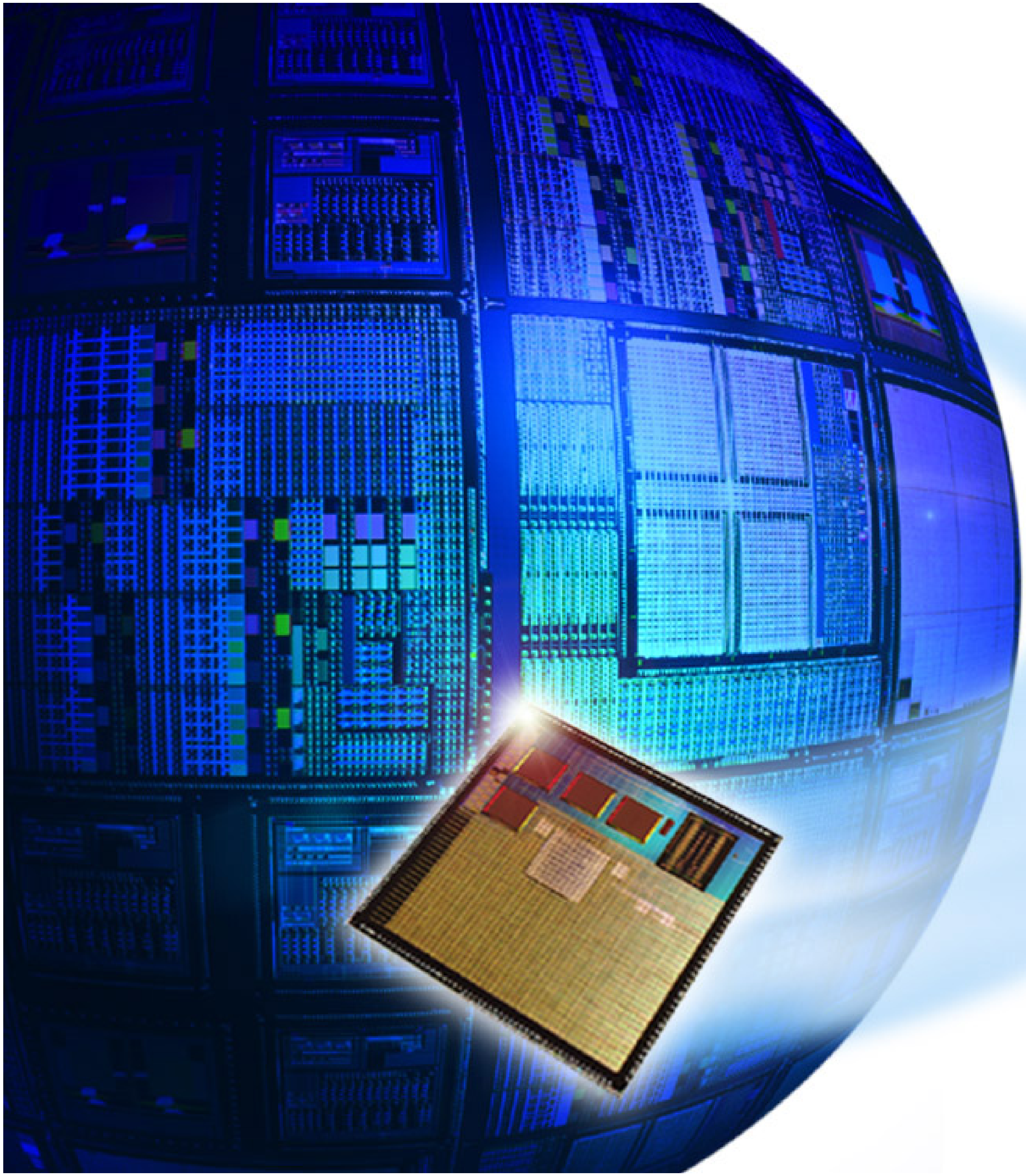

The Effect of DFE Error Propagation

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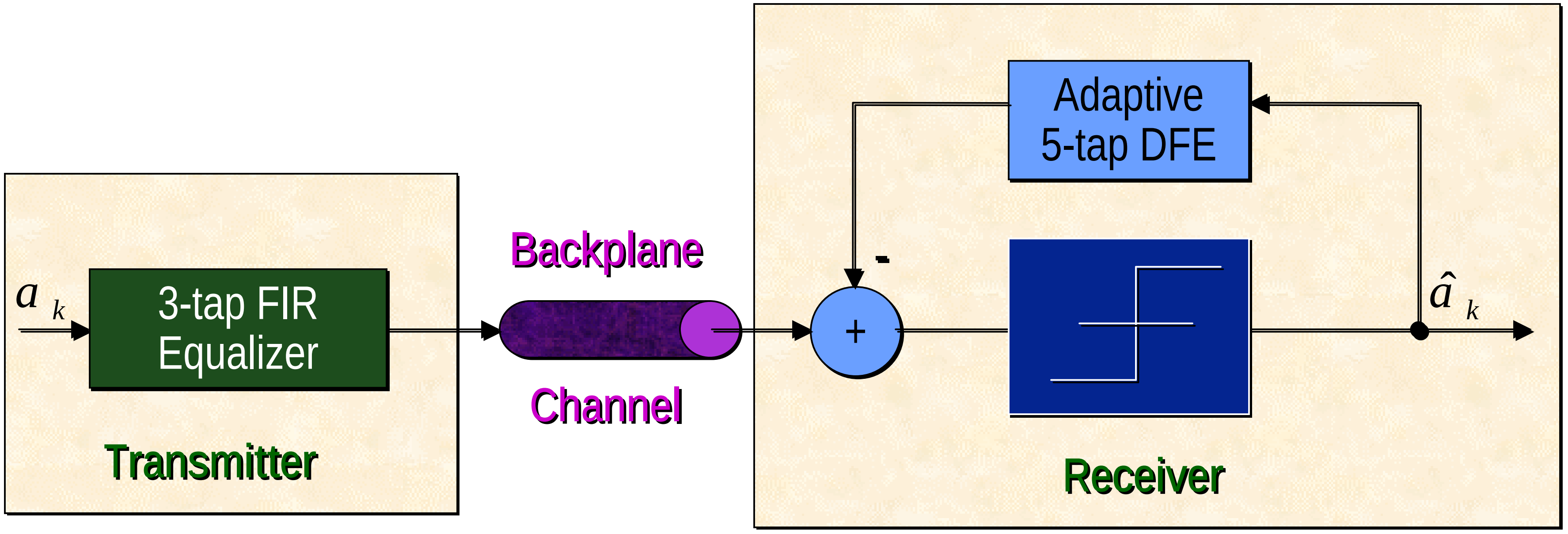
LSI Logic

DFE Error Propagation

- **A single bit error will produce a very long burst of errors with a high probability if a DFE with large number of taps (or large tap weights) is used.**
- **The effect of DFE error propagation on 10GBASE-KR channels is evaluated in this study.**

System Model

- **1/T=10.3125GHz**
- **TX: 3-tap transmit FIR**
- **RX: 5-tap DFE**



Simulation Overview

- **Analytic model is used to get slicer SNR at optimal sampling point.**
  - **Includes**
    - **Intersymbol Interference**
    - **Tx Jitter**
    - **Electronics (White) Noise**
    - **Crosstalk**
  - **Does Not Include**
    - **Receiver Sensitivity**
    - **Duty Cycle Distortion**
    - **Other Sources of DJ**

Simulation Overview (Continued)

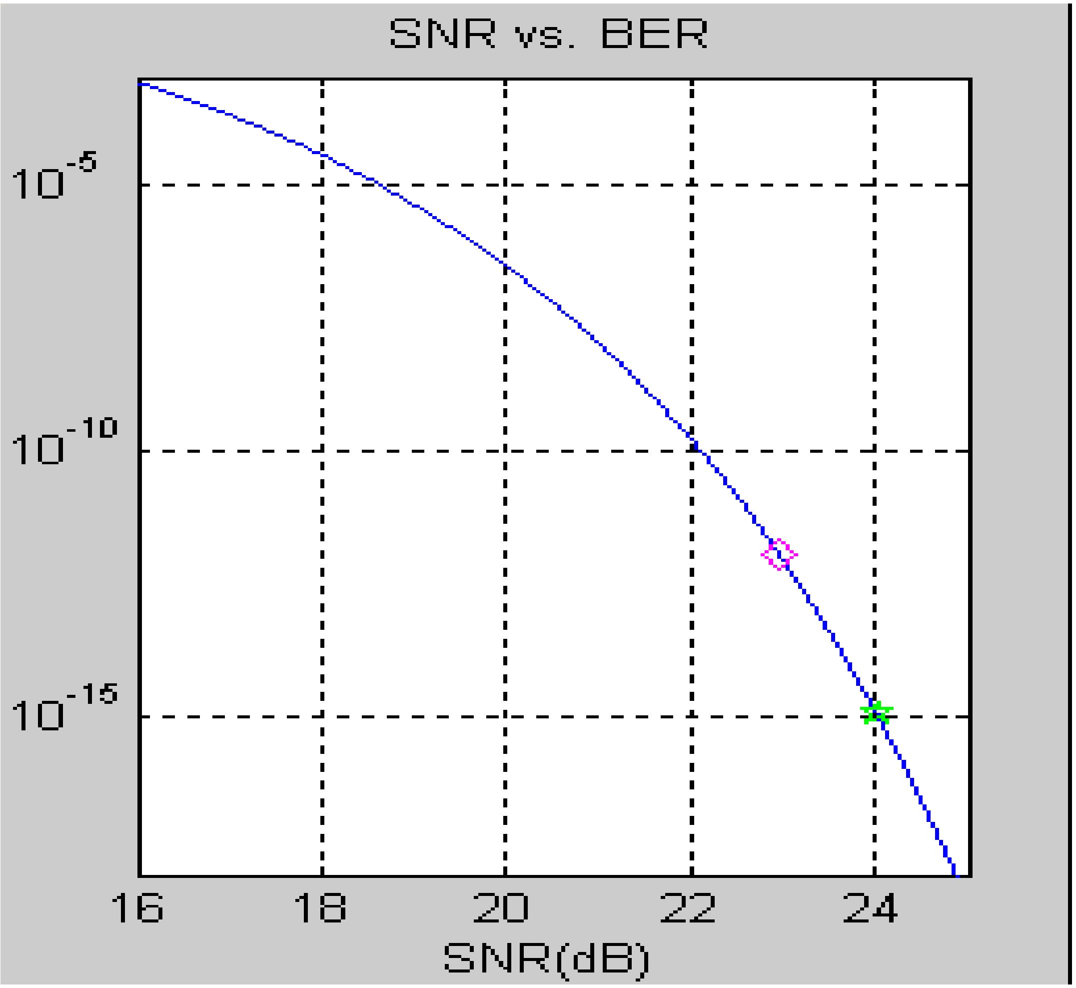
- **The effect of one near-end crosstalk aggressor (the worst one) is considered.**
- **A simple RC model with pole at 0.75*baud rate is used for the transmitter.**
- **Mellitz capacitor-like package model included on both transmitter and receiver.**
- **Only DJ is from ISI.**
  - **No DCD, PJ included**
- **0.013UI σ RJ added.**
- **Signal-To-Electronics Noise Ratio 42dB.**
- **3-tap FIR and DFE tap values are ideal.**

BER vs. SNR

$$SNR = \frac{d_{\min}^2}{\sigma^2}$$

$$\Pr_{err} \approx \frac{1}{2} erfc\left(\frac{\sqrt{SNR}}{2\sqrt{2}}\right)$$

- BER can be calculated based on SNR.



DFE Error Propagation

- $\{y_1 y_2 y_3 y_{4\ldots} y_n\}$: received signal.
  - y_1 is the oldest bit.
- $p(y_i|E)$: the probability of the detection of y_i is wrong after error pattern E happened.
  - $p(y_2|\{y_1\})$: the probability of y_2 wrong when y_1 is in error.
  - $p(y_i|\{y_1,y_2\})$: the probability of y_i wrong when y_1 and y_2 are both in error, $i > 2$.
- $p(y_i|E)$ can be calculated from SNR obtained by our analytic simulator.
  - The previous error pattern E can be modeled by flipping the corresponding feedback tap weight signs.

DFE Error Propagation $p(y_i|E)$:

BP	Tyco1	Tyco2	Tyco3	Tyco4	Tyco5	Tyco6	Tyco7
BER w/o E	3.02E-24	1.02E-24	1.41E-18	4.25E-32	3.10E-40	3.13E-20	2.17E-26
p(y2/y1)	2.17E-03	4.58E-03	1.94E-02	6.30E-12	8.80E-09	7.59E-02	1.15E-01
p(y3/y1)	9.79E-04	9.34E-04	1.07E-05	2.54E-04	3.43E-25	1.52E-17	3.96E-04
p(y4/y1)	3.07E-06	6.62E-06	5.02E-06	5.47E-08	1.80E-30	2.88E-15	1.13E-06
p(y5/y1)	3.06E-24	1.33E-22	1.63E-15	9.26E-20	1.31E-13	6.33E-07	1.46E-16
p(y6/y1)	3.16E-24	1.02E-24	4.77E-18	1.11E-31	1.86E-37	7.78E-18	1.44E-10

BP	Mi2	Mi3	Mi4	Mi5	Mo2	Mo3	Mo4	Mo5
BER w/o E	2.24E-19	7.02E-19	1.01E-18	1.48E-19	5.66E-21	1.60E-20	6.45E-21	2.91E-21
p(y2/y1)	6.00E-04	2.18E-03	1.93E-03	2.85E-03	7.28E-05	1.08E-05	6.64E-05	1.45E-04
p(y3/y1)	1.98E-04	5.15E-04	1.18E-04	6.20E-05	5.19E-04	1.78E-04	3.34E-04	1.19E-04
p(y4/y1)	2.65E-19	7.31E-19	3.87E-18	2.68E-19	1.67E-19	3.36E-19	6.08E-19	1.47E-17
p(y5/y1)	2.26E-19	7.86E-19	2.69E-17	3.04E-19	1.19E-17	3.08E-18	1.80E-19	4.12E-18
p(y6/y1)	4.96E-16	9.44E-15	8.78E-15	2.68E-18	6.52E-19	8.28E-18	1.02E-18	6.04E-21

BP	B1	B12	B20	M1	M20	T1	T12	T20
BER w/o E	5.62E-22	2.06E-20	1.52E-17	4.83E-16	6.46E-18	5.10E-11	5.52E-09	3.47E-07
p(y2/y1)	2.57E-01	1.07E-01	3.16E-02	9.39E-02	2.53E-03	3.07E-01	2.12E-01	1.44E-01
p(y3/y1)	1.77E-02	1.84E-04	7.58E-15	3.24E-06	2.31E-09	9.71E-02	1.83E-04	1.63E-04
p(y4/y1)	4.59E-16	7.57E-20	9.33E-16	3.78E-04	7.13E-10	5.67E-10	4.07E-07	1.94E-03
p(y5/y1)	6.80E-09	1.09E-08	1.00E-08	3.13E-09	2.93E-08	1.51E-04	4.32E-05	4.20E-05
p(y6/y1)	5.82E-22	5.14E-16	3.83E-15	3.64E-15	2.11E-14	1.41E-08	4.64E-06	8.84E-06

Average Run Length of Errors

- $p(rll=i)$: probability of a burst error with run length equal to i.
- rll_{max}: The maximum error run length to be considered.
  - It is related with the total number of DFE taps and the tap weights.
  - $rll_{max}=17$ in our simulation.

Average Run Length of Errors (cont.)

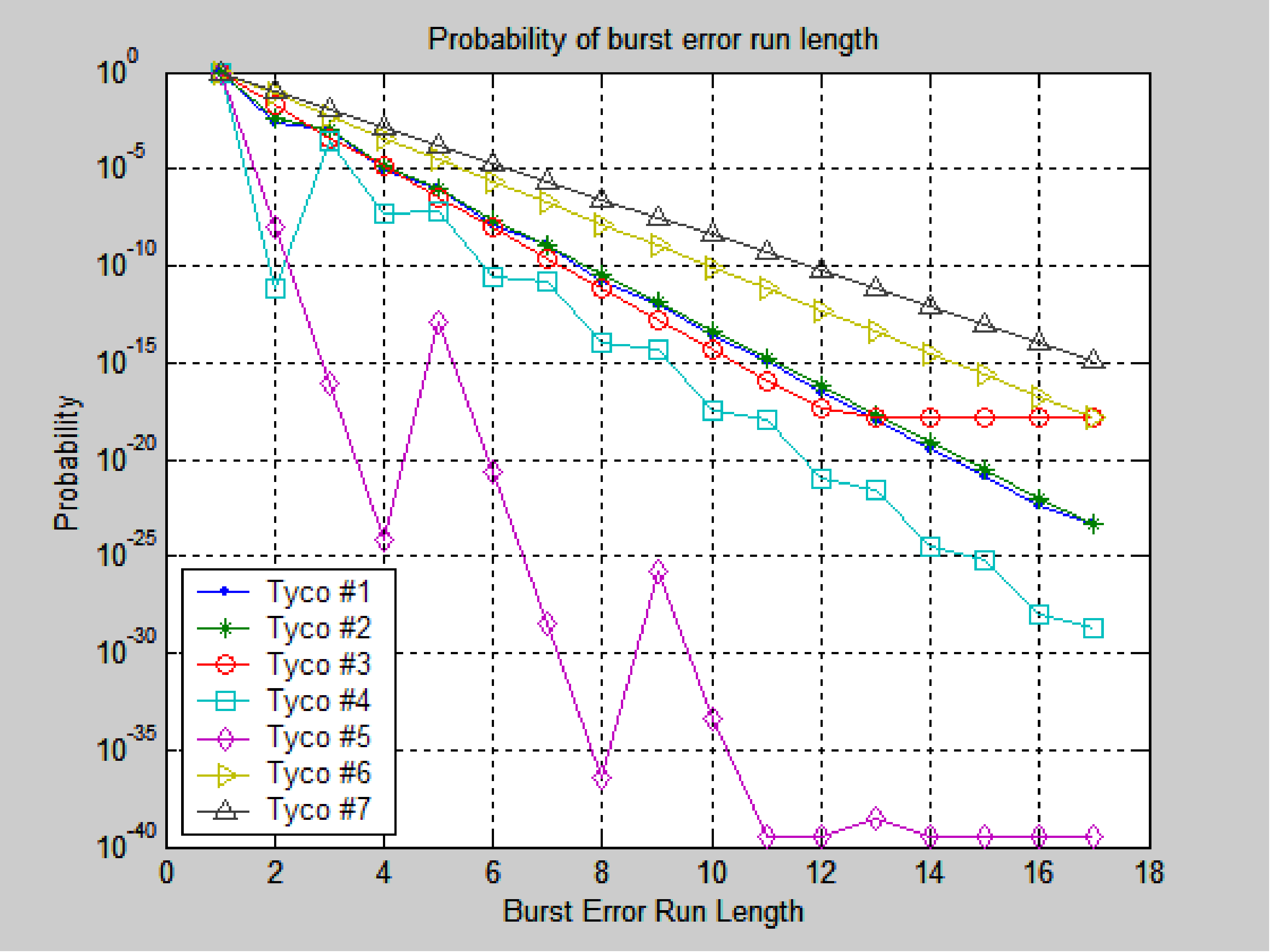
$$p(rll=1)=\prod_{i=2}^{rll_{\max}+1}(1-p(y_i\,|\,y_1))$$

$$p(rll=2)=p(y_2\,|\,y_1)\cdot\prod_{i=3}^{rll_{\max}+2}(1-p(y_i\,|\,\{y_1,y_2\}))$$

$$p(rll=3)=\underbrace{p(y_2\,|\,y_1)\cdot p(y_3\,|\,\{y_1,y_2\})\cdot\prod_{i=4}^{rll_{\max}+3}(1-p(y_i\,|\,\{y_1,y_2,y_3\}))}_{P(E_3=\{111\})}$$

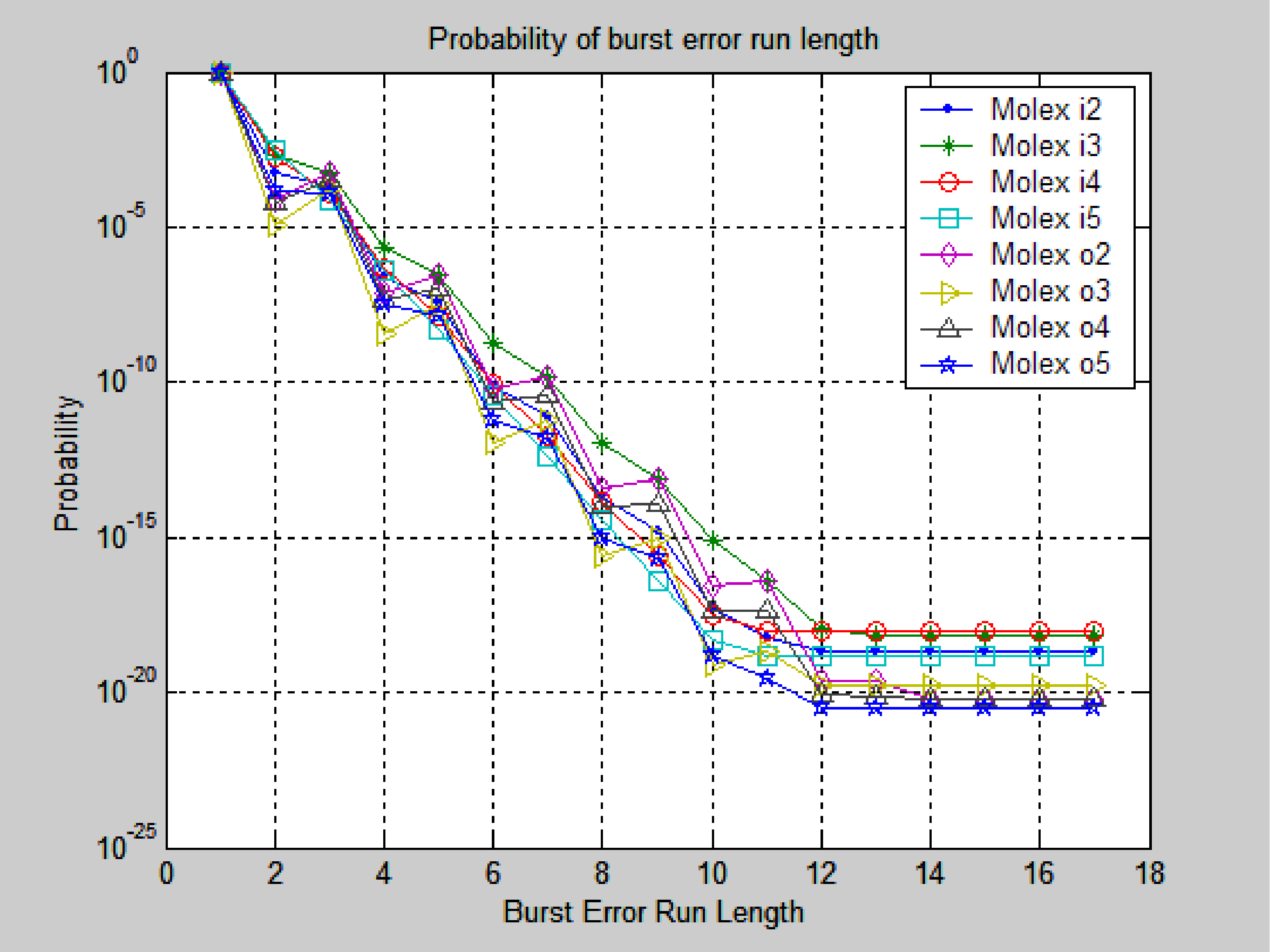
$$+\underbrace{(1-p(y_2\,|\,y_1))\cdot p(y_3\,|\,y_1)\cdot\prod_{i=4}^{rll_{\max+3}}(1-p(y_i\,|\,\{y_1,y_3\}))}_{P(E_3=\{101\})}$$

p(*rll*=*i*): Tyco



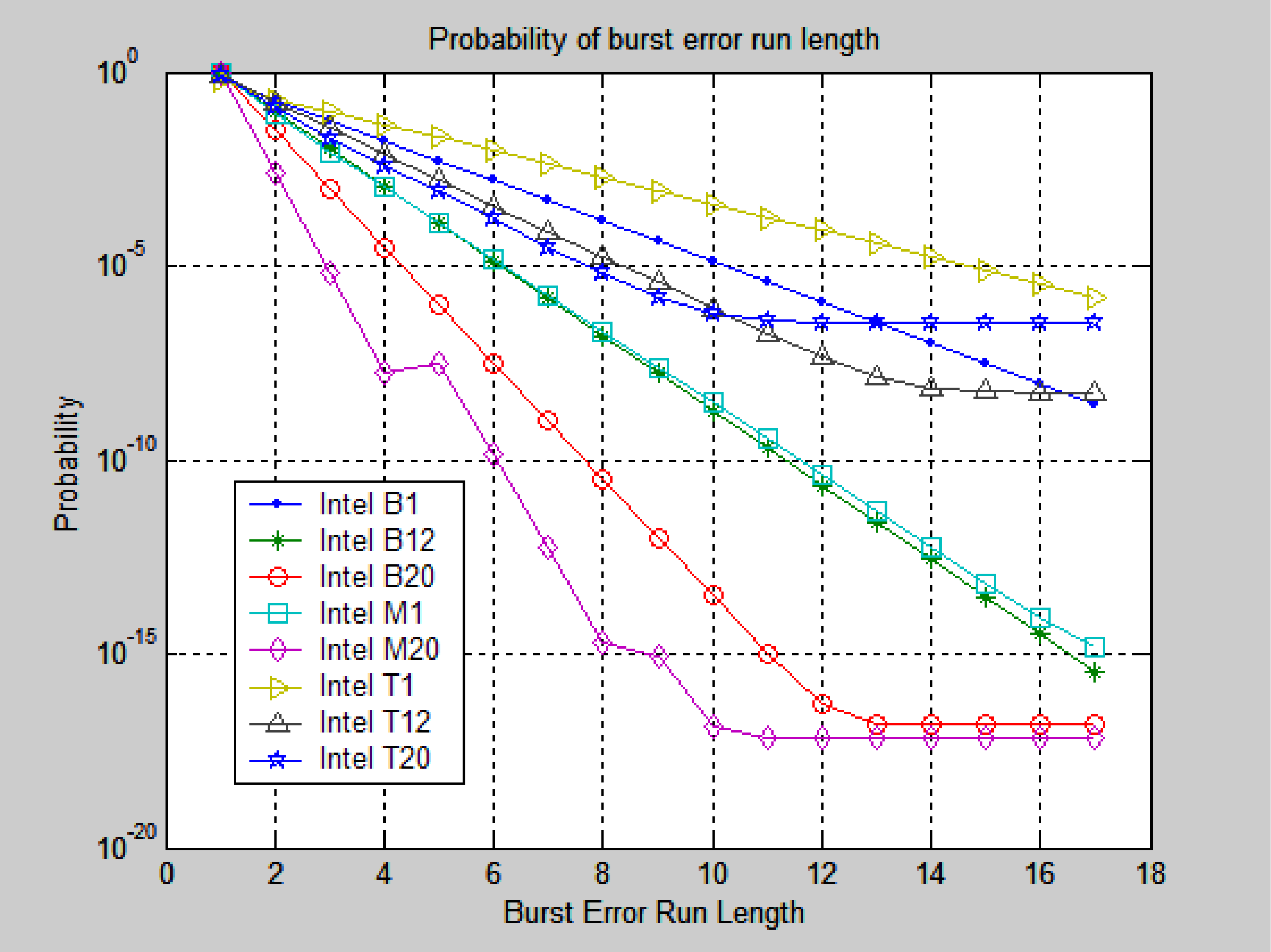
Probability of burst error run length
Probability
Burst Error Run Length
10^0
10^-5
10^-10
10^-15
10^-20
10^-25
10^-30
10^-35
10^-40
0
2
4
6
8
10
12
14
16
18
Tyco #1
Tyco #2
Tyco #3
Tyco #4
Tyco #5
Tyco #6
Tyco #7

$p(rll=i)$: Molex



Probability of burst error run length
Probability
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
0
2
4
6
8
10
12
14
16
18
Burst Error Run Length
Molex i2
Molex i3
Molex i4
Molex i5
Molex o2
Molex o3
Molex o4
Molex o5

$p(rll=i)$: Intel


Probability of burst error run length
Probability
Burst Error Run Length
10^0
10^-5
10^-10
10^-15
10^-20
0
2
4
6
8
10
12
14
16
18
Intel B1
Intel B12
Intel B20
Intel M1
Intel M20
Intel T1
Intel T12
Intel T20

BER with DFE Error Propagation

- n: length of considered code length.
- p_1 is the probability that a bit in error when all previous bits are error free.
  - It can be calculated by analytic simulator.
  - BER considering DFE error propagation is bigger than p_1 due to the error propagation.
- $p(e_i)$: probability that i bits in error among n code bits.
- $W(E)$: weight of error pattern E .
  - $W(101)=2$.

BER with DFE Error Propagation (cont.)

$$p(e_1) = n \cdot p_1 \cdot \prod_{i=2}^{rll_{\max}+1} (1 - p(y_i \mid y_1)) \cdot (1 - p_1)^{n - rll_{\max} - 1}$$

$$= n \cdot p_1 \cdot p(rll = 1) \cdot (1 - p_1)^{n - rll_{\max} - 1}$$

$$p(e_2) = p(2\,bit\;burst\;error) + p(2\,separated\;errors) \approx p(2\,bit\;burst\;error)$$

$$p(2\,separated\;errors) = n \cdot (n - rll_{\max} - 1) \cdot p_1^{\,2} \cdot (1 - p_1)^{n - 2rll_{\max} - 2} \cdot p^2(rll = 1)$$

$$p(2\,bit\;burst\;error) = p(burst\;error = \{11\}) + p(burst\;error = \{101\}) + p(burst\;error = \{1001\}) + \cdots$$

$$p(burst\;error = \{11\}) = n \cdot p_1 \cdot p(rll = 2) \cdot (1 - p_1)^{n - rll_{\max} - 2}$$

$$p(burst\;error = \{101\}) = n \cdot p_1 \cdot p(rll = 3, E = \{101\}) \cdot (1 - p_1)^{n - rll_{\max} - 3}$$

BER with DFE Error Propagation (cont.)

- **Block error rate**

$$p_{block} \approx \sum_{i=1}^{\infty} p(i\ bit\ burst\ error)$$

$$\approx \sum_{i=1}^{rll_{\max}} n \cdot p_1 \cdot p(rll = i) \cdot (1 - p_1)^{n - rll_{\max} - i}$$

- **Bit error rate (BER)**

$$p_{bit} = \sum_{i=1}^{rll_{\max}} \sum_{\text{all } E} p(rll = i, E) \cdot W(E) \cdot p_1 \cdot (1 - p_1)^{n - rll_{\max} - i}$$

BER with FEC Coding

- **Burst error correcting capability =L.**
- **Block error rate**

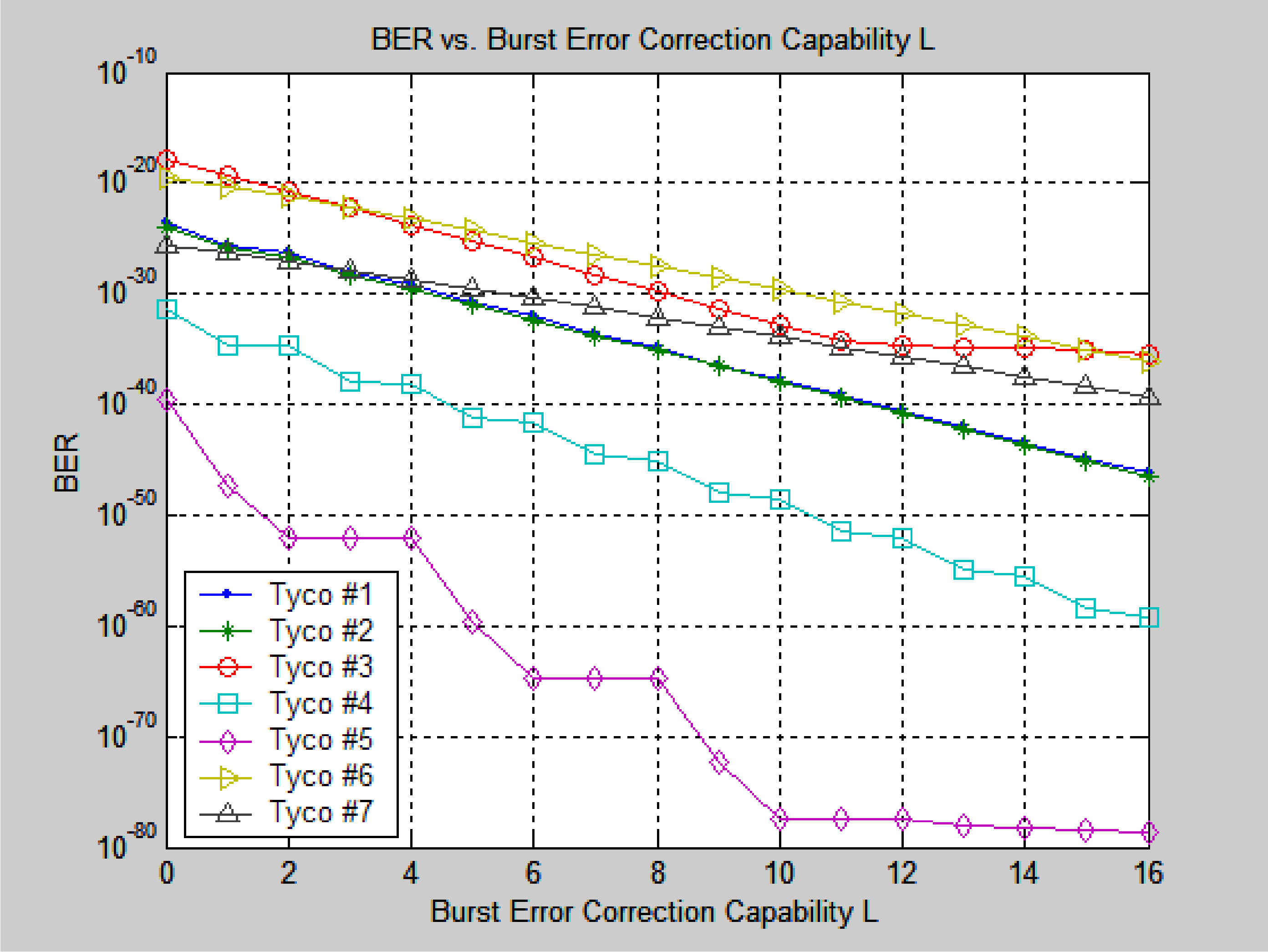
$$p'_{block} = \sum_{i=L+1}^{rll_{\max}} p(i\ bit\ burst\ error)$$

$$= \sum_{i=L+1}^{rll_{\max}} n \cdot p_1 \cdot p(rll = i) \cdot (1 - p_1)^{n - rll_{\max} - i}$$

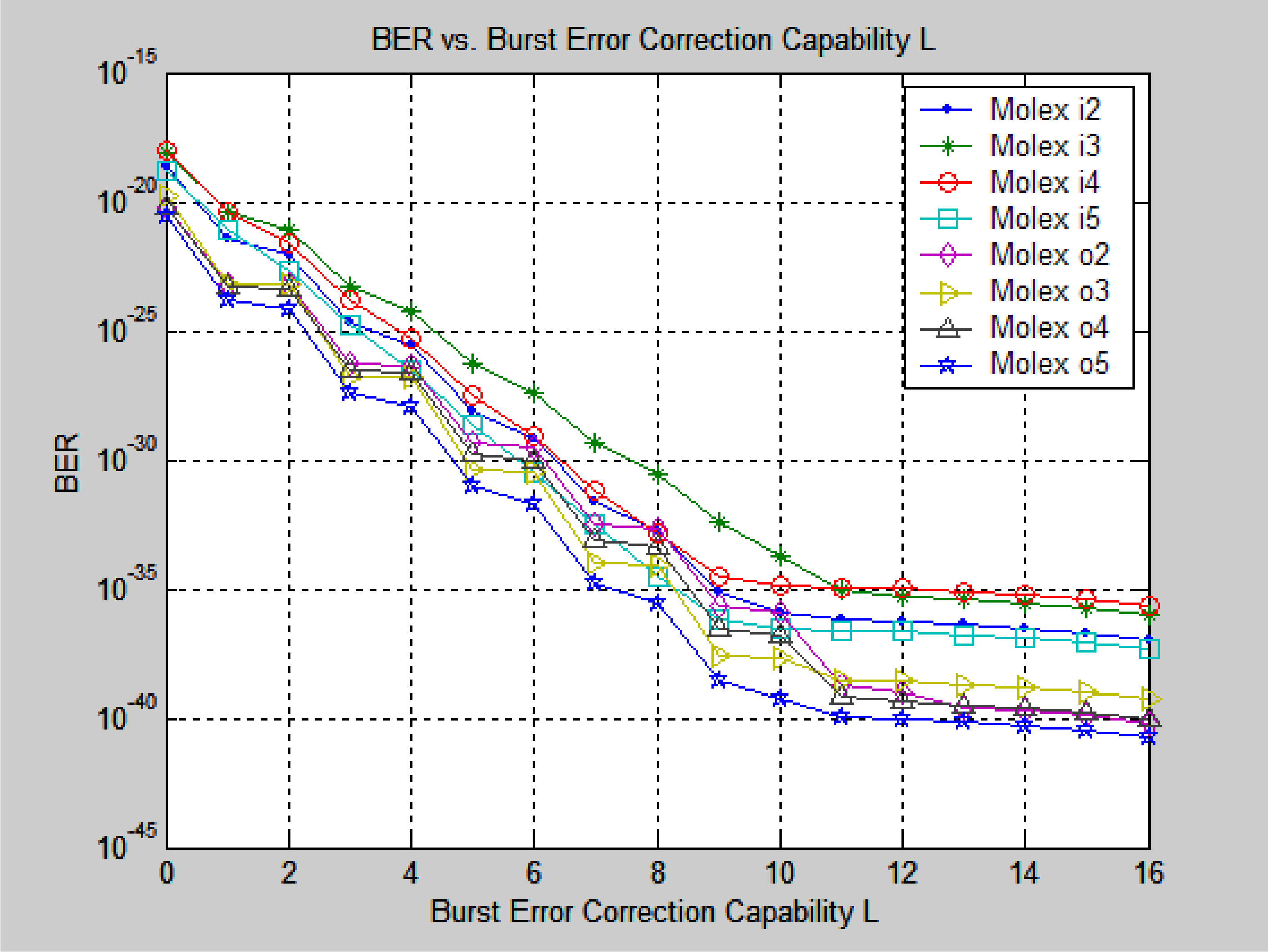
- **Bit error rate**

$$p'_{bit} = \sum_{i=L+1}^{rll_{\max}} \sum_{\text{all } E} p(rll = i, E) \cdot W(E) \cdot p_1 \cdot (1 - p_1)^{n - rll_{\max} - i}$$

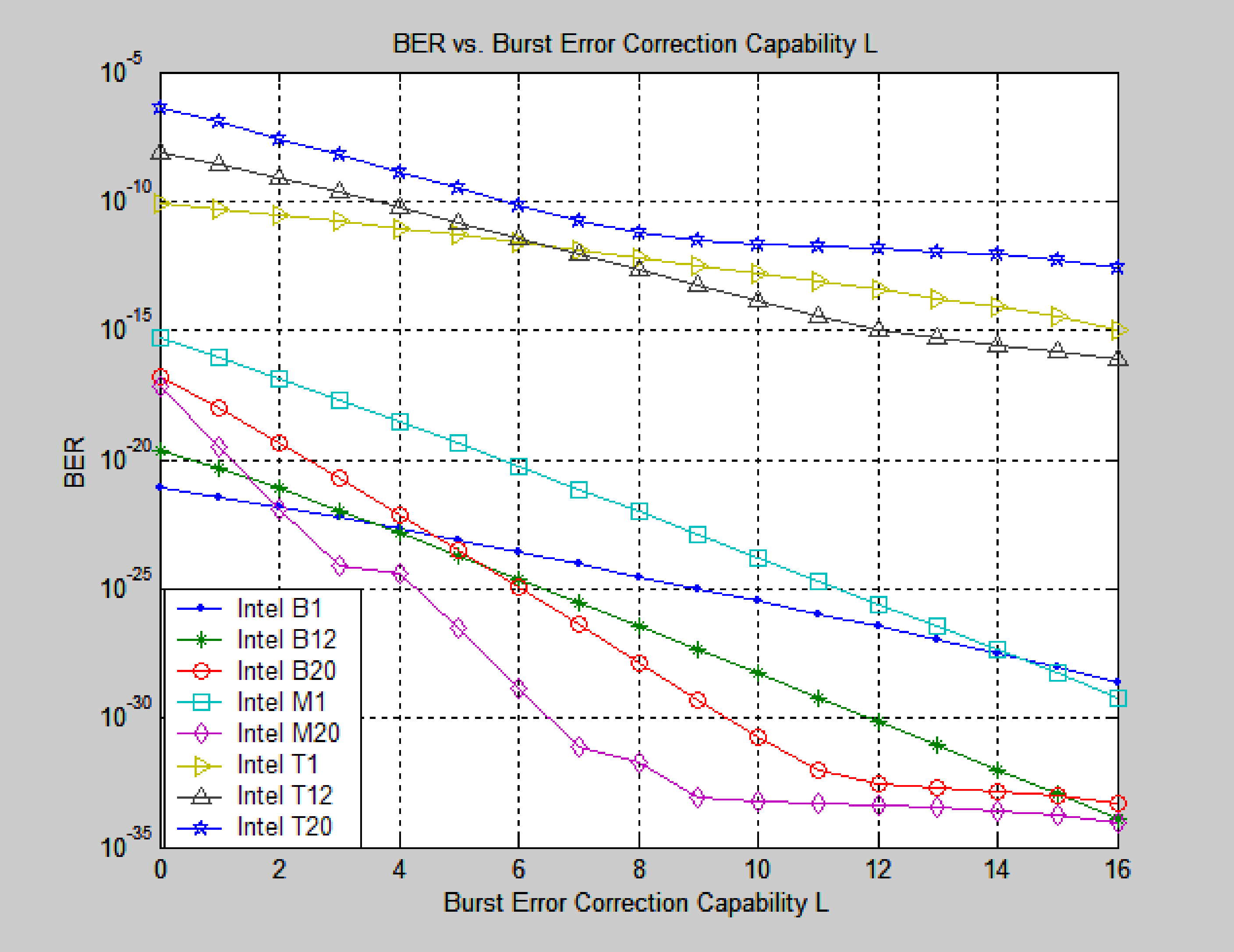
BER improvement: Tyco



BER improvement: Molex



BER improvement: Intel



Mean Time To False Packet Acceptance (MTTFPA)

- The Ethernet CRC32 has considerable error detection capability
  - Hamming distance of 4
  - Detect any 3 bits in error in packet
  - Also detect any 32-bit burst or any two 8-bit burst in packet
- The self-synchronous Scrambler in 64B66B has error propagation properties that compromise the burst error detection capabilities of the Ethernet CRC32.
- Because 64B66B still has a uniform 4-bit Hamming protection, a conservative estimate can be made: MTTFPA > expected time for 4 or more errors.
- For non-bursty channel, the probability of getting 4 or more errors is quite low. Hence, MTTFPA is still acceptable. (walker_1_0300.pdf)
- However, due to DFE error propagation the probability of getting 4 or more errors is quite high, $P(e_4) = n \cdot p_1 \cdot p(rll = 4) \cdot (1 - p_1)^{n - rll_{\max} - 4}$ which causes unacceptable MTTFPA.
  - MTTPFA > 1.12 day ($p_1=10^{-12}$, 10.3Gbps, $p(rll=4)=10^{-3}$)

Conclusions

- To obtain acceptable MTTFPA, stronger error detection/correction capability is needed.
  - Extra CRC (like szczepanek-01-0905.pdf)
  - FEC before descrambler (like ganga_02_0905.pdf)
  - Or both.
- Furthermore, burst error FEC can improve performance margins.