# Shannon's capacity analysis of GEPOF for technical feasibility assessment

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# Agenda

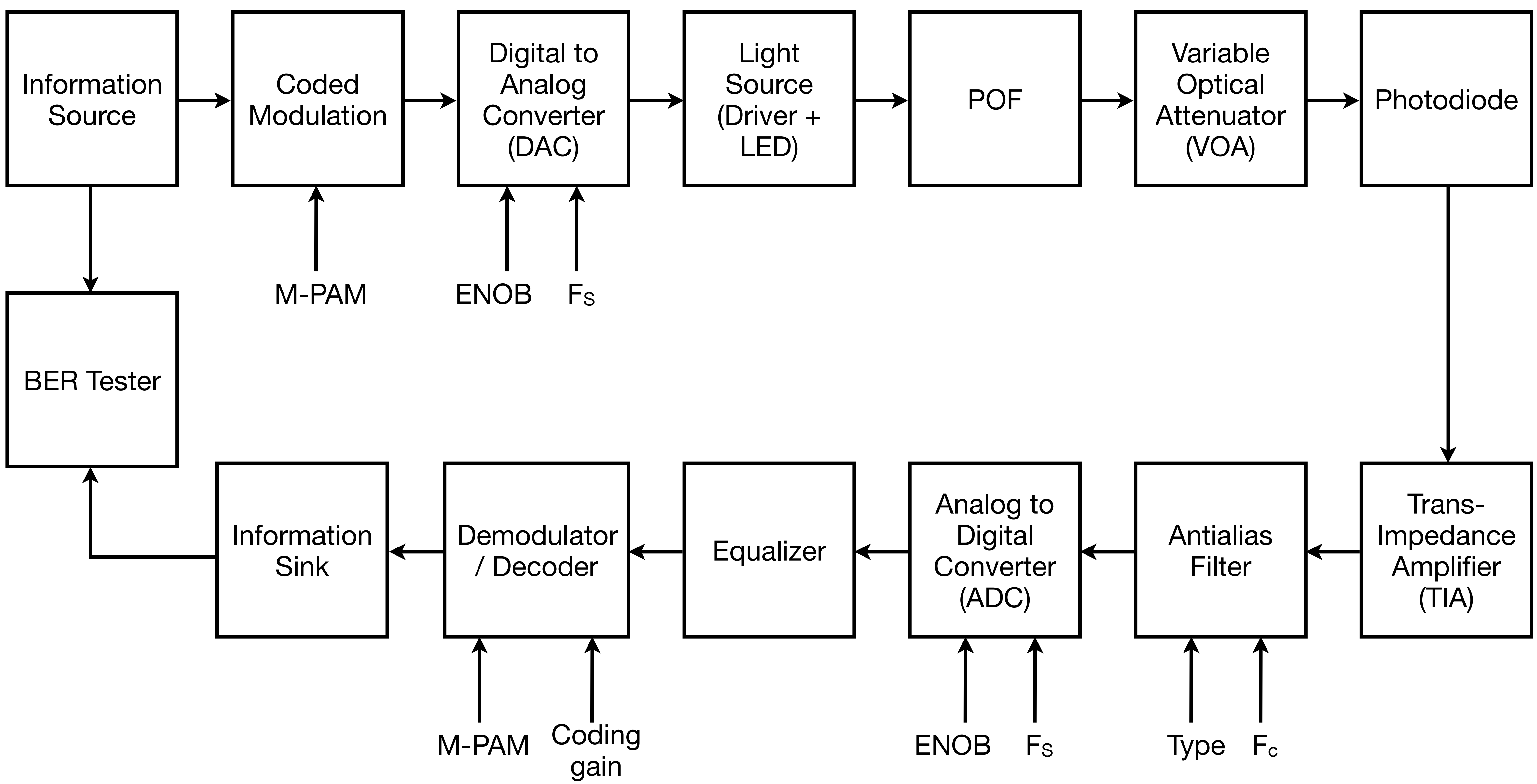
- Objectives
- Information theory model
- Shannon's capacity evaluation method
- Technical feasibility based on Shannon's capacity
- Is the MMSE,u DFE technically feasible?
- Shannon's capacity evaluation with Tomlinson-Harashima Precoding (THP)
- Technical feasibility based on Shannon's capacity for THP
- Conclusions

# Objectives

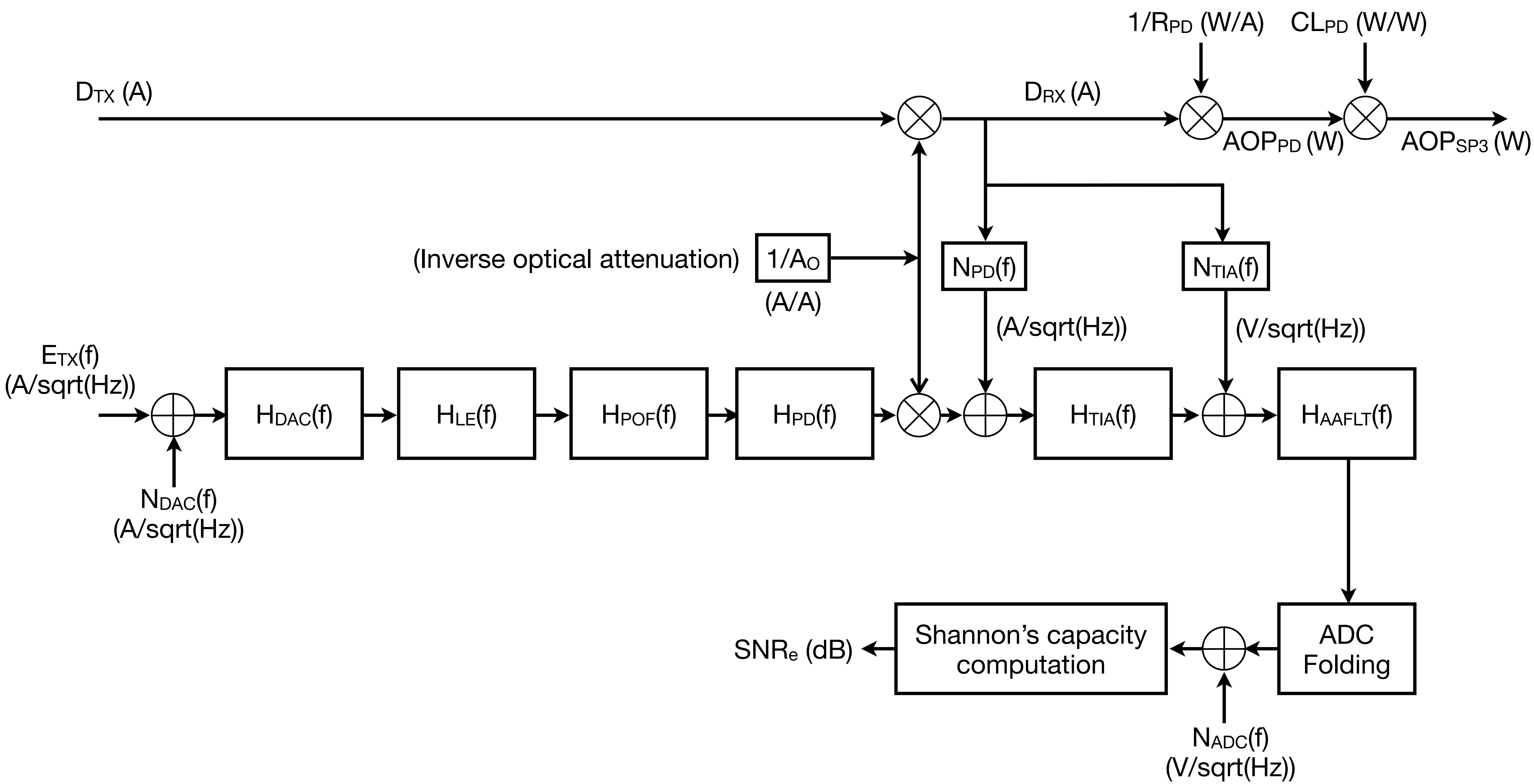
- To provide a technical feasibility study of GEPOF based on Shannon's capacity
- To demonstrate the receiver sensitivity requirements for the three link models presented in [perezaranda_03_0514_linkbudget] are technically feasible

# Information theory model

# Reference model



# Information theory model



# Information theory model

- The model is built in the electrical domain (v.s. optical domain), since it represents an IM/DD optical communication system
- Final sensitivity in terms of optical power at SP3 is derived by using the photodiode responsitivity and optical coupling efficiency
- Two signal paths are considered, DC and AC, both closely related by the Extinction Ratio (ER) and the modulation scheme
- Noise model:
  - Noise sources coming from Photodiode and TIA are vey well modeled as additive gaussian colored noise, equivalent to AWGN filtered by a linear system
  - Quantization noises in DAC and ADC are artificially modeled as additive white gaussian noises derived from the ENOB specification; non-linearities of DAC and ADC are not explicitly considered and they are summarized into ENOB term ➤ worst case analysis
  - No impulse noise and RFI noise are considered, since it is an optical system; the system is immune to any EM interference surrounding it in a good implementation
- Transfer functions $H_{DAC}(f)$, $H_{LE}(f)$, $H_{POF}(f)$, $H_{PD}(f)$ and $H_{AAFLT}(f)$ are normalized for DC gain of 0 dB
- Transfer function $H_{TIA}(f)$ preserves the trans-impedance conversion factor between input current and output voltage, therefore no DC gain normalized

# Information theory model

- Variables:
  - $E_{TX}(f)$: TX electrical current spectral density referred to photodiode output
  - $D_{TX}$: average TX electrical current referred to photodiode output
  - $D_{RX}$: average RX electrical current at the output of photodiode
  - $R_{PD}$: Photodiode responsivity (A/W)
  - $CL_{PD}$: optical coupling loss between POF and photodiode active area
  - $AOP_{PD}$: average optical power coupled to photodiode
  - $AOP_{SP3}$: average optical power at SP3 (output of POF)
  - $A_O$: optical attenuation
  - $N_{DAC}(f)$: DAC noise spectral density in terms of electrical current referred to photodiode output as a function of ENOB
  - $N_{ADC}(f)$: ADC noise spectral density in terms of electrical voltage as a function of ENOB
  - $N_{PD}(f)$: photodiode shot noise spectral density as electrical current at the photodiode output
  - $N_{TIA}(f)$: TIA output voltage noise spectral density
  - $H_{DAC}(f)$, $H_{LE}(f)$, $H_{POF}(f)$, $H_{PD}(f)$ and $H_{AAFLT}(f)$: transfer functions of DAC, light emitter, POF, photodiode and anti-alias filter, respectively
  - $H_{TIA}(f)$: transfer function of the trans-impedance amplifier

# Transmission model - modulation

- Let us assume the TX communication signal be a uniformly distributed M levels PAM, that take values from the set {-M+1, -M+3, …, +M-3, +M-1}, where M is a positive integer ≥ 2

- The crest-factor of TX signal is given by:

$$CF_{TX} = \sqrt{3\frac{M-1}{M+1}}$$

- Under non-negative optical power constraint, the optical channel input is limited in power peak, instead of average power like in an electrical cable

- Based on that, the TX electrical current spectral density is given by:

$$E_{TX}(f) = D_{TX}\frac{ER-1}{ER+1}\frac{1}{CF_{TX}}\frac{1}{\sqrt{F_S/2}} \quad \text{(A/sqrt(Hz))}$$

where ER is the extinction ratio and $F_S$ is the symbol rate.

# Transmission model - DAC

- DAC magnitude transfer function:

$$H_{DAC}(f) = \left| \mathrm{sinc}\left( \frac{f}{F_S} \right) \right|; \quad where \quad \mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

- DAC noise spectral density:

$$N_{DAC}(f) = D_{TX} \frac{ER-1}{ER+1} \frac{1}{\sqrt{3}} \frac{1}{2^{ENOB_{DAC}}} \frac{1}{\sqrt{F_S/2}} \quad \text{(A/sqrt(Hz))}$$

- RC bandwidth limitation of DAC implementation is not considered, because its effect should be negligible by design compared to other bandwidth limitations like LED or fiber.

# PHY receiver analog front-end model

- Automatic Gain Control (AGC) is implemented by the trans-impedance amplifier (TIA) to avoid overloading under realistic low voltage supply ➤ trans-impedance is controlled as a function of the photocurrent
- Depending on the implementation and maximum achievable TIA trans-impedance, a subsequent PGA block could be considered between TIA and ADC, provided the PGA is well designed in terms of noise figure to get from PD and TIA the most relevant noise sources
- An antialias filter is implemented before ADC sampling to reduce the TIA out of band noise folding inside the Nyquist band
- ADC works at symbol rate, provided that a timing recovery circuit gets optimal sampling phase in ADC for maximum SNR
- As it is typical, a margin over the ADC full scale has to be provided in real implementations to cope with baseline wander and to avoid saturations. Let be ξ the minimum used portion of the input ADC full scale
- Based on that, the ADC noise can be modeled as:

$$N_{ADC}(f) = \frac{1}{\xi}\frac{1}{\sqrt{3}}\frac{1}{2^{ENOB_{ADC}}}\frac{1}{\sqrt{F_S/2}} \quad \text{(V/sqrt(Hz))}$$

# Shannon’s capacity evaluation method

# Shannon's capacity evaluation

- Let's define $N_{RX}(f)$ and $S_{RX}(f)$, as the spectral densities at the ADC output, after folding and quantization, of the noise and the communication signal, respectively; both are defined between DC and $F_S/2$
- Let's define the effective SNR ($SNR_e$) at the ADC output as the SNR that provides the same communication capacity to the channel defined by $S_{RX}(f)$ and $N_{RX}(f)$

$$C = F_S \log_2\left(1 + SNR_e\right) \quad \text{bits/s}$$

$$SNR_e = \exp\left\{\frac{1}{F_S}\int_{-\frac{F_S}{2}}^{\frac{F_S}{2}} \ln\left(1 + \frac{S_{RX}(f)}{N_{RX}(f)}\right)\cdot df - 1\right\}$$

- $SNR_e$ is equal to the $SNR_d$ provided in detector by an ideal infinite length filters unbiased MMSE DFE in an ISI channel
  - Ideal stands for there is no error propagation
  - Degradation due to finite length filters constraint is not considered

# Shannon's capacity evaluation

- The minimum $SNR_d$ required for M-PAM scheme can be well approximated by

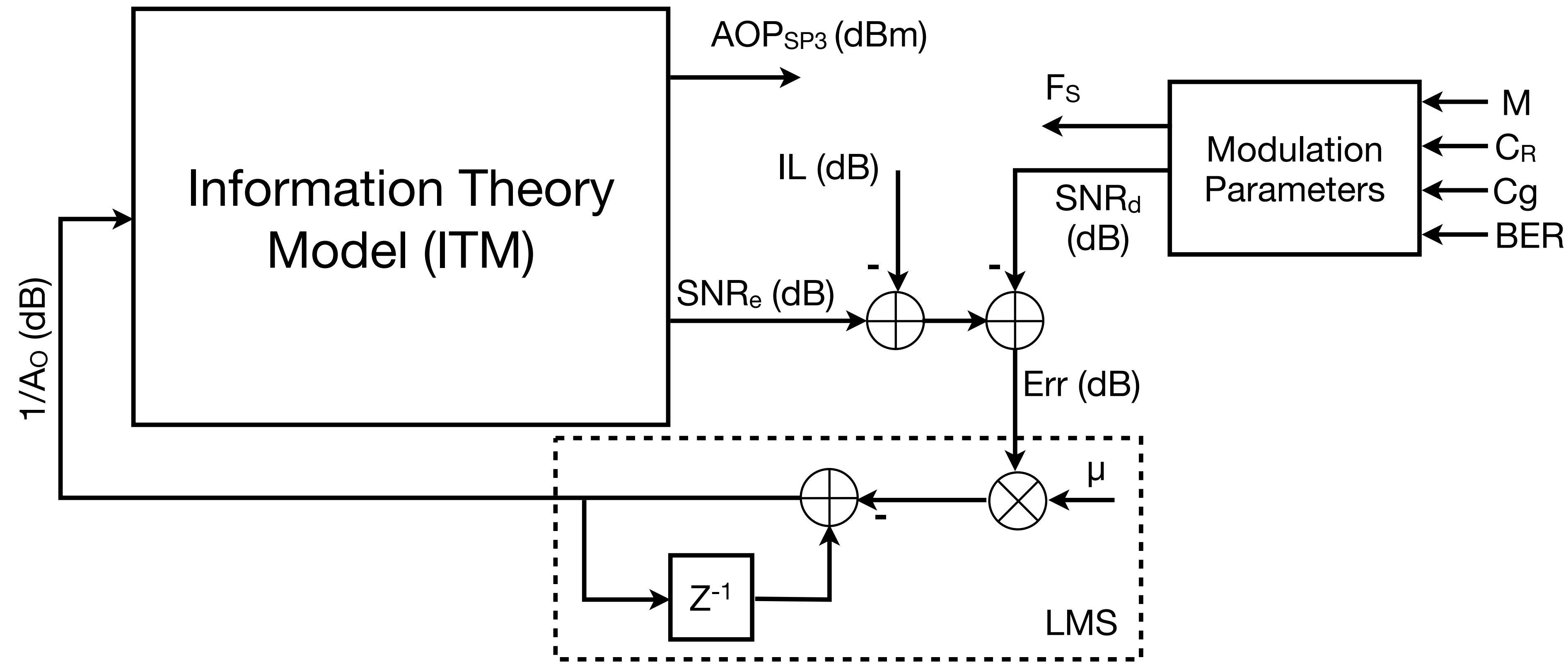
$$SNR_d = \frac{2}{3}\frac{M^{2\cdot C_R}-1}{C_G}\left(\mathrm{erfc}^{-1}\left(\frac{M\log_2(M)}{M-1}\cdot P_b\right)\right)^2$$

where

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-t^2}\,dt$$

- AWGN is assumed in detector after noise whitening and ideal feedback by MMSE,u DFE
- Parameters:
  - M: number of levels of PAM scheme; $M \geq 2$
  - $C_R$: code-rate of FEC scheme; $C_R < 1$ for coded schemes, $C_R = 1$ for uncoded scheme
  - $C_G$: coding gain respect to an uncoded scheme; $C_G > 1$ for coded schemes, $C_G = 1$ for uncoded scheme. Coding gain is defined for a given $P_b$ in non-asymptotic codes
  - $P_b$: bit error probability (i.e. BER for our purposes) after decoding

# Shannon's capacity evaluation




- The ITM is evaluated in several iterations in a LMS loop to find the minimum $AOP_{SP3}$ (i.e. optical receiver sensitivity), provided the coded modulation parameters ($F_S$, M, $C_R$, $C_G$)
- IL (dB) is included to take into account the PHY implementation losses
- Min $AOP_{SP3}$ has to result smaller than that established in the link budget analysis to conclude the system is technically feasible

# Technical feasibility based on Shannon's capacity

# Parameters for Shannon's capacity evaluation

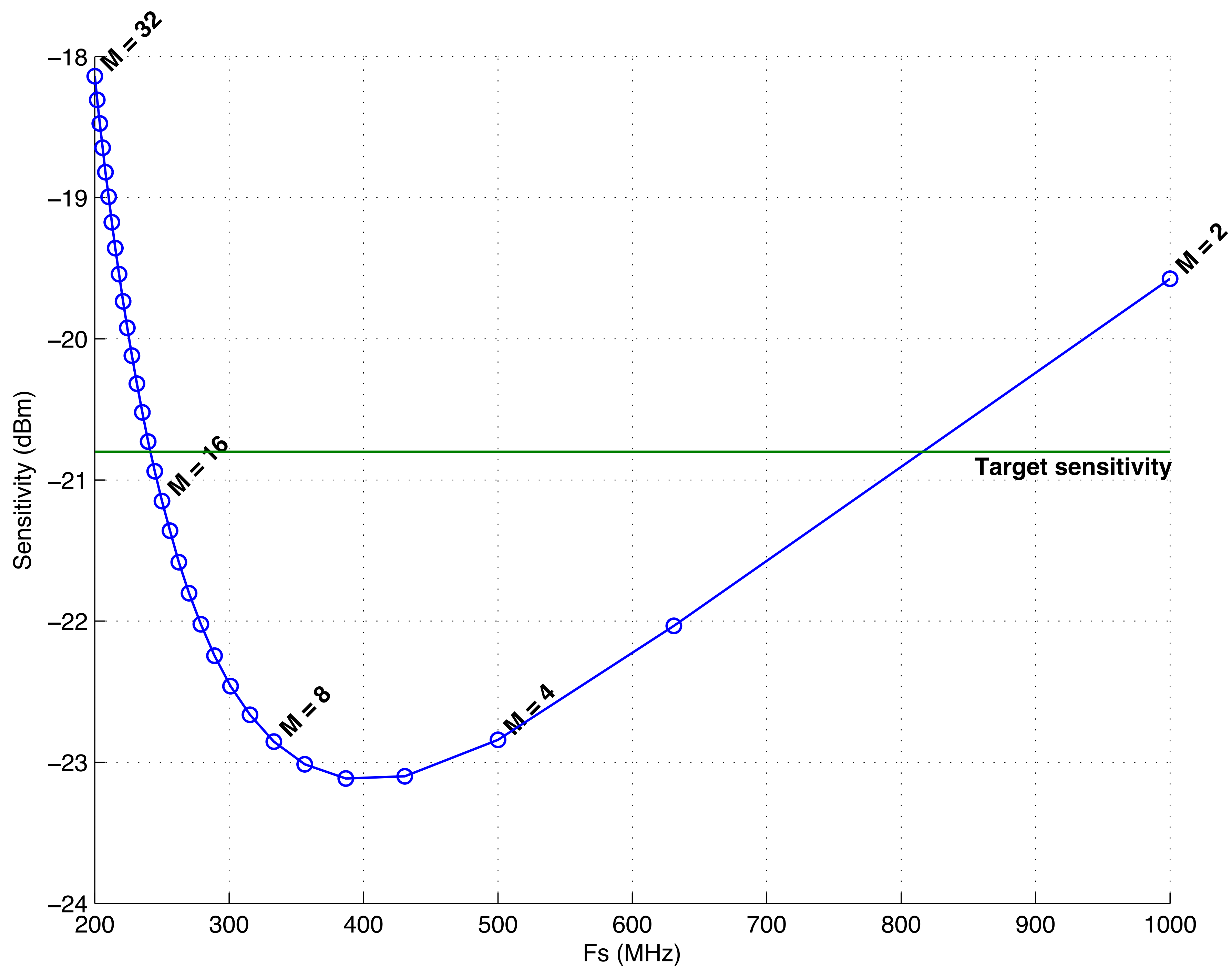
- Bit-rate $B_R$ = 1Gbps
- The analysis shall be carried out moving $F_S$ as a function of M, provided that $B_R$ is fixed. $F_S$ is calculated as:

$$F_S = \frac{B_R}{C_R \cdot \log_2\left(M\right)}$$

- BER $\leq 10^{-12}$
- Junction temperature = 100 ºC
- Antialias filter: 4th order Butterworth with bandwidth $F_{-3dB} = 0.6 \cdot F_S$
- AGC target: ξ = 70% of ADC full range
- Photodiode: Φ = 400 um; $C_{PD} \simeq$ 2pF, $R_{PD}$ = 0.5 A/W and $C_L$ = 3 dBo (see [perezaranda_02_0514_rxcharacteristics])
- TIA is designed accordingly to PD, $F_S$ and technology limitations (see [perezaranda_02_0514_rxcharacteristics])
- POF responses are as in [perezaranda_01_0514_pofresponse]
- Light emitter is a 650nm LED qualified for MOST150 with ER = 10dBo (see [perezaranda_01_0514_txcharacteristics])

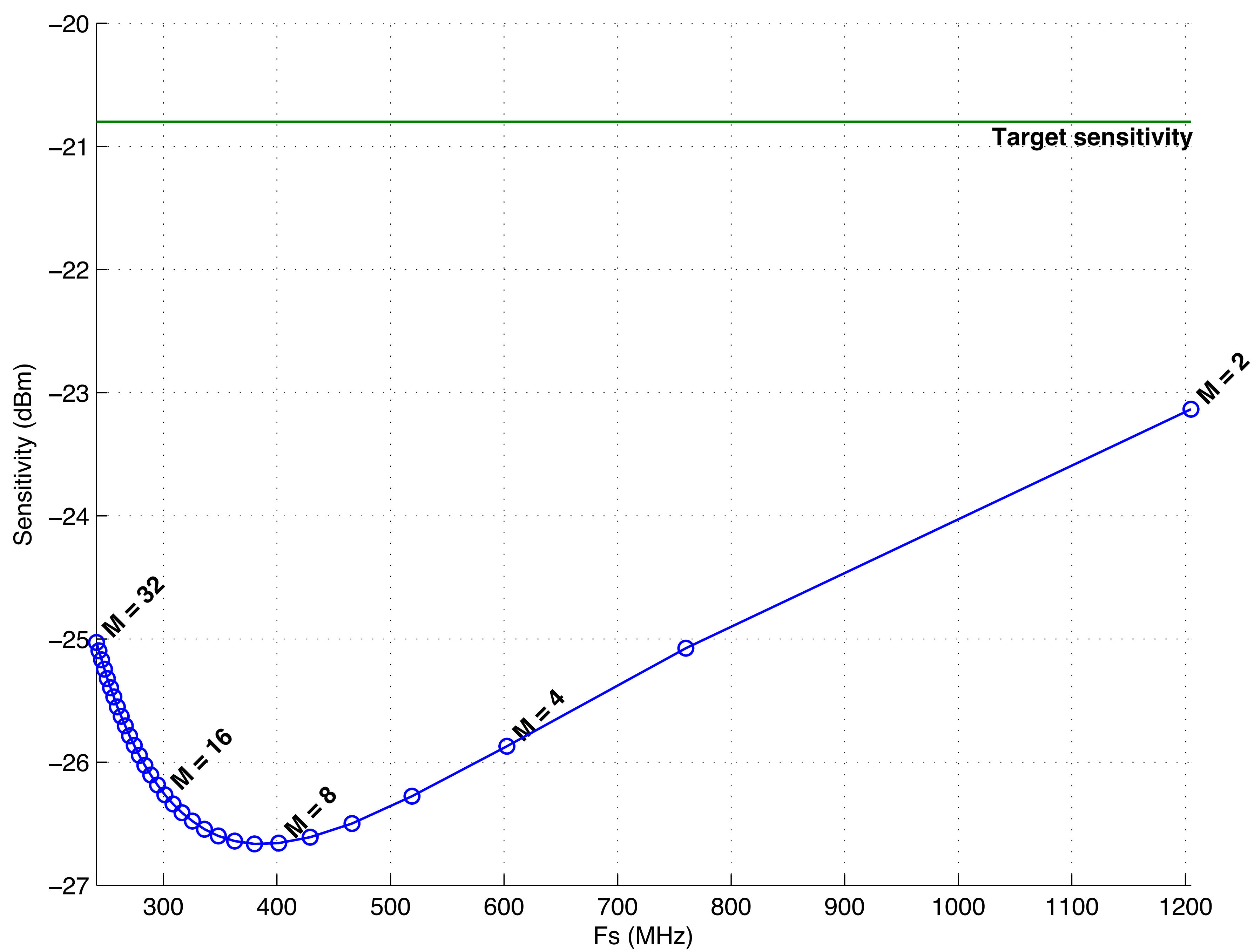
# Technical feasibility for 15 m of POF

- Uncoded scheme, ideal ADC and DAC (ENOB = ∞), IL = 0 dB



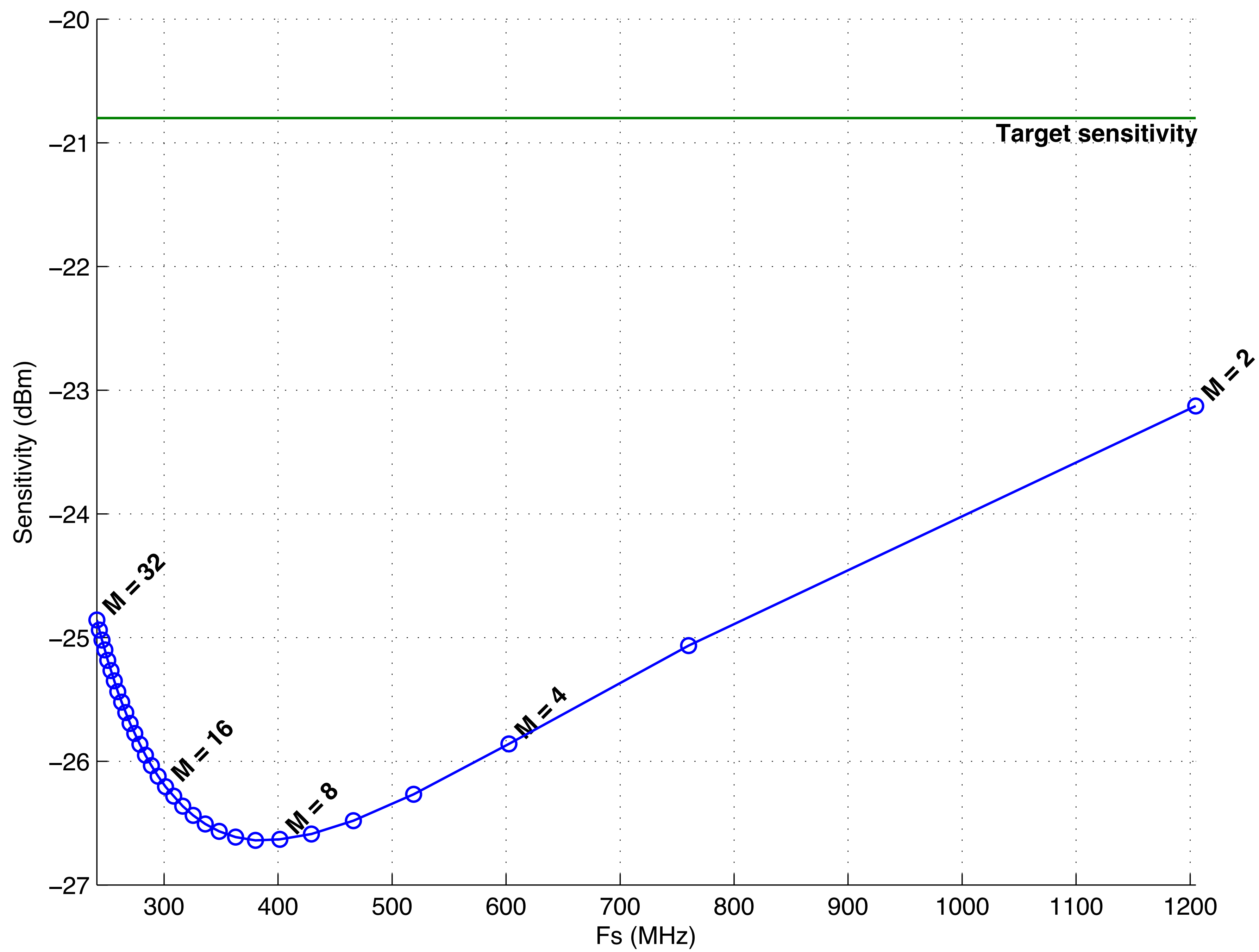
# Technical feasibility for 15 m of POF

- Low cost FEC $C_R = 0.83$, $C_G = 6.7$ dB, ideal ADC and DAC (ENOB = ∞), IL = 0 dB



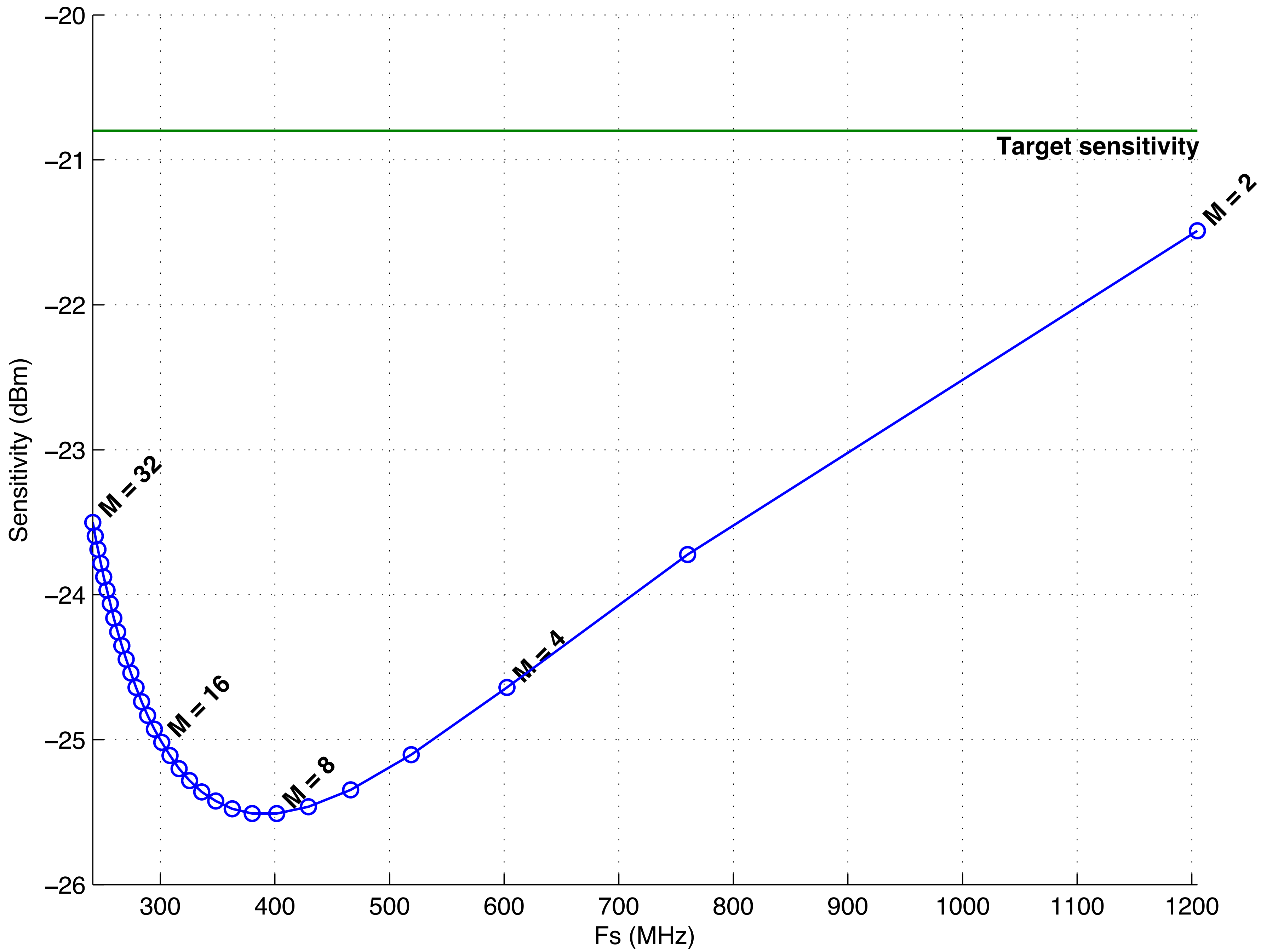
# Technical feasibility for 15 m of POF

- Low cost FEC $C_R$ = 0.83, $C_G$ = 6.7 dB, $DAC_{ENOB}$ = 8, $ADC_{ENOB}$ = 8, IL = 0 dB



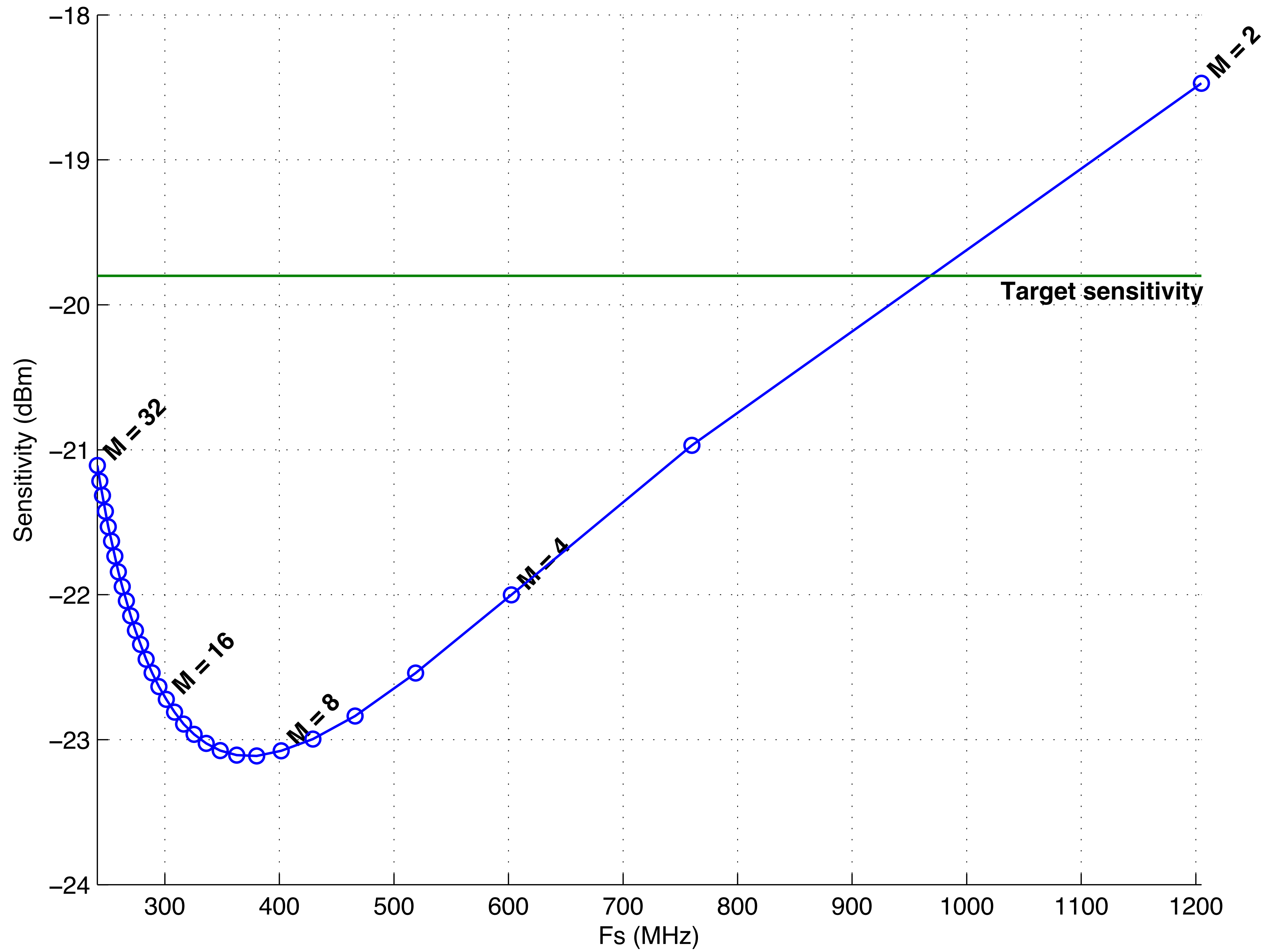
# Technical feasibility for 15 m of POF

- Low cost FEC $C_R = 0.83$, $C_G = 6.7$ dB, $DAC_{ENOB} = 8$, $ADC_{ENOB} = 8$, IL = 2 dB



# Technical feasibility for 50 m of POF

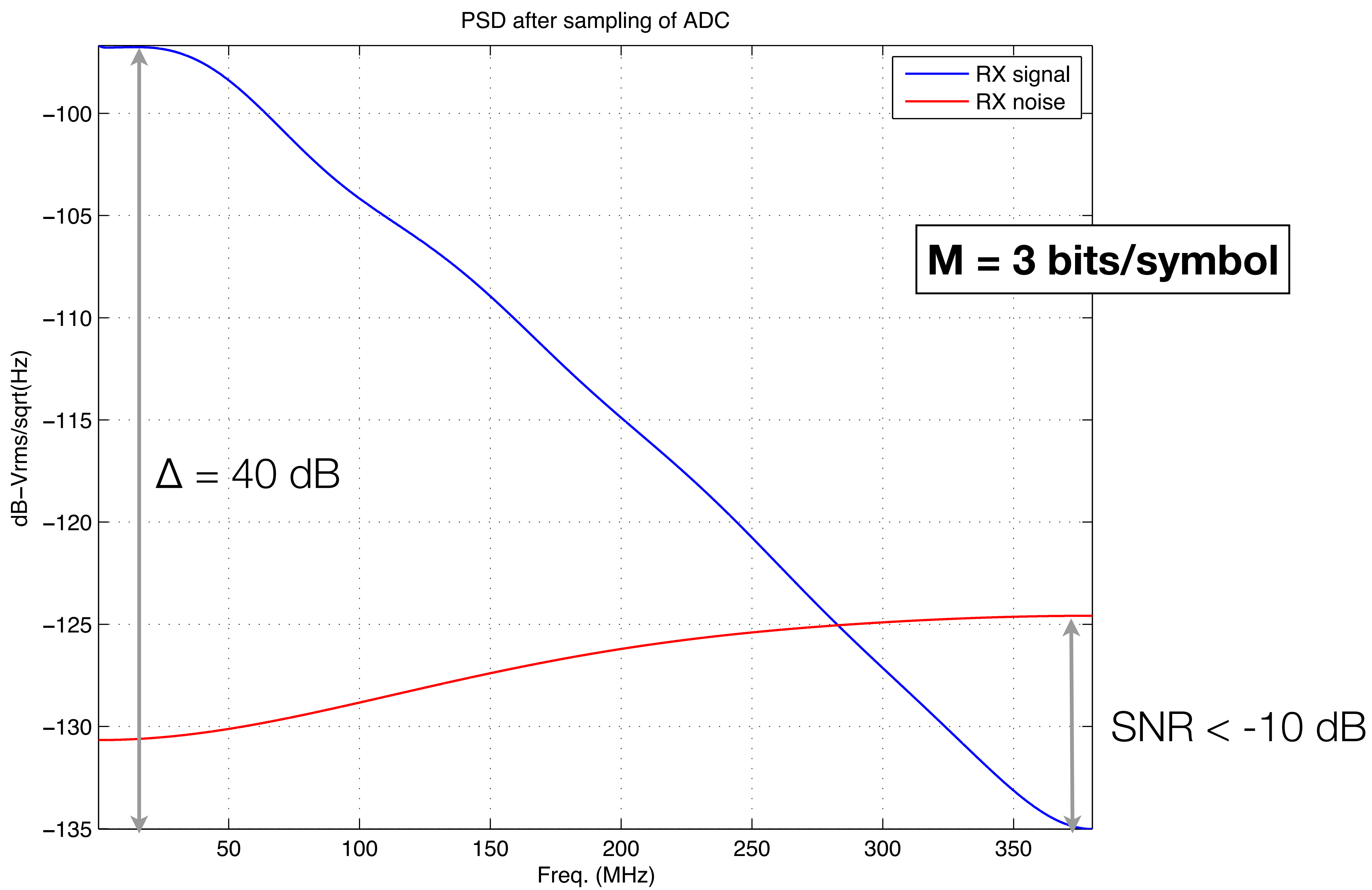
- Low cost FEC $C_R = 0.83$, $C_G = 6.7$ dB, $DAC_{ENOB} = 8$, $ADC_{ENOB} = 8$, IL = 2 dB



# Is the MMSE,u DFE technically feasible?

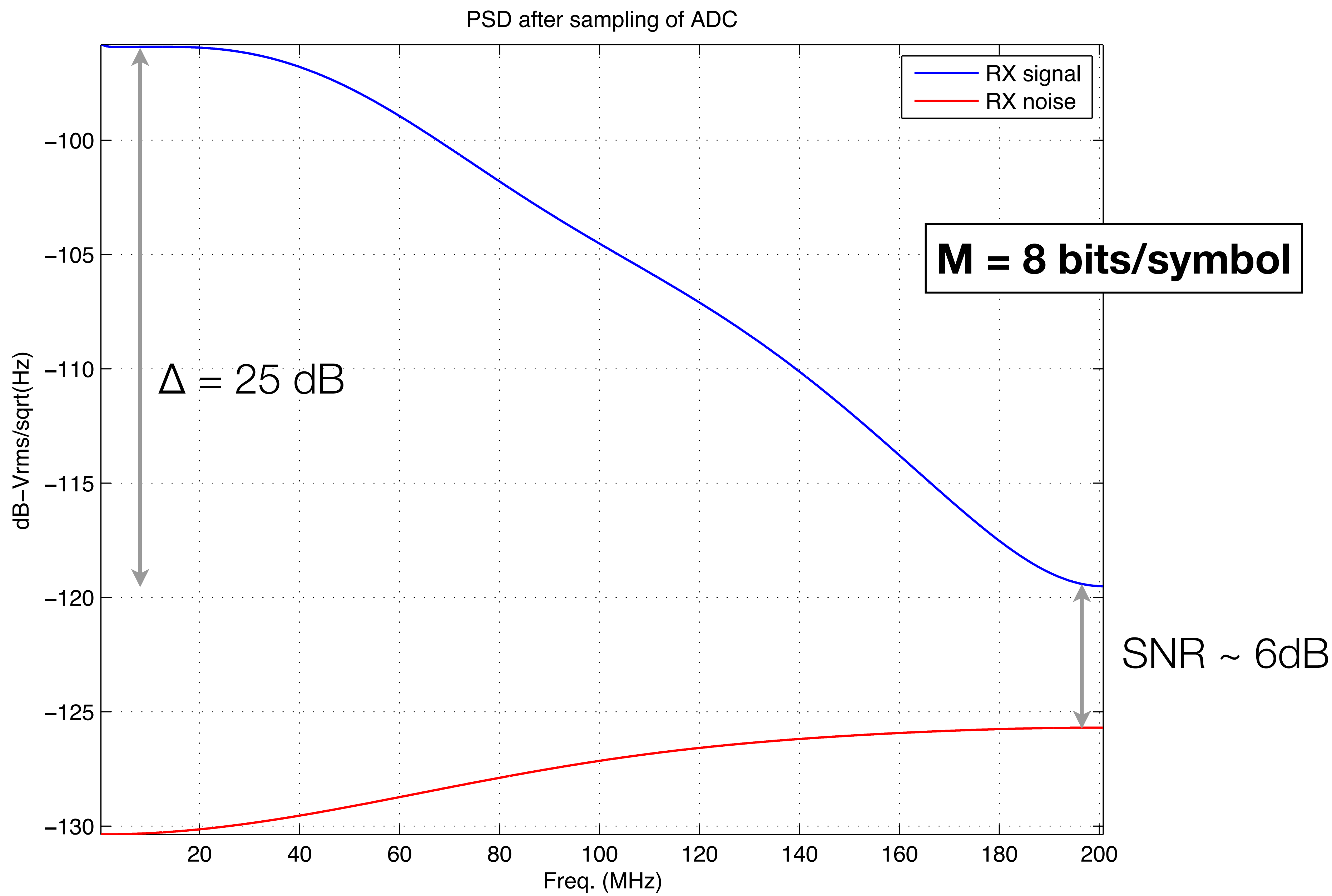
# MMSE,u DFE feasibility - channel response

- Let's examine the channel response and the noise spectral density at the output of ADC for several values of M at the sensitivity point
- Scenario: $C_R$ = 0.83, $C_G$ = 6.7 dB, $DAC_{ENOB}$ = 8, $ADC_{ENOB}$ = 8, IL = 2 dB, POF length = 50 m



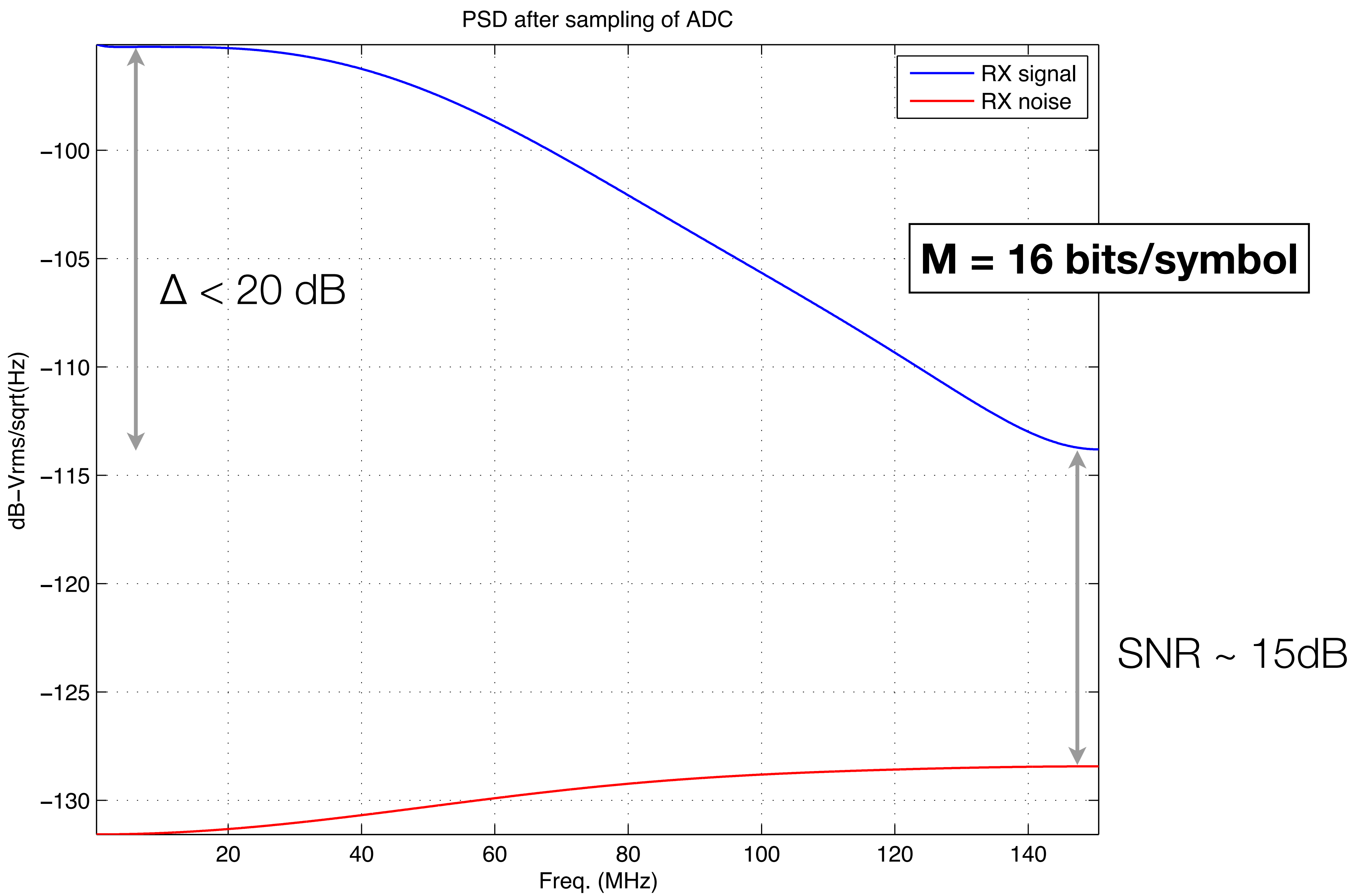
# MMSE,u DFE feasibility - channel response

- Scenario: $C_R$ = 0.83, $C_G$ = 6.7 dB, $DAC_{ENOB}$ = 8, $ADC_{ENOB}$ = 8, IL = 2 dB, POF length = 50 m



# MMSE,u DFE feasibility - channel response

- Scenario: $C_R$ = 0.83, $C_G$ = 6.7 dB, $DAC_{ENOB}$ = 8, $ADC_{ENOB}$ = 8, IL = 2 dB, POF length = 50 m



# MMSE,u DFE feasibility - analysis and conclusions

- For low spectral efficiencies (e.g. M = 3)
  - The most band limiting elements in the signal chain like LED and POF are minimum phase ➤ very long DFE feedback filter with a lot of significant taps
  - Estimated FBF length: 18 taps
  - Number of states of feedback: $3^{18} = 387420489$
  - DFE **error propagation (EP)** is going to be a problem although M is low
  - Burst oriented FEC would be required increasing significantly complexity and latency
  - SOVA-MLSE is not feasible in terms of computational complexity due to the huge number of trellis states
- For high spectral efficiency (e.g. M = 16)
  - Estimated FBF length ~7 taps
  - Number of states of feedback: $16^{7} = 268435456$
  - DFE EP is also going to be a problem although feedback has been reduced
  - Same arguments that before for SOVA-MLSE
- **Tomlinson-Harashima Precoding** solves the DFE EP problem, +20 years used by the industry (from voiceband modems to 10GBase-T).
- **Following slides give the technical feasibility analysis for MMSE-THP**

# Shannon’s capacity evaluation with THP

# THP transmission model and capacity

- The crest-factor of TH precoded TX signal is given by:

$$CF_{TX} = \sqrt{3}$$

- Precoding loss is calculated as:

$$\Gamma_{THP} = \sqrt{\frac{M^2}{M^2 - 1}}$$

- Based on that, the TX electrical current spectral density is given by:

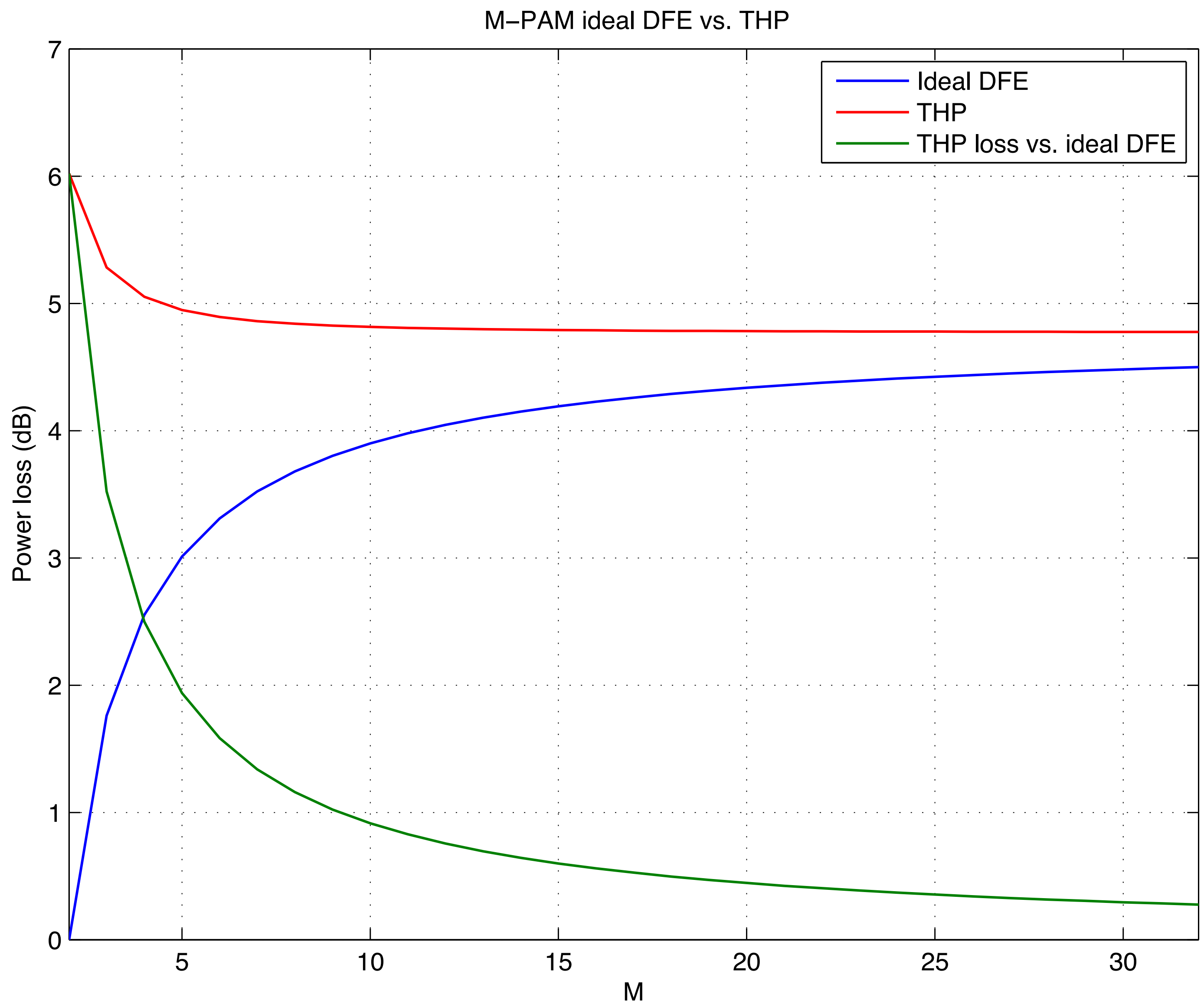
$$E_{TX}(f) = D_{TX} \frac{ER-1}{ER+1} \frac{1}{\sqrt{3}} \frac{1}{\Gamma_{THP}} \frac{1}{\sqrt{F_S/2}} \quad \text{(A/sqrt(Hz))}$$

- The minimum $SNR_d$ required for M-PAM scheme can be upper bounded by next equation, considering modulo loss and detection over an infinite lattice

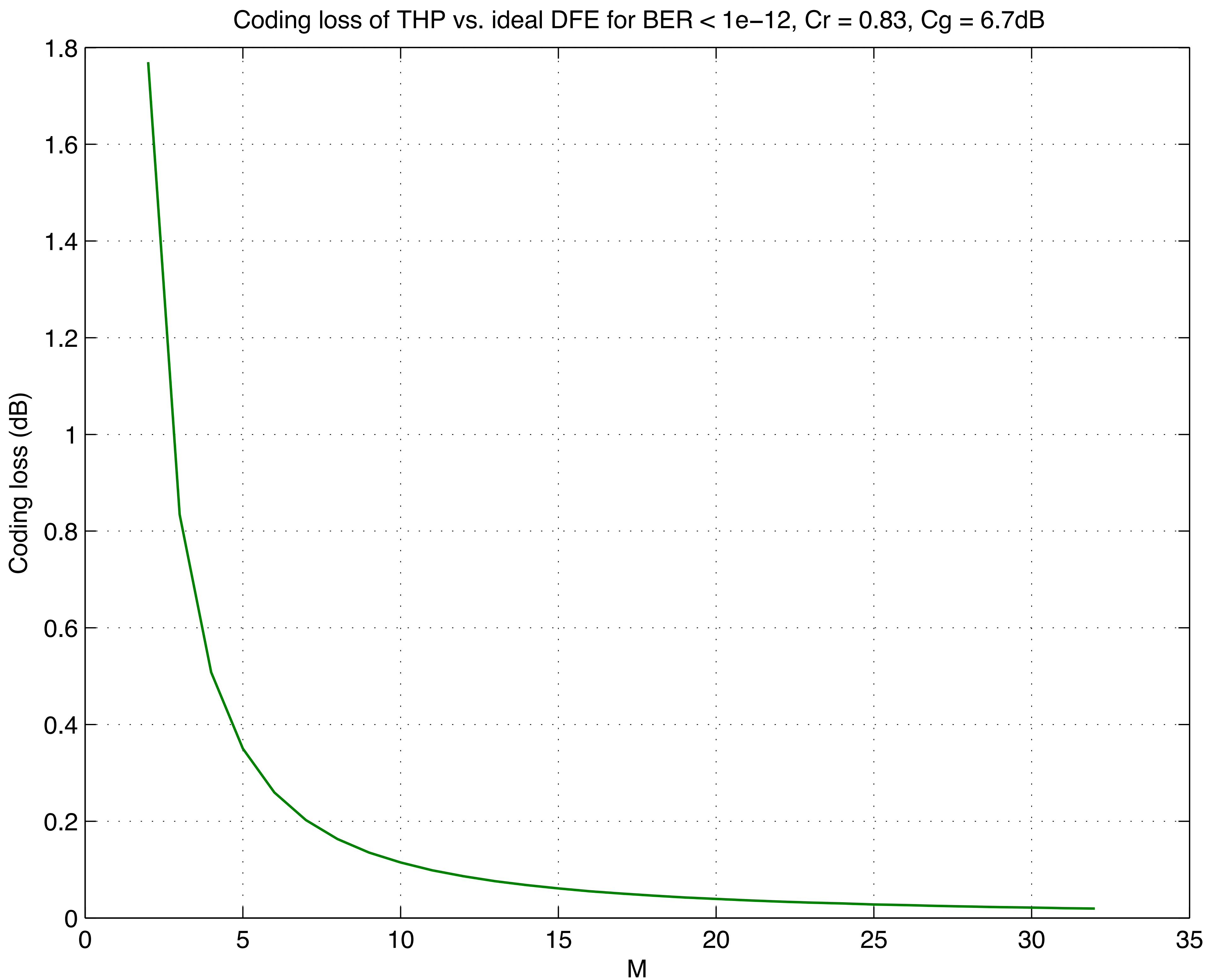
$$SNR_d = \frac{2}{3} \frac{M^{2 \cdot C_R}}{C_G} \left( \mathrm{erfc}^{-1} \left( \log_2 (M) P_b \right) \right)^2$$

# THP power penalty in optical channels

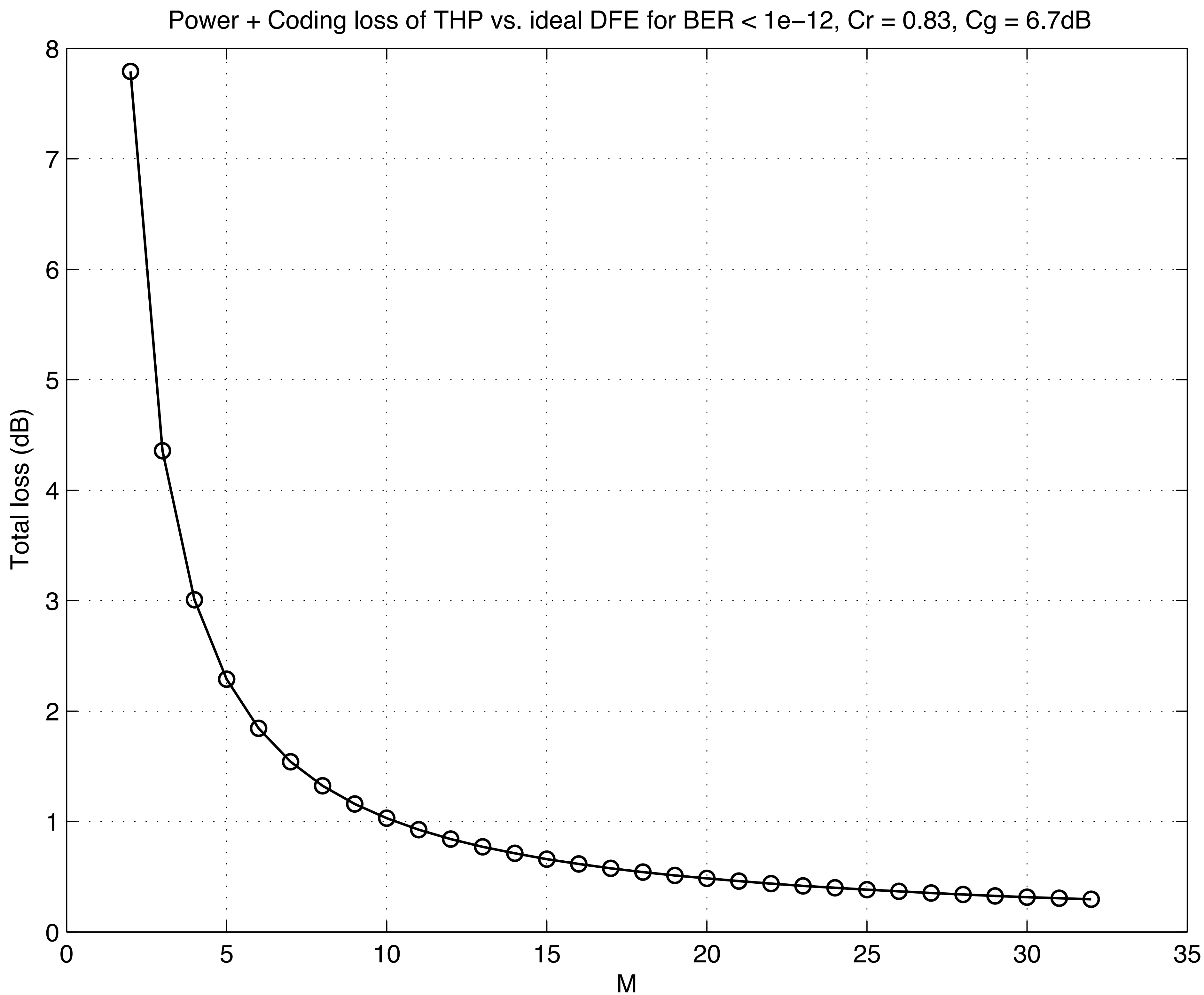
- The penalty of THP vs. non-precoded PAM in terms of $E_{TX}$ as a function of number of PAM levels M is provided in next plot
- Penalty is caused by the crest factor increase as well as precoding loss
- As can be seen, this penalty is very important for low spectral efficiency schemes



# THP coding penalty



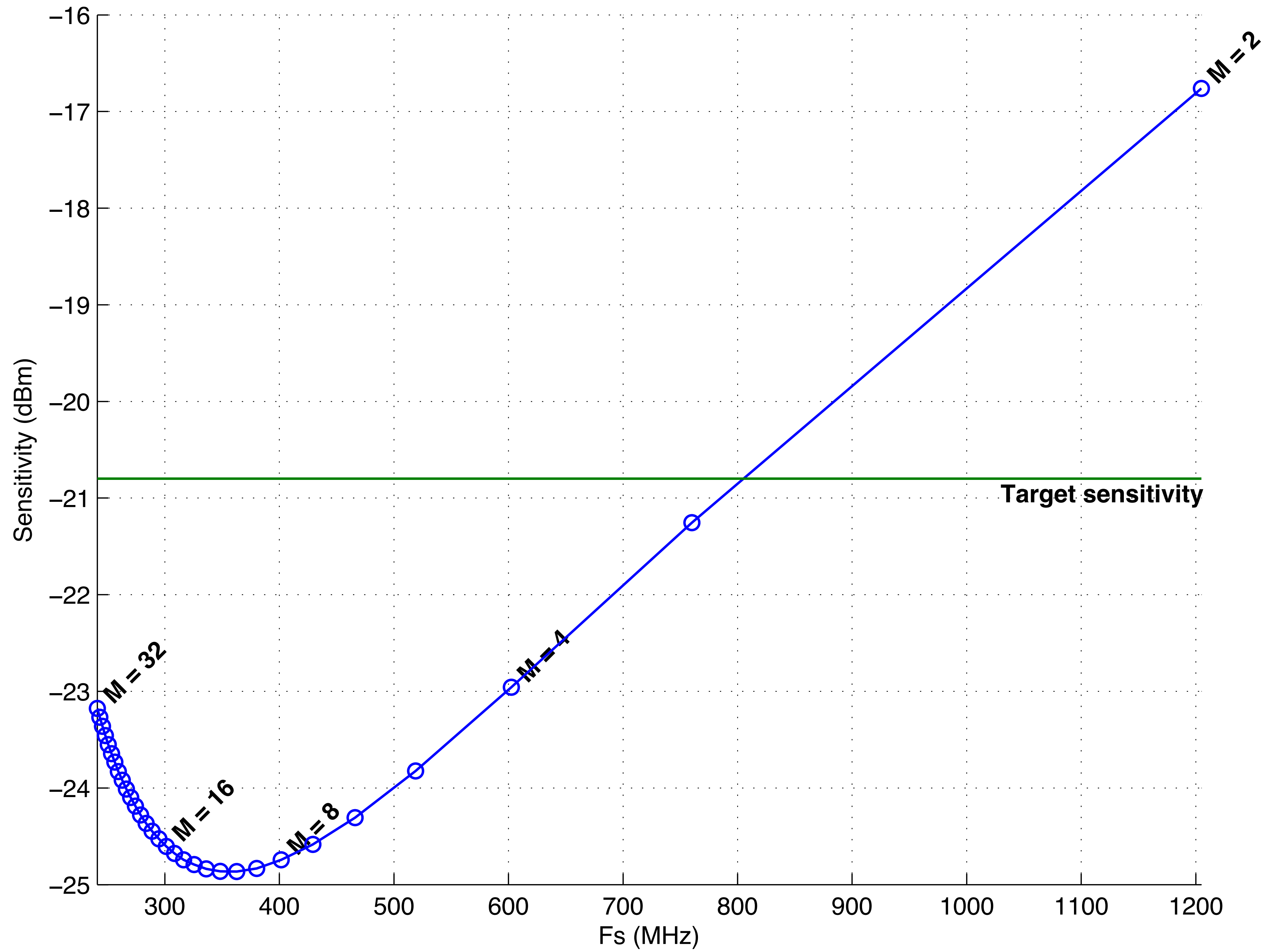
# THP power + coding penalty in optical channels



# Technical feasibility based on Shannon's capacity for THP

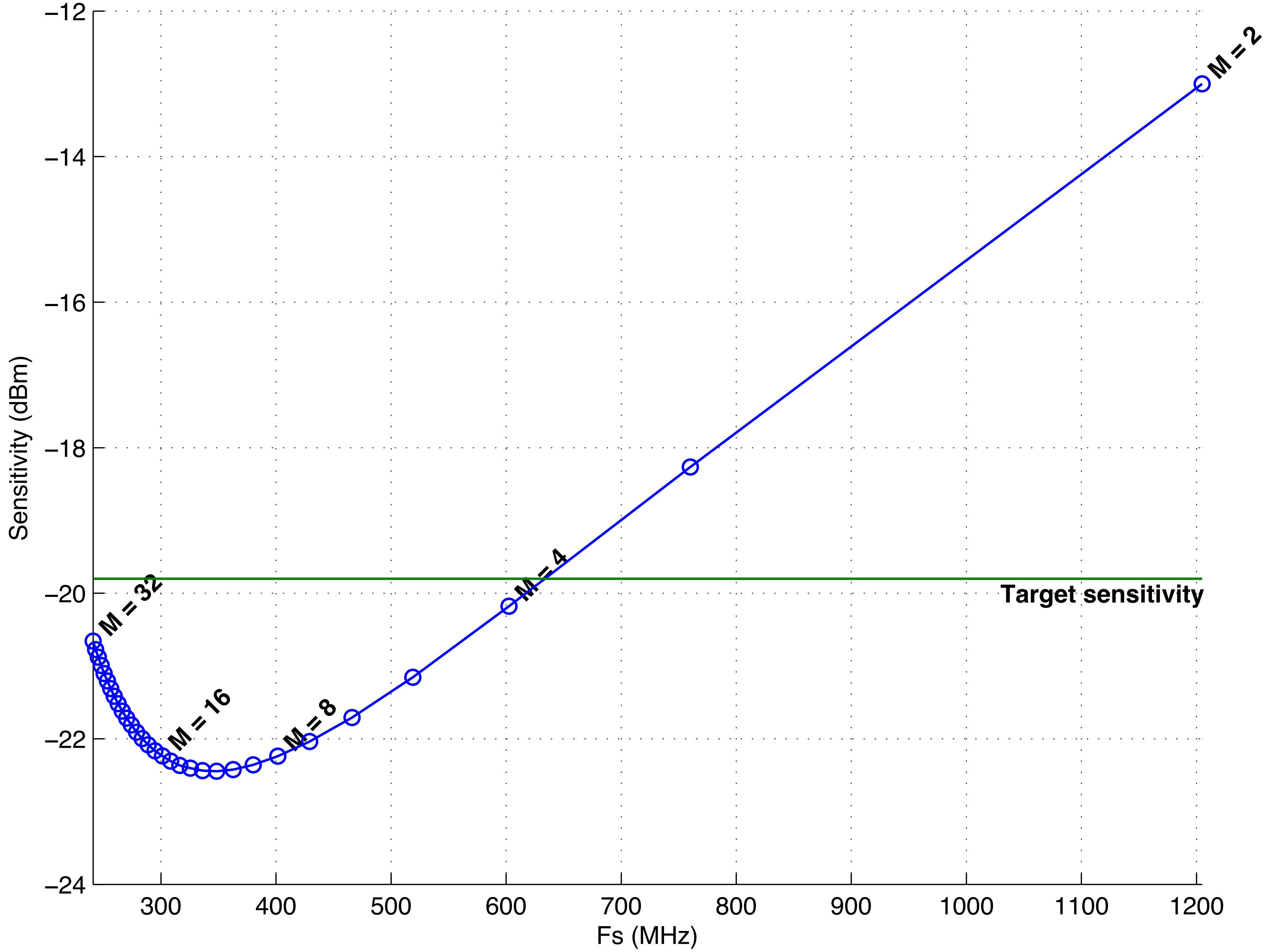
# Technical feasibility for 15 m of POF (THP)

- Low cost FEC $C_R$ = 0.83, $C_G$ = 6.7 dB, $DAC_{ENOB}$ = 8, $ADC_{ENOB}$ = 8, IL = 2 dB



# Technical feasibility for 50 m of POF (THP)

- Low cost FEC $C_R = 0.83$, $C_G = 6.7$ dB, $DAC_{ENOB} = 8$, $ADC_{ENOB} = 8$, IL = 2 dB



# Conclusions

- The Shannon's capacity based analysis shows the GEPOF is technically feasible, being considered
  - worst case ambient conditions,
  - realistic technology limits,
  - implementation losses
- Obtained margins respect to minimum required RX sensitivity for worst-case link budget analysis reported in [perezaranda_04_0514_linkbudget]
  - Automotive application: 15m POF + 4 inline connectors ➤ margin = ~4 dBo
  - Automotive application: 40m POF w/o inline connector ➤ margin > 2 dBo
  - Consumer application: 50m POF with 1 inline connector ➤ margin > 3 dBo
- Shannon says to us GEPOF is feasible

Knowledge Development POF

# Questions?