Efficient Estimation of Bit Error Rates and Eye Diagrams in Equalizer Enhanced Links

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Bit Error Rate Estimation in 10Gb/s LANs

- Monte Carlo simulations are prohibitively long at a BER of $10^{-12}$:
  - At least 100 errors for reliable BER estimation for an AWGN Channel
    - Need to transmit at least $10^{14}$ bits
  - Requires about 2.78 hours of real-time data at 10Gb/s
- Need to resort to BER estimation techniques
**Method 1: Gaussian Approx. Approach**

- **Model the ISI and the noise term together as AWGN**
- Assume 1’s and 0’s to be equally likely and independent
- Estimate means and standard deviations
  - at the zero rail \((m_0, \sigma_0)\)
  - at the one rail \((m_1, \sigma_1)\)
- Optimum Bit Error Rate (BER) and Threshold are then given by:

\[
P_e = \frac{1}{2} \text{erfc} \left( \frac{1}{\sqrt{2}} \left[ \frac{m_1 - m_0}{\sigma_1 + \sigma_0} \right] \right) \quad D = \frac{m_0\sigma_1 + m_1\sigma_0}{\sigma_1 + \sigma_0}
\]

- **But in practice, input to decision device is the sum of Gaussians**
  - each corresponding to one combination of neighboring bits
  - Hence BER estimate is not correct
Method 2: ISI Pattern Approach

- Model input to the decision device as a sum of Gaussians
  - Mean of each Gaussian depends on the adjacent bits

- For each transmitted bit find the ISI pattern (or Gaussian) to which it belongs
- Estimate means and standard deviations of each Gaussian
- Bit Error Rate at a threshold $D$ and channel span $L$ is given by:

$$P_e = \frac{1}{2^L} \sum_{i \in S_0} \text{erfc} \left( \frac{D - m_i}{\sqrt{2}\sigma_i} \right) + \frac{1}{2^L} \sum_{i \in S_1} \text{erfc} \left( \frac{m_i - D}{\sqrt{2}\sigma_i} \right)$$

- **Complexity of Method 2 results in Long simulation times**
  - increases exponentially with channel memory
  - Also requires large number of bits to reliably estimate mean & std. deviation
Proposed Method: ISI Statistics Analytically

- Assume that there is no error propagation in the DFE
  - Usually valid at BERs lower than $10^{-5}$; we are operating at even lower BERs
  - Then $\hat{x}_n = x_{n-\Delta}$
    - $x_n$ denotes the transmitted bit for the nth bit period and
    - $\Delta$ is an appropriately chosen delay

- **Entire system up to CDR is a linear filter with known coefficients!!**
  - System is from $x_n$ to $w_n$
  - Ideal equalizer coefficients are determined based on channel response
Proposed Method contd.

• Thus the Input to the threshold device is:

\[ w_n = d_{\Delta} x_{n-\Delta} + r_n + \nu_n; \quad r_n = \sum_{i \neq \Delta} d_i x_{n-i} \]

  - Where \( r_n \) corresponds to the residual ISI and \( \nu_n \) is the additive noise

• **Probability Density Function, \( \hat{P}(r_n) \), of ISI can be computed analytically**
  - Since its Characteristic Function is a direct function of above coefficients
  - And the input alphabet statistics

• And so the minimum BER and optimum Threshold are given by:

\[ P_e = \frac{1}{2} \int_{-\delta/2}^{\delta/2} p(y) \text{erfc} \left( \frac{d_{\Delta}/2 - y}{\sqrt{2}\sigma} \right) dy; \quad D = \frac{d_{\Delta} + \sum_{i \neq \Delta} d_i}{2} \]

• Where \( \delta = \sum_{i \neq \Delta} |d_i| \)
**Results: ISI Statistics**

- **Two feed-forward taps and One feedback tap**
- Probability Density Function of ISI at input to slicer
- Non-Impulsive nature of ISI PDF indicates significant residual ISI
- Non-Gaussian nature of ISI evident

Increasing SNR

Red for zeros
Blue for ones
Results: ISI Statistics

- Five feed-forward taps and Five feedback taps
- Non-Gaussian ISI at low TX powers
- Negligible ISI at high TX powers

Increasing SNR

Red for zeros
Blue for ones
Results: BER Comparison

2 FF Taps + 1 FB Tap case:
- Single Gaussian Approx. (Method 1) deviates significantly with fewer equalizer coefficients
- Multiple Gaussian Approach (Method 2) better but still deviates
  - Since ISI is assumed to have contributions from only two adjacent bits

5 FF Taps + 5 FB Taps case:
- Estimates from all three approaches agree at high sensitivities
- Methods 1 & 2 still deviate at low sensitivities: ISI is not Gaussian in this regime

Proposed method is about 1000x times faster than Method 2 for all transmit powers!!
Estimation of Deterministic Eye Diagram

- For each sampling phase, noise-free input to slicer: 
  \[ w_n = d_{\Delta} x_{n-\Delta} + \sum_{i \neq \Delta} d_i x_{n-i} \]
- For each value of \( x_{n-\Delta} \) (zero)
  - Maximum value of ISI = sum of all positive \( d_i \)
  - Minimum value of ISI = sum of all negative \( d_i \)
  - When \( x_{n-\Delta} = 1 \), we need to add \( d_{\Delta} \) to each max/min value
  - Both can be exactly computed
- Can be used to find optimum sampling instant and threshold also
Contour Plots

- Proposed technique can be used to quickly explore the equalizer design space via Contour Plots
- ISI Penalty = additional RX sensitivity required to achieve BER of $10^{-12}$
Conclusions

• An efficient BER estimation method has been proposed

• Advantages:
  ➢ **Accurate:**
    • More accurate than other methods when significant ISI is present
    • At least as accurate as other methods when ISI is negligible
    • Can even be applied at the input to the equalizer with accurate results
  ➢ **complexity that increases linearly with channel memory**
    • As opposed to exponential complexity of Method 2
  ➢ **about 1000x faster than other techniques**
  ➢ **Independent of the equalizer adaptation technique**

• Permits easy estimation of the Deterministic Eye
  ➢ Can also find optimum sampling instant and threshold

• Permits quick exploration of the equalizer design space