10GBASE-T Coding and Modulation: 128-DSQ + LDPC

IEEE P802.3an Task Force
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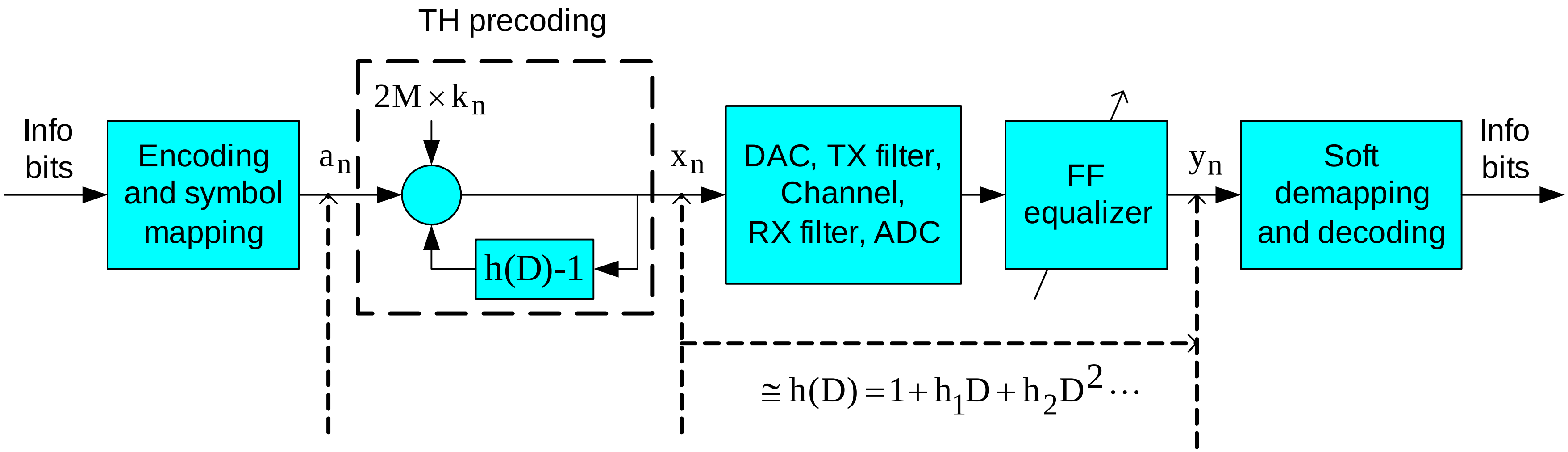
Precoding system and definition of SNR



$a(D)$

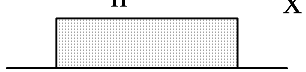
$$x(D) = \frac{a(D) + 2Mk(D)}{h(D)}$$

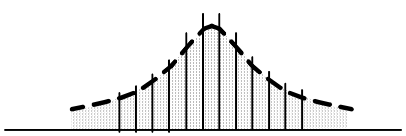
$$y(D) = a(D) + 2Mk(D) + w(D)$$

σ_w^2

$$a_n \in M\text{-PAM} = \{\pm 1, \pm 3, \cdots \pm (M-1)\}$$

$$-M \le x_n < M: \quad E_x = (2M)^2/12$$

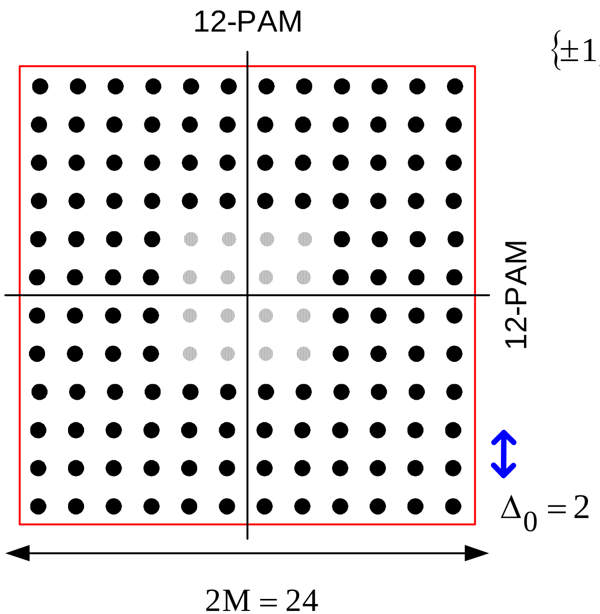




Signal-to-noise ratio $\text{SNR} = E_x/\sigma_w^2$

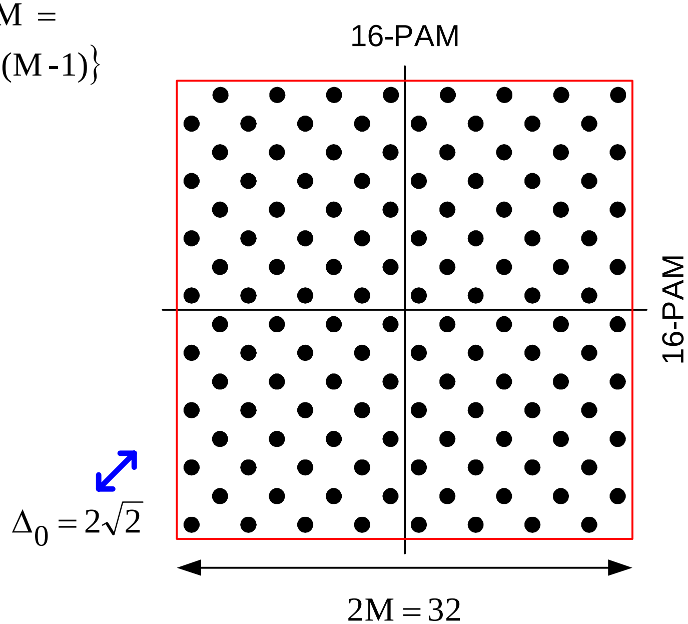
2-D constellations for modulation rates in 800+ Mb range

12-PAM^2 (with or w/o hole)



$$M\text{-PAM} = \{\pm 1, \pm 3, \cdots \pm (M-1)\}$$

128-DSQ (Double SQuare)



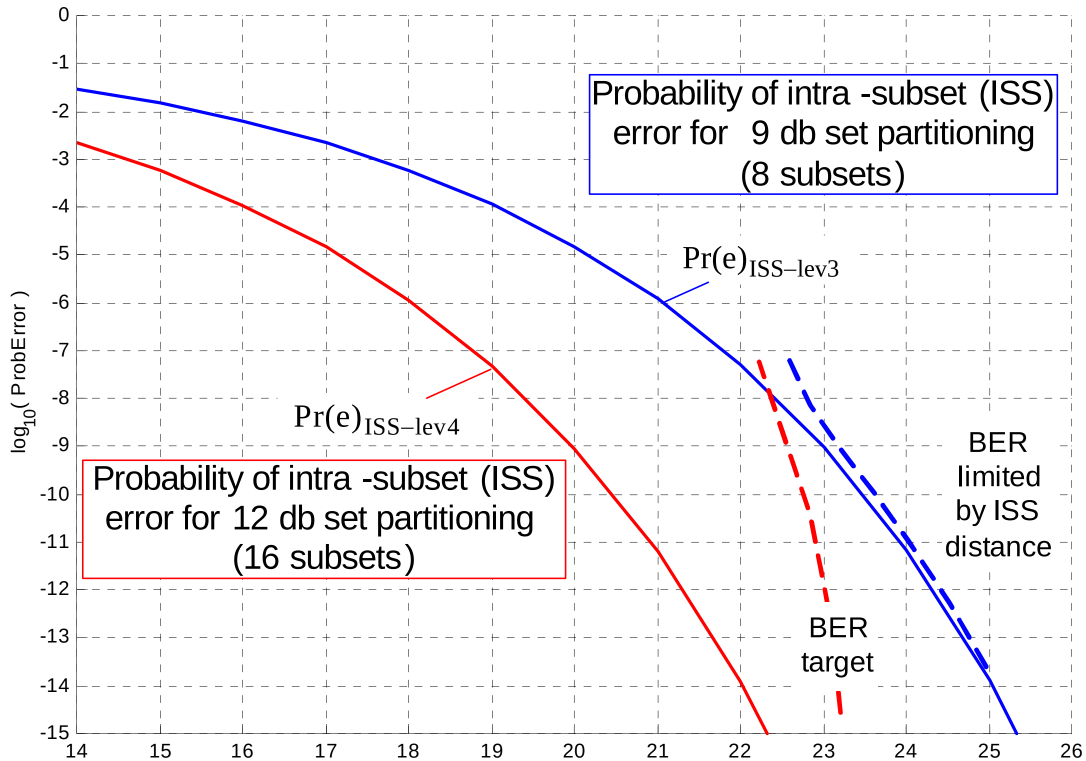
Signal energy per dimension at precoder output $E_x = (2M)^2/12$

$$\boxed{E_x / \Delta_0^2 = 48/4 = 12}$$

$$\boxed{E_x / \Delta_0^2 = (256/3)/8 = 10.666}$$

-0.5115 dB

128-DSQ: probability of intra-subset errors



$$\mathrm{SNR} = E_x / \sigma_w^2 \quad [\mathrm{dB}]$$

$$\mathbf{128-DSQ}: \; M = 16; \; E_x = (2M)^2/12; \; \Delta_0^2 = 8$$

$$\Delta_3^2 = 8\Delta_0^2: \; \Pr(e)_{ISS-lev3} = \frac{1}{2} \times 4 \times Q\left(\frac{\Delta_3}{2\sigma_w}\right); \; \Delta_4^2 = 16\Delta_0^2: \; \Pr(e)_{ISS-lev4} = \frac{1}{2} \times 4 \times Q\left(\frac{\Delta_4}{2\sigma_w}\right)$$

This confirms the need for 12 dB set partitioning

Coding, modulation, framing: two variants

Variant I: 128-DSQ + LDPC(1024,821)

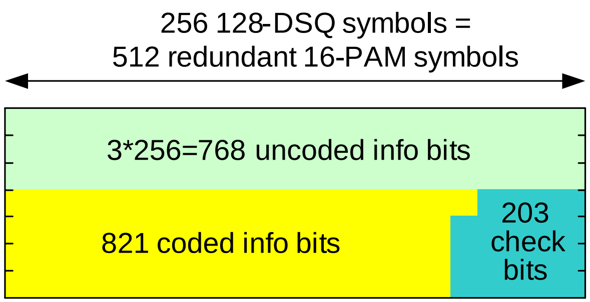
- LDPC coding weak w.r.t. to uncoded-bit-only error performance
- Code rate 3.1035 bit/dim
- Framing example: 1 frame = 8 code blocks → modulation rate 821.51 Mbaud, 0.29% OH for synch and aux. channel.

Variant II: 128-DSQ + LDPC(1024,809)

- Stronger LDPC coding better matched to uncoded-bit-only error performance
- Code rate 3.0801 bit/dim (-0.0234 bit/dim vs. 0.14 dB gain)
- Framing example: 1 frame = 1 code block → modulation rate 833.33 Mbaud (25 MHz x 100/3) , 1.07% OH for synch and aux. channel.

Coding, modulation, framing: variant I

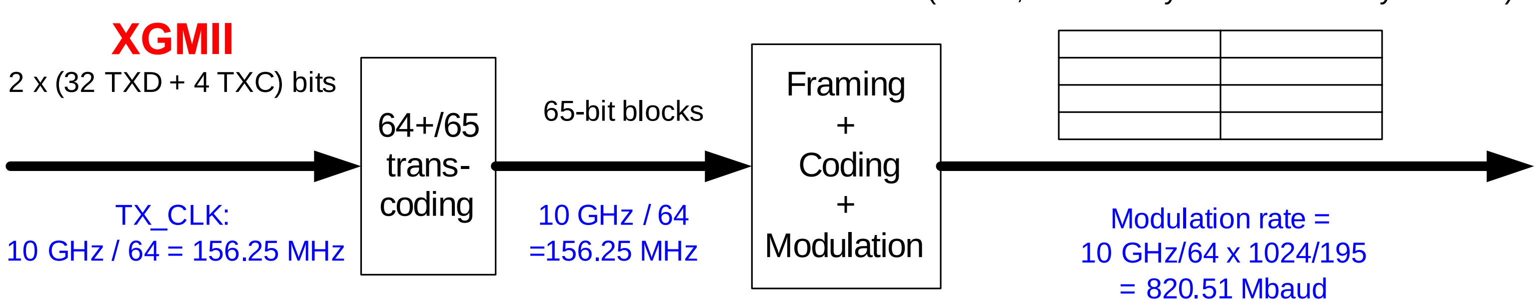
128-DSQ modulation with 12 dB set partitioning (into 16 2-D subsets) and (1024,821) LDPC coding



Code block = 1589 info bits encoded into 512 PAM symbols (3.1035 bit/dim)

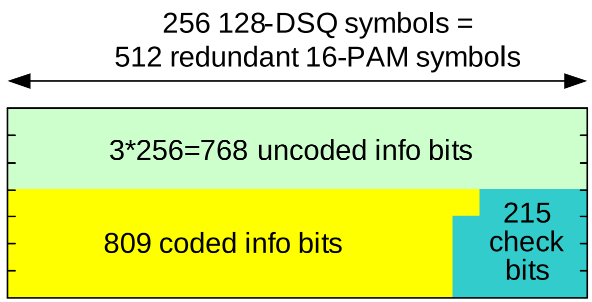
Framing example

10GBASE-T Frame =
8 code blocks over four pairs = 8 x 1589 bits =
195 x 65-bit blocks + 37 overhead bits
(0.29%, for frame synch and auxiliary channel)



Coding, modulation, framing: variant II

128-DSQ modulation with 12 dB set partitioning (into 16 2-D subsets) and (1024,809) LDPC coding

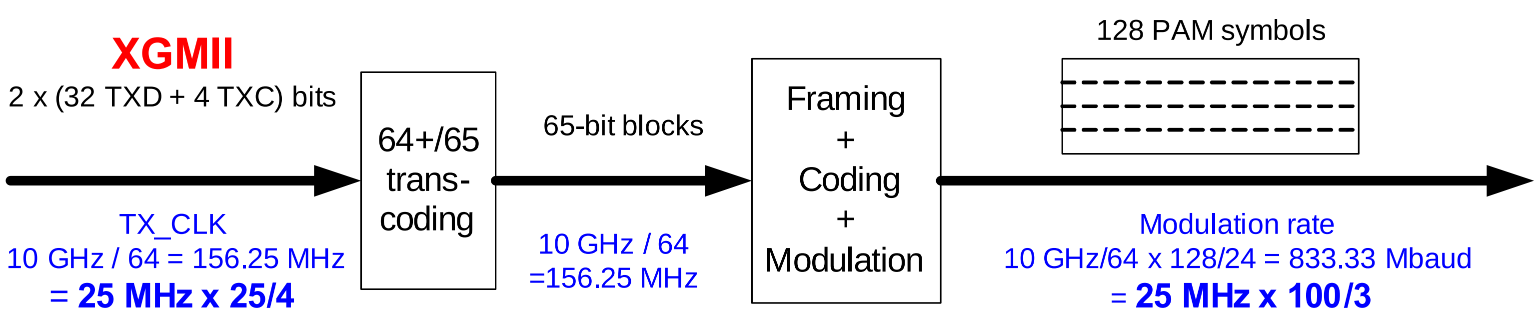


Code block = 1577 info bits encoded into 512 PAM symbols (3.0801 bit/dim)

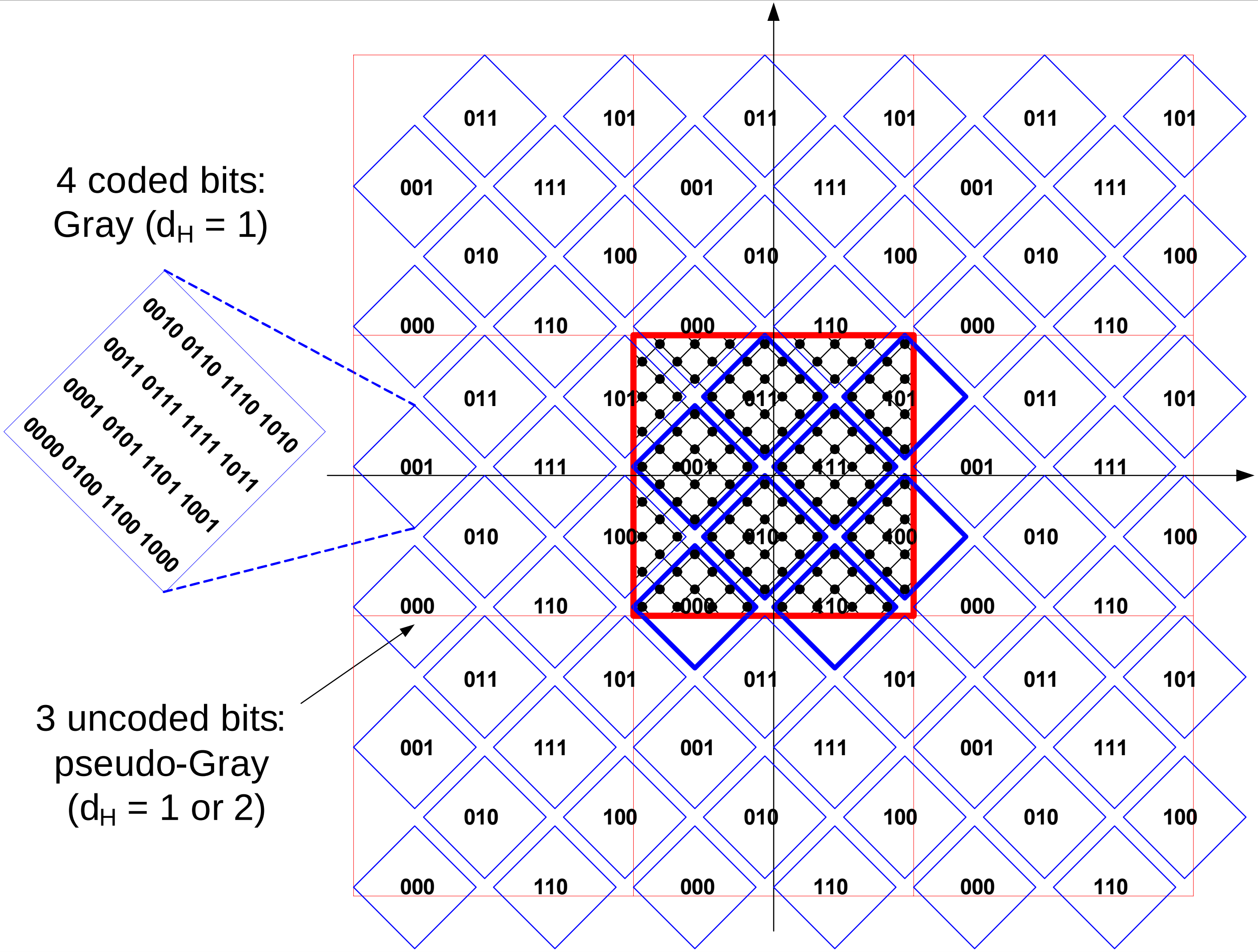
Framing example

10GBASE-T Frame =

1 Code block over four pairs: 1577 bits =
24 x 65-bit blocks + 17 overhead bits
(1.077%, for frame synch and auxiliary channel)

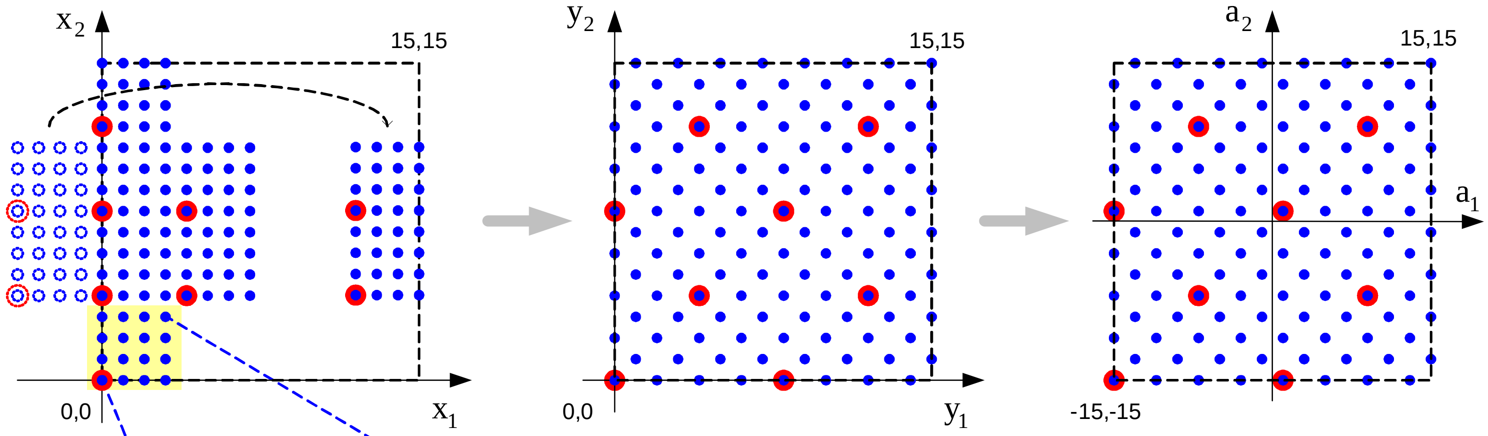


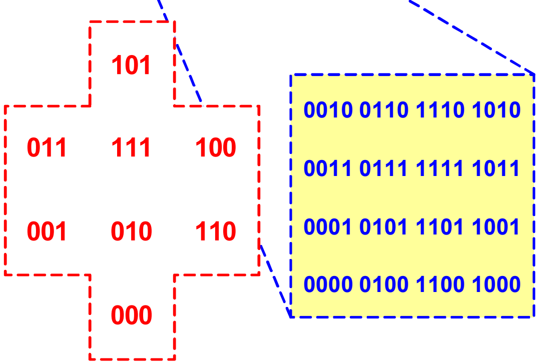
128-DSQ bit mapping: 3 uncoded bits, 4 coded bits



Basic 128-DSQ with cyclic precoding extensions

128-DSQ bit mapping





$u_1\ u_2\ u_3$
3 uncoded bits
pseudo-Gray
mapped
(d_H = 1 or 2)

$c_1\ c_2\ c_3\ c_4$
4 coded bits
Gray mapped
(d_H = 1)

$$\textbf{Step 1}: \quad 0 \le (x_i = 8x_i^3 + 4x_i^2 + 2x_i^1 + x_i^0) \le 15, \quad i = 1,2$$

$$x_1^3 = \bar{u}_1 \,\&\, u_3 \qquad x_2^3 = (u_2 \,\&\, u_3) \vee (u_1 \,\&\, \bar{u}_2)$$
$$x_1^2 = u_1 \oplus u_3 \qquad x_2^2 = u_2 \oplus u_3$$

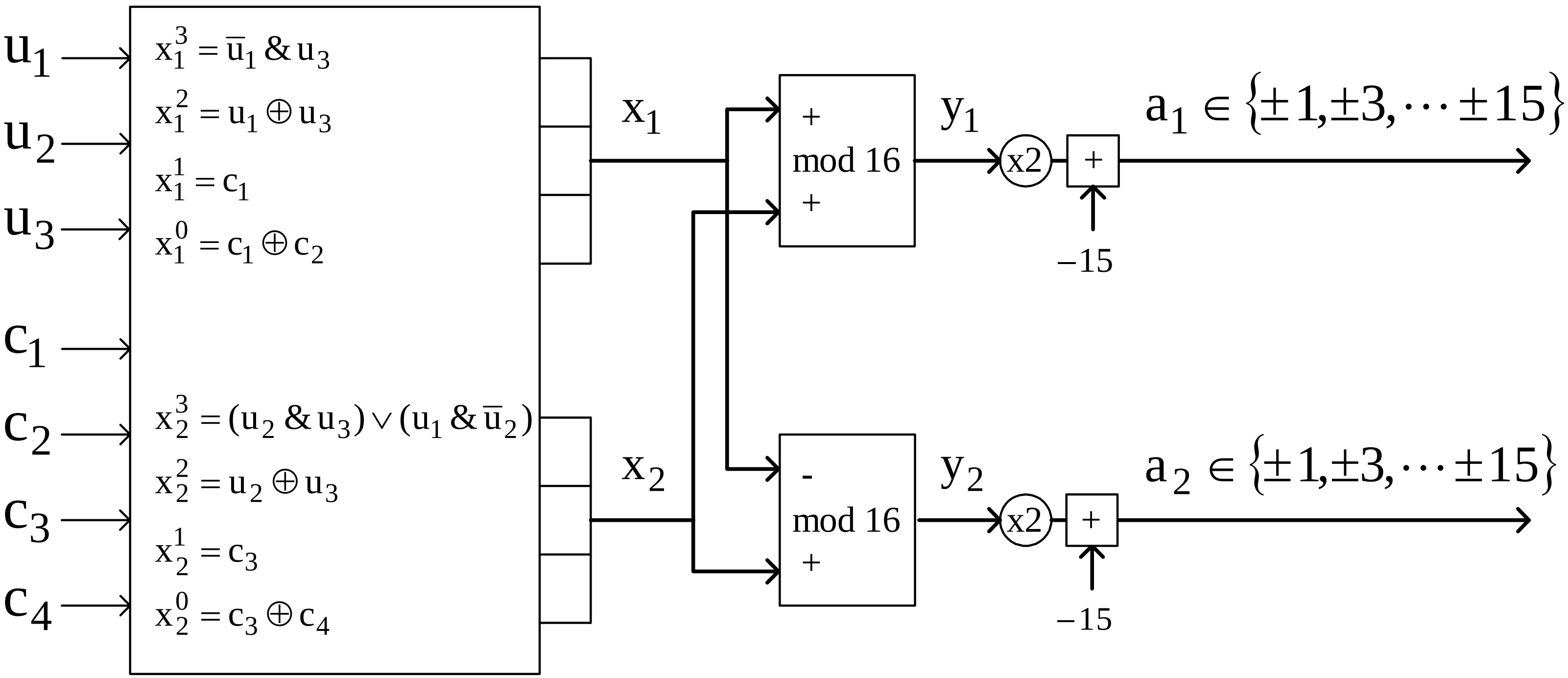
$$x_1^1 = c_1 \qquad x_2^1 = c_3$$
$$x_1^0 = c_1 \oplus c_2 \qquad x_2^0 = c_3 \oplus c_4$$

$$\textbf{Step 2}: \quad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}}^{\mathbf{R}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \bmod 16$$

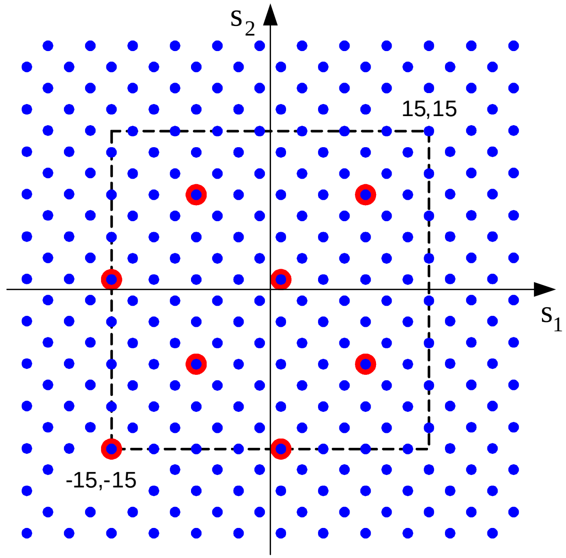
$$\textbf{Step 3}: \quad \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 2\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} 15 \\ 15 \end{bmatrix}$$

128-DSQ bit mapping: implementation

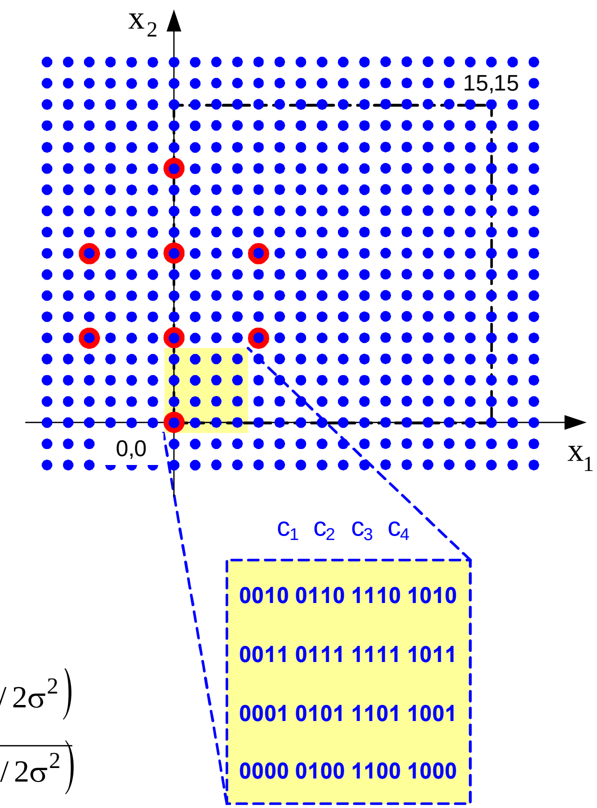
7-bit label **two 16-PAM symbols**



128-DSQ soft demapping: 4 coded bits



$$\overbrace{\begin{bmatrix} 0.5 & -0.5 \\ 0.5 & 0.5 \end{bmatrix}}^{\mathbf{R}^{-1}} \begin{bmatrix} (s_1+15)/2 \\ (s_2+15)/2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



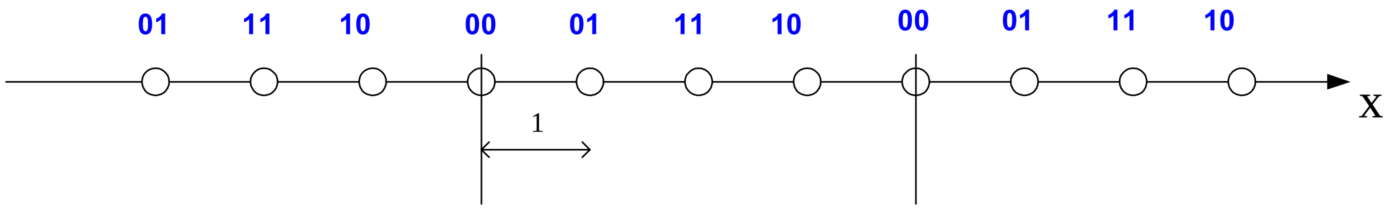
Extended constellation points caused by precoding

$$\mathrm{llrb}(x) = \ln \frac{\sum_k \exp\left(-[x-(4k+0)]^2/2\sigma^2\right) + \exp\left(-[x-(4k+1)]^2/2\sigma^2\right)}{\sum_k \exp\left(-[x-(4k+2)]^2/2\sigma^2\right) + \exp\left(-[x-(4k+3)]^2/2\sigma^2\right)}$$

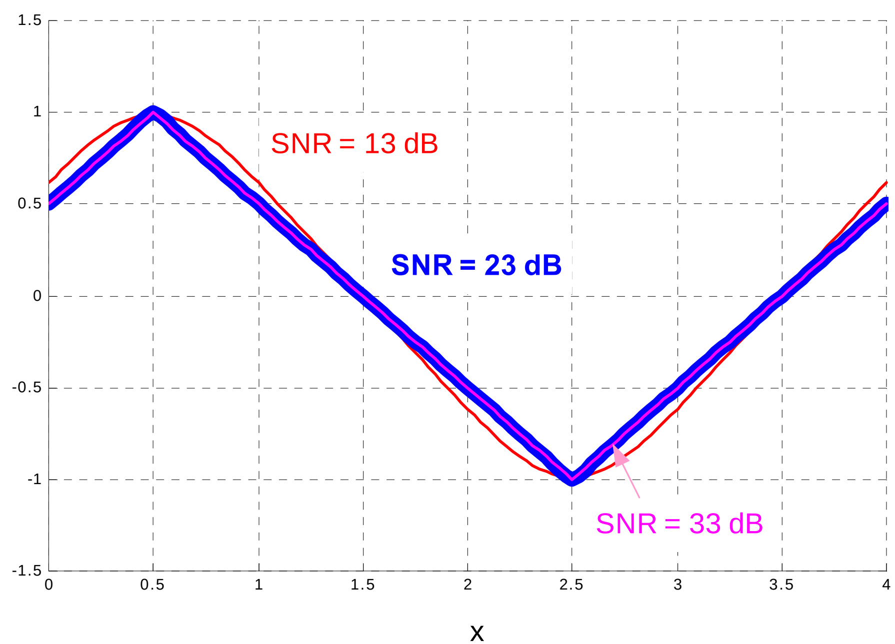
$$\log \frac{\Pr(c_1=0/x_1)}{\Pr(c_1=0/x_1)} = \mathrm{llrb}(x_1 \mod 4) \qquad \log \frac{\Pr(c_2=0/x_1)}{\Pr(c_2=0/x_1)} = \mathrm{llrb}(x_1+1 \mod 4)$$

$$\log \frac{\Pr(c_3=0/x_2)}{\Pr(c_3=0/x_2)} = \mathrm{llrb}(x_2 \mod 4) \qquad \log \frac{\Pr(c_4=0/x_2)}{\Pr(c_4=0/x_2)} = \mathrm{llrb}(x_2+1 \mod 4)$$

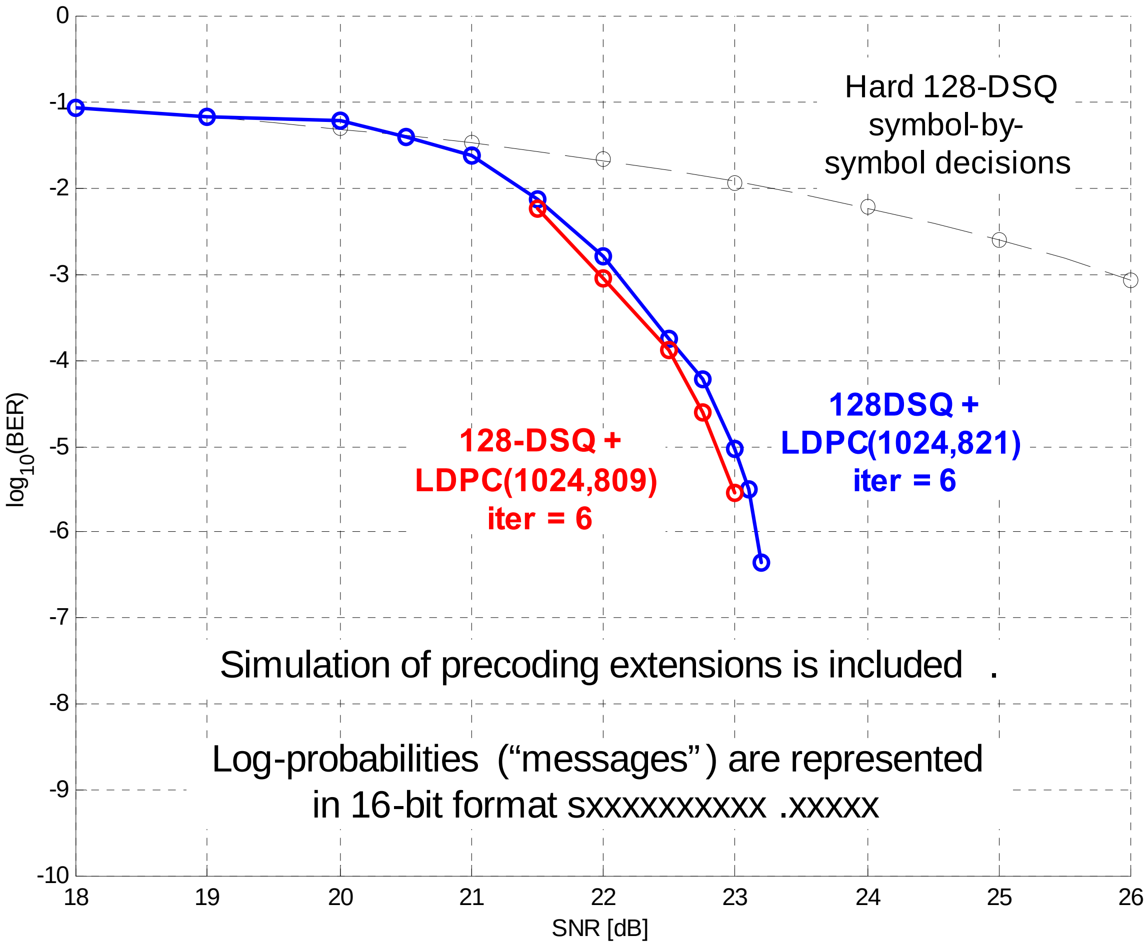
The function llrb(x)



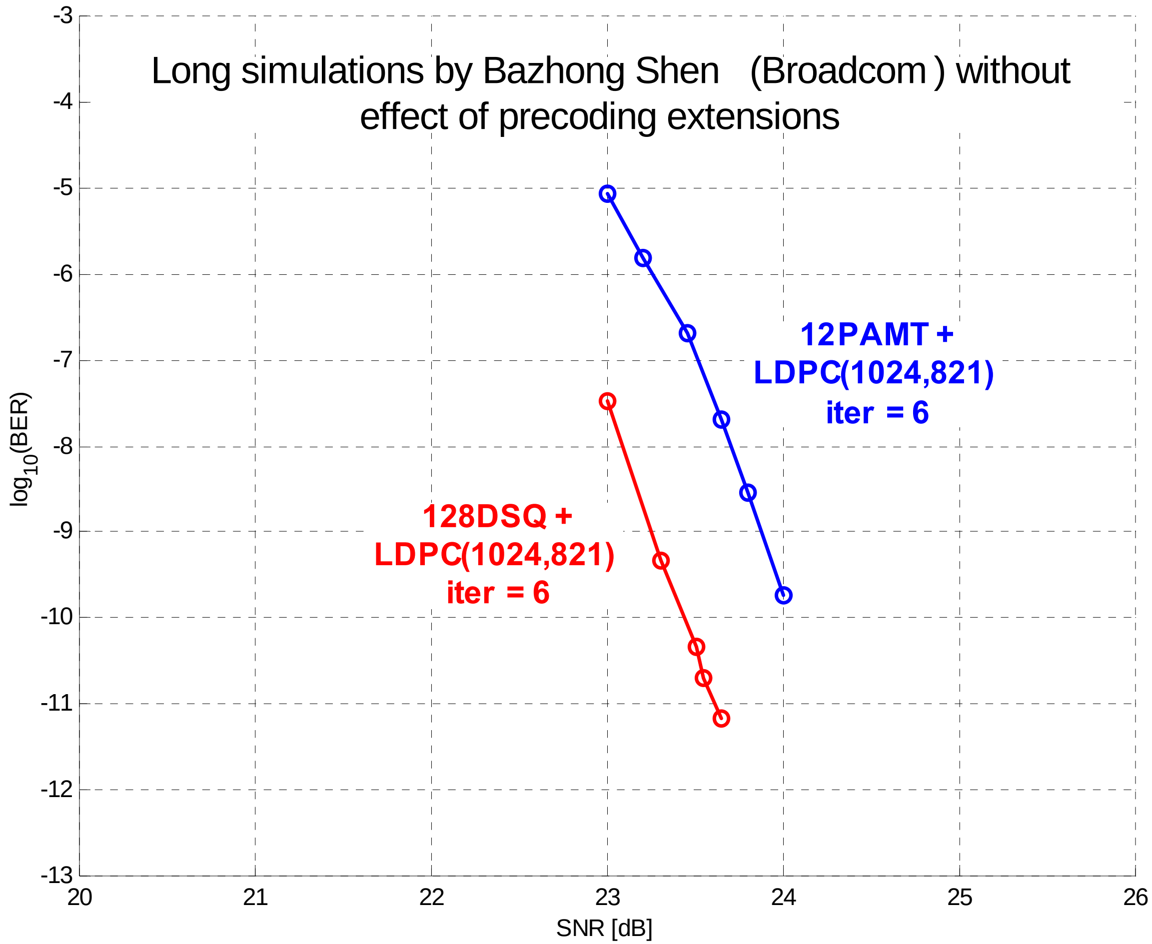
$$\mathrm{llrb}(x) = \ln \frac{\sum_k \exp\left(-[x-(4k+0)]^2/2\sigma^2\right) + \exp\left(-[x-(4k+1)]^2/2\sigma^2\right)}{\sum_k \exp\left(-[x-(4k+2)]^2/2\sigma^2\right) + \exp\left(-[x-(4k+3)]^2/2\sigma^2\right)} \cong \frac{1}{\sigma^2}\begin{cases} x+0.5 : & 0 \le x \le 0.5 \\ 1.5-x : & 0.5 \le x \le 2.5 \\ x-3.5 : & 2.5 \le x \le 4 \end{cases}$$



128-DSQ + LDPC performance



12-PAM-T and 128-DSQ + LDPC performance



Conclusions

- 128-DSQ constellation is the natural in-between 8-PAM and 16-PAM modulation
- Bit mapping, precoding, metric calculation, subset decoding … all based on simple logic and power-of-two based arithmetic
- LDPC(1024,809) coding appears to be preferable over LDPC(1024,821) …
- and allows for simple framing and clock generation.