A Unified Method of Calculating PMD-induced Pulse Broadening

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Three Independent Random Distributions

$$p(x) == \frac{2\, e^{-x^2 \beta}\, x^{2\alpha-1}\, \beta^{\alpha}}{\Gamma(\alpha)}$$

x = Fiber PMD value

For N cables $\beta \to N\beta$

$\alpha \to N\alpha$

$$p(\tau_d) == \frac{e^{-\frac{\tau_d^2}{2\sigma_d^2}} \sqrt{\frac{2}{\pi}}\, \tau_d^2}{\sigma_d^3}$$

Maxwell distribution for DGD

$$p(\theta) == \frac{1}{2} \sin(\theta)$$

Distribution for alignment of polarization vectors

The Gamma Distribution for Fiber PMD

$$p(x) == \frac{2\, e^{-x^2 \beta} x^{2\alpha-1} \beta^{\alpha}}{\Gamma(\alpha)}$$

For N cable Segments: $\beta \to N\beta$, $\alpha \to N\alpha$

Experiment: $\alpha == 0.979$, $\beta == 48.6$ gives $PMD_Q == 0.2$ (N=20 cables, method 1)

(S.A. Jacobs et al, Electronics Letters **33**, 619 (1997))

Our choice: $\alpha == 1.$

$$\beta == \frac{2.}{PMD_Q^2}$$

Calculating the Probability Distribution of Pulse Broadening

$$\sigma_\Delta == \frac{1}{2} \sin(\Theta)\ \tau_d$$

$$p(\sigma_\Delta) == \left\{2 \int_{2\sigma_\Delta}^{\infty} \int_0^{\infty} p(x)\, p(\tau_d)\, p(\Theta(\sigma_\Delta, \tau_d)) \sec(\Theta(\sigma_\Delta, \tau_d))\, dx\, d\tau_d\right\}$$

$$p(\sigma_\Delta) == \frac{4^{\alpha+2}\, \beta^\alpha\, K_{\alpha-1}\left(8\sqrt{\frac{\beta\sigma_\Delta^2}{L\pi}}\right) \left(\frac{\pi\beta L}{\sigma_\Delta^2}\right)^{\frac{1-\alpha}{2}} \sigma_\Delta}{L\,\pi\,\Gamma(\alpha)}$$

Comparison with Previous Work

Dave Dolfi: Assuming $p(x) == \delta(x - \mathrm{PMD}_Q)$

$$p(\sigma_\Delta) == \frac{4\, e^{-\frac{2\,\sigma_\Delta^2}{\sigma_d^2}}\, \sigma_\Delta}{\sigma_d^2}$$

Our results: Assuming $p(x) == \frac{2\, e^{-x^2 \beta}\, x^{2\alpha-1}\, \beta^\alpha}{\Gamma(\alpha)}$

$$p(\sigma_\Delta) == \frac{4^{\alpha+2}\, \beta^\alpha\, K_{\alpha-1}\left(8\sqrt{\frac{\beta\sigma_\Delta^2}{L\pi}}\right) \left(\frac{\pi\beta L}{\sigma_\Delta^2}\right)^{\frac{1-\alpha}{2}} \sigma_\Delta}{L\,\pi\,\Gamma(\alpha)}$$

Assumptions

- Gamma distribution for fiber characteristics, based on experiment, with number of links N = L / (10 km)
- FWHM of pulse is 1/(signaling rate)
- Horizontal eye closure is fractional change in FWHM
- Power penalty using Poole formula, Gaussian shape, multiplied by empirical factor of 2, i. e.

$$dBQ[PMD] == 2\ AB^2\ (1 - \gamma)\ \gamma\ \Delta t^{2}$$

$$A == 25\ \text{(Gaussian pulse)}$$

Calculating Maximum Eye Closure

FWHM of pulse = 1 / (signaling rate)

$$\sigma_0 == \frac{0.425532}{B}$$

(NB: Dave Dolfi took $\sigma_0 = 1/B$)

$$\text{Eye closure} = \frac{\Delta\sigma}{\sigma_0} == \frac{\sigma_{\Delta_0}^2}{2\,\sigma_0^2} == 2.76125 \quad B^2\,\sigma_{\Delta_0}^2$$

Solve numerically:

$$\int_{\sigma_{\Delta_0}}^{\infty} p\,(\sigma_\Delta)\ d\sigma_\Delta == 3.\ \times 10^{-8}$$

(2 seconds per year outage)

Numerical Results

PMD_Q (ps/sqrt(km))	L (km)	Eye closure (UI)	Eye closure (UI) (David Dolfi method with σ_0=0.426/B)	Power Penalty (dB)
0.5	40	0.0092	0.0100	0.167
0.5	50	0.0129	0.0125	0.233
0.5	60	0.0144	0.0150	0.260
1.0	**40**	**0.0368**	**0.0400**	**0.666**
1.0	50	0.0519	0.0500	0.939
1.0	60	0.0576	0.0600	1.043
1.5	40	0.0828	0.0900	1.499
1.5	50	0.1168	0.1125	2.114
1.5	60	0.1298	0.1350	2.349
2.0	40	0.1472	0.1600	2.664
2.0	50	0.2076	0.2000	3.758
2.0	60	0.2307	0.2400	4.176