A Furukawa Company

Issues with Modeling Mode-Partition Noise (MPN) in VCSEL-MMF Links

Kasyapa Balemarthy
Robert Lingle Jr.

IEEE 802.3 Next Generation 40 Gb/s and 100 Gb/s Optical Ethernet Study Group, July 2012, San Diego

Issues with MPN Modeling and the Spreadsheet

- **Mode partition noise is a dominant impairment that limits reach of 25Gbps links**

- **Imperative that the MPN model is “correct” or at least “accurate enough”**
  - Eliminate glaring errors

- **Issues in MPN modeling:**
  - ISI scaling :
    - Implicit normalization in the Ogawa-Agrawal model for MPN
    - ISI scaling of MPN standard deviation in the spreadsheet
    - Consistency between RIN and MPN treatment in the spreadsheet
  - Bit pattern choice to model worst-case performance
    - As per spreadsheet philosophy

ISI Scaling of MPN and RIN in the spreadsheet

- **Ogawa-Agrawal (OA) model used to compute MPN penalty**
  - Spreadsheet simply borrows this expression

- **Ambiguity stems from lack of explicit description of normalization in the original OA-model formulation**

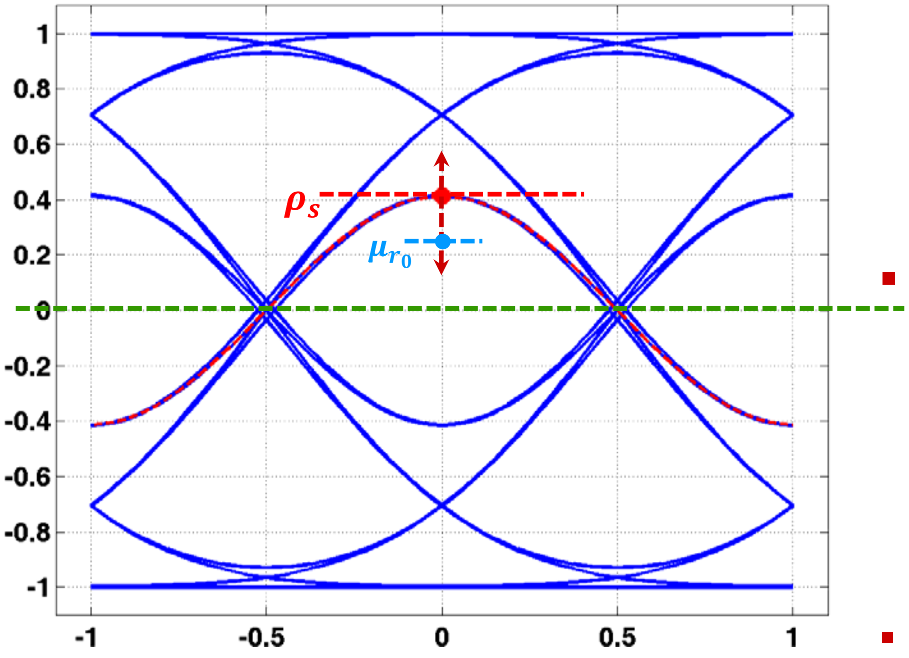
- **We adopt a first-principles approach and analytically derive the penalty for the link with MPN and RIN (including the OA model)**

- **We analytically show that scaling factors for RIN and MPN penalty computation should be the total ISI**

- **However, the OA-model already does implicit scaling of MPN with a part of the ISI**

- **So MPN in the spreadsheet requires scaling only by the rest of the ISI and NOT the total ISI as has been proposed**

ISI, MPN and RIN: the Big Picture



- ρ_s: **ISI with a "single-mode VCSEL"**
  - Received waveform due to "single-mode VCSEL"
  - Only ISI due to pulse-broadening due to CD (+ modal BW + TX/RX BW) present
  - Implicit starting point of Ogawa-Agrawal model → will establish this next
- μ_{r_0}: **ISI with a multi-moded VCSEL**
  - $\mu_{r_0} = \rho_s \rho_m$, where the factor ρ_m scales the ISI in single-mode case to that of a MM source
  - ρ_m: additional ISI due to delays induced by the wavelength dependent multiple VCSEL modes
- **Spreadsheet directly computes total ISI = μ_{r_0} for the inner-most eye**
- **Contributions to variance of received sample r_0:**
  - $\sigma^2_{r_0}$: due to MPN
  - $\sigma^2_{RIN-OMA}$: due to RIN
  - σ^2_{th}: due to thermal noise

One-step ISI Computation: Approach 1

- **Approach 1 used in the spreadsheet → total ISI**
- **RX waveform:** $r_c(t) = \sum_k x_k h_c(t - kT)$
  - x_k: k^{th} transmit bit, T : bit period
  - End-to-end impulse response: $h_c(t)$

$$h_c(t) = Q\left(\frac{t - T/2}{\sigma_c}\right) - Q\left(\frac{t + T/2}{\sigma_c}\right) \qquad Q(x) = \frac{1}{2}\operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right) \qquad \sigma_c = \frac{T_c}{C_1} \cdot \frac{\sqrt{0.6 \log 10}}{2\pi}$$

$$T_c^2 = T_{TX}^2 + C_1^2\left[\frac{1}{BW_{CD,c}^2} + \frac{1}{BW_{ME}^2} + \frac{0.5}{BW_{RX}^2}\right] \qquad C_1 = \frac{\sqrt{0.6 \log 10}}{2\pi} \cdot \left[Q^{-1}(0.1) - Q^{-1}(0.9)\right] = 479.5\text{ns} \cdot \text{MHz}$$

  - T_{TX}: TX rise-time, BW_{RX}: receiver bandwidth, BW_{ME}: modal bandwidth
  - $BW_{CD,c}$: chromatic dispersion bandwidth of the link

$BW_{CD,c} = 0.187/(DL\sigma_\lambda)$ σ_λ: VCSEL RMS spectral-width

- **$h_c(t)$ includes ISI due to:**
  - Pulse broadening due to chromatic dispersion
  - Delays induced by the wavelength dependent multiple VCSEL modes
  - Modal bandwidth
  - TX rise-time, RX bandwidth

Two-Step ISI Computation: Approach 2

A Furukawa Company

- Approach 2 is required for the Ogawa-Agrawal model
- **RX waveform:** $r_s(t) = \sum_k x_k h_s(t - kT)$**, End-to-end impulse response:** $h_s(t)$

$$h_s(t) = Q\left(\frac{t - T/2}{\sigma_s}\right) - Q\left(\frac{t + T/2}{\sigma_s}\right)$$

- **All link parameters same as in approach 1, except for** $BW_{CD,c}$ **being replaced by** $BW_{CD,s}$**:**

$BW_{CD,s} = 0.187/(DL(\Delta\lambda))$ $\quad \Delta\lambda \approx (B\lambda_0^2)/c$: spectral-width of NRZ TX pulse-shape

- $h_s(t)$ **includes all the ISI contributions from Approach 1, but does not include ISI due to delays induced by the wavelength dependent multiple VCSEL modes**

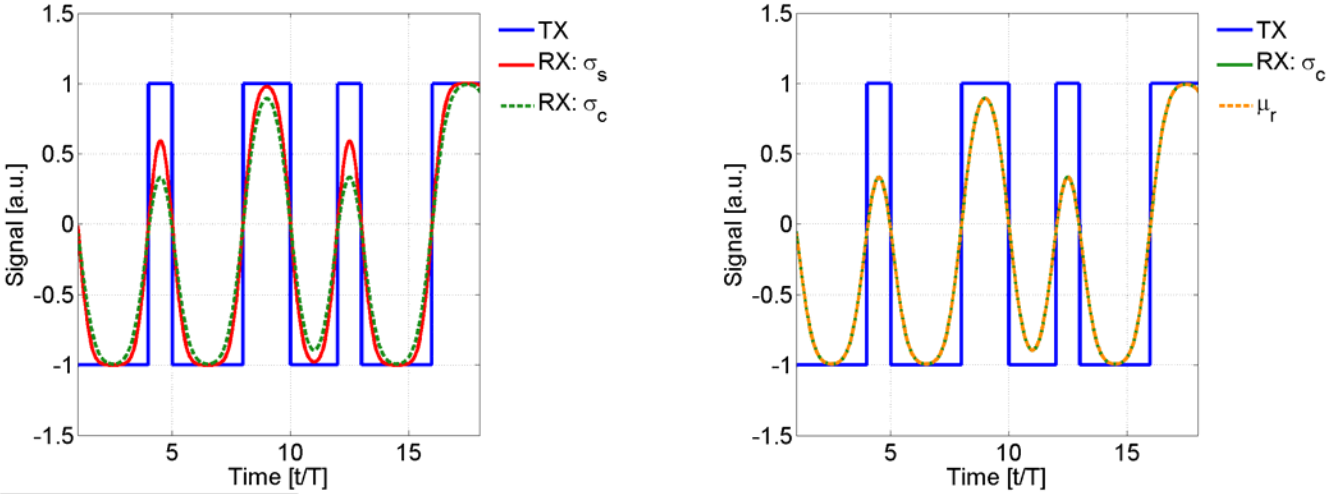
- **This remaining ISI factor is generated by applying the OA-model to the above RX waveform (to get mean and variance of composite RX waveform):**

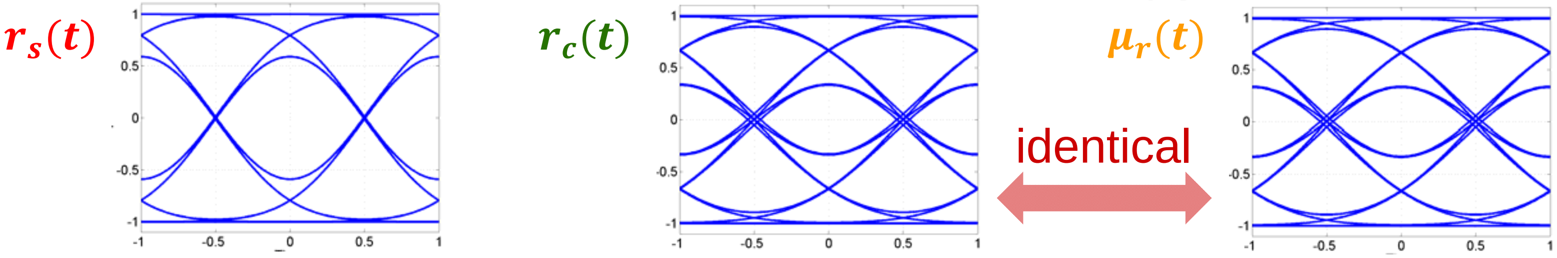
$$r(t) = \sum_{i=1}^{N} a_i\, r_s(t - \tau_i) \qquad \mu_r(t) = \sum_{i=1}^{N} \mu_i\, r_s(t - \tau_i) \qquad \sigma_r^2(t) = k_{MPN}^2 \left[\sum_{i=1}^{N} \mu_i\, r_s^2(t - \tau_i) - \mu_r^2(t)\right]$$

  - VCSEL has N modes: $i = 1, \ldots, N$ with instantaneous mode power a_i, mean mode power μ_i and center wavelengths λ_i with $\tau_i = DL[\lambda_i - \lambda_0]$
  - $\boldsymbol{\mu_r(t)}$ **is independent of** $\boldsymbol{k_{MPN}}$**, only** $\boldsymbol{\sigma_r(t)}$ **is proportional to** $\boldsymbol{k_{MPN}}$

- **Therefore,** $\mu_r(t)$ **includes all the ISI terms in Approach 1 and should be identical to** $r_c(t)$

Validation with continuous Gaussian VCSEL Spectrum for a 200m link





- **ISI due to delays induced by wavelength dependent VCSEL modes can be significant (drop from red curve to green curve)**
- $\mu_r(t) = r_c(t)$ → **ISI from Approaches 1 and 2 match (green & orange curves)**
- **Validation establishes that the starting point for the OA-model is $h_s(t)$ and NOT $h_c(t)$**
- **Caution: if we had computed the mean RX waveform $\mu_r(t)$ starting from $h_c(t)$ instead of $h_s(t)$ we would have over-estimated the ISI → incorrect**

Total Penalty (ISI, MPN, RIN)

- **Given $h_s(t)$, the bit sequence and the VCSEL spectrum, OA prescribes how to compute $\mu_r(t)$ and $\sigma_r^2(t)$: the mean and variance of the received waveform**
    - At the optimum sampling instant τ_0, $\mu_{r_0} = \mu_r(\tau_0)$, $\sigma_{r_0} = \sigma_r(\tau_0)$
- **How do μ_{r_0} and σ_{r_0} impact the link penalty? How is RIN incorporated?**

- **The RX sample can be modeled as:** $r_k = S\mu_{r_0} + n_{th,k} + Sn_{RIN-OMA,k} + Sn_{MPN-OMA,k}$
    - All three noise sources are assumed to be zero-mean, white Gaussian

      S: OMA
        - Thermal noise $n_{th,k} \sim N\left(0, \sigma_{th}^2\right)$; $N(\mu, \sigma^2)$: Gaussian random variable with mean μ & var. σ^2
        - RIN (normalized to OMA) $n_{RIN-OMA,k} \sim N(0, \sigma_{RIN-OMA}^2)$
        - MPN (normalized to OMA) $n_{MPN-OMA,k} \sim N\left(0, \sigma_{r_0}^2\right)$

- **System Q** $(= Q_{opt})$ **given by:** $Q_{opt}^2 = \dfrac{S^2\mu_{r_0}^2}{\sigma_{th}^2 + S^2\sigma_{RIN-OMA}^2 + S^2\sigma_{r_0}^2}$

  $BER = Q(Q_{opt})$

  $Q(x) = \dfrac{1}{2}\operatorname{erfc}\left(\dfrac{x}{\sqrt{2}}\right)$

- **System Q without any ISI, MPN or RIN:** $Q_{opt}^2 = \dfrac{S_0^2}{\sigma_{th}^2}$
    - Link model (here) is: $r_k = S_0 + n_{th,k}$
- **Therefore, can prove the total link penalty (ISI + MPN + RIN) to be**

$$P_{ISI+MPN+RIN} = 10\log_{10}\left(\frac{S}{S_0}\right) = -5\log_{10}\left[\mu_{r_0}^2 - Q_{opt}^2\left(\sigma_{RIN-OMA}^2 + \sigma_{r_0}^2\right)\right]$$

Separate ISI, MPN, RIN Penalties

- **Can separate out individual penalties as follows:**

$$P_{ISI+MPN+RIN} = -10\log_{10}\left[\mu_{r_0}\right] - 5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{RIN-OMA}^2 + \sigma_{r_0}^2}{\mu_{r_0}^2}\right)\right]$$

$$P_{ISI} = -10\log_{10}\left[\mu_{r_0}\right]$$

$$P_{MPN} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{r_0}^2}{\mu_{r_0}^2}\right)\right] \qquad P_{RIN} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{RIN-OMA}^2}{\mu_{r_0}^2}\right)\right]$$

$$P_{cross} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{RIN-OMA}^2 + \sigma_{r_0}^2}{\mu_{r_0}^2}\right)\right] - P_{RIN} - P_{MPN}$$

$$P_{ISI+MPN+RIN} = P_{ISI} + P_{MPN} + P_{RIN} + P_{cross}$$

- **P_{cross} is not a phenomenological penalty**
  - Simply arises from the fact that the total penalty is decomposed into individual penalties
  - Ensures that the noise sources add in quadrature (variances add)

Scaling of RIN and MPN Penalties in the Spreadsheet

ofs
A Furukawa Company

$$P_{RIN} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{RIN-OMA}^2}{\mu_{r_0}^2}\right)\right] \qquad P_{MPN} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{r_0}^2}{\mu_{r_0}^2}\right)\right]$$

- **Both RIN and MPN should be normalized by total ISI** $= \mu_{r_0}$
- **Spreadsheet uses inner-most eye to estimate ISI**
  - Recall that spreadsheet uses the one-step computation of total ISI from $h_c(t)$
  - Total ISI can be shown to be $\mu_{r_0} = 2h_c(0) - 1 = 1 - 4Q(T/[2\sigma_c]\,)$
- **RIN: Spreadsheet does normalize RIN std. dev. by total ISI** $= \mu_{r_0}$ **→ correct**

Scaling of MPN Penalty in the Spreadsheet

$$P_{MPN} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{r_0}^2}{\mu_{r_0}^2}\right)\right]$$

- **The correct scaling factor for the MPN std. dev. σ_{r_0} is the total ISI = μ_{r_0}**
- **We established that the starting point for the Ogawa-Agrawal model is $h_s(t)$ and NOT $h_c(t)$**
    - So the inner-most eye has ISI $\rho_s = 2h_s(0) - 1 = 1 - 4Q(T/[2\sigma_s]\,)$
- **The OA-model approximates the inner-most eye by $\rho_s cos(\pi Bt)$ and assumes a continuous Gaussian VCSEL spectrum**
    - In this case, we can prove that: $\boldsymbol{\mu_{r_0} = \rho_s\rho_m}$, $\boldsymbol{\rho_m = e^{-\beta^2/2}}$, $\boldsymbol{\sigma_{r_0}^2 = \frac{\rho_s^2 k_{MPN}^2}{2}\left[1 - e^{-\beta^2}\right]^2}$ where $\boldsymbol{\beta = \pi BDL\sigma_\lambda}$
- **In contrast, the OA-model (and current spreadsheet) formula is**

$$P_{MPN-OA} = -5\log_{10}\left[1 - Q_{opt}^2\sigma_{MPN-OA}^2\right] \qquad \sigma_{MPN-OA}^2 = \frac{k_{MPN}^2}{2}\left[1 - e^{-\beta^2}\right]^2$$

- **So OA-model (& spreadsheet) implicitly normalizes $\boldsymbol{\sigma_{r_0}}$ by $\boldsymbol{\rho_s}$ to get $\boldsymbol{\sigma_{MPN-OA}}$ which is only part of the required normalization factor $\boldsymbol{\mu_{r_0} = \rho_s\rho_m}$**
    - Strictly speaking they assume $\rho_s = 1$ which is equivalent to scaling by ρ_s
- **Therefore, MPN std. dev. in spreadsheet requires scaling by $\boldsymbol{\rho_m = e^{-\beta^2/2}}$ to get the correct MPN penalty**
- **If we were to normalize σ_{MPN-OA} by μ_{r_0}, we would have effectively normalized σ_{r_0} by $\rho_s^2\rho_m$ → double counts ρ_s → not correct**

Summary of ISI Scaling factor treatment in the IEEE Spreadsheet

A Furukawa Company

- **Analytically proved that the scaling for RIN in the spreadsheet is correct**
- **MPN treatment in spreadsheet is consistent with the Ogawa-Agrawal model**
- **Mathematically proved that the MPN std. dev. in the OA model (and current version of spreadsheet) $\boldsymbol{\sigma_{MPN-OA}}$ needs to be scaled by $\boldsymbol{\rho_m = e^{-\beta^2/2}}$ to get correct MPN penalties**
  - ρ_m is nothing but the additional ISI due to the delays induced by the wavelength dependent VCSEL modes
  - Normalizing σ_{MPN-OA} by μ_{r_0}, will double count ρ_s and will result in wrong MPN penalties
- **Shown that while in general both RIN and MPN penalties require the same scaling factors (= total ISI), these factors should be different in the spreadsheet due to how various variances are defined and partially pre-normalized**

Bit Patterns

- **For links without mode partition noise, it is well-known that the worst-case ISI pattern is the isolated ‘1’ pattern: “000010000”**
  - Corresponds to the inner-most eye

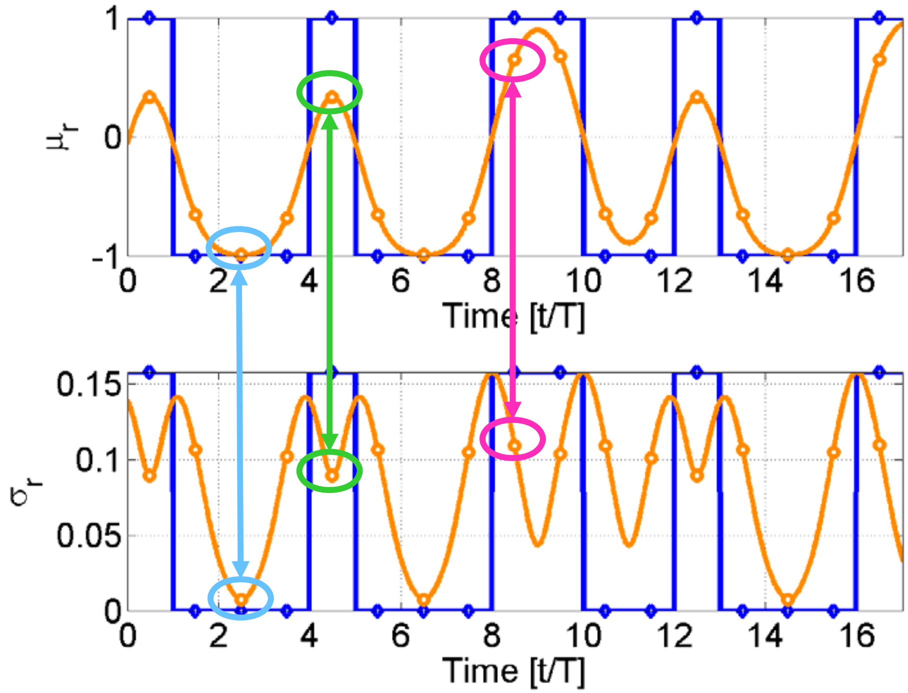
- **The OA-model (and current spreadsheet) uses the inner-most eye to compute the MPN penalty**

- **It has been suggested that the worst-case ISI pattern for links with MPN is not the isolated ‘1’ pattern but the so-called “transition pattern”: “000011111”**

- **Since the transition pattern does not have the worst-case ISI, it is likely that its total penalty may still be lower than that of the isolated ‘1’ pattern**

- **We check the validity of the above claim by evaluating the BER curves for all possible ISI patterns of a given length and estimating the total penalty**

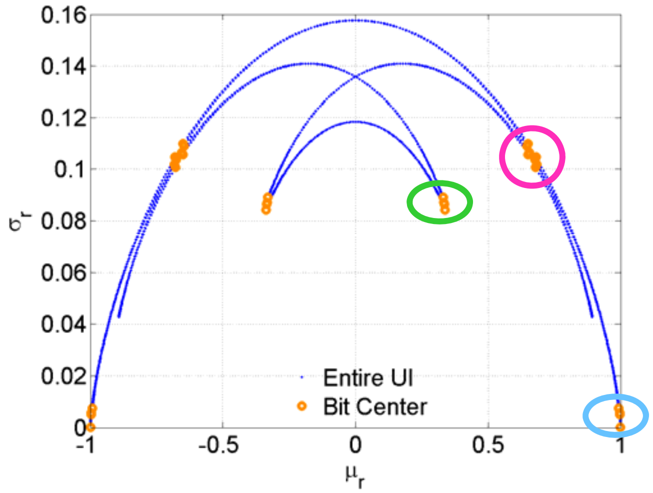
Mean and Standard Deviation of RX waveform with MPN

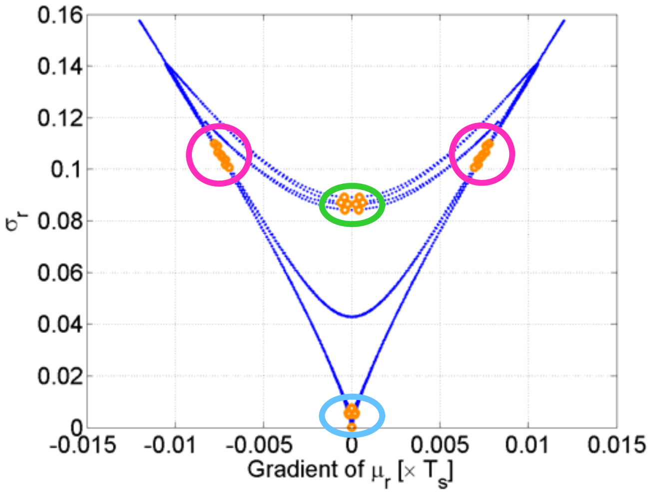


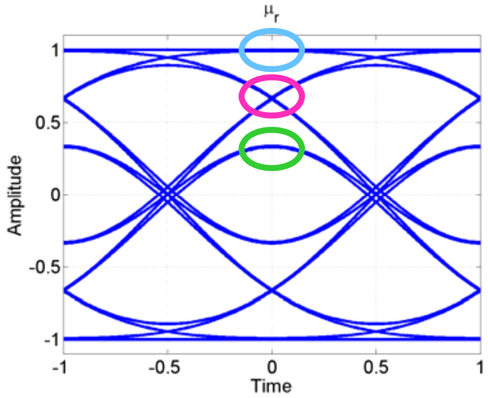
- $\mu_r(t)$ and $\sigma_r(t)$ synchronized with transmit bit sequence
- Samples at center of bit are marked

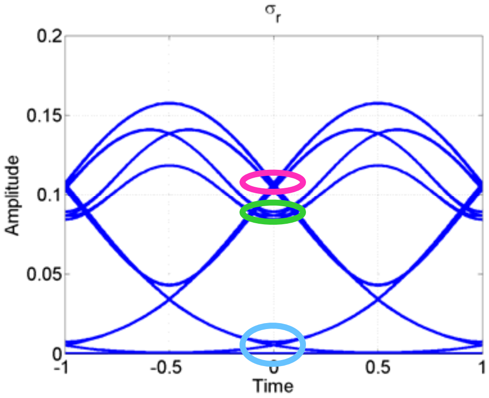
- **Bits with extremely low ISI (high $|\mu_r|$) have low std. dev. (σ_r)**
  - Blue ovals → best-case ISI patterns
  - Consistent with Petar's slide 12 from June 29, 2012 MPN call (MMF ad hoc)
- **But there are exceptions depending on the ISI pattern**
  - Green ovals have higher ISI but lower σ_r than Pink ovals
  - Green ovals → worst-case ISI patterns
  - Pink ovals → "transition case" (as termed in July 6, 2012 MPN call)

Correlation between ISI (μ_r) and std. dev. (σ_r)



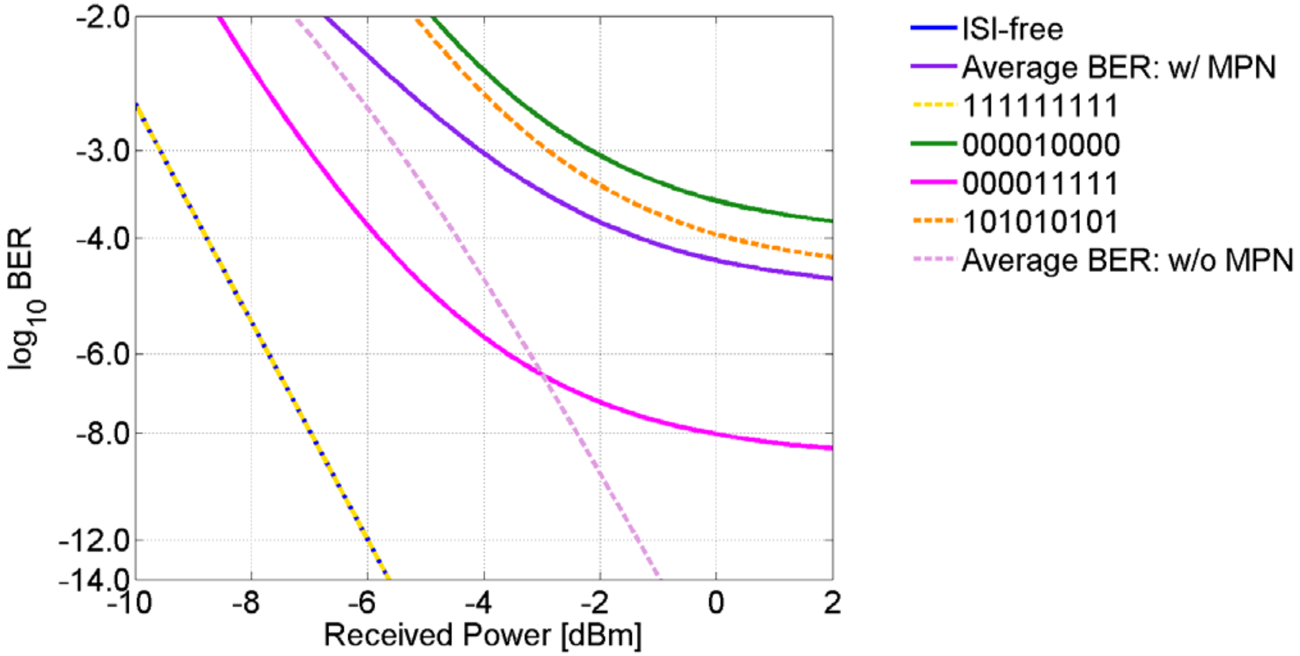






- **The previous observations are borne out by the above $\sigma_r \; vs \; \mu_r$ plot**
- **ISI patterns with higher RX waveform slope imply higher σ_r → consistent with Petar's conclusion although the difference between the best-case ISI pattern and the worst-case ISI pattern is dramatic even though have ~0 gradient**

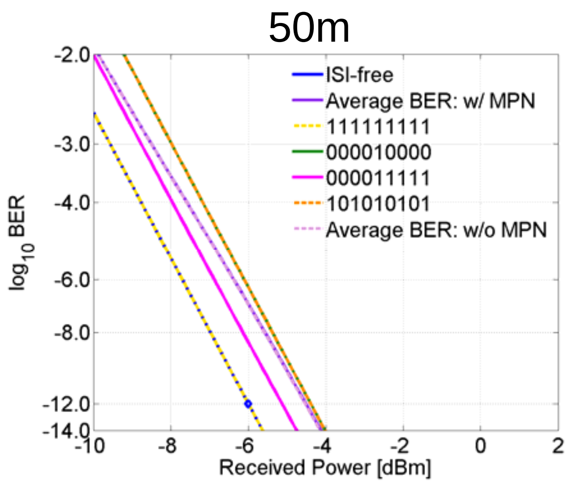
Worst-case ISI Pattern Determination
200m link length (to exaggerate effects)

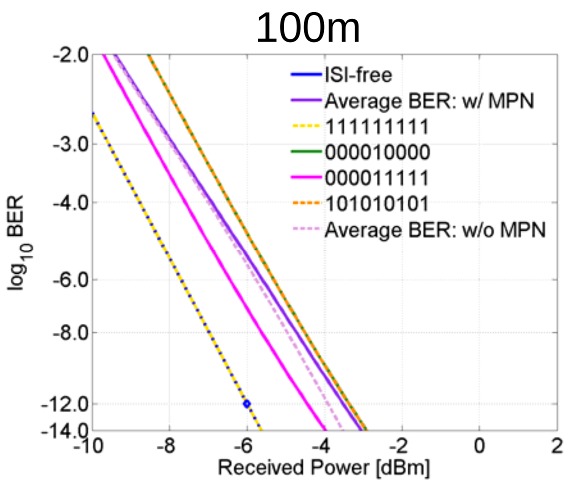


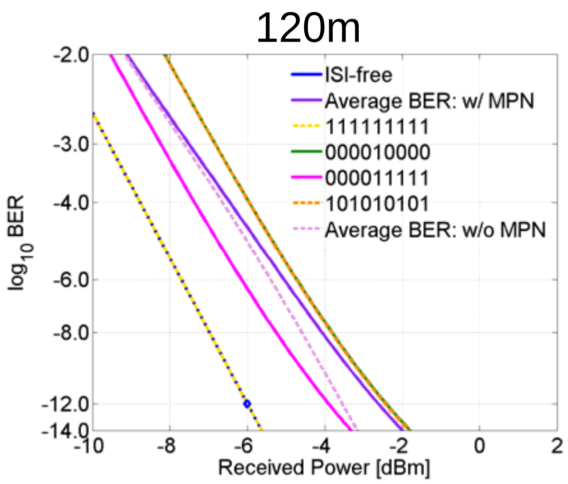
- Average BER w/ MPN upper-bounded by the BER for the “000010000” pattern (green curve) → “000010000” is the worst-case pattern
- The transition pattern “000011111” (pink) is not even close to the avg. BER
- The “111111111” has the same performance as the ISI-free link → “111111111” is the best-case pattern (expected, yellow dashed curve)
- BER floor type behavior observed at 200m → MPN adversely impacts links

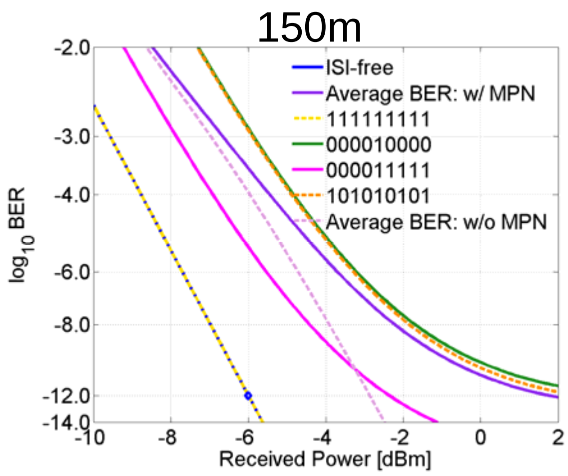
Worst-case ISI Pattern Determination at other link lengths

A Furukawa Company





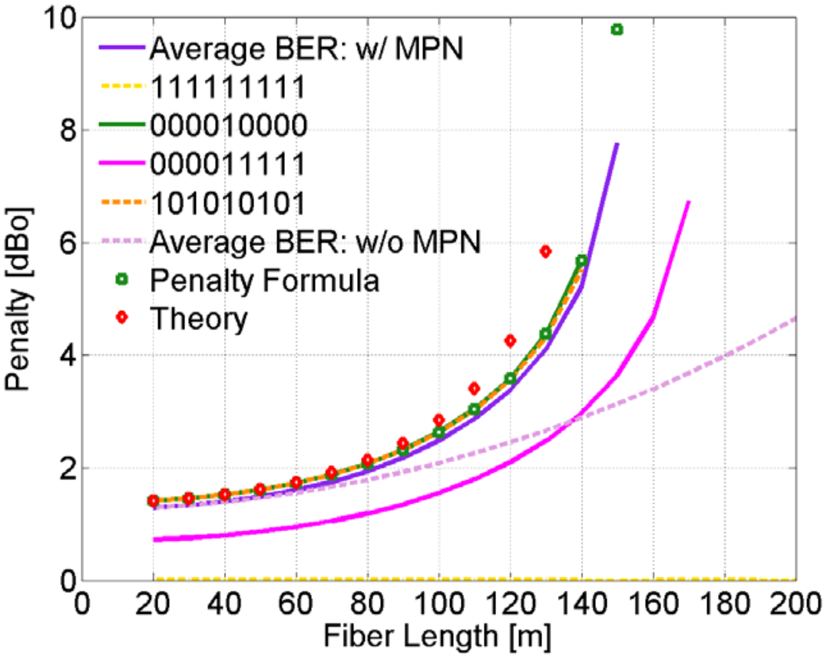




- **“000010000” is the worst-case pattern over all link lengths of interest**
- **“101010101” matches the worst-case BER for link lengths shorter than ~120m**
- **“000011111” is not even close to being the worst-case pattern**
  - Is actually better than the average BER without MPN for link lengths short than ~120m!

Total Penalty at BER=10^{-12}

- Calculations made with OA model without normalization for the straight and dashed line plots.
- “Penalty Formula” is noted in second bullet below
- “Theory” refers to O-A model with additional ISI as proposed in lingle_01_0512 and on *ad hoc* calls, while maintaining the infinite Gaussian spectrum.



- **Can estimate penalty from BER curves → worst-case pattern: “000010000”**
- **Can also compute penalty from** $\text{Penalty} = -5\log_{10}\left[\mu_{r_0}^2 - Q_{opt}^2\sigma_{r_0}^2\right]$ **(derived from BER exp.)**
  - Verified for worst-case ISI pattern (green squares overlap green curve)
- **Can also estimate** μ_r **and** σ_r^2 **for worst-case model from “modified” OA-model:**

$$\mu_r = \rho_s e^{-\beta^2/2} \qquad \sigma_r^2 = \frac{\rho_s^2 k_{MPN}^2}{2}\cdot\left[1-e^{-\beta^2}\right]^2 \qquad \rho_s = 1 - 4Q\left(\frac{T}{2\sigma_s}\right)$$

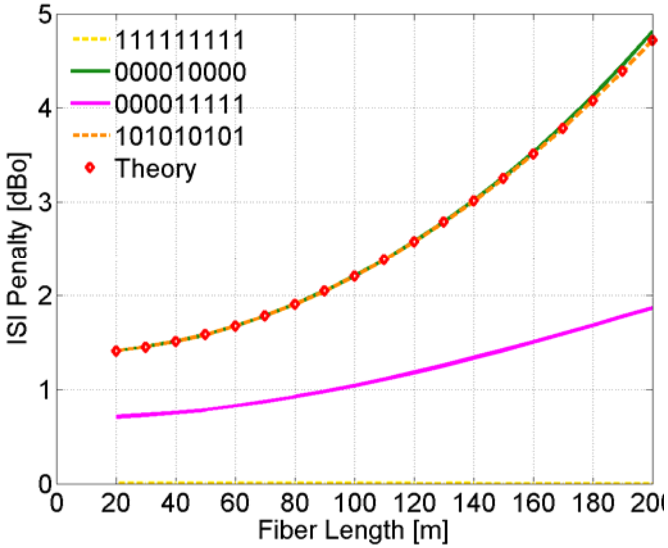
  - Verified: “Theory” red diamonds overlap green curve, but deviations do exist due to cosine approximation used by the OA-model instead of $\mathrm{erfc}(\cdot)$ based responses

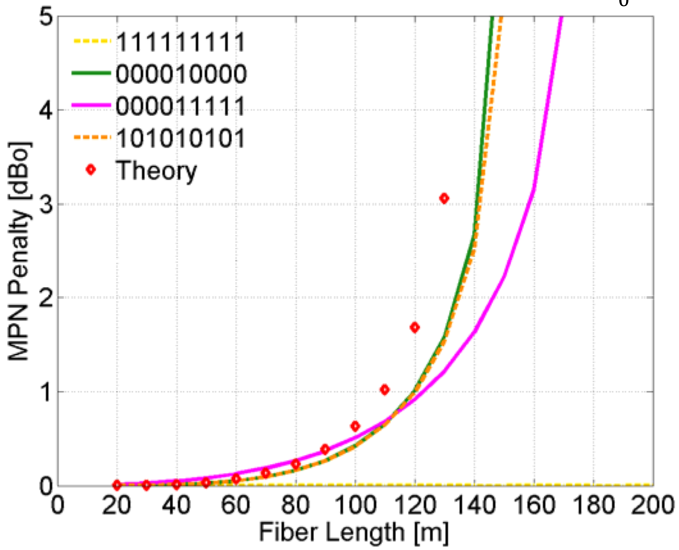
ISI and MPN Penalty for different ISI patterns

- **Total penalty** $\text{Penalty} = -5\log_{10}\left[\mu_{r_0}^2 - Q_{opt}^2\sigma_{r_0}^2\right]$ **can be decomposed into separate penalties:**

$$P_{ISI} = -10\log_{10}\mu_{r_0}$$

$$P_{MPN} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_{r_0}^2}{\mu_{r_0}^2}\right)\right]$$





- **ISI penalty of “000010000” is significantly higher than “000011111”**
- **MPN penalty of “000010000” is marginally lower than “000011111” in the ~0-110m range**
- **However, the ISI penalty gap more than compensates the MPN penalty gap such that “000010000” is still the worst-case ISI pattern**

Summary of ISI Pattern Evaluation

- **ISI patterns with higher RX waveform slope implies high σ_r**
  - "000010000" has higher ISI but lower σ_r than "000011111"

- **ISI penalty of "000010000" is significantly higher than "000011111" but the MPN penalty of "000010000" is marginally lower than "000011111" in the ~0-110m range**

- **However, the total penalty is still the largest for the isolated '1' pattern "000010000"**

- **Worst-case pattern is still "000010000" and NOT "000011111"**

Conclusions

- **Starting point of link-level MPN simulations using the full O-A model and arbitrary waveforms should be the channel response with a single-moded VCSEL (σ_s) and not the response based on different VCSEL modes having different wavelengths (σ_c)**
  - Otherwise will over-estimate the ISI → incorrect
- **ISI patterns with higher RX waveform slope implies high σ_r**
  - “000010000” has higher ISI but lower σ_r than “000011111”
- **Worst-case pattern is still “000010000” and NOT “000011111”**
  - Both from a BER and penalty (at 10^{-12}) point of view
- **Penalty formula** $\mathrm{Penalty} = -5\log_{10}\left[\mu_r^2 - Q_{opt}^2\sigma_r^2\right]$ **has been verified**
  - Can be decomposed into ISI and MPN penalties

$$P_{ISI} = -10\log_{10}\mu_r \qquad P_{MPN} = -5\log_{10}\left[1 - Q_{opt}^2\left(\frac{\sigma_r^2}{\mu_r^2}\right)\right]$$

- **The significantly higher ISI penalty of “000010000” compared to “000011111” more than compensates for its marginally lower MPN penalty over the ~0-110m range**
  - Beyond 110m, even the MPN penalty of “000010000” is worse than that of “000011111”
- **OA-model (and spreadsheet) MPN std. dev. requires normalization by ρ_m which is the additional ISI due to a multi-moded VCSEL**
  - For the Gaussian spectrum, $\rho_m = e^{-\beta^2/2}$
- **OA-model (and spreadsheet) after above scaling correction may still over-estimate penalty longer lengths due to cosine approximation: more study required**

Simulation Parameters

- **The only link impairments considered are:**
  - Transmit rise-time, Receiver bandwidth, Modal bandwidth, Chromatic dispersion and Mode partition noise (and of course, thermal noise)
- **Parameters:**
  - Bit rate = 25.781Gbps,
  - $T_{TX,20\%-80\%} = 19ps$
  - $BW_{RX} = 20.5GHz$
  - $BW_m = 4700MHz \cdot km$
  - $D = -108.68ps/nm - km,$
  - $k_{MPN} = 0.3$
  - $\sigma_\lambda = 0.6$nm
  - Fiber length = 200m (for initial plots, but later fiber length is varied)
- **All results are for a continuous Gaussian VCSEL spectrum**
- **Transmit sequence: Use de Bruijn bit sequence (DBBS)**
  - PRBS does not have all zero ISI combination
  - I^{th} order DBBS has all possible I-bit ISI combinations exactly once
  - Use 9^{th} order DBBS