A method for evaluating channels

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100 Gb/s Backplane and Copper Study Group
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Supporters

- Brian Misek, Avago Technologies
- Rich Mellitz, Intel

Background

- IEEE 802.3ba™-2010 introduced new methods for evaluating channels
  - Pulse response amplitude, 85.8.3.3
  - Integrated crosstalk noise (ICN), 85.10.7
- The influence of channel insertion loss deviation (ILD) and return loss on link performance was recognized by not rigorously quantified

Objectives

- Extend the ICN calculations to quantify the impact of channel ILD and return loss
- For the study group, provide a method for estimating the suitability of channels for use at some bit rate
  - Justify reach objectives
- For the (prospective) task force, provide a framework for the definition of channel performance requirements

The basic method consists of 4 steps

1. Predict the available signal at the receiver given the channel and an assumed transmitter
2. Compute an equivalent noise that represents the interference from nearby channels (crosstalk) and residual inter-symbol interference (ISI)
3. Apply an estimate of the additional SNR loss corresponding to “real world” implementations of the receiver and transmitter
4. Compute the SNR and compare it to a target intended to provide a symbol error ratio (SER) better than 10^{-12}

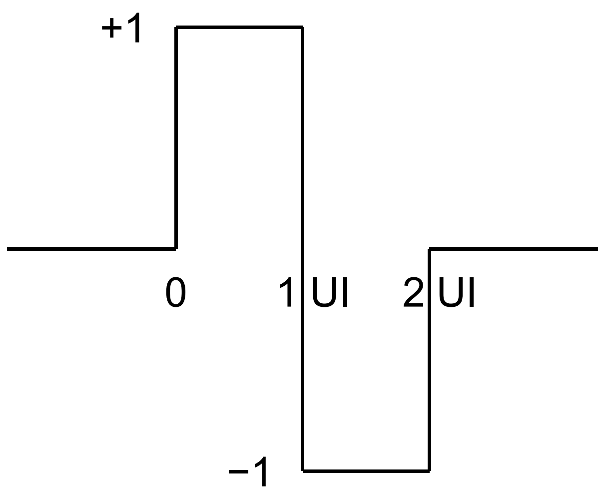
This method is described and justified by text and equations. Equations are derived in Appendix A.

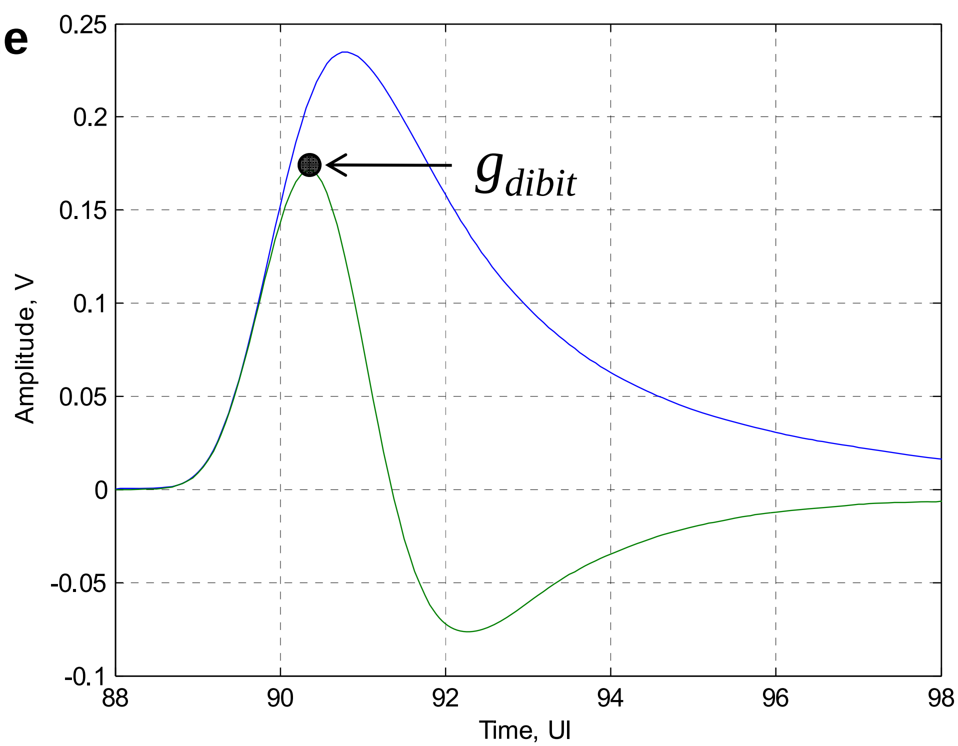
Incidental values, which may changed without changing the method, are tabulated in Appendix B.

Signal measure

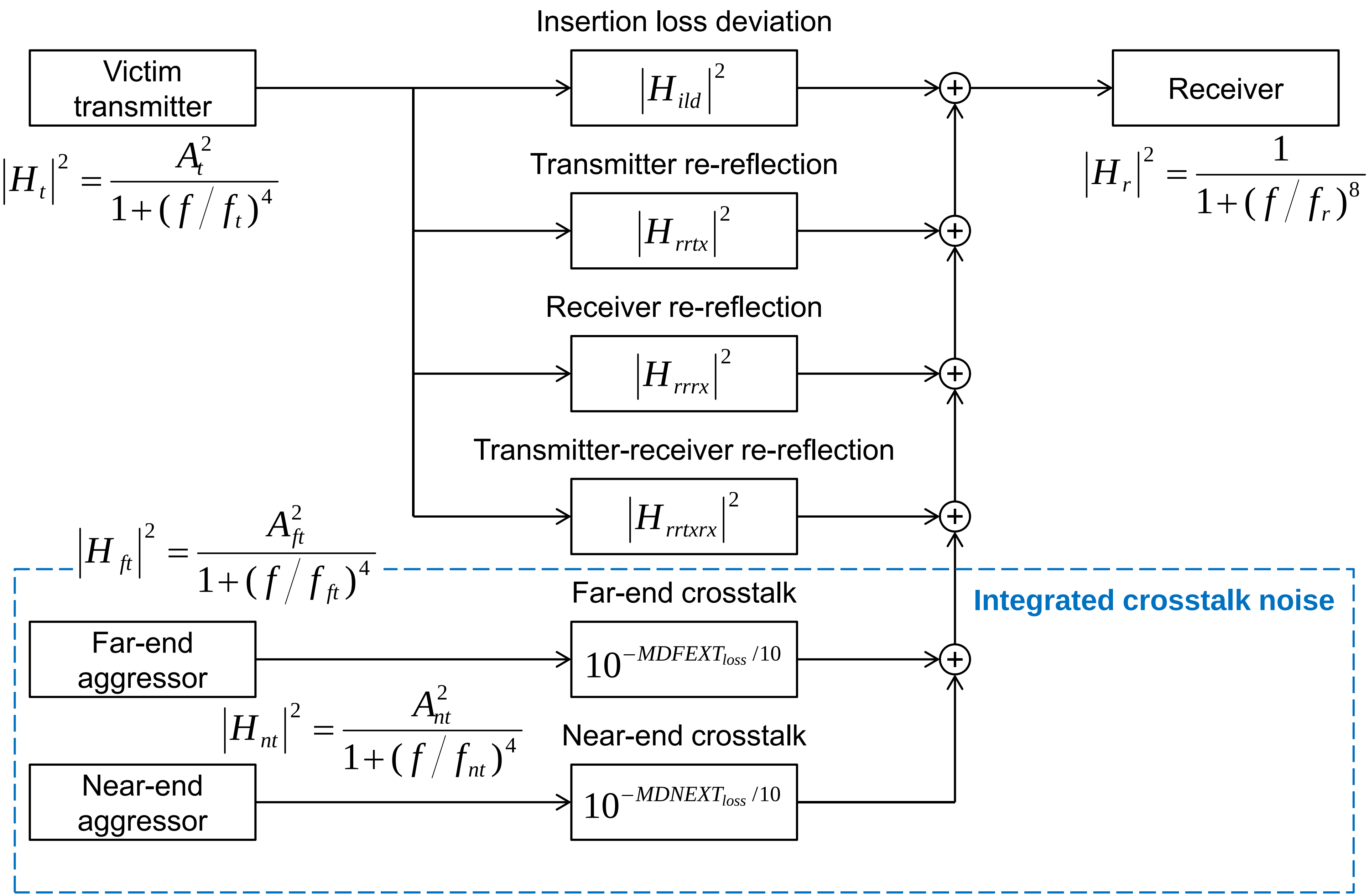
- The signal measure is the dibit amplitude which is the product of the transmitted amplitude and the dibit gain
- Considers the influence of pre-cursor ISI from the first following bit
  - Not correctable by the decision feedback equalizer
- Includes representations of the transmitter and receiver filters

The dibit gain is the peak of response to the stimulus:





Sources of noise



Insertion loss deviation (ILD) noise

- Most cleanly designed channels with low reflections have a transfer function which may be modeled as:

$$H = e^x$$

$$\log(H_{fit}) = x \cong \gamma_0 + \gamma_1\sqrt{f} + \gamma_2 f + \gamma_4 f^2$$

$$\gamma_i = \alpha_i + j\beta_i$$

- Since most channels will have this basic characteristic, it is reasonable to expect that transmitters and receivers are designed to equalize it
- Deviations from the transfer function model will represent unexpected perturbations that may be difficult to equalize
- The difference between the actual transfer function and the best fit to the model may be considered to be error term whose power can be added to the total noise

$$H_{ild} = H - H_{fit}$$

- The best fit is the one that minimizes H_{ild} in the least mean squares sense and this is a fit weighted by H

Best fit calculation

$$F_w = \begin{bmatrix} H(f_1) & H(f_1)\sqrt{f_1} & H(f_1)f_1 & H(f_1)f_1^2 \\ H(f_2) & H(f_1)\sqrt{f_1} & H(f_2)f_2 & H(f_2)f_2^2 \\ \vdots & \vdots & \vdots & \vdots \\ H(f_N) & H(f_N)\sqrt{f_N} & H(f_N)f_N & H(f_N)f_N^2 \end{bmatrix} \qquad y = \begin{bmatrix} H(f_1)\log(H(f_1)) \\ H(f_2)\log(H(f_2)) \\ \vdots \\ H(f_N)\log(H(f_N)) \end{bmatrix}$$

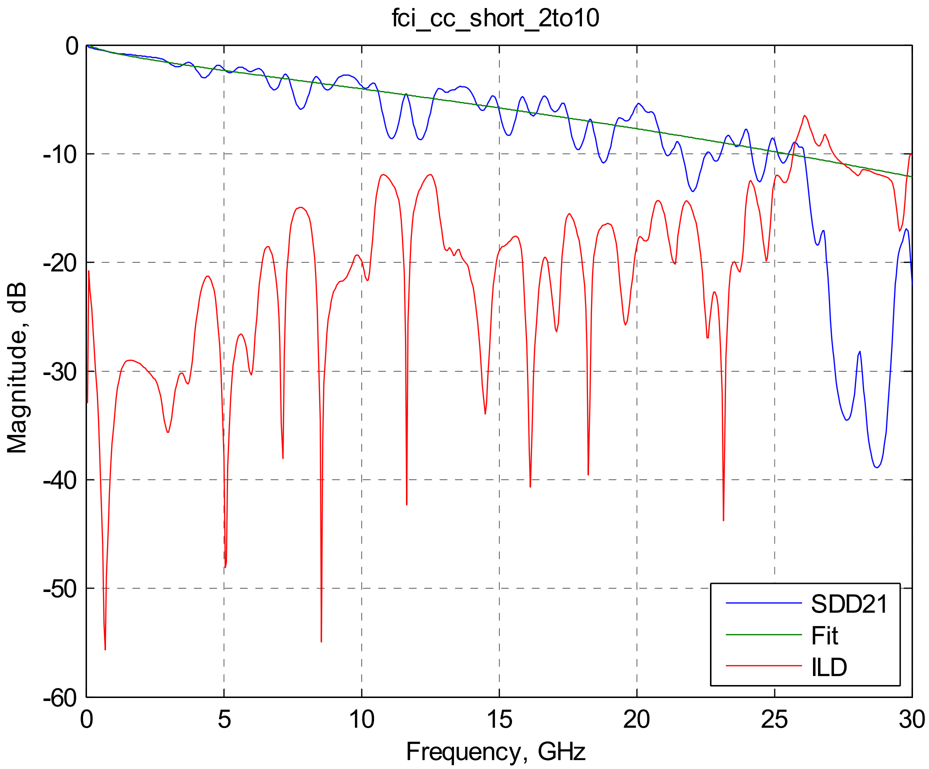
$$\gamma = \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \gamma_2 \\ \gamma_4 \end{bmatrix}$$

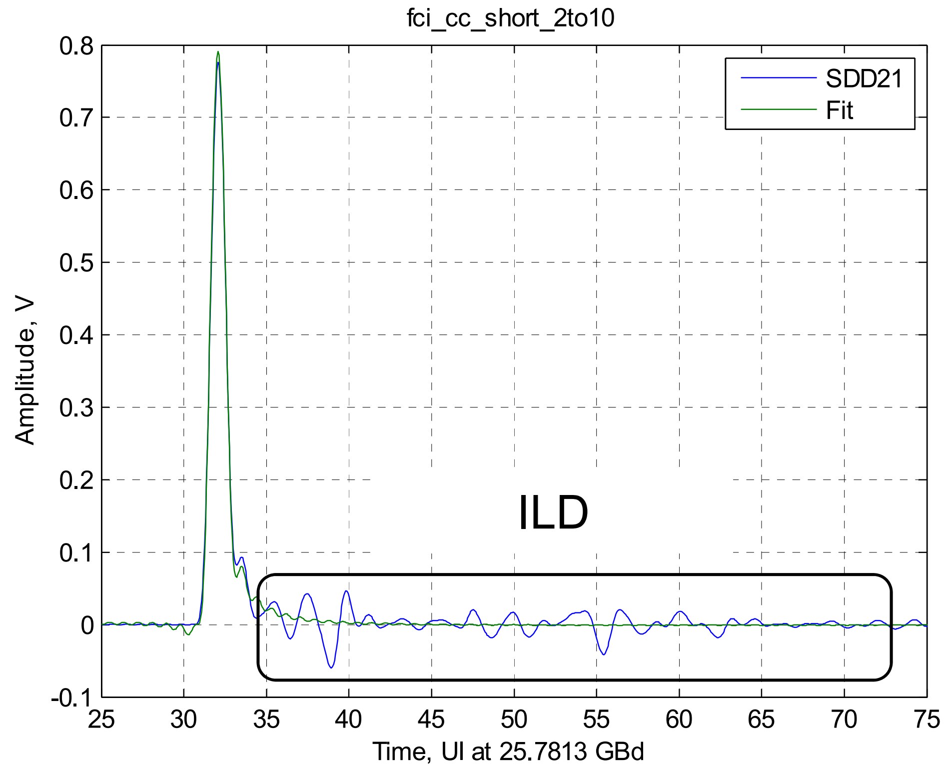
$$\gamma^{lms} = (F_w^T F_w)^{-1} F_w^T y$$

$$x^{lms} = \gamma_0^{lms} + \gamma_1^{lms}\sqrt{f} + \gamma_2^{lms} f + \gamma_4^{lms} f^2 \qquad H_{fit} = e^{x^{lms}}$$

- To avoid undue influence from signals which contribute little noise, the range of the fit is limited such that $f_N \leq f_{max}$
- The value of $\log(H(f))$ is ambiguous but the form used here “unwraps” the phase
  - The magnitude may be fit independently from the phase by substituting the magnitude of $H(f)$ for $H(f)$ in the expressions above to yield α^{lms}
  - The unwrapped phase may then be fit directly to yield β^{lms}

ILD noise example

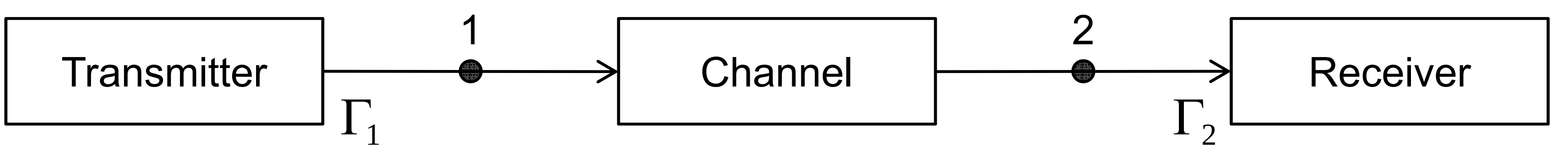


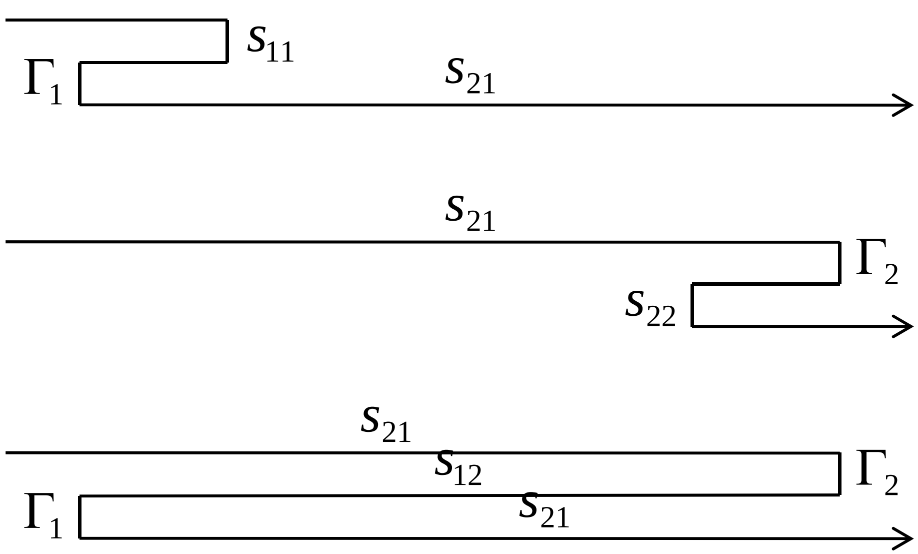


Re-reflection interference (noise)

- Transmitter, receiver, and channel return loss influence the transfer function of the assembled link

$$S = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$$





<u>Transmitter re-reflection</u>

$$H_{rrtx} = s_{11}\Gamma_1 s_{21}$$

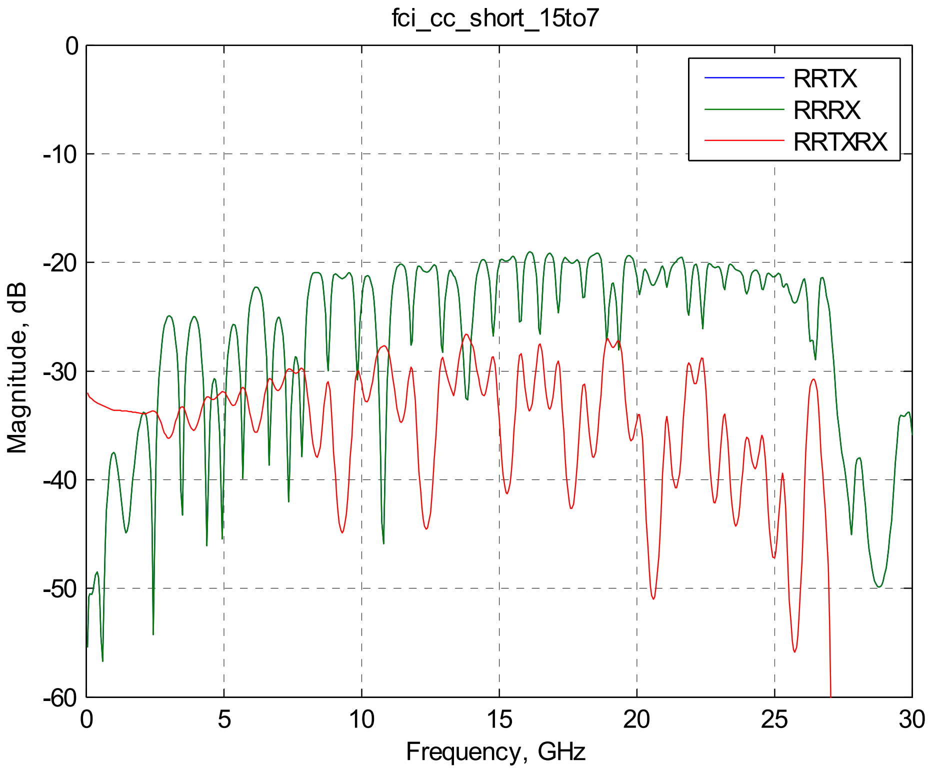
<u>Receiver re-reflection</u>

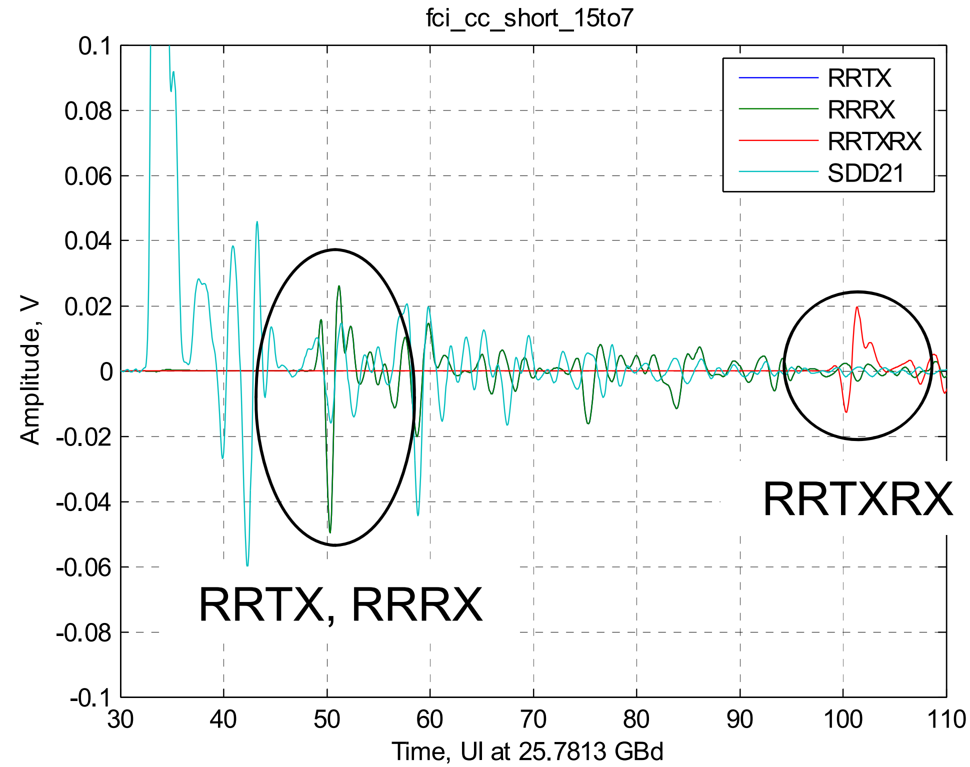
$$H_{rrrx} = s_{21}\Gamma_2 s_{22}$$

<u>Transmitter-receiver re-reflection</u>

$$H_{rrtxrx} = s_{21}\Gamma_2 s_{12}\Gamma_1 s_{21}$$

Re-reflection noise example





Calculating and combining noise

- Calculate the power of each noise term using the ICN integral
- Assume noise sources are statistically independent

$$\sigma_{ch}^2 = \sigma_{fx}^2 + \sigma_{nx}^2 + \sigma_{ild}^2 + \sigma_{rrtx}^2 + \sigma_{rrrx}^2 + \sigma_{rrtxrx}^2$$

σ_{fx}^2	$\sigma_{fx}^2 = 2\Delta f \sum_n W_{ft}(f_n) 10^{-MDFEXT_{loss}/10}$
σ_{nx}^2	$\sigma_{nx}^2 = 2\Delta f \sum_n W_{nt}(f_n) 10^{-MDNEXT_{loss}/10}$
σ_{ild}^2	$\sigma_{ild}^2 = 2\Delta f \sum_n W_t(f_n) \left\|H_{ild}(f_n)\right\|^2$
σ_{rrtx}^2	$\sigma_{rrtx}^2 = 2\Delta f \sum_n W_t(f_n) \left\|H_{rrtx}(f_n)\right\|^2$
σ_{rrrx}^2	$\sigma_{rrtx}^2 = 2\Delta f \sum_n W_t(f_n) \left\|H_{rrrx}(f_n)\right\|^2$
σ_{rrtxrx}^2	$\sigma_{rrtxrx}^2 = 2\Delta f \sum_n W_t(f_n) \left\|H_{rrtxrx}(f_n)\right\|^2$

Weighting functions:

$$W_t(f_n) = \left[\frac{A_t^2}{1+(f_n/f_t)^4}\right]\left[\frac{1}{1+(f_n/f_r)^8}\right]P(f_n)$$

$$W_{ft}(f_n) = \left[\frac{A_{ft}^2}{1+(f_n/f_{ft})^4}\right]\left[\frac{1}{1+(f_n/f_r)^8}\right]P(f_n)$$

$$W_{nt}(f_n) = \left[\frac{A_{nt}^2}{1+(f_n/f_{nt})^4}\right]\left[\frac{1}{1+(f_n/f_r)^8}\right]P(f_n)$$

$$P(f) = \frac{1}{f_b}\left[\frac{\sin(\pi f/f_b)}{\pi f/f_b}\right]^2$$

Implementation penalty

- Numerous other effects will limit the achievable SNR but are difficult to predict without detailed simulation
- These effects include but are not limited to:
  - Transmitter jitter
  - Clock recovery error and jitter
  - Minimum slicer overdrive
  - Thermal noise and receiver noise figure
  - Baseline wander
  - Limitations on the accuracy of practical equalizers
  - "Real world" limitations on bandwidth, linearity, etc. of components
- Margin needs to built into the SNR estimate to allow for these effects

Amplitude penalty	**Noise penalty**
$A_s = A_{dibit} g_{ip}$	$\sigma_n^2 = \sigma_{ch}^2 + \sigma_{ip}^2$

Target SNR

- The SNR may now be computed as $A_s \,/\, \sigma_n$
- The SNR that is required for NRZ to provide a SER better than 10^{-12} may be calculated using the inverse error function
  - SNR_2 is approximately 7.03
- For a fixed differential output voltage, multi-level modulation (L-PAM):
  - yields a reduction in the spacing between adjacent levels
  - yields somewhat smaller RMS interference amplitude
  - allows more bits per symbol hence lower symbol rates

Symbol rate	**Target SNR**
$f_b = R/\log_2(L)$	$SNR_L = SNR_2\sqrt{\frac{3}{L^2-1}}$

- Note that inner signal levels have a higher propensity for error and in the limit of large L this effectively doubles the SER
  - For an SER of 10^{-12}, this could increase the target SNR by 0.1 dB

Comparison to Salz SNR

- Salz SNR represents an upper bound on the performance of decision feedback equalizers
  - Readily computed from the channel insertion loss to crosstalk ratio (ICR)
  - However, practical equalizers will not perform as well
- Salz SNR is not very sensitive to pre-cursor ISI, ILD, and re-reflection
  - These can be significant error terms in this analysis
  - 10GBASE-KR ICR limit assumes a 3 dB SNR penalty due to ILD (and re-reflection?)
  - When considered in detail using this method, the penalty may actually be much greater
- Salz SNR should also be adjusted by an implementation penalty
  - Implementation penalty is explicitly included by this method

Conclusions

- Link operation at less than a given symbol error ratio can be estimated from a determination of the SNR
- The SNR can be derived from characteristics which are represented by collections of scattering parameters
- Reasonable suggestions for these calculations have been shown
- The authors recommend:
  - The Study Group use this method, including reasonable refinements, as part of the process to determine channel reach objectives
  - Should a Task Force be formed, use this method (and refinements) as the framework for defining channel performance requirements

Appendix A

Derivation of formulae

Dibit amplitude

The signal measure is dibit amplitude which is the product of the peak-to-peak transmitted amplitude and the dibit gain.

$$A_{dibit} = A_t g_{dibit} \qquad \text{A.1}$$

A dibit is a signal which consists of a unit positive pulse 1 unit interval (UI) long followed by a unit negative pulse 1 UI long. The dibit gain is the peak of the response of a system to a dibit. In this case, the system consists of the transmitter low pass filter, which accounts for the transmitter rise time, the channel, and the receiver low pass filter. Dibit gain can be calculated with a time domain simulator or in the frequency domain by performing the integral:

$$dibit(\tau) = 2\int_0^{\infty} \left[2j \frac{\sin^2(\pi f / f_b)}{\pi f} \right] H_t(f) SDD21(f) H_r(f) e^{j2\pi f \tau} df \qquad \text{A.2}$$

Dibit amplitude, continued

If the transmitter and receiver low pass filters are real, i.e. zero phase, then the peak of the dibit will occur near:

$$\tau_e = \mathrm{imag}\left[\frac{\gamma_1}{2\sqrt{0.38 f_b}} + \gamma_2 + 2\gamma_4 (0.38 f_b)\right] + 0.5 \qquad \text{A.3}$$

The coefficients γ_i are derived as shown on slide 9. The units of τ_e are unit intervals.

When computed this way, the dibit gain will usually be accurate to better than 0.5 dB. For better accuracy, values at τ_e ± 0.2 UI can be computed and the peak found by quadratic fitting.

Insertion loss deviation (ILD) noise

The objective is to find the set of coefficients γ^{lms} that minimize the mean squared error between H and e^x.

$$\varepsilon_n = H(f_n) - e^{x(f_n)} \qquad \text{A.4}$$

$$x^{lms} = \gamma_0^{lms} + \gamma_1^{lms}\sqrt{f} + \gamma_2^{lms} f + \gamma_4^{lms} f^2 \qquad \text{A.5}$$

Begin by linearizing e^x around γ^{lms}.

$$e^x \cong e^{x^{lms}} + \sum_i (\gamma_i - \gamma_i^{lms}) \left.\frac{\partial e^x}{\partial \gamma_i}\right|_{\gamma^{lms}} \qquad \text{A.6}$$

Expansion of Equation A.6 yields Equation A.7 and Equation A.8.

$$e^x \cong e^{x^{lms}} + (\gamma_0 - \gamma_0^{lms})e^{x^{lms}} + (\gamma_1 - \gamma_1^{lms})e^{x^{lms}}\sqrt{f} + (\gamma_2 - \gamma_2^{lms})e^{x^{lms}} f + (\gamma_4 - \gamma_4^{lms})e^{x^{lms}} f^2 \qquad \text{A.7}$$

$$e^x \cong e^{x^{lms}} + (\gamma_0 + \gamma_1\sqrt{f} + \gamma_2 f + \gamma_4 f^2)e^{x^{lms}} - (\gamma_0^{lms} + \gamma_1^{lms}\sqrt{f} + \gamma_2^{lms} f + \gamma_4^{lms} f^2)e^{x^{lms}} \qquad \text{A.8}$$

Insertion loss deviation (ILD) noise, continued

If the model is a good representation of the channel over the frequency range of interest and the error is small, it can be assumed that:

$$H \cong e^{x^{lms}} \quad \text{A.9}$$

$$\log(H) \cong x^{lms} \quad \text{A.10}$$

Substituting Equations A.9 and A.10 into Equation A.8:

$$e^{x} \cong H(f) + (\gamma_0 + \gamma_1\sqrt{f} + \gamma_2 f + \gamma_4 f^2)H(f) - \log(H(f))H(f) \quad \text{A.11}$$

Substituting A.11 into the expression for the fit error Equation A.4 yields:

$$\varepsilon_n \cong \log(H(f_n))H(f_n) - (\gamma_0 + \gamma_1\sqrt{f_n} + \gamma_2 f_n + \gamma_4 f_n^2)H(f_n) \quad \text{A.12}$$

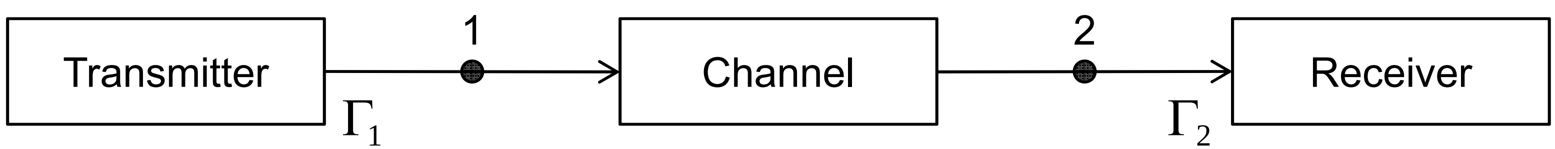
Equation A.12 is the basis for determining the best fit coefficients γ^{lms} as shown on slide 9.

Re-reflection noise

So far, the channel transfer function has been assumed to be SDD21 which is measured under the condition of the transmitter and receiver are terminated by the reference impedance. When terminated with a transmitter with output reflection coefficient Γ_1 and a receiver with input reflection coefficient Γ_2, the transfer function from port 1 to port 2 becomes:

$$H_{12} = \frac{s_{21}}{1-\left(\Gamma_1 s_{11} + \Gamma_2 s_{22} + \Gamma_1 \Gamma_2 s_{21} s_{12} - \Gamma_1 \Gamma_2 s_{11} s_{22}\right)} \quad \text{A.13}$$

$$S = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$$



Re-reflection noise, continued

Given that 1/(1−x) is approximately 1+x when |x| << 1, the transfer function is approximately:

$$H_{12} \cong s_{21}\left(1+\Gamma_1 s_{11}+\Gamma_2 s_{22}+\Gamma_1\Gamma_2 s_{21}s_{12}-\Gamma_1\Gamma_2 s_{11}s_{22}\right) \qquad \text{A.14}$$

Under this assumption, the magnitude of the denominator is close to 1 so these reflection terms have negligible effect on the dibit gain. Note that the last term in Equation A.14 is the product of the of second and third terms, which are assumed to be small, and may be safely ignored. In addition, knowing that s_{21} = s_{21} yields:

$$H_{12} \cong s_{21}+s_{11}s_{21}\Gamma_1+s_{21}s_{22}\Gamma_2+s_{21}^3\Gamma_1\Gamma_2 \qquad \text{A.15}$$

The last three terms are called the transmitter re-reflection, receiver re-reflection, and transmitter-receiver re-reflection respectively. An intuitive interpretation of these terms is given on slide 11.

Re-reflection noise computation

It would be possible to compute the true transfer function if all of the S-parameters for the channel and the complex reflection coefficients of the transmitter and receiver were known. In general, the provider of the channel will only have access to limits on the magnitude of the reflection coefficients.

Appendix B

Parameters for calculations

General parameters

Parameter	Symbol	Value
Symbol rate, GHz	f_b	Variable
Victim differential output amplitude, mV peak	A_t	400
Victim transmitter 3 dB bandwidth, GHz	f_t	0.55 x f_b
Far-end disturber differential output amplitude, mV peak	A_{ft}	400
Far-end disturber 3 dB bandwidth, GHz	f_{ft}	0.55 x f_b
Near-end disturber differential output amplitude, mV peak	A_{nt}	600
Near-end disturber 3 dB bandwidth, GHz	f_{nt}	1.00 x f_b
Receiver 3 dB bandwidth, GHz	f_r	0.75 x f_b
Maximum frequency for transfer function fit, GHz	f_{max}	0.75 x f_b
Implementation amplitude penalty, V/V	g_{ip}	0.667
Implementation noise penalty, mV	σ_{ip}	$0.024A_t\sqrt{g_{dibit}}$

Transmitter and receiver reflection coefficients

Parameter	Symbol	Value
Transmitter DC reflection coefficient, V/V	g_{01}	0.161
Transmitter return loss reference frequency, GHz	f_1	1.25 x f_b
Receiver DC reflection coefficient, V/V	g_{02}	0.161
Receiver return loss reference frequency, GHz	f_2	1.25 x f_b

$$\left|\Gamma_1\right|^2 = \frac{g_{01}^2 + (f/f_1)^2}{1 + (f/f_1)^2} \quad \text{B.1}$$

$$\left|\Gamma_2\right|^2 = \frac{g_{02}^2 + (f/f_2)^2}{1 + (f/f_2)^2} \quad \text{B.2}$$