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ISI, RIN and MPN Modeling: Some Clarifications

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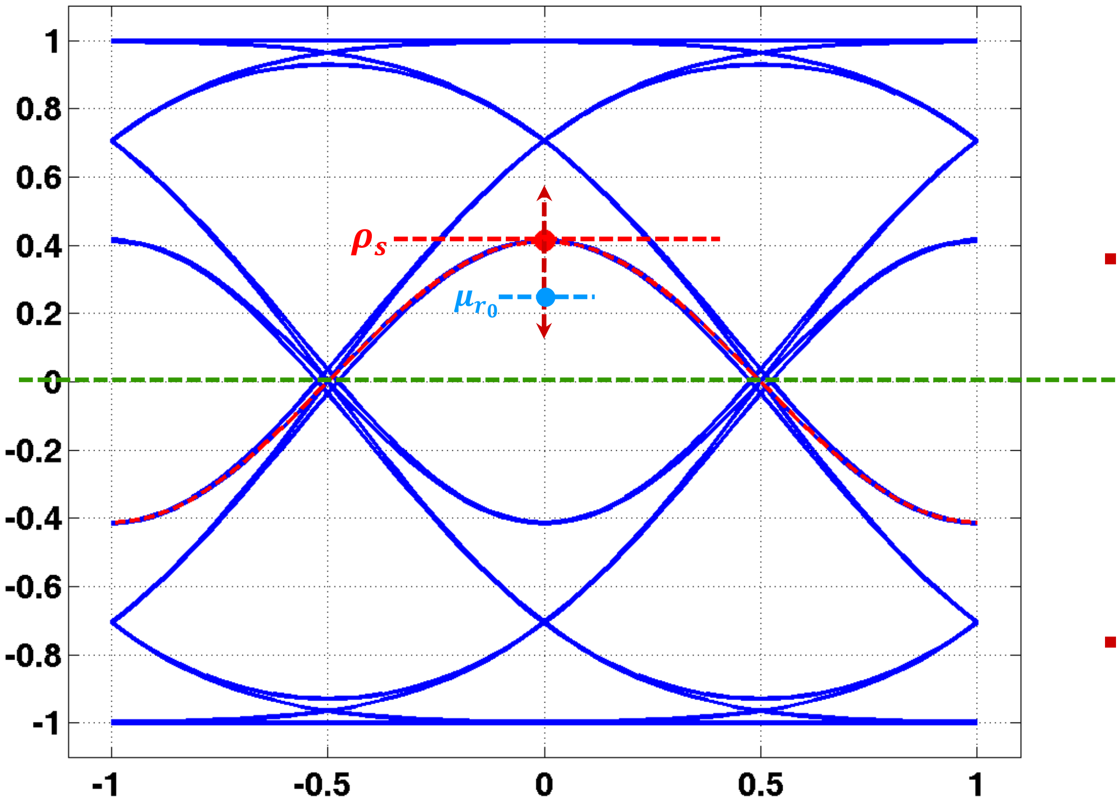
ISI, MPN and RIN: the Big Picture



- ρ_s: **ISI with a "single-mode VCSEL"**
  - Worst-case received waveform due to "single-mode VCSEL" $\approx \rho_s \cos(\pi B t)$
  - Red-curve in eye-diagram
  - Implicit in Ogawa-Agrawal model
- μ_{r_0}: **ISI with a multi-moded VCSEL**
  - $\mu_{r_0} = \rho_s \rho_m$, where the factor ρ_m scales the ISI in single-mode case to that of a MM source
  - ρ_m: additional ISI due to multi-mode VCSEL
- **Contributions to variance of received sample r_0:**
  - $\sigma_{r_0}^2$: due to MPN
  - $\sigma_{RIN-OMA}^2$: due to RIN
  - σ_{th}^2: due to thermal noise

ISI, MPN, RIN Penalties

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- **Model:** $r_k = S\mu_{r_0} + n_{th,k} + S n_{RIN-OMA,k} + S n_{MPN-OMA,k}$

$$n_{th,k} \sim \mathcal{N}(0, \sigma_{th}^2)$$
$$n_{RIN-OMA,k} \sim \mathcal{N}(0, \sigma_{RIN-OMA}^2)$$
$$n_{MPN-OMA,k} \sim \mathcal{N}(0, \sigma_{r_0}^2) \qquad S\text{: OMA}$$

- **Total Penalty (ISI + MPN + RIN):** $P_{ISI+MPN+RIN} = -5\log_{10}\left[\mu_{r_0}^2 - Q_{opt}^2\left(\sigma_{RIN-OMA}^2 + \sigma_{r_0}^2\right)\right]$
  - Q_{opt}: system Q

- **Can separate out penalties:** $P_{ISI+MPN+RIN} = P_{ISI} + P_{MPN} + P_{RIN} + P_{cross}$

$$P_{ISI} = -10\log_{10}\left[\mu_{r_0}\right]$$

$$P_{MPN} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{r_0}^2}{\mu_{r_0}^2}\right)\right]$$

$$P_{RIN} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{RIN-OMA}^2}{\mu_{r_0}^2}\right)\right]$$

$$P_{cross} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{RIN-OMA}^2 + \sigma_{r_0}^2}{\mu_{r_0}^2}\right)\right] - P_{RIN} - P_{MPN}$$

Scaling of RIN and MPN Penalties

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$$P_{RIN} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{RIN-OMA}^2}{\mu_{r_0}^2}\right)\right] \qquad P_{MPN} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{r_0}^2}{\mu_{r_0}^2}\right)\right]$$

- **Both RIN and MPN should be normalized by total ISI** $= \mu_{r_0} = \rho_s\rho_m$
- **RIN: Spreadsheet does normalize RIN std. dev. by total ISI** $= \mu_{r_0}$ **→ correct**
- **Mode Partition Noise:**
  - Ogawa-Agrawal model for MPN already normalizes σ_{r_0} by ρ_s
  - Consistent with OA-model, spreadsheet uses $\sigma_{MPN-OA} = \sigma_{r_0}/\rho_s$ → not correct
  - Therefore, MPN std. dev. in the spreadsheet σ_{MPN-OA} requires scaling by ρ_m
  - If we were to normalize σ_{MPN-OA} by μ_{r_0}, we would have effectively normalized $\sigma_{r_0}$by $\rho_s^2\rho_m$ → double counts ρ_s
- **spreadsheet Summary:**
  - RIN treatment accurate in spreadsheet
  - MPN std. dev. requires scaling by ρ_m (additional ISI due to multi-moded nature of VCSEL), identified in lingle_01_0512 and following.

Conclusions

- **Have re-derived ISI, MPN and RIN penalties from first principles to resolve issues related to correct scaling factors in the spreadsheet**

- **Shown that the scaling factor for RIN in the spreadsheet is correct**

- **MPN treatment in spreadsheet is consistent with Ogawa-Agrawal model**

- **Shown that the MPN std. dev. in the OA model (and current version of spreadsheet) σ_{MPN-OA} needs to be further scaled by ρ_m, the additional ISI due to the multi-moded nature of the VCSEL**
  - With the mode continuum approximation and Gaussian VCSEL spectrum, it can be shown that $\rho_m = e^{-\beta^2/2}$ where $\beta = \pi BDL\sigma_\lambda$

- **Shown that while in general both RIN and MPN penalties require the same scaling factors (= total ISI), these factors should be different in the spreadsheet due to how various variances are defined and partially pre-normalized**

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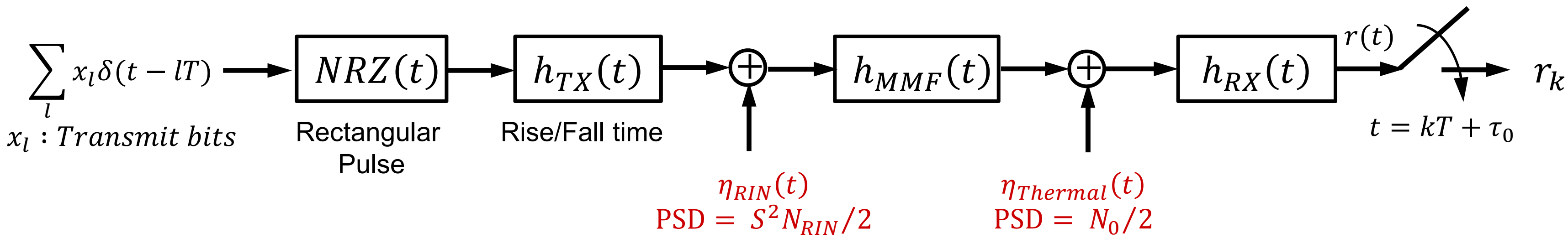
Detailed Analysis

Link Model



- **Received waveform given by:** $r(t) = S\sum_l x_l h(t - lT) + n_{th}(t) + n_{RIN}(t)$ **where:**
  - S is the OMA
  - End-to-end link response: $h(t) = NRZ(t) \star h_{TX}(t) \star h_{MMF}(t) \star h_{RX}(t)$
  - Thermal noise: $n_{th}(t) = \eta_{thermal}(t) \star h_{RX}(t)$
  - RIN: $n_{RIN}(t) = \eta_{RIN}(t) \star h_{MMF}(t) \star h_{RX}(t)$
- **spreadsheet uses Gaussian approximations for the filters** $h_{TX}(t), h_{MMF}(t)$ **and** $h_{RX}(t)$**and so the end-to-end link response is:**

$$h(t) = Q\left(\frac{t - T/2}{\sigma_c}\right) - Q\left(\frac{t + T/2}{\sigma_c}\right) \qquad \sigma_c = \frac{T_c}{C_1} \cdot \frac{\sqrt{0.6 \log 10}}{2\pi} \qquad Q(t) = \frac{1}{2}\operatorname{erfc}\left(\frac{t}{\sqrt{2}}\right)$$

$$T_c^2 = T_{TX}^2 + C_1^2\left[\frac{1}{BW_{CD}^2} + \frac{1}{BW_{ME}^2} + \frac{0.5}{BW_{RX}^2}\right] \quad C_1 = \frac{\sqrt{0.6 \log 10}}{2\pi} \cdot [Q^{-1}(0.1) - Q^{-1}(0.9)] = 479.5\text{ns} \cdot \text{MHz}$$

  - T_{TX} is the 10%-90% rise-time of the transmit pulse
  - BW_{CD}, BW_{ME} are the chromatic and modal bandwidths of the fiber
  - BW_{RX} is the receiver bandwidth

Inter-symbol Interference

- **Received samples:** $r_k = S\sum_l h_l x_{k-l} + n_{th,k} + n_{RIN,k}$

- **spreadsheet approximates end-to-end link response by 3 T-spaced taps:**

$$r_k \approx S \cdot [h_0 x_k + h_1 x_{k-1} + h_{-1} x_{k+1}] + n_{th,k} + n_{RIN,k} \qquad h_k = h(kT)$$

- **Assuming $x_k = 1$, the received sample corresponding to the worst-case ISI is given by:**

$$r_k \approx S \cdot [h_0 - h_1 - h_{-1}] + n_{th,k} + n_{RIN,k}$$

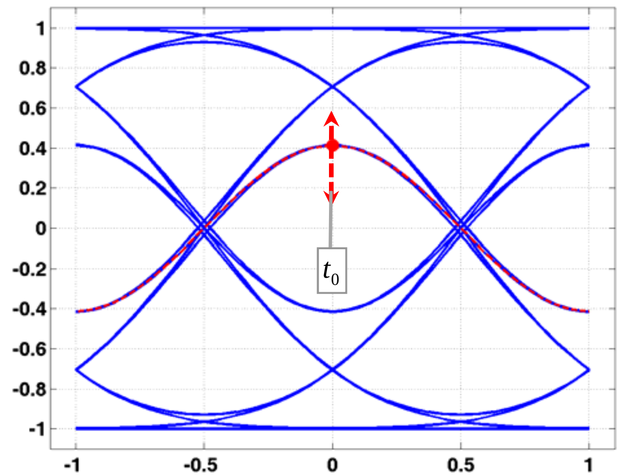
  - where S is the OMA

- **Note that** $h_0 = 1 - 2Q\left(\frac{T}{2\sigma_c}\right)$ $\qquad h_1 = h_{-1} = Q\left(\frac{T}{2\sigma_c}\right) - Q\left(\frac{3T}{2\sigma_c}\right) \approx Q\left(\frac{T}{2\sigma_c}\right) = \frac{1-h_0}{2}$

- **Therefore, the worst-case received sample without MPN is:**

$$r_k \approx S\rho_0 + n_{th,k} + n_{RIN,k} \qquad \rho_0 = 2h_0 - 1$$

Mode Partition Noise Modeling



- **Define ρ_s = worst-case ISI with single-moded VCSEL (Normalized to OMA S)**
- **Can approximate worst-case received waveform by** $\rho_s\cos(\pi Bt)$ (red curve)
- **Received sample due to VCSEL mode i is given by** $\rho_i = \rho_s\cos(\pi BDL[\lambda_i - \lambda_0])$
  - Mode i of the VCSEL induces a delay $\Delta t_i = DL[\lambda_i - \lambda_0]$
- **Therefore, worst-case received sample with multi-moded VCSEL is:**

$$r_0 = \sum_i a_i \rho_i \quad \rho_i = \rho_s\cos(\pi BDL[\lambda_i - \lambda_0])$$

  - a_i : relative VCSEL mode powers
- r_0 **fluctuates due to variations in relative VCSEL mode powers → MPN model**

$$r_0 \approx \mu_{r_0} + n_{MPN-OMA} \quad n_{MPN-OMA} \sim \mathcal{N}\left(0, \sigma_{r_0}^2\right)$$

MPN modeling contd.

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- **Assume continuum of VCSEL modes:**

$$r_0 = \int_{-\infty}^{\infty} a(\lambda)\rho(\lambda)d\lambda, \qquad \rho(\lambda) = \rho_s \cos(\pi BDL[\lambda - \lambda_0])$$

$$\mu_{r_0} = \int_{-\infty}^{\infty} \bar{a}(\lambda)\rho(\lambda)d\lambda, \qquad \sigma_{r_0}^2 = k_{MPN}^2 \left[\int_{-\infty}^{\infty} \bar{a}(\lambda)\rho^2(\lambda)d\lambda - \mu_{r_0}^2\right]$$

- **Assume Gaussian VCSEL spectrum:**

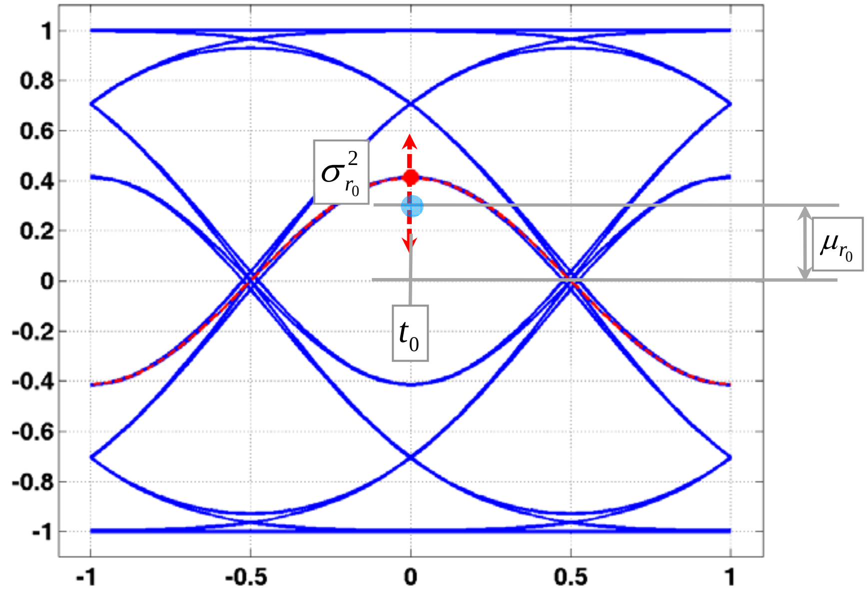
$$\bar{a}(\lambda) = \frac{1}{\sqrt{2\pi\sigma_\lambda^2}} \exp\left[-\frac{1}{2}\left(\frac{\lambda - \lambda_0}{\sigma_\lambda}\right)^2\right], \qquad \sigma_\lambda\text{: RMS spectral−width}$$

- **Mean and variance of received sample given by:**

$$\mu_{r_0} = \rho_s\rho_m, \qquad \rho_m = \exp\left[-\frac{\beta^2}{2}\right], \qquad \sigma_{r_0}^2 = \frac{\rho_s^2 k_{MPN}^2}{2}\left[1 - e^{-\beta^2}\right]^2, \qquad \beta = \pi BDL\sigma_\lambda$$

MPN Modeling contd.

- **What does $\mu_{r_0} = \rho_s \rho_m = \rho_s e^{-\beta^2/2} \leq \rho_s$ mean?**



- **Explanation:**
  - Red dot: received sample with single-moded VCSEL
  - Blue dot: received sample with multi-moded VCSEL with mean mode powers
  - $\rho_m = e^{-\beta^2/2}$ denotes the drop from the red-dot to the blue-dot → additional ISI induced due to multi-moded nature of VCSEL
- **Note that $\mu_{r_0} = \rho_s \rho_m = \rho_s e^{-\beta^2/2} \approx \rho_0 = 2h_0 - 1$ = total ISI with a multi-moded VCSEL**

Relative Intensity Noise

- **RIN power spectral density at the receiver:** $S_{n_{RIN}}(f) = S^2|G(f)|^2 N_{RIN}/2$
  - where $g(t) = h_{MMF}(t) \star h_{RX}(t)$

- **Easy to show that**

$$g(t) = \frac{1}{\sqrt{2\pi\sigma_g^2}} \exp\left[-\frac{1}{2}\left(\frac{t}{\sigma_g}\right)^2\right] \qquad \sigma_g = \frac{\sqrt{0.6\log 10}}{2\pi} \cdot \sqrt{\frac{1}{BW_{CD}^2} + \frac{1}{BW_{ME}^2} + \frac{0.5}{BW_{RX}^2}}$$

- **RIN variance can be computed from:**

$$\sigma_{RIN}^2 = \int_{-\infty}^{\infty} S_{n_{RIN}}(f)df = \frac{S^2 N_{RIN}}{2}\int_{-\infty}^{\infty}|G(f)|^2 df = \frac{S^2 N_{RIN}}{2}\int_{-\infty}^{\infty}|g(t)|^2 dt$$

- **Straight-forward to show that RIN variance is**

$$\sigma_{RIN}^2 = S^2\sigma_{RIN-OMA}^2, \qquad \sigma_{RIN-OMA}^2 = \frac{N_{RIN}}{4\sqrt{\pi}\sigma_g} \Rightarrow n_{RIN,k} = S \cdot n_{RIN-OMA,k}$$

  - $\sigma_{RIN-OMA}^2$ is the RIN variance normalized to OMA

ISI + MPN + RIN Penalty

- **Worst-case received sample with MPN is:** $r_k = Sr_0 + n_{th,k} + Sn_{RIN-OMA,k}$
- **But the MPN model gives:** $r_0 = \mu_{r_0} + n_{MPN-OMA,k}$
- **Therefore, final model for received sample is:**

$$r_k = S\mu_{r_0} + n_{th,k} + Sn_{RIN-OMA,k} + Sn_{MPN-OMA,k}$$

- **All three noise sources are assumed to zero-mean, white Gaussian:**
  - Thermal noise $n_{th,k} \sim \mathcal{N}\left(0, \sigma_{th}^2\right)$
  - RIN (normalized to OMA) $n_{RIN-OMA,k} \sim \mathcal{N}\left(0, \sigma_{RIN-OMA}^2\right)$
  - MPN (normalized to OMA) $n_{MPN-OMA,k} \sim \mathcal{N}\left(0, \sigma_{r_0}^2\right)$
- **System Q** $(= Q_{opt})$ **given by:** $Q_{opt}^2 = \dfrac{S^2\mu_{r_0}^2}{\sigma_{th}^2 + S^2\sigma_{RIN-OMA}^2 + S^2\sigma_{r_0}^2}$
- **System Q without any ISI, MPN or RIN:** $Q_{opt}^2 = \dfrac{S_0^2}{\sigma_{th}^2}$
  - Link model (here) is: $r_k = S_0 + n_{th,k}$
- **Therefore, total link penalty (ISI + MPN + RIN) =**

$$P_{ISI+MPN+RIN} = 10\log_{10}\left(\frac{S}{S_0}\right) = -5\log_{10}\left[\mu_{r_0}^2 - Q_{opt}^2\left(\sigma_{RIN-OMA}^2 + \sigma_{r_0}^2\right)\right]$$

Separate ISI, MPN, RIN Penalties

- **Can separate out individual penalties as follows:**

$$P_{ISI+MPN+RIN} = -10\log_{10}[\mu_{r_0}] - 5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{RIN-OMA}^2 + \sigma_{r_0}^2}{\mu_{r_0}^2}\right)\right]$$

$$P_{ISI} = -10\log_{10}[\mu_{r_0}]$$

$$P_{MPN} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{r_0}^2}{\mu_{r_0}^2}\right)\right]$$

$$P_{RIN} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{RIN-OMA}^2}{\mu_{r_0}^2}\right)\right]$$

$$P_{cross} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{RIN-OMA}^2 + \sigma_{r_0}^2}{\mu_{r_0}^2}\right)\right] - P_{RIN} - P_{MPN}$$

$$P_{ISI+MPN+RIN} = P_{ISI} + P_{MPN} + P_{RIN} + P_{cross}$$

RIN Penalty and ISI Scaling

- **RIN penalty:**

$$P_{RIN} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{RIN-OMA}^2}{\mu_{r_0}^2}\right)\right]$$

- **Requires scaling of RIN std. dev. by total ISI** $\mu_{r_0} = \rho_s\rho_m$
  - $\mu_{r_0} = \rho_s\rho_m = \rho_0 = 2h_0 - 1$

- **Consistent with the current version of the spreadsheet**

- **spreadsheet accurately captures RIN penalty**

MPN Penalty and ISI Scaling

- **MPN penalty:**

$$P_{MPN} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{r_0}^2}{\mu_{r_0}^2}\right)\right]$$

$$\mu_{r_0} = \rho_s\rho_m, \qquad \rho_m = e^{-\beta^2/2}, \qquad \sigma_{r_0}^2 = \frac{\rho_s^2 k_{MPN}^2}{2}\left[1 - e^{-\beta^2}\right]^2$$

- **Requires scaling of r_0 std. dev. by total ISI $\mu_{r_0} = \rho_s\rho_m$**
- **Scaling treatment of MPN penalty same as that of RIN penalty**
  - Both std. devs. should be scaled by the total ISI μ_{r_0}

MPN Penalty and ISI Scaling contd.

- **However, Ogawa-Agrawal model (and current version of spreadsheet) uses**

$$P_{MPN-OA} = -5\log_{10}\left[1 - Q_{opt}^2\sigma_{MPN-OA}^2\right] \qquad \sigma_{MPN-OA}^2 = \frac{k_{MPN}^2}{2}\left[1 - e^{-\beta^2}\right]^2$$

- **Effectively uses** $\sigma_{MPN-OA} = \sigma_{r_0}/\rho_s$ **instead of** σ_{r_0}/μ_{r_0}

- **Therefore, scaling is currently done with ISI due to a single-moded VCSEL,** ρ_s **→ not with the total ISI** μ_{r_0}
  - OA-model (and so spreadsheet) ignores the additional ISI induced by multi-moded VCSEL, ρ_m

- **Require additional scaling of** σ_{MPN-OA} **by** $\rho_m = e^{-\beta^2/2}$ **in OA model (and current version of spreadsheet) to get correct MPN penalty**

- **Implies that scaling factor should be different for RIN and MPN in the current spreadsheet because of how various quantities are defined and partially pre-normalized**

Conclusions

- **Have re-derived ISI, MPN and RIN penalties from first principles to resolve issues related to correct scaling factors in the spreadsheet**
- **Shown that the scaling factor for RIN in the spreadsheet is correct**
- **MPN treatment in spreadsheet is consistent with Ogawa-Agrawal model**
- **Shown that the MPN std. dev. in the OA model (and current version of spreadsheet) σ_{MPN-OA} needs to be scaled by $\rho_m = e^{-\beta^2/2}$ to get correct MPN penalties**
- **Shown that while in general both RIN and MPN penalties require the same scaling factors (= total ISI), these factors should be different in the spreadsheet due to how various variances are defined and partially pre-normalized**