A method for evaluating channels

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100 Gb/s Backplane and Copper Study Group
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Supporters

• Brian Misek, Avago Technologies
• Rich Mellitz, Intel
Background

- IEEE 802.3ba™-2010 introduced new methods for evaluating channels
  - Pulse response amplitude, 85.8.3.3
  - Integrated crosstalk noise (ICN), 85.10.7
- The influence of channel insertion loss deviation (ILD) and return loss on link performance was recognized by not rigorously quantified
Objectives

• Extend the ICN calculations to quantify the impact of channel ILD and return loss
• For the study group, provide a method for estimating the suitability of channels for use at some bit rate
  – Justify reach objectives
• For the (prospective) task force, provide a framework for the definition of channel performance requirements
The basic method consists of 4 steps

1. Predict the available signal at the receiver given the channel and an assumed transmitter
2. Compute an equivalent noise that represents the interference from nearby channels (crosstalk) and residual inter-symbol interference (ISI)
3. Apply an estimate of the additional SNR loss corresponding to “real world” implementations of the receiver and transmitter
4. Compute the SNR and compare it to a target intended to provide a symbol error ratio (SER) better than $10^{-12}$

This method is described and justified by text and equations. Equations are derived in Appendix A.

Incidental values, which may changed without changing the method, are tabulated in Appendix B.
Signal measure

• The signal measure is the dibit amplitude which is the product of the transmitted amplitude and the dibit gain
• Considers the influence of pre-cursor ISI from the first following bit
  – Not correctable by the decision feedback equalizer
• Includes representations of the transmitter and receiver filters

The dibit gain is the peak of response to the stimulus:
**Sources of noise**

- **Victim transmitter**
  \[ |H_t|^2 = \frac{A_t^2}{1 + (f/f_t)^4} \]

- **Transmitter re-reflection**
  \[ |H_{rrtx}|^2 \]

- **Receiver re-reflection**
  \[ |H_{rrrx}|^2 \]

- **Transmitter-receiver re-reflection**
  \[ |H_{rrtxr}|^2 \]

- **Far-end aggressor**
  \[ |H_{nt}|^2 = \frac{A_{nt}^2}{1 + (f/f_{nt})^4} \]

- **Near-end aggressor**
  \[ |H_{nt}|^2 = \frac{A_{nt}^2}{1 + (f/f_{nt})^4} \]

**Far-end crosstalk**
\[ 10^{-MDFEXTloss/10} \]

**Near-end crosstalk**
\[ 10^{-MDNEXTloss/10} \]

**Integrated crosstalk noise**

\[ |H_r|^2 = \frac{1}{1 + (f/f_r)^8} \]
Insertion loss deviation (ILD) noise

• Most cleanly designed channels with low reflections have a transfer function which may be modeled as:

\[ H = e^x \]

\[ \log(H_{fit}) = x \cong \gamma_0 + \gamma_1\sqrt{f} + \gamma_2f + \gamma_4f^2 \]

\[ \gamma_i = \alpha_i + j\beta_i \]

• Since most channels will have this basic characteristic, it is reasonable to expect that transmitters and receivers are designed to equalize it

• Deviations from the transfer function model will represent unexpected perturbations that may be difficult to equalize

• The difference between the actual transfer function and the best fit to the model may be considered to be error term whose power can be added to the total noise

\[ H_{ild} = H - H_{fit} \]

• The best fit is the one that minimizes \( H_{ild} \) in the least mean squares sense and this is a fit weighted by \( H \)
Best fit calculation

\[
F_w = \begin{bmatrix}
H(f_1) & H(f_1)\sqrt{f_1} & H(f_1)f_1 & H(f_1)f_1^2 \\
H(f_2) & H(f_1)\sqrt{f_1} & H(f_2)f_2 & H(f_2)f_2^2 \\
\vdots & \vdots & \vdots & \vdots \\
H(f_N) & H(f_N)\sqrt{f_N} & H(f_N)f_N & H(f_N)f_N^2
\end{bmatrix}
\]

\[
y = \begin{bmatrix}
H(f_1)\log(H(f_1)) \\
H(f_2)\log(H(f_2)) \\
\vdots \\
H(f_N)\log(H(f_N))
\end{bmatrix}
\]

\[
\gamma = \begin{bmatrix}
\gamma_0 \\
\gamma_1 \\
\gamma_2 \\
\gamma_4
\end{bmatrix}
\]

\[
\gamma^{lms} = (F_w^T F_w)^{-1} F_w^T y
\]

\[
x^{lms} = \gamma_0^{lms} + \gamma_1^{lms} \sqrt{f} + \gamma_2^{lms} f + \gamma_4^{lms} f^2
\]

\[
H_{fit} = e^{x^{lms}}
\]

• To avoid undue influence from signals which contribute little noise, the range of the fit is limited such that \(f_N \leq f_{max}\)

• The value of \(\log(H(f))\) is ambiguous but the form used here “unwraps” the phase
  – The magnitude may be fit independently from the phase by substituting the magnitude of \(H(f)\) for \(H(f)\) in the expressions above to yield \(\alpha^{lms}\)
  – The unwrapped phase may then be fit directly to yield \(\beta^{lms}\)
ILD noise example

A method for evaluating channels
Re-reflection interference (noise)

- Transmitter, receiver, and channel return loss influence the transfer function of the assembled link.

\[
S = \begin{bmatrix}
    s_{11} & s_{12} \\
    s_{21} & s_{22}
\end{bmatrix}
\]

Transmitter-receiver re-reflection
\[
H_{rrtxrx} = s_{21} \Gamma_2 s_{12} \Gamma_1 s_{21}
\]

Transmitter re-reflection
\[
H_{rrtx} = s_{11} \Gamma_1 s_{21}
\]

Receiver re-reflection
\[
H_{rrrx} = s_{21} \Gamma_2 s_{22}
\]
Re-reflection noise example

A method for evaluating channels
Calculating and combining noise

- Calculate the power of each noise term using the ICN integral
- Assume noise sources are statistically independent

\[ \sigma_{ch}^2 = \sigma_{fx}^2 + \sigma_{nx}^2 + \sigma_{ild}^2 + \sigma_{rrtx}^2 + \sigma_{rrrx}^2 + \sigma_{rrtxrx}^2 \]

<table>
<thead>
<tr>
<th>( \sigma_{fx}^2 )</th>
<th>( \sigma_{fx}^2 = 2\Delta f \sum_n W_{ft}(f_n)10^{-MDFEXT_{loss}/10} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{nx}^2 )</td>
<td>( \sigma_{nx}^2 = 2\Delta f \sum_n W_{nt}(f_n)10^{-MDNEXT_{loss}/10} )</td>
</tr>
<tr>
<td>( \sigma_{ild}^2 )</td>
<td>( \sigma_{ild}^2 = 2\Delta f \sum_n W_t(f_n)</td>
</tr>
<tr>
<td>( \sigma_{rrtx}^2 )</td>
<td>( \sigma_{rrtx}^2 = 2\Delta f \sum_n W_t(f_n)</td>
</tr>
<tr>
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</tr>
<tr>
<td>( \sigma_{rrtxrx}^2 )</td>
<td>( \sigma_{rrtxrx}^2 = 2\Delta f \sum_n W_t(f_n)</td>
</tr>
</tbody>
</table>

Weighting functions:

\[ W_t(f_n) = \left[ \frac{A_t^2}{1 + (f_n/f_t)^4} \right] \left[ \frac{1}{1 + (f_n/f_r)^8} \right] P(f_n) \]

\[ W_{ft}(f_n) = \left[ \frac{A_{ft}^2}{1 + (f_n/f_{ft})^4} \right] \left[ \frac{1}{1 + (f_n/f_r)^8} \right] P(f_n) \]

\[ W_{nt}(f_n) = \left[ \frac{A_{nt}^2}{1 + (f_n/f_{nt})^4} \right] \left[ \frac{1}{1 + (f_n/f_r)^8} \right] P(f_n) \]

\[ P(f) = \frac{1}{f_b} \left[ \frac{\sin(\pi f/f_b)}{\pi f/f_b} \right]^2 \]
Implementation penalty

• Numerous other effects will limit the achievable SNR but are difficult to predict without detailed simulation
• These effects include but are not limited to:
  – Transmitter jitter
  – Clock recovery error and jitter
  – Minimum slicer overdrive
  – Thermal noise and receiver noise figure
  – Baseline wander
  – Limitations on the accuracy of practical equalizers
  – “Real world” limitations on bandwidth, linearity, etc. of components
• Margin needs to built into the SNR estimate to allow for these effects

<table>
<thead>
<tr>
<th>Amplitude penalty</th>
<th>Noise penalty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_s = A_{dibit} g_{ip}$</td>
<td>$\sigma_n^2 = \sigma_{ch}^2 + \sigma_{ip}^2$</td>
</tr>
</tbody>
</table>
Target SNR

- The SNR may now be computed as $A_s / \sigma_n$
- The SNR that is required for NRZ to provide a SER better than $10^{-12}$ may be calculated using the inverse error function
  - $SNR_2$ is approximately 7.03
- For a fixed differential output voltage, multi-level modulation (L-PAM):
  - yields a reduction in the spacing between adjacent levels
  - yields somewhat smaller RMS interference amplitude
  - allows more bits per symbol hence lower symbol rates

<table>
<thead>
<tr>
<th>Symbol rate</th>
<th>Target SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_b = R/\log_2(L)$</td>
<td>$SNR_L = SNR_2 \sqrt{\frac{3}{L^2 - 1}}$</td>
</tr>
</tbody>
</table>

- Note that inner signal levels have a higher propensity for error and in the limit of large $L$ this effectively doubles the SER
  - For an SER of $10^{-12}$, this could increase the target SNR by 0.1 dB
Comparison to Salz SNR

• Salz SNR represents an upper bound on the performance of decision feedback equalizers
  – Readily computed from the channel insertion loss to crosstalk ratio (ICR)
  – However, practical equalizers will not perform as well
• Salz SNR is not very sensitive to pre-cursor ISI, ILD, and re-reflection
  – These can be significant error terms in this analysis
  – 10GBASE-KR ICR limit assumes a 3 dB SNR penalty due to ILD (and re-reflection?)
  – When considered in detail using this method, the penalty may actually be much greater
• Salz SNR should also be adjusted by an implementation penalty
  – Implementation penalty is explicitly included by this method
Conclusions

• Link operation at less than a given symbol error ratio can be estimated from a determination of the SNR
• The SNR can be derived from characteristics which are represented by collections of scattering parameters
• Reasonable suggestions for these calculations have been shown
• The authors recommend:
  – The Study Group use this method, including reasonable refinements, as part of the process to determine channel reach objectives
  – Should a Task Force be formed, use this method (and refinements) as the framework for defining channel performance requirements
Appendix A

Derivation of formulae
Dibit amplitude

The signal measure is dibit amplitude which is the product of the peak-to-peak transmitted amplitude and the dibit gain.

\[ A_{\text{dibit}} = A_t g_{\text{dibit}} \quad \text{A.1} \]

A dibit is a signal which consists of a unit positive pulse 1 unit interval (UI) long followed by a unit negative pulse 1 UI long. The dibit gain is the peak of the response of a system to a dibit. In this case, the system consists of the transmitter low pass filter, which accounts for the transmitter rise time, the channel, and the receiver low pass filter. Dibit gain can be calculated with a time domain simulator or in the frequency domain by performing the integral:

\[ \text{dibit}(\tau) = 2 \int_{0}^{\infty} \left[ 2 j \frac{\sin^2(\pi f / f_b)}{\pi f} \right] H_t(f) SDD21(f) H_r(f) e^{j2\pi f \tau} df \quad \text{A.2} \]
Dibit amplitude, continued

If the transmitter and receiver low pass filters are real, i.e. zero phase, then the peak of the dibit will occur near:

\[
\tau_e = \text{imag} \left[ \frac{\gamma_1}{2\sqrt{0.38f_b}} + \gamma_2 + 2\gamma_4 (0.38f_b) \right] + 0.5
\]

The coefficients \( \gamma_i \) are derived as shown on slide 9. The units of \( \tau_e \) are unit intervals.

When computed this way, the dibit gain will usually be accurate to better than 0.5 dB. For better accuracy, values at \( \tau_e \pm 0.2 \) UI can be computed and the peak found by quadratic fitting.
Insertion loss deviation (ILD) noise

The objective is to find the set of coefficients $\gamma^{lms}$ that minimize the mean squared error between $H$ and $e^x$.

$$\varepsilon_n = H(f_n) - e^{x(f_n)}$$  \hspace{1cm} (A.4)

$$x^{lms} = \gamma_0^{lms} + \gamma_1^{lms} \sqrt{f} + \gamma_2^{lms} f + \gamma_4^{lms} f^2$$  \hspace{1cm} (A.5)

Begin by linearizing $e^x$ around $\gamma^{lms}$.

$$e^x \cong e^{x^{lms}} + \sum_i (\gamma_i - \gamma_i^{lms}) \frac{\partial e^x}{\partial \gamma_i} \bigg|_{\gamma^{lms}}$$  \hspace{1cm} (A.6)


$$e^x \cong e^{x^{lms}} + (\gamma_0 - \gamma_0^{lms})e^{x^{lms}} + (\gamma_1 - \gamma_1^{lms})e^{x^{lms}} \sqrt{f} + (\gamma_2 - \gamma_2^{lms})e^{x^{lms}} f + (\gamma_4 - \gamma_4^{lms})e^{x^{lms}} f^2$$  \hspace{1cm} (A.7)

$$e^x \cong e^{x^{lms}} + (\gamma_0 + \sqrt{f} + \gamma_2 f + \gamma_4 f^2)e^{x^{lms}} - (\gamma_0^{lms} + \gamma_1^{lms} \sqrt{f} + \gamma_2^{lms} f + \gamma_4^{lms} f^2)e^{x^{lms}}$$  \hspace{1cm} (A.8)
Insertion loss deviation (ILD) noise, continued

If the model is a good representation of the channel over the frequency range of interest and the error is small, it can be assumed that:

\[ H \cong e^{x_{lms}} \quad \text{A.9} \]

\[ \log(H) \cong x_{lms} \quad \text{A.10} \]

Substituting Equations A.9 and A.10 into Equation A.8:

\[ e^x \cong H(f) + (\gamma_0 + \gamma_1 \sqrt{f} + \gamma_2 f + \gamma_4 f^2)H(f) - \log(H(f))H(f) \quad \text{A.11} \]

Substituting A.11 into the expression for the fit error Equation A.4 yields:

\[ \varepsilon_n \cong \log(H(f_n))H(f_n) - (\gamma_0 + \gamma_1 \sqrt{f_n} + \gamma_2 f_n + \gamma_4 f_n^2)H(f_n) \quad \text{A.12} \]

Equation A.12 is the basis for determining the best fit coefficients $\gamma_{lms}$ as shown on slide 9.
Re-reflection noise

So far, the channel transfer function has been assumed to be SDD21 which is measured under the condition of the transmitter and receiver are terminated by the reference impedance. When terminated with a transmitter with output reflection coefficient $\Gamma_1$ and a receiver with input reflection coefficient $\Gamma_2$, the transfer function from port 1 to port 2 becomes:

$$H_{12} = \frac{s_{21}}{1 - \left(\Gamma_1 s_{11} + \Gamma_2 s_{22} + \Gamma_1 \Gamma_2 s_{21} s_{12} - \Gamma_1 \Gamma_2 s_{11} s_{22}\right)}$$  \hspace{1cm} A.13

![Channel Diagram]

A method for evaluating channels
Re-reflection noise, continued

Given that \(1/(1-x)\) is approximately \(1+x\) when \(|x| << 1\), the transfer function is approximately:

\[
H_{12} \approx s_{21} \left( 1 + \Gamma_1 s_{11} + \Gamma_2 s_{22} + \Gamma_1 \Gamma_2 s_{12} - \Gamma_1 \Gamma_2 s_{11} s_{22} \right)
\]  \hspace{1cm} \text{A.14}

Under this assumption, the magnitude of the denominator is close to 1 so these reflection terms have negligible effect on the dibit gain. Note that the last term in Equation A.14 is the product of the of second and third terms, which are assumed to be small, and may be safely ignored. In addition, knowing that \(s_{21} = s_{21}\) yields:

\[
H_{12} \approx s_{21} + s_{11} s_{21} \Gamma_1 + s_{21} s_{22} \Gamma_2 + s_{21}^3 \Gamma_1 \Gamma_2 \]  \hspace{1cm} \text{A.15}

The last three terms are called the transmitter re-reflection, receiver re-reflection, and transmitter-receiver re-reflection respectively. An intuitive interpretation of these terms is given on slide 11.
Re-reflection noise computation

It would be possible to compute the true transfer function if all of the S-parameters for the channel and the complex reflection coefficients of the transmitter and receiver were known. In general, the provider of the channel will only have access to limits on the magnitude of the reflection coefficients.
Appendix B

Parameters for calculations
## General parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol rate, GHz</td>
<td>$f_b$</td>
<td>Variable</td>
</tr>
<tr>
<td>Victim differential output amplitude, mV peak</td>
<td>$A_t$</td>
<td>400</td>
</tr>
<tr>
<td>Victim transmitter 3 dB bandwidth, GHz</td>
<td>$f_t$</td>
<td>$0.55 \times f_b$</td>
</tr>
<tr>
<td>Far-end disturber differential output amplitude, mV peak</td>
<td>$A_{ft}$</td>
<td>400</td>
</tr>
<tr>
<td>Far-end disturber 3 dB bandwidth, GHz</td>
<td>$f_{ft}$</td>
<td>$0.55 \times f_b$</td>
</tr>
<tr>
<td>Near-end disturber differential output amplitude, mV peak</td>
<td>$A_{nt}$</td>
<td>600</td>
</tr>
<tr>
<td>Near-end disturber 3 dB bandwidth, GHz</td>
<td>$f_{nt}$</td>
<td>$1.00 \times f_b$</td>
</tr>
<tr>
<td>Receiver 3 dB bandwidth, GHz</td>
<td>$f_r$</td>
<td>$0.75 \times f_b$</td>
</tr>
<tr>
<td>Maximum frequency for transfer function fit, GHz</td>
<td>$f_{max}$</td>
<td>$0.75 \times f_b$</td>
</tr>
<tr>
<td>Implementation amplitude penalty, V/V</td>
<td>$g_{ip}$</td>
<td>0.667</td>
</tr>
<tr>
<td>Implementation noise penalty, mV</td>
<td>$\sigma_{ip}$</td>
<td>$0.024 A_t \sqrt{g_{dibit}}$</td>
</tr>
</tbody>
</table>
Transmitter and receiver reflection coefficients

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitter DC reflection coefficient, V/V</td>
<td>$g_{01}$</td>
<td>0.161</td>
</tr>
<tr>
<td>Transmitter return loss reference frequency, GHz</td>
<td>$f_1$</td>
<td>$1.25 f_b$</td>
</tr>
<tr>
<td>Receiver DC reflection coefficient, V/V</td>
<td>$g_{02}$</td>
<td>0.161</td>
</tr>
<tr>
<td>Receiver return loss reference frequency, GHz</td>
<td>$f_2$</td>
<td>$1.25 f_b$</td>
</tr>
</tbody>
</table>

\[
|\Gamma_1|^2 = \frac{g_{01}^2 + (f/f_1)^2}{1 + (f/f_1)^2}\]

\[
|\Gamma_2|^2 = \frac{g_{02}^2 + (f/f_2)^2}{1 + (f/f_2)^2}\]

B.1

B.2

A method for evaluating channels