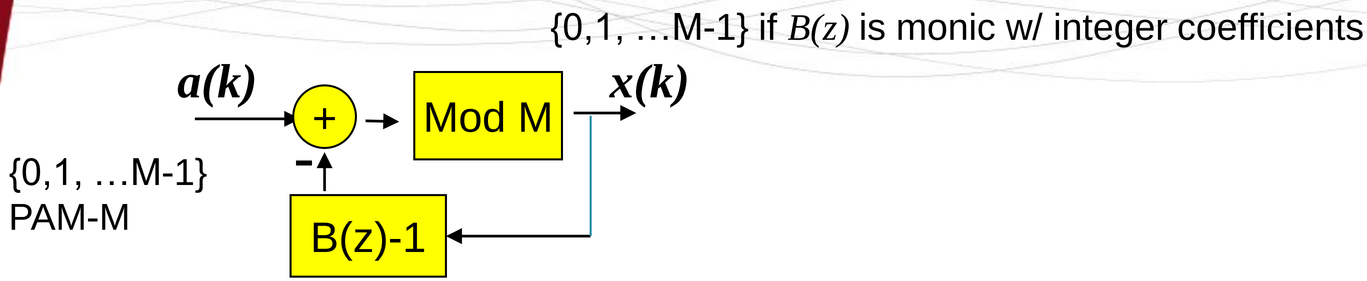


- Most of the PR literature lumps TX and RX filtering in with the ‘real physical channel’ and calls this net ‘THE CHANNEL’
- In general, want the net noise at the Data Detector input to be white
  - And for DFEs, want the ‘signal’ to be minimum phase
  - In general, don’t want the TX Linear Filter to be ‘the PR target’ (like 1+D, etc)

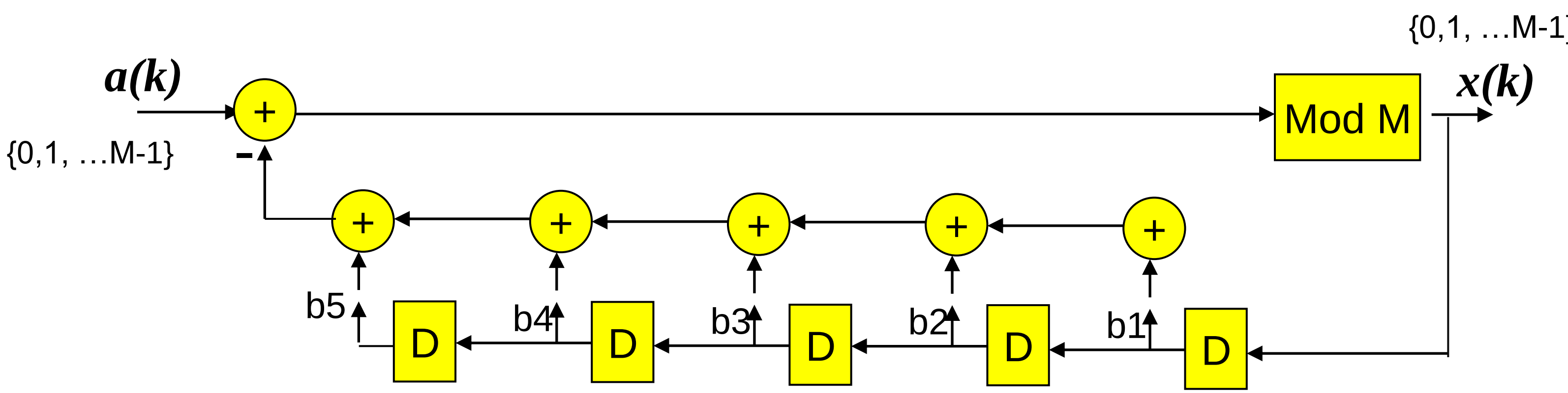
What is a Partial Response Precoder?

- A PR precoder is always a special non-linear recursive 'filter'
  - Note, if not non-linear, then just another linear TX filter, like TX emphasis, etc
- Precoder == $1/B(D)$ mod-M
  - Without loss of generality, define PAM-M to be the integers {0,1, …, M-1}
    - We include a TX level shifter such that actually transmit DC free
  - Where $B(D)$ is the net equalized channel (including all TX and RX linear filtering)
  - Mod-M for PAM-M   (so for NRZ M=2, which is Boolean arithmetic, + == XOR)
  - $B(D)$ is a monic polynomial (coefficient of D^0 is 1)
- For true (simple) partial response, all coefficients of $B(D)$ are INTEGERS
  - So a PAM-M input stream creates only a PAM-M output stream (no expansion)
- More general precoders with non-integer coefficients are usually called Tomlinson-Harishima precoders
  - They're 'more interesting', but they require a full precision TX DAC and they expand the TX range by M/(M-1)
  - Used in many communication systems, including 10GBASE-T
  - Beyond the scope of this discussion, and not usually called 'Partial Response'

Partial Response Integer Precoder *1/B(z)* mod-M for PAM-M



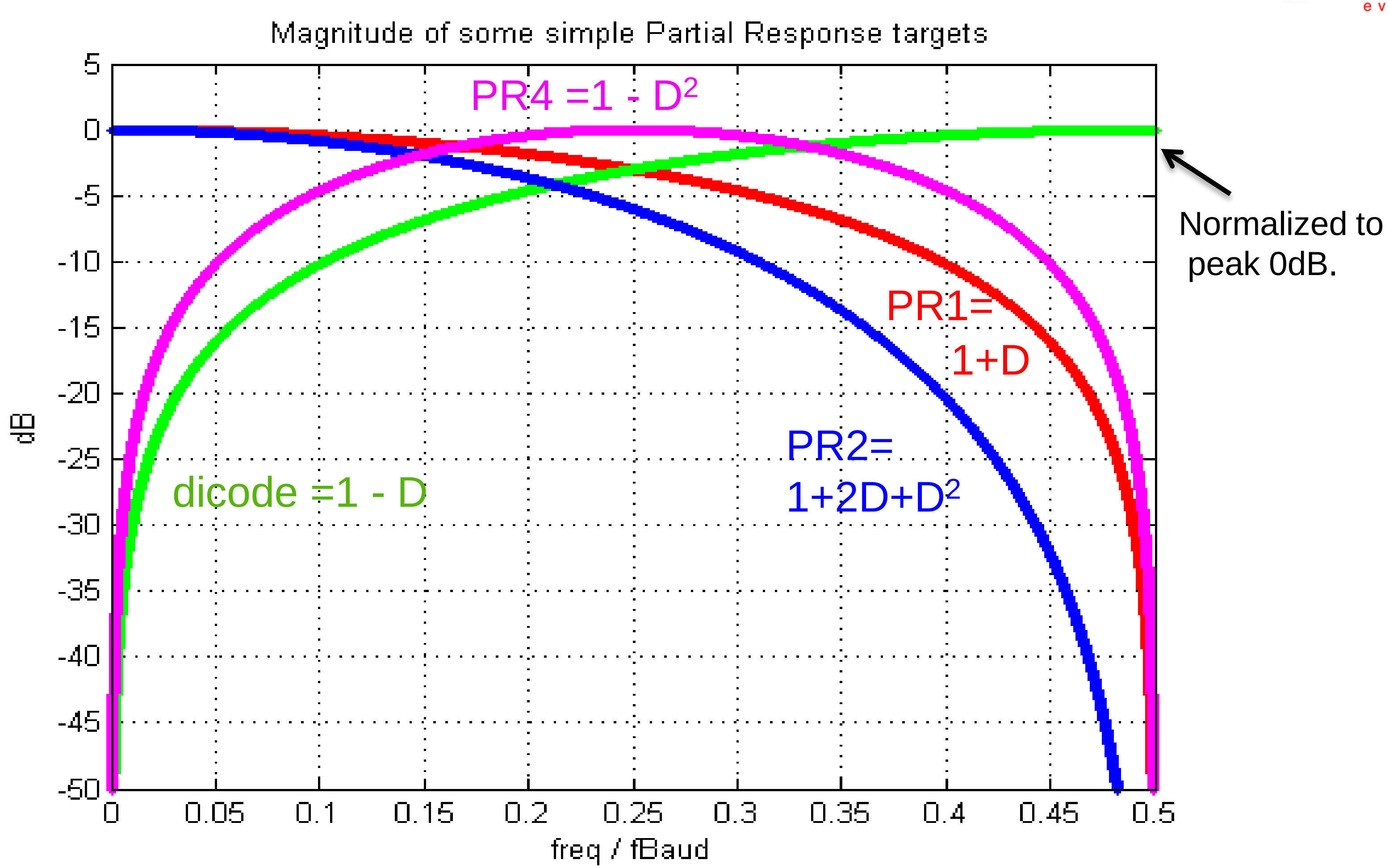
- This is the linear IIR filter *1/B(z)*, except for the non-linear Modulo-M operation
- Below the block *[B(z)-1]* is expanded to show the similarity to a canonic IIR filter
- Very easy to implement with parallelism and pipelining, even for very high speeds, because 'loop unwinding' is simple



What's with all the Partial Response 'Names'?

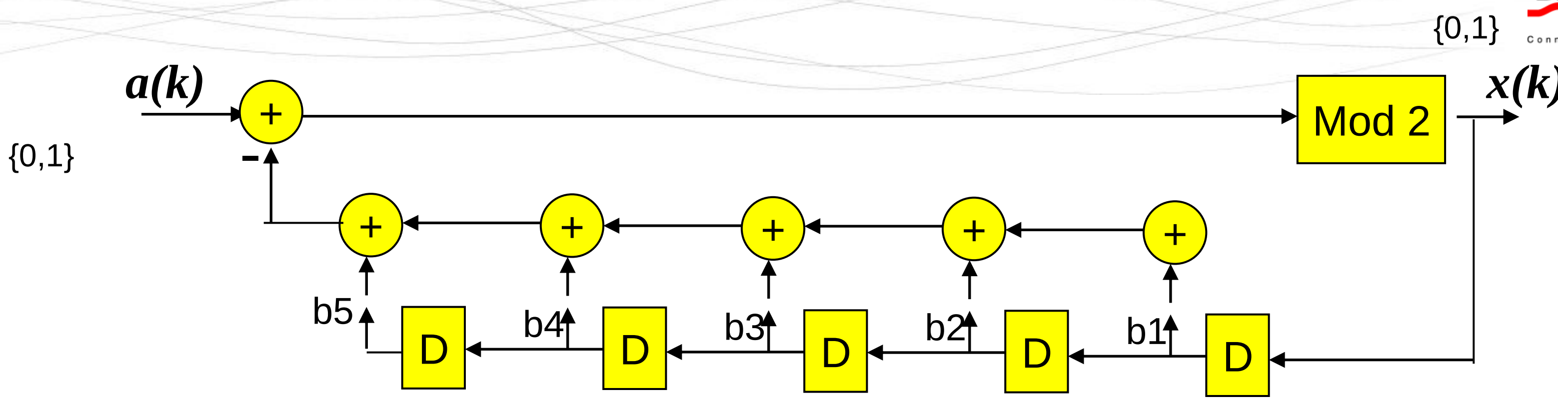
- DEFINITION; The polynomial $B(D)$ (note $D == z^{-1}$) is generally a description of the net channel including linear filtering in the TX and filtering and equalization in the RX (before a decision device)
  - Includes any linear system bandwidth restrictions in the TX, whether intended or not
  - Includes any deliberate TX filter (aka pre-emphasis)
  - Includes any RX linear system bandwidth restrictions (whether intended or not)
  - Includes any deliberate RX CTF
  - Includes any Feed Forward Equalizer (FFE)
  - Includes any linear system bandwidth restrictions from sampling in time
- When you write first, you have a lot of leeway to name things as you wish!
  - (1+D), One zero at Nyquist. PR class I, PR1, Duo-binary
  - (1-D), One zero at DC. Di-code. (No class! not much interest to backplane)
  - $(1+2D+D^2)$ = (1+D)*(1+D). Two zeros at Nyquist. PR class II, PR2
  - $(1-D^2)$ = (1+D)*(1-D). One zero at Nyquist and one zero at DC. PR class IV, PR4
  - Unclear how many simple polynomials were 'named'
  - See "Generalization of a Technique for Binary Data Communication," E.R. Kretzmer, *IEEE Tran Comm,* COM-14, pp. 67-68, Feb., 1966
- PROPOSAL: Just call out the polynomial. It's simple and unambiguous

Magnitudes (dB) of some Simple Partial Response Targets

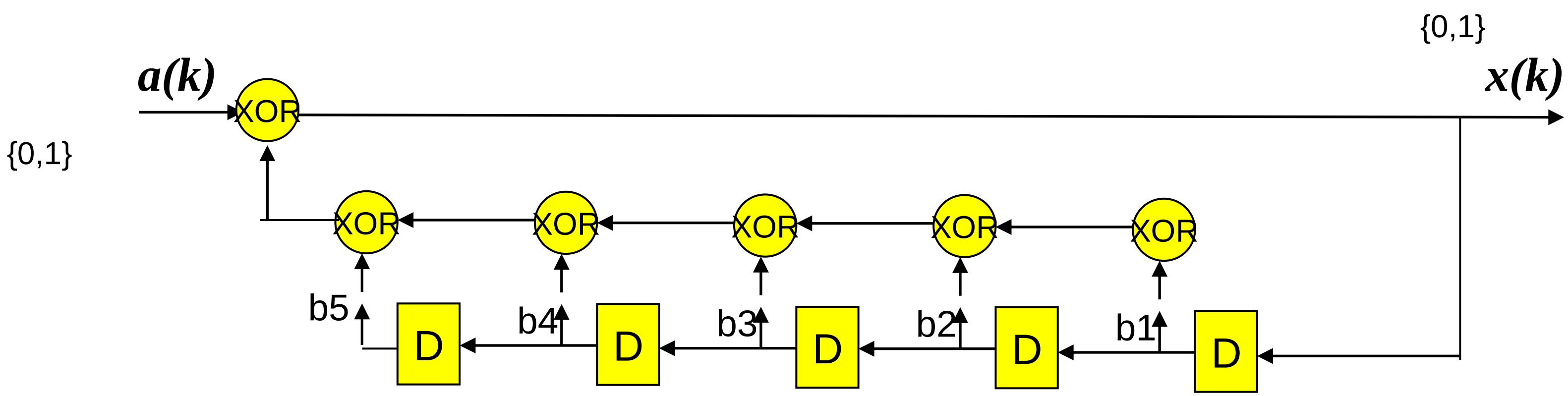


- Generally choose the 'PR Target' which makes the noise the 'whitest'
- So match to the Signal to Noise spectrum, not just the signal spectrum
  - Amplitude scaling is a 'free variable' in picking the best match

NRZ with Partial Response Integer Precoder *1/B(z)* Mod-2

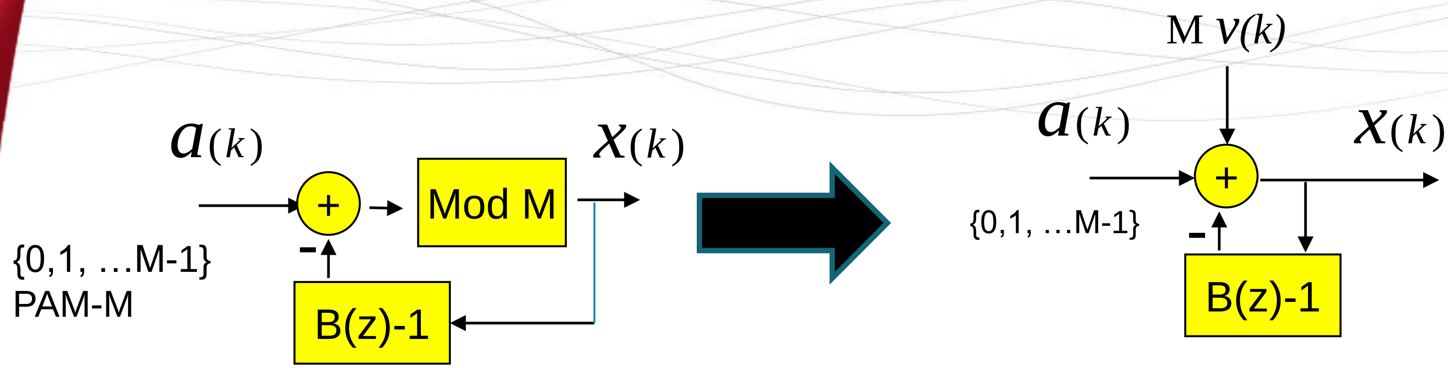


- For NRZ, Mod-M is Mod-2
- Mod-2 is exactly Boolean algebra, where += = XOR, and subtraction == addition
- Mod-2 (and XOR) distributes over addition sums, so can be redrawn as



- Which is exactly a self-synchronizing scrambler for polynomial *B(z)!*
- So for NRZ, in addition to the 'real data scrambler', we have a pre-coder which is just another scrambler! Why is this useful?

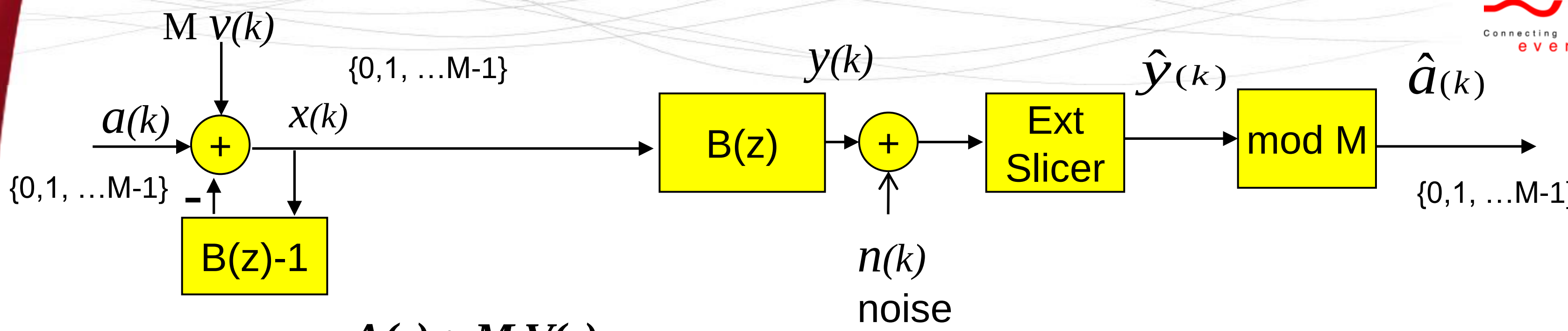
Linear Analysis of the Non-Linear Precoder 1/B(z) mod-M



$$X(z) = [\,A(z) + M\,V(z)] \,/\, B(z)$$

- **Most engineers have little training or experience with non-linear systems**
- **Analysis is much easier (for us) if we can somehow 'linearize' the problem**
- **The Modulo-M function can only add or subtract integer multiples of M**
- **Postulate the hypothetical input sequence $M\,v(k)$, where $v(k)$ is a sequence of integers**
- **The non-linear 'in $A(z)$' precoder is now a linear IIR filter on the 'hypothetical net input' $[A(z) + M\,V(z)]$**
- **This helps us see a simple RX that is enabled by this precoder;**

PR Extended Slicer RX for *1/B(z)* Mod-M Pre-coder



$$X(z) = \frac{A(z) + M\,V(z)}{B(z)}$$

$$Y(z) = X(z)\,B(z) = A(z) + M\,V(z)$$

$$y(k) = a(k) + M\,v(k)$$

$M\,v(k)$ is the hypothetical input

$a(k) = \{\,y(k)\,\}$ Mod-**M**

After Mod-M, the true input $a(k)$ is recovered

- **The block *B(z)* is the net of all the linear filtering including TX, channel, RX, and FFE**
- **The Extended Slicer's dynamic range is extended as needed for the polynomial *B(z)*, but it only outputs integers**
- **d_min=1 at the extended slicer, exactly the same as a DFE for channel *B(z)***
- **So the asymptotic (high SNR, low BER) error event rates are the same**
- **But the PR extended slicer can NOT propagate errors, while DFEs can and do**
- **No DFE expense nor any DFE error propagation**

How does a PR Precoder change the Power Spectrum of the Transmitted Signal?

- E.g., consider ***B(D)*** = 1+D (aka duo-binary, aka PR1) with uniform distribution and 'white' PAM-M input to the precoder
  - Without loss of generality, when discussing spectrum and correlation we take out the DC shift inherent in the non-negative integer definition of PAM-M
  - So consider a DC balanced PAM-M; $a'(k) = a(k) - (M-1)/2$
  - Then we have Uncorrelated inputs; $E\{a'(k)\, a'(k-m)\} = c\,\delta(m)$
- For NRZ case, the precoder output ***y(k)*** is the running sum of the input taken Mod-2
  - The output is {0,1} based on "Is the running sum even or odd?"
  - So clearly $y(k)$ is independent of the whole prior sequence $\{y(k-1), y(k-2), \ldots\}$
  - So the 'DC Free' version $y'(k)$ is also white and uniform
  - So the TX power spectrum is unchanged by the precoder! It remains white (flat)
- Easily generalizes to any integer coefficient ***B(D)*** and any M PAM-M
  - An uncorrelated uniform input creates an uncorrelated uniform output
    - A 'random white' input creates a 'random white' output

How to compare the 'Bandwidths' of Partial Response vs. PAM-M systems?

- Confusing comparisons abound
  - E.g., comparing the spectrum at the Net Channel Output (deep inside the RX) with a spectrum at the TX output (or TX Line code out)
  - Frequent discussion of the 'lower BW' of Partial Response, etc.
- It's only fair to compare Bandwidths at the same point in the two systems
  - IF we consider the normal case of random white input PAM-M data, then
  - The TX spectrums of Partial Response with PAM-M vs. only PAM-M are the same, as both remain white
  - The spectrum in the RX at the extended slicer of the PR system is identical to the spectrum at the input to a PAM-M DFE for $B(D)$ (before the FBF is subtracted)
    - Both spectrums are colored by the net channel $B(D)$

Is Partial Response 'like PAM-M'?

- Integer PR polynomials have the effect of creating integer levels at the output of the 'net channel'
  - E.g., consider PAM-2 input levels {0,1} sent through the $B(D)=$ 1+D channel.
    - The output of the net channel takes on values {0,1,2}
    - Its easy to show that levels {0 and 2} occur with probability 1/4 each, while level {1} has probability 1/2
    - Note that an efficient (maxentropic) PAM-3 system would have all levels occur with probability 1/3 each, in order to maximize information content
    - So the three levels at the output of the 1+D channel are not 'the same' as PAM-3
- It's only fair to compare 'the number of levels' at the same point in the two systems
  - The TX Partial Response with PAM-M has exactly M levels, the same as PAM-M w/o any partial response
  - If we compare a PR system with precoder $1/B(D)$ *mod-M* with a simple PAM-M system with a PAM-M DFE for $B(D)$, we find exactly the same levels at the input to the Extended Slicer as we do at the input to the DFE (before the FBF is subtracted)

Things You've Heard About Partial Response & PAM-M

What they Mean

- Partial Response has lower Bandwidth than NRZ (or PAM-M)?
  - No. PR precoders map random white inputs into random white outputs, so the spectrum remains white
- Partial Response is like (or is) Multi-level modulation PAM-M?
  - No. PR precoders don't change the PAM-M levels
  - Comparison inside the RX shows the same levels as in a comparable DFE
- Partial Response 'Class X' is polynomial … ?
  - The historical list of polynomials of interest was given
  - Lets call out the polynomial itself to avoid confusion
- Partial Response precoders 'Invert the channel'?
  - PR precoders are non-linear
  - Precoders can be thought of as 'inverting the net channel', but only in a certain Mod-M (not linear over the real numbers) fashion

References on Partial Response and Precoding

- **Partial Response references**
  - A. Lender, "The duobinary technique for high-speed data transmission," *IEEE Tran Communications Electronics,* vol. 82, pp. 214-218, May 1963
  - E. Kretzmer, "Generalization of a technique for binary data communication," *IEEE Tran Communication Technology*, vol. COM—14, pp. 67-68, Feb 1966
  - P. Kabal and S. Pasupathy, "Partial-Response Signaling", *IEEE Tran. Communications*, vol COM-23 no. 9, pp. 921-934, Sept 1975
- **General (non-linear and non-integer) Precoding references**
  - M. Tomlinson, "New automatic equalizer employing modulo arithmetic," Electron. Letters, vol. 7, pp. 138-139, Mar 1971
  - H. Harishima and H. Miyakawa, "Matched-transmission technique for channels with intersymbol interference," IEEE Tran Comm., vol. COM-20, pp. 272-273, Aug 1972