Shedding Some Light on Coding Gain

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• **Ideal** capacity–achieving system for AWGN channel
  
  - Capacity \( C_{\text{bit/dim}} = \frac{1}{2} \log_2 (1 + \text{SNR}) \rightarrow \text{SNR}_{\text{dB}} \approx 6 \times n \text{ bit/dim} \)
  
  - Example: PAM-8 \( \rightarrow \) 3 bit/dim requires \( \text{SNR}_{\text{dB}} \approx 18 \text{dB} \)

• **System “A”** employing **practical modulation & coding**
  
  - Rate \( R_{\text{bit/dim}}(“A”) = \frac{1}{2} \log_2 (1 + \frac{\text{SNR}}{G(“A”)}) \), \( G > 1 \) (‘gap to capacity’) expresses additional SNR required by practical scheme to achieve same rate as capacity-achieving scheme
  
  - Uncoded modulation: \( G_{\text{dB}} \approx 9 \text{ dB @ BER} = 10^{-6}, \ 12.3 \text{dB @ BER} = 10^{-12} \)
    
    - Maximum possible coding gain is 7.5dB \( 10^{-6} \) @ and 10.8dB @ \( 10^{-12} \)

• **Precoding (Tx) + whitened matched filter (Rcv)**
  
  - Any linear channel can be transformed into an AWGN-like channel
  
  - Small penalty if “water-pouring” Tx spectrum not used
Gap to Capacity for PAM (AWGN channel)

• Approximately independent of the number of PAM levels (M large)
  – “normalized”
    \[ SNR_{\text{norm}} \equiv \frac{SNR}{(M^2-1)} \]
  – Defines baseline curve for any M-PAM

• Example: PAM-8
  – Requires \( 12.25 + 6.02 \times 3 \)
    = 30.3 dB to achieve \( 10^{-12} \) error rate
    (uncoded)

\[ SNR = gap + 6.02 \times \log_2 M \] (dB)
Achievable Coding Gain

(slides adapted from “Advanced Downstream Physical Layer for Cable Systems (AdDnPhy)” by G. Ungerboeck presented in Irvine on March 10, 2000)

Capacity
AWGN channel w/o shaping

Uncoded modulation w/o shaping

BER = 10^{-6}
coding gain typically specified @ 10^{-6}

9 dB

1.5 dB

7.5 dB

3.3 dB

≈ 0.8 dB

≈ 0.9 dB

≈ 0.9 dB

4.2 dB

TCM (16-state 4-dim)

TCM + RS (hard or erasure decoding)

TCM + RS with iterative TCM-RS-TCM-RS ...

decoding

Turbo TCM (TTCM) or other concatenated schemes

with iterative (“turbo”) decoding

Practical high-rate coding schemes w/o shaping
Coding Gain of Proposed LDPC Scheme*

- Proposed scheme maps 2747 bits to 1024 symbols
  - Effective bits/symbol = \((2747/1024) = 2.68 \text{ bits/sym}\)
  - Approximately “PAM-6.5”

- Coding gain 5.8dB @ BER=10\(^{-6}\), 8.7dB @ 10\(^{-12}\)

- Note: The fact that PAM symbols are simultaneously transmitted over four pairs does not make the code “4-D”
  - Overall error rate will be 4x the error rate of each “1-D” pair

* Proposed by Intel in November 2003 Plenary
Coding Alternatives

- More traditional approaches can offer code gains similar to LDPC

- Concatenated block + convolutional codes widely used
  - Reed-Solomon, Viterbi, block/convolutional interleaver
  - Decoding complexity well understood
  - Code performance well proven over several decades of use
  - No “error floor” issues

- Application-driven latency requirements must also be considered in the coding choice
Example 1: Traditional Concatenated TCM+RS

• PAM-8, 16-state 4-D TCM + RS(253,245)
  – Each wire carries 1 dimension (PAM-8) with 2.663 effective bits/sym
  • Approx equal to rate of LDPC proposal
  – Non-iterative, single pass
  – \( F_{\text{baud}} = 938.8\text{Msps} \)
  – Each 253 byte RS block contains 184 11-bit 4D symbols
  – Estimated coding gain assumes ideal interleaving

*Illustrative example meant to match the LDPC rate, not intended as a proposal
Example 2: Concatinated RS + Binary CC (RSCC) with Iterative Decoding

System Considered for Optical Application

RS encoder
RS(255,K)
\( R_{rs} = K/255 \)
\( T = (255-K)/2 \)

RS byte interleaver
16 x 255 = 4080 RS bytes
= 32640 bits

Binary CC encoder
Rate-1/2 punctured to
\( R_{cc} = m/n \)

RS decoding

\( RS(255,243), T=6, R_{rs} = 0.9529; \quad R_{cc} = 9/10 \rightarrow R_c = 0.8573 \ (14.26\%) \)

Iterative decoding
1. SIHO CC decoding by VA with \( a \ priori \) information LLA and channel information LLR
2. De-interleaving and HIHO RS decoding
3. Successfully decoded RS codewords considered reliable, update LLA ; go to 1.
Serially concatenated RS and binary CC (RSCC) with iterative decoding

RS(255,243), T=6 + rate-9/10 64-state CC : $R_c=0.8575$

Computed BER from measured RS byte error rate after 1st CC decoding (assuming ideal byte interleaving)

Simulated BER: 1st iteration

Coding Gain = 6dB @ BER=10^{-6}

9.1dB @ 10^{-12}
Conclusions

• The true coding gain offered by the “LDPC PAM-8” scheme is about 5.8dB @ BER=10^{-6}, 8.7dB @ 10^{-12}

• Similar coding gain may be achieved in other ways with well understood performance and complexity
  – Straight forward TCM+RS achieves about 7.1 dB @ BER=10^{-12}
  – Iterative CC+RS achieves over 9dB @ BER=10^{-12}

• A decision on the coding scheme for 10GBASE-T must be made on the basis of real coding gains versus carefully evaluated decoding complexity
  – Latency must also be considered