Updated estimates of ISI power penalties, applied to 10GBASE-LRM.

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Outline

- Motivation and introduction
- ISI in the time domain.

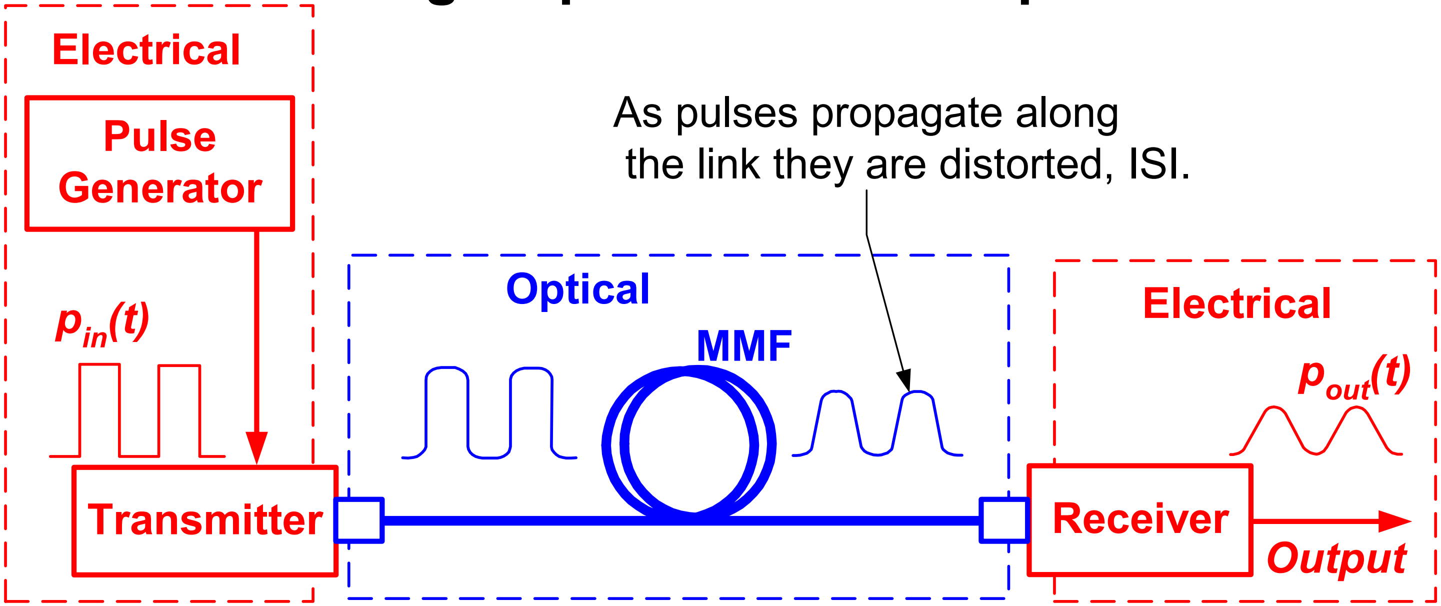
  (ISI = Inter-Symbol Interference [1].)
- ISI in the frequency domain
- Compare ISI in the 2 domains + spreadsheet [2]
- Changes in ISI power penalties
- Conclusions.

Motivation and introduction

- ISI - dominant source of bandwidth limitations
- No apparent sources - ISI from 1st principles to final results.
- Provide details: math & intuitive.
- Update theory for enhanced generality & accuracy.

ISI in the time domain

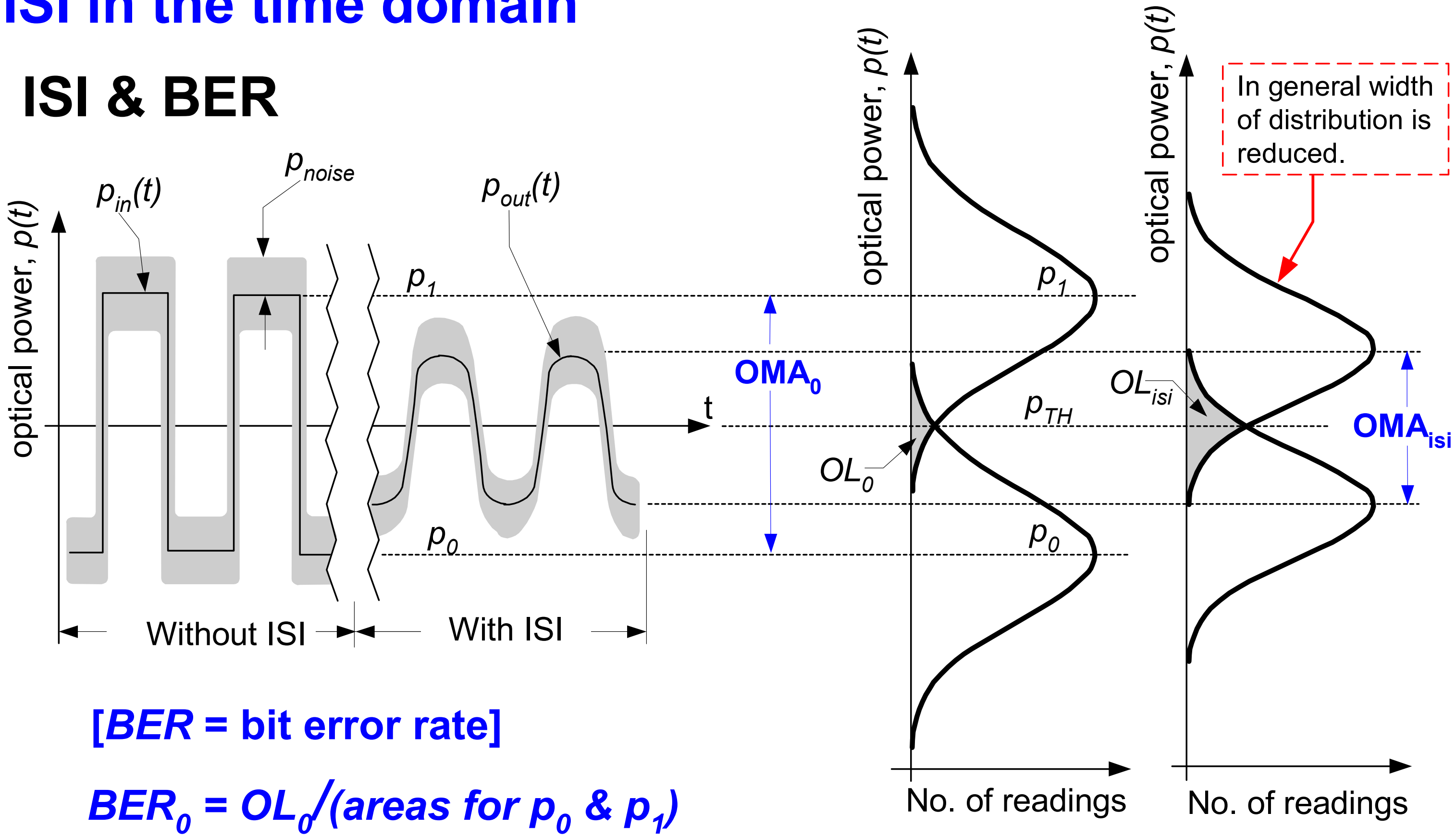
Signal path for a MMF optical link



- Not all components are shown
- Each component has a time response. Pulses are distorted.
- Shape of $p_{out}(t)$ is different from shape of $p_{in}(t)$.
- Amplitude of $p_{out}(t)$ is usually less than that of $p_{in}(t)$.

ISI in the time domain

ISI & BER



In general width of distribution is reduced.

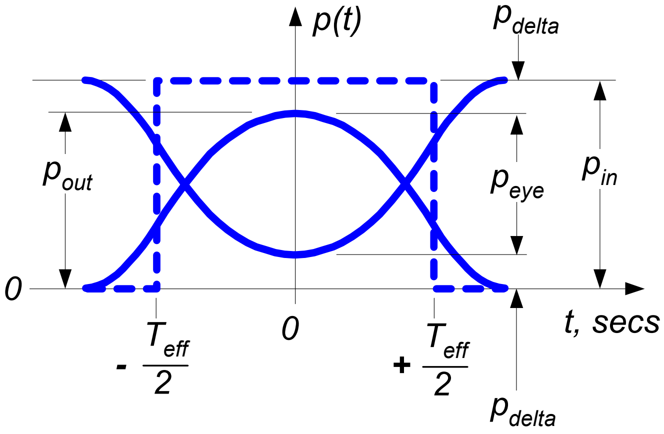
[*BER* = bit error rate]

$BER_0 = OL_0/(\text{areas for } p_0 \,\&\, p_1)$

$BER_{isi} = OL_{isi}/(\text{areas for } p_0 \,\&\, p_1)$

$P_{isi} = 10*\log_{10}(OMA_0/OMA_{isi})$, dB ← **[ISI power penalty]**

ISI in the time domain



T_{eff} - pulse width, secs. p - pulse power in W.

$P_{isi} = 10*\log_{10}(OMA_0 / OMA_{isi})$

$= 10*\log_{10}(p_{in} / p_{eye})$, dB.

Normalize p_{in} & p_{out} so that $p_{in} = 1$.

$P_{isi} = 10*\log_{10}[(1/(2*p_{out} - 1)]$, dB.

Starting point, first principles. Convolution Theorem [4-5].

distortions (source, fiber receiver) [3]

$$p_{out}(t) = \int_{-\infty}^{\infty} p_{in}(u) \cdot h(t-u) \cdot du$$

variable of integration, secs.

time

no distortions

(several math steps. See Appendix & ref [6].)

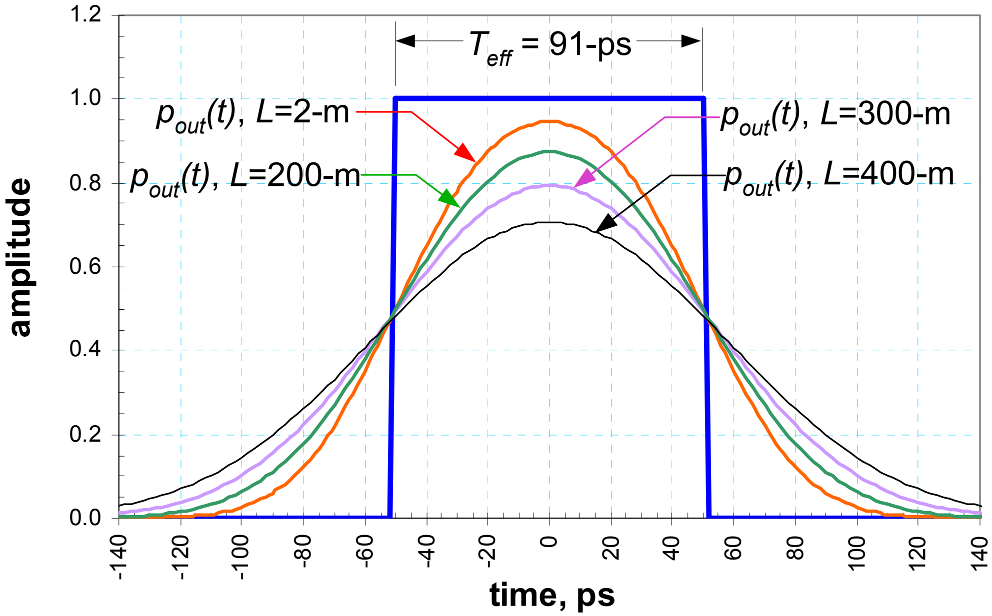
$$p_{out} = erf\left[\frac{2.564}{2 \cdot \sqrt{2}} \cdot \left(\frac{T_{eff}}{T_c}\right)\right]$$

10% to 90% rise time due to all the components in the optical channel.

Error function

ISI in the time domain, an example.

- 10 Gb/s = data rate, 100-ps = bit period.
- 9-ps = duty cycle distortion [includes amplifier phase distortions (transmitter & receiver)].
- Effective time width, T_{eff} = 100 - 9 = 91-ps.
- MMF: 2000 MHz*km & 50/125μm.
- Default rise times for transmitter and receiver (spreadsheet [2])



ISI in the frequency domain

Starting point, first principles. Convolution Theorem.

transmitter

fiber

$$P_{out}(\nu) = \boxed{P_{in}(\nu)} \cdot \boxed{P_s(\nu)} \cdot \boxed{P_f(\nu)} \cdot \boxed{P_r(\nu)}$$

frequency, Hz

initiating pulse,
no distortions

receiver

Simple: Components are just multipliers in the frequency domain.

[several math steps. See Appendix.]

SD of time response
for the optical channel

$$p_{out}(t=0) = \int_{-\infty}^{+\infty} \left\{ \frac{\sin(\pi \cdot \nu \cdot T_{eff})}{(\pi \cdot \nu)} \right\} \cdot \left\{ \exp\left[-\frac{(2 \cdot \pi \cdot \nu \cdot \boxed{\sigma_c})^2}{2} \right] \right\} \cdot d\nu$$

Evaluate numerically. Compare with time domain result.

Compare ISI in the 2 domains + spreadsheet [2]

L, m = 2		T_c, ps = 66.43	
T_{eff}, ps	P_{isi}, dB freq. domain	P_{isi}, dB time domain	P_{isi}, dB spread-sheet
70	1.894	1.891	1.897
80	1.222	1.220	1.224
91	0.749	0.746	0.750
100	0.493	0.492	0.494

L, m = 200		T_c, ps = 83.81	
T_{eff}, ps	P_{isi}, dB freq. domain	P_{isi}, dB time domain	P_{isi}, dB spread-sheet
70	3.651	3.650	3.655
80	2.535	2.534	2.538
91	1.727	1.725	1.729
100	1.262	1.262	1.264

L, m = 300		Tc, ps = 101.43	
T_{eff}, ps	P_{isi}, dB freq. domain	P_{isi}, dB time domain	P_{isi}, dB spread-sheet
70	6.066	6.061	6.073
80	4.247	4.244	4.252
91	3.015	3.009	3.018
100	2.310	2.308	2.313

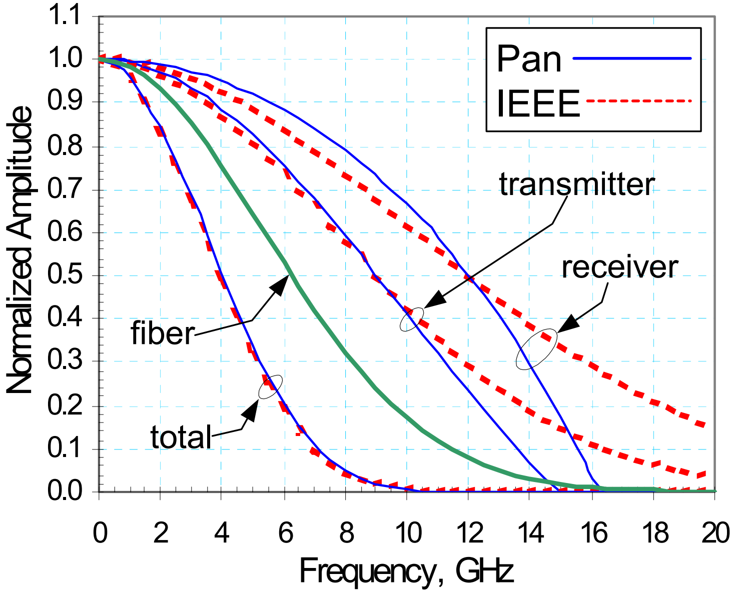
L, m = 400		Tc, ps = 121.90	
T_{eff}, ps	P_{isi}, dB freq. domain	P_{isi}, dB time domain	P_{isi}, dB spread-sheet
70	11.147	11.149	11.167
80	6.996	6.996	7.004
91	4.913	4.909	4.918
100	3.829	3.829	3.832

Summary of the differences

L, m =	2	200	300	400
T_{eff}, ps	max. dB diff.	max. dB diff.	max. dB diff.	max. dB diff.
70	0.005	0.005	0.012	0.020
80	0.004	0.004	0.008	0.008
91	0.004	0.005	0.009	0.009
100	0.002	0.002	0.005	0.004

(Differences are negligible compared to the total power budget of 7.3-dB [12].)

Changes in ISI power penalties



L = 300-m T_{eff} = 91-ps

B_s = 15-GHz B_r = 16-GHz

Pan P_{isi} = 2.71-dB

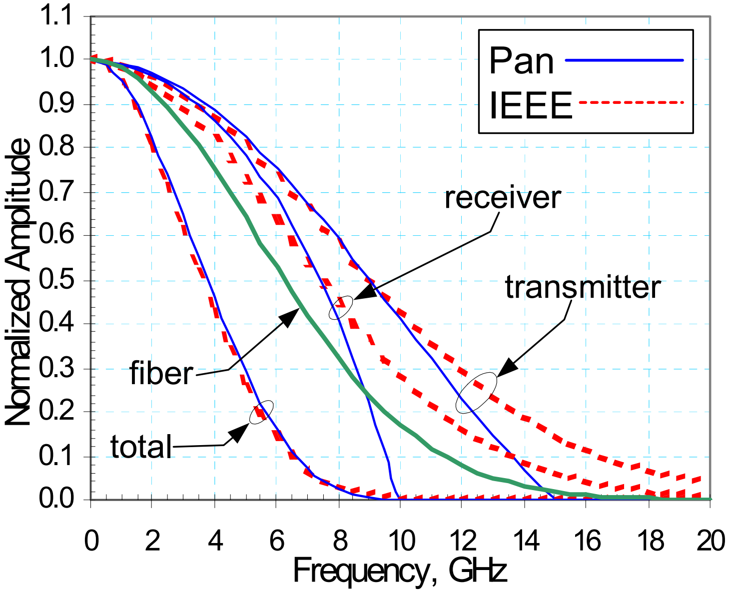
IEEE P_{isi} = 3.01-dB

L = 300-m T_{eff} = 91-ps

B_s = 15-GHz B_r = 10-GHz

Pan P_{isi} = 3.61-dB

IEEE P_{isi} = 4.07-dB



Note: For "Pan" transmitter & receiver functions (not Gaussian) see ref. [10], equations (16) & (19).

Conclusions.

- Details of ISI power penalties. Intuitive. Time & frequency domains.
- Comparisons: (1) time domain, (2) frequency domain, (3) IEEE spread sheet model [2]. Differences were negligible.
- Equivalence established between time and frequency domains. Advantages of time domain: intuitive & scopes.
- Advantages of frequency domain: (1) Each component need not be Gaussian. (2) More effects can be added.

$$P_{out}(\nu) = \boxed{P_{in}(\nu) \cdot P_s(\nu) \cdot P_f(\nu) \cdot P_r(\nu)}$$

Need not be Gaussian & include more terms.

- Improved accuracy is anticipated.

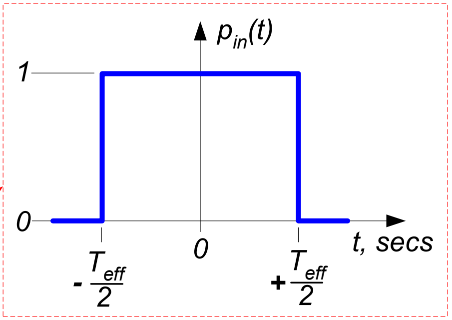
Appendix 1

Supporting details are given here which are not part of presentation.

Starting point, basic principles, Convolution Theorem:

$$p_{out}(t) = \int_{-\infty}^{\infty} p_{in}n(u)\cdot h(t-u)\cdot du \tag{1}$$

$$p_{in}(t) = \begin{cases} 1, & -T_{eff}/2 \le t \le T_{eff}/2 \\ 0, & elsewhere \end{cases} \tag{2}$$



Assume time response for optical link is Gaussian [6]:

SD of the time response for optical channel (transmitter, MMF, & receiver).

$$h(t') = \frac{1}{\boxed{\sigma_c}\sqrt{2\cdot\pi}}\cdot\exp\left[-\frac{1}{2}\cdot\left(\frac{\boxed{t'}}{\sigma_c}\right)^2\right] \tag{3}$$

time [to be substituted in (4)]

Substitute (2) & (3) into (1). Set *t'* = *t* - *u*:

$$p_{out}(t) = \frac{1}{\sigma_c\sqrt{2\cdot\pi}}\cdot\int_{-T_{eff}/2}^{+T_{eff}/2}\exp\left[-\frac{1}{2}\cdot\left(\frac{t-u}{\sigma_c}\right)^2\right]\cdot du \tag{4}$$

10% to 90% rise time

$$\boxed{T_c} = 2.564\cdot\sigma_c \tag{5}$$

Appendix 2

Change variables to that error function, *erf,* can be used [7]:

$$\int_a^b \exp\left[-w^2\right]\cdot dw = \frac{\sqrt{\pi}}{2}\cdot\left[erf(b)-erf(a)\right] \tag{6}$$

Change from *u* to *w*:

$$w^2 = \frac{1}{2}\cdot\left(\frac{t-u}{\sigma_c}\right)^2$$

Use (5):

$$w = \frac{2.564}{\sqrt{2}}\cdot\left(\frac{t-u}{T_c}\right) \tag{7}$$

Change limits of integration in (4) using (7):

$$a = \frac{2.564}{\sqrt{2}}\cdot\left(\frac{t+T_{eff}/2}{T_c}\right) = \frac{2.564}{2\cdot\sqrt{2}}\cdot\left(\frac{2t+T_{eff}}{T_c}\right) \qquad b = \frac{2.564}{2\cdot\sqrt{2}}\cdot\left(\frac{2t-T_{eff}}{T_c}\right) \tag{8}$$

Use (7) to change from *du* to *dw*:

$$du = \frac{-\sqrt{2}\cdot T_c}{2.564}\cdot dw \tag{9}$$

Substitute (5), (7), (8), & (9) into (4):

$$p_{out}(t) = \frac{2.564}{T_c\cdot\sqrt{2\cdot\pi}}\cdot\int_a^b \exp\left[-w^2\right]\cdot\left[\frac{-\sqrt{2}\cdot T_c}{2.564}\right]\cdot dw \tag{10}$$

Appendix 3

Cancel constants & replace minus sign with an exchange of limits of integration:

$$p_{out}(t) = \frac{1}{\sqrt{\pi}} \cdot \int_b^a \exp\left[-w^2\right] \cdot dw \tag{11}$$

Use (6) & (8) to evaluate (13). Note limits of integration:

$$p_{out}(t) = \frac{1}{2} \cdot \left\{ erf\left[\frac{2.564}{2 \cdot \sqrt{2}} \cdot \left(\frac{2 \cdot t + T_{eff}}{T_c}\right)\right] - erf\left[\frac{2.564}{2 \cdot \sqrt{2}} \cdot \left(\frac{2 \cdot t - T_{eff}}{T_c}\right)\right]\right\} \tag{12}$$

Evaluate (12) at t = 0 [see sketch for (2)] and use identity erf(-x) = - erf(x) [8]:

$$p_{out}(t=0) = erf\left[\frac{2.564}{2 \cdot \sqrt{2}} \cdot \left(\frac{T_{eff}}{T_c}\right)\right] \tag{13}$$

This is the desired result for the time domain with all the details which helps intuitive understanding.

Frequency domain. Here is a similar analysis. Start from first principles. See slide "ISI in the frequency domain".

$$P_{out}(\nu) = P_{in}(\nu) \cdot P_s(\nu) \cdot P_f(\nu) \cdot P_r(\nu) \tag{14}$$

Appendix 4

Fourier transform of $P_{out}(\nu)$ gives equivalent time domain, $p_{out}(t)$, which is desired:

$$p_{out}(t) = \int_{-\infty}^{+\infty} P_{out}(\nu)\cdot\exp(-2\cdot\pi\cdot i\cdot\nu\cdot t)\cdot d\nu \qquad (15)$$

For a non-return to zero (NRZ) wave form, worst case $P_{in}(\nu)$ [10]:

$$P_{in}(\nu) = \frac{\sin(\pi\cdot\nu\cdot T_{eff})}{(\pi\cdot\nu)} \qquad (16)$$

Assume optical path, $P_c(\nu)$, can be combined: $P_c(\nu) = P_s(\nu)*P_f(\nu)*P_r(\nu)$. Assume that Pc(n) in the time domain has a Gaussian distribution which can be converted to the frequency domain with a Fourier transform:

$$P_c(\nu) = \int_{-\infty}^{+\infty}\left[\frac{1}{\sqrt{2\cdot\pi}\cdot\sigma_c}\cdot\exp\left(-\frac{t^2}{2\cdot\sigma_c^2}\right)\right]\cdot\exp(-2\cdot\pi\cdot i\cdot\nu\cdot t)\cdot dt \qquad (17)$$

Resultant SD, sec., for the transmitter, fiber, & receiver in time domain.

Use Euler formula in (17), $e^{iy} = \cos(y) + i*\sin(y)$

$$\begin{aligned} P_c(\nu) = &\int_{-\infty}^{+\infty}\left[\frac{1}{\sqrt{2\cdot\pi}\cdot\sigma_c}\cdot\exp\left(-\frac{t^2}{2\cdot\sigma_c^2}\right)\right]\cdot\cos(2\cdot\pi\cdot\nu\cdot t)\cdot dt + \\ &i\cdot\int_{-\infty}^{+\infty}\left[\frac{1}{\sqrt{2\cdot\pi}\cdot\sigma_c}\cdot\exp\left(-\frac{t^2}{2\cdot\sigma_c^2}\right)\right]\cdot\sin(2\cdot\pi\cdot\nu\cdot t)\cdot dt \end{aligned} \qquad (18)$$

Appendix 5

The term in (18) with sin(y) is odd: Integration over a symmetrical range in 0. The cos(y) term has been integrated [5]:

$$P_c(\nu) = \exp\left[-\frac{(2\cdot\pi\cdot\nu\cdot\sigma_c)^2}{2}\right] \tag{19}$$

From (14), (15), (16) & (19):

$$p_{out}(t=0) = \int_{-\infty}^{+\infty}\left\{\frac{\sin(\pi\cdot\nu\cdot T_{eff})}{(\pi\cdot\nu)}\right\}\cdot\left\{\exp\left[-\frac{(2\cdot\pi\cdot\nu\cdot\sigma_c)^2}{2}\right]\right\}\cdot d\nu \tag{20}$$

(20) can be evaluated numerically to determine P_{isi}.

References 1

[1] “Optical Receiver Performance Evaluation,” Application Note: HFAN-3.0.2, Rev 0; 03/03, Maxim Integrated Products, web site: http://www.maxim-ic.com/, March 2003.

[2] The spreadsheet model for the 10Gigabit Ethernet Link Model is located at http://www.ieee802.org/3/efm/public/tools/EFM0_0_2.7.xls.

[3] “Bandwidth and Rise Time Calculations for Digital Multimode Fiber-Optic Data Links,” Brown, G., J. Lightwave Technol., Vol. 10, No. 5, pp. 672 - 678, May 1992.

[4] “An Introduction to Statistical Communication Theory,” Thomas, J. B., New York: Wiley, 1969, p. 143.

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[6] “Review of the 10Gigabit Ethernet Link Model,” Cunningham, D. and Dawe, P., ONIDS 2002 White Paper, http://literature.agilent.com/litweb/pdf/5988-5908EN.pdf.

[7] “Advanced Engineering Mathematics, 2nd Ed.,” Kreyszig, E., New York: Wiley, 1967, p. 509.

[8] “Handbook of Mathematical Functions,” Eds. Abramowitz, M., and Stegun, I., New York: Dover, 1965.

References 2

[9] "Proposal to Modify the ISI Penalty calculation in the current GbE Spreadsheet Model," David Dolfi, web site: http://www.ieee802.org/3/10G_study/public/email_attach/new_isi.pdf, April 6, 2000.

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[11] "Signals and Systems: Continuous and Discrete, 3rd Ed.," New York: Macmillan, 1993.

[12] "802.3aeTM IEEE Standard for Information technology — Telecommunications and information exchange between systems— Local and metropolitan area networks—Specific requirements Part 3: Carrier Sense Multiple Access with Collision Detection (CSMA/CD) Access Method and Physical Layer Specifications," IEEE, 2002.