How to convert 2-port S-parameters to 4-port S-parameters

Introduction

Figure 1 below represents an N-port coupled differential line system. All links 1:N are coupled differential ports. This document presents the single ended 2-port VNA (Vector Network Analyzer) measurement technique and computation of mixed mode (differential/common mode) scattering parameters for any arbitrary coupled differential link. Additionally, mixed mode scattering parameter measurement and computation will be shown for the cross coupling (FEXT, NEXT) between an arbitrary victim link and a single aggressor link.

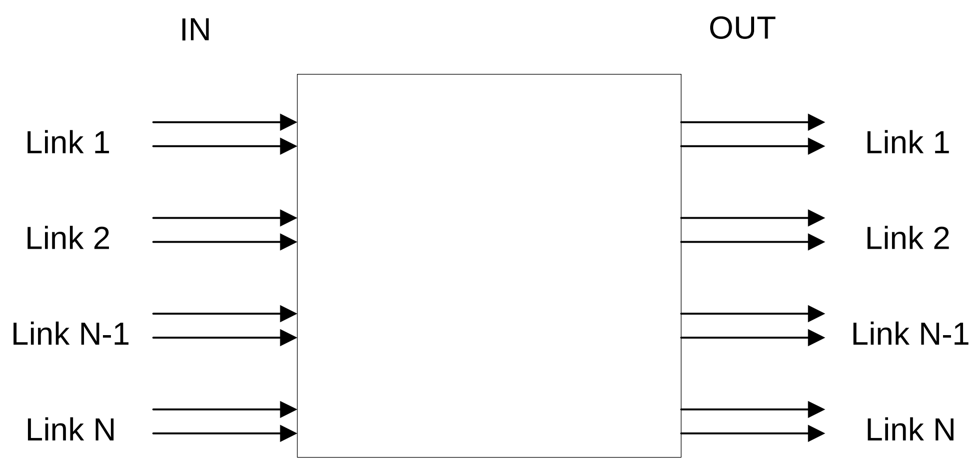

Figure 1. N-Port Coupled Differential Line System

Consider a single Link i from Figure 1 shown below as a 4 port system (4 terminals with respect to ground) in the power flow diagram of Figure 2. Link i is represented as pair of lines, one line between terminals **x**-**z** and second line between terminals **y**-**w**. These two lines can be coupled. Incident and reflected power waves are given the traditional notation of **a** and **b** respectively.

The familiar single ended S-parameters for line **x**-**z** are shown as (Sxx, Sxz, Szx, Szz) and for line **y**-**w**, (Syy, Syw, Swy, Sww). Since the two lines are coupled, scattering parameters can be defined relating the incident and reflected waves of ports that are not coupled through transmission lines. From every incident port **x**,**y**,**w**,**z** there exists 2 additional S-parameter terms. Thus there are eight additional S-parameter terms to be combined with the familiar 8 transmission line S-parameter terms to form a complete single ended 4x4 scattering matrix $\mathbf{S}_s$. The single ended system is described by equation 1 & 2 below.

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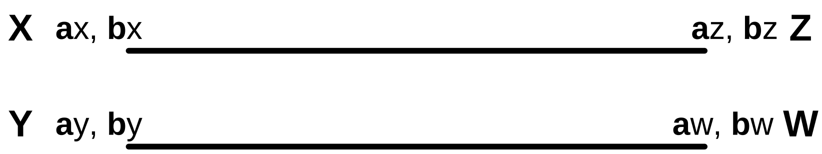
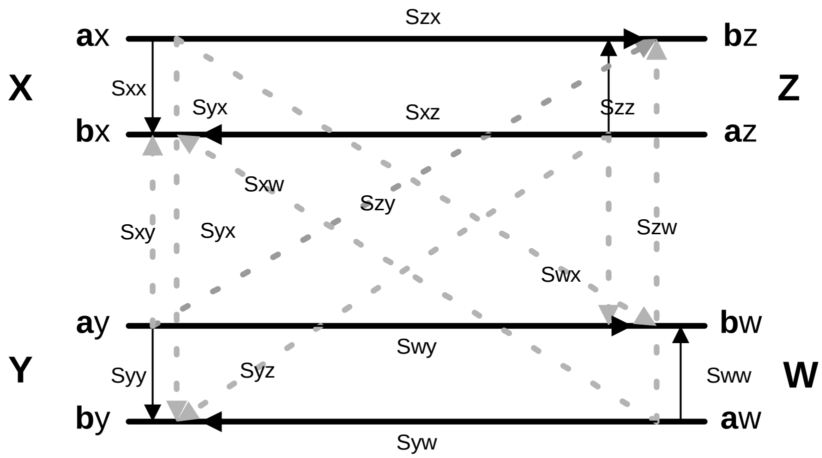


Physical Coupled Line



Power Flow Graph

Figure 2. Single Ended Scattering Parameters for a Coupled Pair

$$\mathbf{b_s} = \mathbf{S_s}\mathbf{a_s} \qquad \text{Eq1}$$

$$\begin{pmatrix} b_x \\ b_y \\ b_z \\ b_w \end{pmatrix} = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} & S_{xw} \\ S_{yx} & S_{yy} & S_{yz} & S_{yw} \\ S_{zx} & S_{zy} & S_{zz} & S_{zw} \\ S_{wx} & S_{wy} & S_{wz} & S_{ww} \end{pmatrix} \cdot \begin{pmatrix} a_x \\ a_y \\ a_z \\ a_w \end{pmatrix} \qquad \text{Eq2}$$

Mixed mode scattering parameters are comprised of the differential mode, common mode, common to differential and differential to common mode scattering parameters. They can be defined in terms of the single ended scattering matrix elements defined in $\mathbf{S_s}$. If we assume that the link is driven with an odd-mode differential wave then **ax** = -**ay**, **aw** = -**az**, for even mode and **ax** = **ay**, **aw** = **az** for common mode, then:

$$b_{d1} = \frac{1}{\sqrt{2}} \cdot (b_x - b_y) \quad a_{d1} = \frac{1}{\sqrt{2}} \cdot (a_x - a_y)$$
$$b_{d2} = \frac{1}{\sqrt{2}} \cdot (b_z - b_w) \quad a_{d2} = \frac{1}{\sqrt{2}} \cdot (a_z - a_w)$$
$$b_{c1} = \frac{1}{\sqrt{2}} \cdot (b_x + b_y) \quad a_{c1} = \frac{1}{\sqrt{2}} \cdot (a_x + a_y)$$
$$b_{c2} = \frac{1}{\sqrt{2}} \cdot (b_z + b_w) \quad a_{c2} = \frac{1}{\sqrt{2}} \cdot (a_z + a_w)$$

Eq3

$$\begin{pmatrix} b_{d1} \\ b_{d2} \\ b_{c1} \\ b_{c2} \end{pmatrix} = \begin{pmatrix} S_{d1d1} & S_{d1d2} & S_{d1c1} & S_{d1c2} \\ S_{d2d1} & S_{d2d2} & S_{d2c1} & S_{d2c2} \\ S_{c1d1} & S_{c1d2} & S_{c1c1} & S_{c1c2} \\ S_{c2d1} & S_{c2d2} & S_{c2c1} & S_{c2c2} \end{pmatrix} \cdot \begin{pmatrix} a_{d1} \\ a_{d2} \\ a_{c1} \\ a_{c2} \end{pmatrix}$$

$$\mathbf{b_m} = \mathbf{S_m} \mathbf{a_m}$$

Eq4

Which can be fully expanded to give:

$$S_{d1d1} = \frac{1}{2}\cdot(S_{XX} - S_{XY} - S_{YX} + S_{YY}) \qquad S_{d1c1} = \frac{1}{2}\cdot(S_{XX} - S_{YX} + S_{XY} - S_{YY})$$

$$S_{d1d2} = \frac{1}{2}\cdot(S_{XZ} - S_{YZ} - S_{XW} + S_{YW}) \qquad S_{d1c2} = \frac{1}{2}\cdot(S_{XZ} - S_{YZ} + S_{XW} - S_{YW})$$

$$S_{d2d1} = \frac{1}{2}\cdot(S_{ZX} - S_{WX} - S_{ZY} + S_{WY}) \qquad S_{d2c1} = \frac{1}{2}\cdot(S_{ZX} - S_{WX} + S_{ZY} - S_{WY})$$

$$S_{d2d2} = \frac{1}{2}\cdot(S_{ZZ} - S_{WZ} - S_{ZW} + S_{WW}) \qquad S_{d2c2} = \frac{1}{2}\cdot(S_{ZZ} - S_{WZ} + S_{ZW} - S_{WW})$$

$$S_{c1d1} = \frac{1}{2}\cdot(S_{XX} + S_{YX} - S_{XY} - S_{YY}) \qquad S_{c1c1} = \frac{1}{2}\cdot(S_{XX} + S_{YX} + S_{XY} + S_{YY})$$

$$S_{c1d2} = \frac{1}{2}\cdot(S_{XZ} + S_{YZ} - S_{XW} - S_{YW}) \qquad S_{c1c2} = \frac{1}{2}\cdot(S_{XZ} + S_{YZ} + S_{XW} + S_{YW})$$

$$S_{c2d1} = \frac{1}{2}\cdot(S_{ZX} + S_{WX} - S_{ZY} - S_{WY}) \qquad S_{c2c1} = \frac{1}{2}\cdot(S_{ZX} + S_{WX} + S_{ZY} + S_{WY})$$

$$S_{c2d2} = \frac{1}{2}\cdot(S_{ZZ} + S_{WZ} - S_{ZW} - S_{WW}) \qquad S_{c2c2} = \frac{1}{2}\cdot(S_{ZZ} + S_{WZ} + S_{ZW} + S_{WW})$$

Eq5

Forward Channel Measurements

With a 2 port VNA (Vector Network Analyzer) one can acquire the single ended scattering parameters and compute the mixed mode parameters using Equation 5.

Figure 3 below illustrates the 6 measurements required for obtaining $\mathbf{S}_s$.

How to convert 2 port S-parameters to 4 port S-parameters

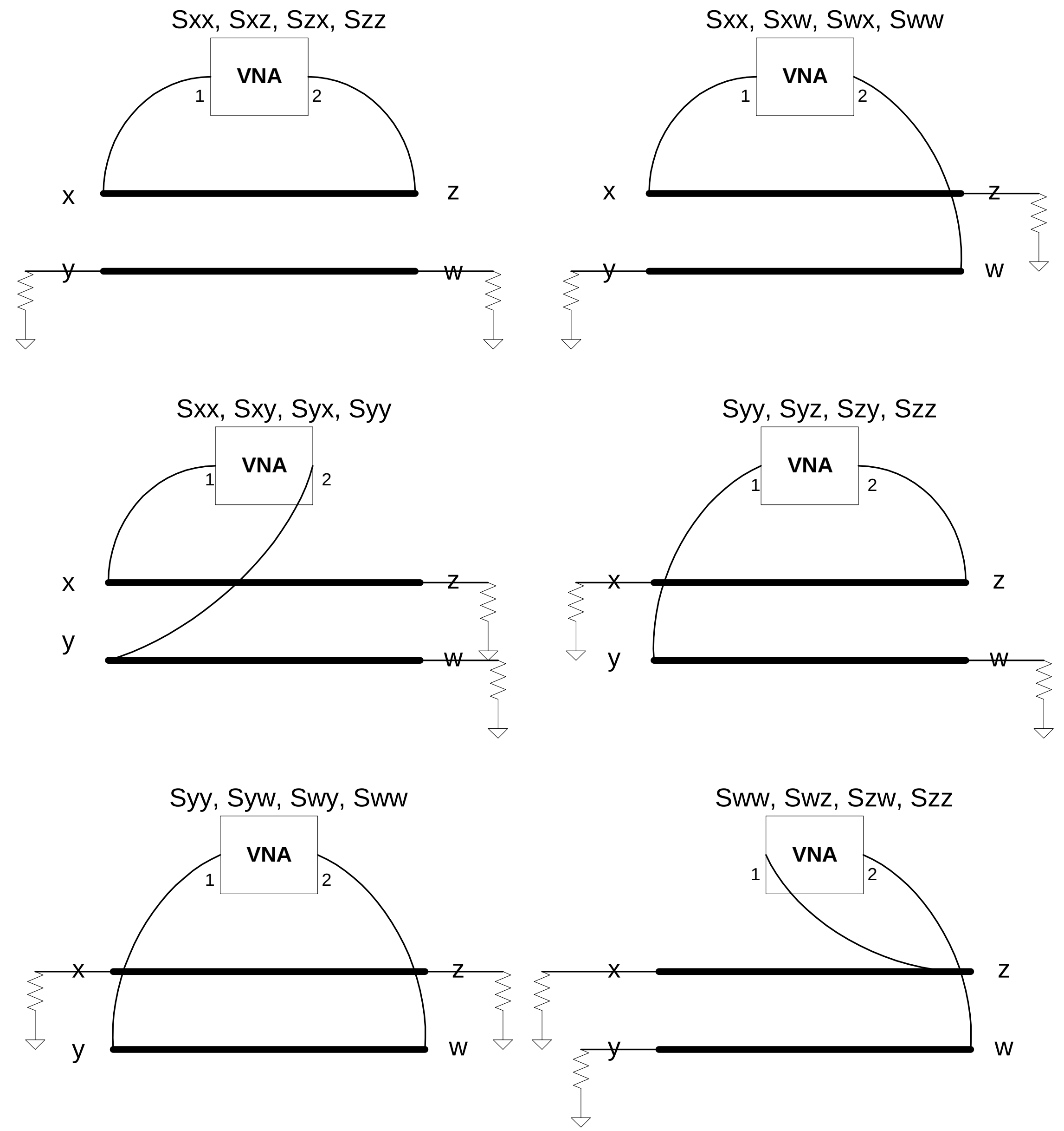


Figure 3. Single Ended S-Parameter Measurements for a Coupled Pair.

Notes:

1. Insure that all unconnected terminals must be terminated with the characteristic impedance of the measuring system. **Important** to note that if these are not calibrated (usually the case) then they can give **large** errors for tightly coupled lines (that is similar coupling from **x** to **z** as **x** to **y**/**w**, however most differential pairs are loosely coupled hence the errors are typically quite low).
2. Measurements must be consistent with respect to port1 and port 2 of the VNA, for example when measuring across terminals **x**,**y** and **z**,**w** port 1 of the VNA is connected to **x**,**z** respectively.

3. There are overlaps that could be removed to lower the number of sweeps, for example Sxx is measured 3 times in figure 3.

NEXT Measurements

Figure 4 (physical port diagram) illustrates a re-definition of the **x**,**w**,**y**,**z** ports that were used to define the mixed mode parameters for two coupled lines. This is done so that equation 5 can be re-used for mixed mode NEXT computation with the same variable names and the only difference is the assignment of the variable names to the measured single ended parameters from the VNA. Note that double-headed arrows are used to show simultaneously transverse and reverse single ended S-parameters. Reflection S-parameters Sxx, Syy, Szz, Sww are not shown but are required for the computation.

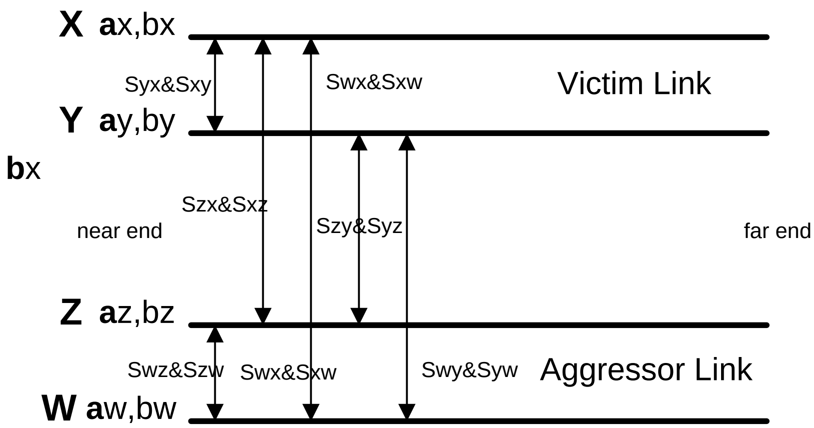


Figure 4. Single Ended NEXT S-parameters

Equations 4 & 5 can now be used (respecting the change of variable/port definitions) to compute the mixed mode NEXT S-parameters.

Figure 5 illustrates the required 6 NEXT single ended measurements.

How to convert 2 port S-parameters to 4 port S-parameters

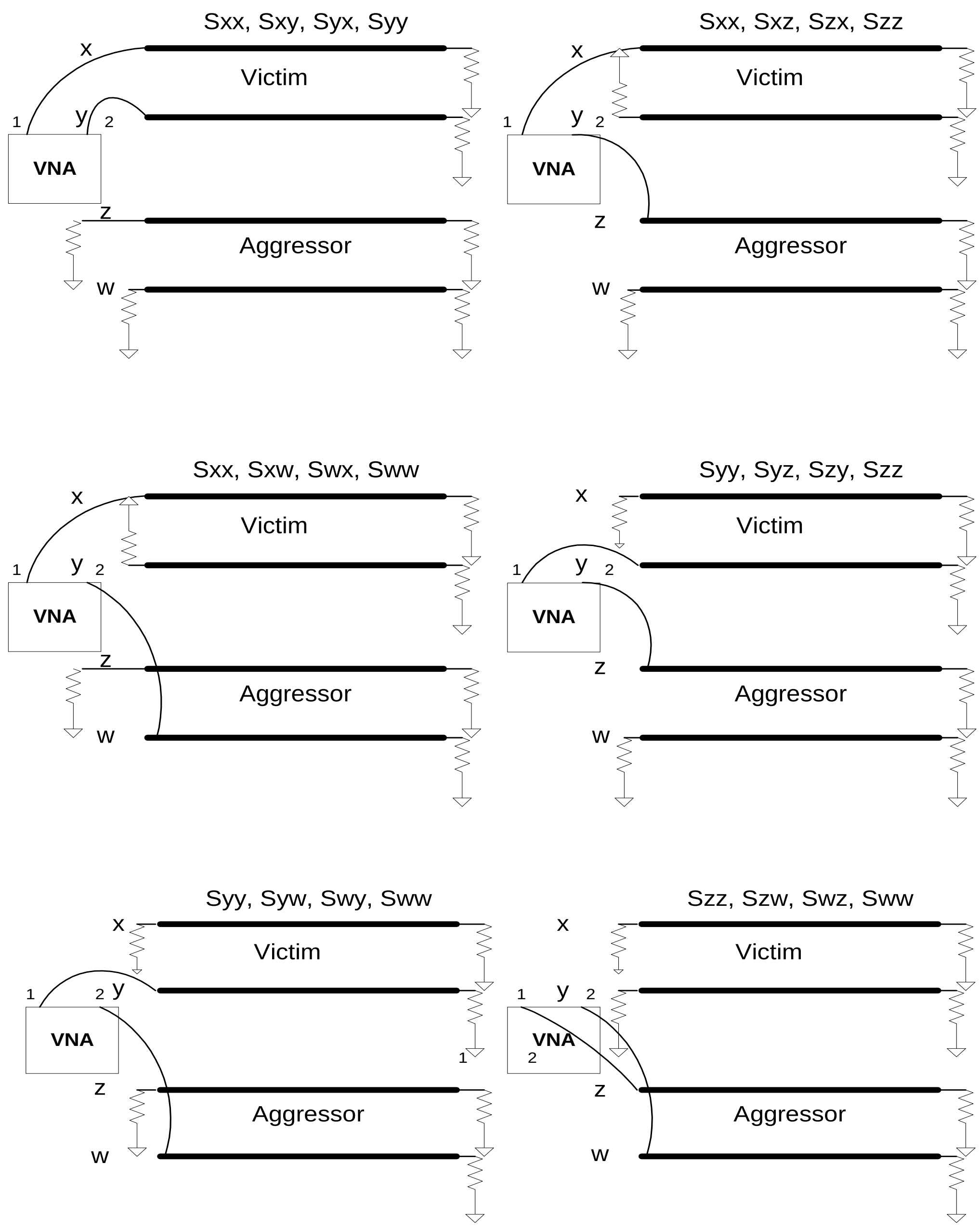


Figure 5. NEXT Single Ended S-Parameter Measurements.

Notes:

1. All far-end terminals of the victim and aggressor must be terminated. **Important** to note that if these are not calibrated (usually the case) then they can give **large** errors for tightly coupled lines however this is not normally the case hence the errors are typically quite low.
2. Strictly, any other unconnected terminals in the system where energy could couple to and reflect back to the VNA port must also be terminated.

Important to note that if these are not calibrated (usually the case) then they can give **large** errors for tightly coupled lines however this is not normally the case hence the errors are typically quite low.

3. There is duplication of measurements that exist between NEXT measurements and the single ended measurements described in Figure 3.
4. There are overlaps that could be removed to lower the number of sweeps, for example Sxx is measured 3 times in figure 5.

FEXT Measurements

Figure 6 illustrates a re-definition of the **x**,**w**,**y**,**z** ports that were used to define the mixed mode parameters for two coupled lines. This is done so that equation 5 can be re-used for mixed mode FEXT computation with the same variable names and the only difference is the assignment of the variable names to the measured single ended parameters from the VNA. Note that double-headed arrows are used to show simultaneously transverse and reverse single ended S-parameters. Reflection S-parameters Sxx, Syy, Szz, Sww are not shown but are required for the computation.

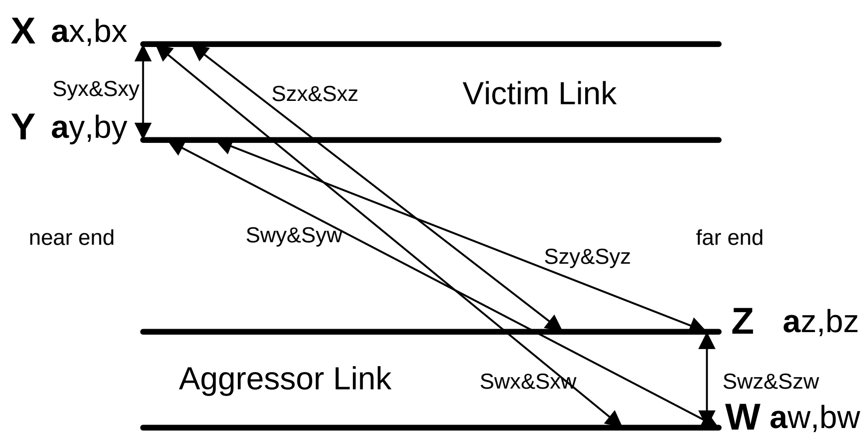


Figure 6. Single Ended FEXT S-parameters

Equations 4 & 5 can now be used (respecting the change of variable/port definitions) to compute the mixed mode FEXT S-parameters.

Figure 7 illustrates the required 6 FEXT single ended measurements.

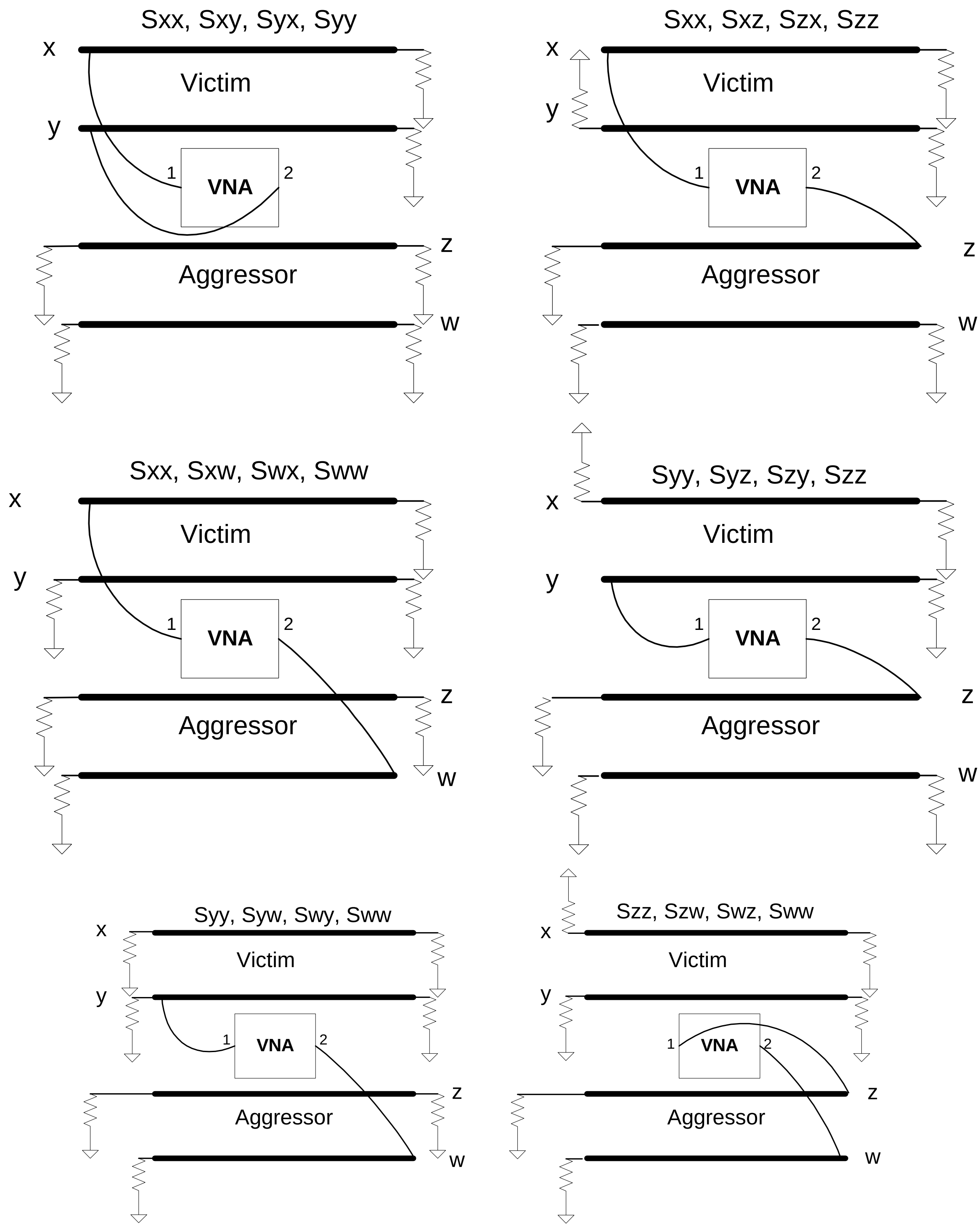


Figure 7. FEXT Single Ended S-Parameter Measurements.

Notes:
1. All far-end terminals of the victim and aggressor must be terminated. **Important** to note that if these are not calibrated (usually the case) then they can give **large** errors for tightly coupled lines however this is not normally the case hence the errors are typically quite low.
2. Strictly, any other unconnected terminals in the system where energy could couple to and reflect back to the VNA port must also be terminated. **Important** to note that if these are not calibrated (usually the case) then they can give **large** errors for tightly coupled lines however this is not normally the case hence the errors are typically quite low.
3. There is duplication of measurements that exist between FEXT measurements and the single ended measurements described in Figure 3.
4. There are overlaps that could be removed to lower the number of sweeps, for example Sxx is measured 3 times in figure 7.

References

D. Bockelman, W. Eisenstadt "Combined Differential and Common-Mode Scattering Parameters: Theory and Simulation," IEEE Transactions on Microwave Theory and Techniques, VOL 43, NO. 7, July 1995.