Statistical modeling of multimode-fiber links: a supplement to the information provided in the release note of 12 October 2004 in relation to Release 1.2 from the University of Cambridge

Jonathan Ingham, Richard Penty and Ian White *University of Cambridge* 12 October 2004

A Introduction

A statistical model has been adopted to generate information which is intended to be representative of challenging multimode-fiber (MMF) links. The principle of an earlier form of this model is described in detail in M. Webster *et al.*, "A statistical analysis of conditioned launch for Gigabit Ethernet links using multimode fiber", *Journal of Lightwave Technology*, vol. 17, no. 9, pp. 1532-1541, September 1999. The purpose of this supplementary document is to provide further description and clarification of the method which has been adopted to generate the files of impulse responses and frequency responses provided by the University of Cambridge in Release 1.2 of October 2004. In particular, the activities of Task 1 of the IEEE 802.3aq Channel Modeling Ad-Hoc have resulted in the decision to remove the step perturbation at the core-cladding interface which existed in earlier versions of the model. In addition, kink perturbations have been added. The form of the kink perturbation was suggested by John Abbott and the radii at which the kinks occur were suggested by Paul Kolesar. The participants of Task 1 are thanked sincerely for their enthusiastic involvement in this work.

B Overview of the statistical model

The essence of the statistical model is to: (i) generate a set of refractive-index profiles which contain perturbations from a near-ideal power-law profile and which are representative of the imperfections found in installed-base MMF; (ii) for each perturbed fiber, calculate the electric-field distributions and propagation delays of the guided modes supported by the fiber; (iii) for each perturbed fiber, calculate the differential modal delay (DMD) and overfilled-launch (OFL) bandwidth-length product (BWL); (iv) for each perturbed fiber, perform a scaling process in order to adjust the DMD such that the fiber is associated with a target worst-case DMD.

C Generation of perturbed refractive-index profiles

Release 1.2 is concerned exclusively with MMF which has a core diameter of 62.5 μm and which is operated at a wavelength of 1300 nm. Therefore, a near-ideal refractive-index profile is defined here as a profile with: (i) a power-law parameter $\alpha = 1.97$; (ii) a core radius $r_{core} = 31.25$ μm; (iii) an axial refractive index $n_{core} = 1.5$; (iv) a cladding refractive index $n_{clad} = 1.474$, which is equivalent to a numerical aperture $\mathrm{NA} = \sqrt{(n_{core}^2 - n_{clad}^2)} = 0.28$.

Five types of perturbation of this near-ideal power-law refractive-index profile are considered. Three of the five types of perturbation, which are described in detail below, are each associated with three possible values. The remaining two perturbation types are each associated with two possible values, i.e. the perturbation is applied or not applied. This leads to $3^3\,2^2 = 108$ distinct refractive-index profiles for the subsequent modeling.

(1) **Deviation of the power-law parameter α from near-ideal within the *inner* region of the fiber core**
In addition to its near-ideal value of 1.97, the power-law parameter α for the inner region of the fiber core ($0 \le r \le r_{core}/2$) may assume values of 1.89 and 2.05. The figures of 1.89 and 2.05 are chosen since for a pure power-law fiber they result in an OFLBWL close to the ISO/IEC 11801 specification of 500 MHz km at 1300 nm.

(2) **Deviation of the power-law parameter α from near-ideal within the *outer* region of the fiber core**
In addition to its near-ideal value of 1.97, the power-law parameter α for the outer region of the fiber core ($r_{core}/2 < r \le r_{core}$) may assume values of 1.89 and 2.05, as for perturbation (1).

For perturbations (1) and (2), the continuity of the refractive-index profile is maintained by adjusting the outer refractive-index profile ($r_{core}/2 < r \le r_{core}$) by a correction factor n_1/n_2, where n_1 is the refractive index at $r = r_{core}/2$ calculated using the inner power-law parameter, and n_2 is the refractive index at $r = r_{core}/2$ calculated using the outer power-law parameter. The refractive index of the cladding ($r > r_{core}$) is unaffected by this adjustment.

(3) **Dip or peak located on the axis of the fiber**
A dip or peak located on the axis of the fiber ($r = 0$) is modeled by a gaussian function which is added to the refractive-index profile. The dip and peak both have a full width at half maximum FWHM = 3 μm and are modeled by the addition of: $n(r) = A\exp(-r^2/\delta^2)$, where $\delta = \mathrm{FWHM}/[2\sqrt{(\log_e 2)}]$ and the amplitude of the perturbation $A = -0.004$ (dip), 0 (no perturbation) or 0.002 (peak).

(4) **Imperfect transition from the fiber core to the fiber cladding**
One type of transition at the core-cladding interface is considered in addition to the ideal: an exponential decay,

C* Generation of perturbed refractive-index profiles *continued

such that for $r > r_0 = 28$ µm: $n(r) = (n_0 - n_{clad})\exp[-\beta(r - r_0)] + n_{clad}$, where n_0 is the refractive index at $r = r_0$ before the perturbation is applied. The decay constant $\beta = 3.0457 \times 10^5$ m^{-1}.

(5) **Kink perturbation**
The kink perturbation suggested by John Abbott is documented in the accompanying file "Abbott Kinks_and_Perturbations.pdf". The amplitude `a_k` = 0.03 and the locations `r_k` are chosen to match the suggestion of Paul Kolesar in the accompanying spreadsheet. The locations are shown in Appendix *A,* which provides a table which relates the fiber identification number (as used in Release 1.2) to the corresponding refractive-index profile settings. *Please note that the numbering differs relative to earlier releases.*

***D* Calculation of modal electric-field distributions and propagation delays**

The 108 perturbed refractive-index profiles generated in part *C* are input separately to a scalar-wave-equation mode solver. This mode solver determines which linearly-polarized (LP) guided modes are supported by a given fiber at a desired wavelength, which is 1300 nm throughout this work. Each LP guided mode is associated with: (i) a cylindrical order $\nu \geq 0$; (ii) a radial order $\mu \geq 1$; (iii) a mode-group order $g = 2\mu + \nu + 1 \geq 3$. Moreover, the mode solver provides: (i) the electric-field distribution $E_{\nu\mu}(r, \phi)$ and (ii) the propagation delay $\tau_{\nu\mu}$ (ns/km) of each LP guided mode supported by the fiber. All modes for which the mode-group order $g > 20$ are ignored from this point onwards. This is equivalent to a mode-dependent loss (MDL) of 0 dB for mode groups with $g \leq 20$ and a MDL of ∞ dB for all other mode groups. No further modeling of MDL is performed.

At this point in the model, each LP guided mode is associated with a unique propagation delay $\tau_{\nu\mu}$. In general, this propagation delay will vary between mode groups and also within mode groups. For the simplicity of the release, the propagation delay $\tau_{\nu\mu}$ of each guided mode is replaced by the arithmetic mean τ_g of the propagation delays for the mode group g to which the guided mode belongs. The values of τ_g form the "modal delay" values provided in the files of impulse responses as part of Release 1.2.

***E* Calculation of DMD and OFLBWL**

(1) **DMD**
The first step in the calculation of the DMD of a fiber is to determine the modal excitation in response to a radially-scanned single-mode fiber (SMF). The SMF output beam is modeled by a gaussian beam for which the electric-field distribution $E_I(r, \phi)$ has a FHWM = 7 µm. For radial offsets of this SMF beam from 0 µm to 30 µm, in increments of 1 µm, $E_I(r, \phi)$ is overlapped with $E_{\nu\mu}(r, \phi)$, according to the overlap integrals of equation (3) in Webster *et al*. This generates a power-coupling coefficient $P_{\nu\mu}$ for each guided mode. The power-coupling coefficients $P_{\nu\mu}$ are then averaged within each mode group, such that the power-coupling coefficient $P_{\nu\mu}$ for each guided mode is replaced by the arithmetic mean P_g of the power-coupling coefficients for the mode group g to which the guided mode belongs. This is performed in order to model the effect of complete mode-mixing *within* mode groups. Note that mode mixing *between* mode groups is not modeled in this work.

Once the power-coupling coefficients P_g have been obtained for each radial offset of the SMF beam, an "intermediate" DMD value may be calculated for each of the 31 offsets using the P_g values in combination with the propagation delays τ_g. This "intermediate" DMD is defined as: $\Sigma N_g P_g \tau_g / \Sigma N_g P_g$ *. Note that although the propagation delays and power-coupling coefficients have been averaged within each mode group g, the multiplicity of modes within each mode group is retained, i.e. the contribution of each mode group g is weighted by the number of modes N_g within the mode group g. The values of $N_g P_g$ form the "power-coupling coefficient" values in the files of impulse responses as part of Release 1.1. Once the "intermediate" DMD has been calculated for each radial offset of the SMF beam, the final DMD is obtained by: (i) subtracting the minimum "intermediate" DMD from the set of 31 values; (ii) extracting the maximum value from the resulting set of normalized "intermediate" DMDs. Note that this definition of DMD is often referred to as *mean* DMD, which distinguishes it from other definitions of DMD, e.g. the definition of DMD described in TIA/EIA-455-220.

(2) **OFLBWL**
To calculate the OFLBWL, the initial step is to calculate the impulse response $h(t)$ for the case of OFL. Since OFL indicates equal excitation of all guided modes, no power-coupling calculation is required. Therefore, $h(t) = \Sigma N_g \delta(t - \tau_g)$ *, where δ is the Dirac delta function. The OFLBWL is then obtained by extracting the –3-dB bandwidth from the corresponding frequency response $H(f) = \Sigma N_g \exp(-j\, 2\pi f \tau_g)$ *, where the delays are set for 1 km of fiber in order for the extracted –3-dB bandwidth to be the OFLBWL.

* The summations run over all mode-group orders g such that: $3 \leq g \leq 20$.

F DMD scaling

Following the procedures of part *E* generates the DMD and OFLBWL for each of the 108 fibers. This information forms the input to the DMD scaling process in which each fiber is considered separately. The essence of this process is to exploit the inverse relationship between DMD and modal bandwidth in order to adjust each fiber, by means of a bandwidth scale factor, to provide a DMD at the target value of 2 ns/km. This bandwidth scale factor is defined as the ratio of the calculated DMD relative to the target DMD, i.e. DMD / (2 ns/km). Therefore, if the DMD is excessive in relation to the target, then the bandwidths obtained from any subsequent calculation for the fiber are scaled *up* by this scale factor. Similarly, if the DMD is insufficient in relation to the target, the bandwidths are scaled *down* in any subsequent consideration of the fiber. The scale factor for the DMD is the reciprocal of the bandwidth scale factor.

Importantly, it is necessary to monitor the effect of the scaling on the OFLBWL. Special care must be taken for those cases where the resulting OFLBWL is below the specification of 500 MHz km. The four cases that may be encountered during DMD scaling of each fiber are now considered.

(1) **DMD ≥ 2 ns/km OFLBWL × DMD / (2 ns/km) ≥ 500 MHz km**
The DMD is excessive in relation to the target or equal to the target. Therefore, the bandwidth scale factor DMD / (2 ns/km) is greater than or equal to unity. The scaled OFLBWL meets the 500 MHz km specification and therefore the fiber is retained with a scaled DMD of 2 ns/km and a bandwidth scale factor of DMD / (2 ns/km).

(2) **DMD ≥ 2 ns/km OFLBWL × DMD / (2 ns/km) < 500 MHz km**
The DMD is excessive in relation to the target or equal to the target. Therefore, the bandwidth scale factor DMD / (2 ns/km) is greater than or equal to unity. The scaled OFLBWL does not meet the 500 MHz km specification. The immediate reaction would be to reject the fiber. However, an alternative is to define a bandwidth scale factor as 500 MHz km / OFLBWL, which results in the fiber being scaled to the OFLBWL specification of 500 MHz km.

(3) **DMD < 2 ns/km OFLBWL × DMD / (2 ns/km) ≥ 500 MHz km**
The DMD is insufficient in relation to the target. Therefore, the bandwidth scale factor DMD / (2 ns/km) is less than unity. The scaled OFLBWL meets the 500 MHz km specification and therefore the fiber is retained with a scaled DMD of 2 ns/km and a bandwidth scale factor of DMD / (2 ns/km).

(4) **DMD < 2 ns/km OFLBWL × DMD / (2 ns/km) < 500 MHz km**
The DMD is insufficient in relation to the target. Therefore, the bandwidth scale factor DMD / (2 ns/km) is less than unity. The scaled OFLBWL does not meet the 500 MHz km specification. The immediate reaction would be to reject the fiber. However, an alternative is to define a bandwidth scale factor as 500 MHz km / OFLBWL, which results in the fiber being scaled to the OFLBWL specification of 500 MHz km.

When the DMD scaling process is complete, each of the 108 fibers is equipped with a unique bandwidth scale factor in readiness for impulse response and frequency response calculations.

G Offset-launch results

The offset-launch results provided in Release 1.2 are then calculated by considering each fiber and determining the power-coupling coefficients for a launch from a radially-offset SMF, in order to model a Gigabit Ethernet mode-conditioning patchcord. As in the DMD scaling process, the output from the SMF is modeled by a gaussian beam with an electric-field distribution which has a FHWM = 7 μm. As before, averaging within mode groups is performed for both the power-coupling coefficients and the propagation delays (for a fiber length of 300 m). Once the frequency responses have been calculated, the bandwidth scale factor from the DMD scaling process is used to scale the frequency responses as necessary. For example, if the bandwidth scale factor is greater than unity, then the calculated frequency responses are "stretched" along the frequency axis appropriately.

It is important to reiterate that the released results do not include any other effects in addition to the intermodal dispersion of the MMF. In particular, intramodal (chromatic) dispersion is not included; neither are filtering effects associated with either the transmitter or the receiver in the MMF link.

***Appendix A* is overleaf…**

Appendix A Table of refractive-index profile settings for the 108 fibers of Release 1.2

The table shown below describes the refractive-index profile settings for each of the 108 fibers of Release 1.2. **Please note that the numbering of the fibers differs relative to earlier releases.**

Descriptions of columns 1 to 6:

COLUMN 1 Fiber identification number from 1 to 108.
COLUMN 2 Power-law parameter α for the inner region of the fiber core ($0 \le r \le r_{core}/2$).
COLUMN 3 Power-law parameter α for the outer region of the fiber core ($r_{core}/2 < r \le r_{core}$).
COLUMN 4 Core-cladding transition setting ("0" denotes no perturbation; "1" denotes exponential decay to cladding).
COLUMN 5 On-axis dip or peak setting ("–1" denotes dip; "0" denotes no perturbation; "1" denotes peak).
COLUMN 6 Radius associated with kink perturbation in μm. N.B. A value of "0" is used to indicate that no kink perturbation is applied.

1	2	3	4	5	6	1	2	3	4	5	6
1	1.89	1.89	0	-1	0	55	1.97	1.97	1	-1	0
2	1.89	1.89	0	-1	11	56	1.97	1.97	1	-1	23
3	1.89	1.89	0	0	0	57	1.97	1.97	1	0	0
4	1.89	1.89	0	0	17	58	1.97	1.97	1	0	11
5	1.89	1.89	0	1	0	59	1.97	1.97	1	1	0
6	1.89	1.89	0	1	23	60	1.97	1.97	1	1	17
7	1.89	1.89	1	-1	0	61	1.97	2.05	0	-1	0
8	1.89	1.89	1	-1	11	62	1.97	2.05	0	-1	13
9	1.89	1.89	1	0	0	63	1.97	2.05	0	0	0
10	1.89	1.89	1	0	17	64	1.97	2.05	0	0	19
11	1.89	1.89	1	1	0	65	1.97	2.05	0	1	0
12	1.89	1.89	1	1	23	66	1.97	2.05	0	1	25
13	1.89	1.97	0	-1	0	67	1.97	2.05	1	-1	0
14	1.89	1.97	0	-1	19	68	1.97	2.05	1	-1	13
15	1.89	1.97	0	0	0	69	1.97	2.05	1	0	0
16	1.89	1.97	0	0	25	70	1.97	2.05	1	0	19
17	1.89	1.97	0	1	0	71	1.97	2.05	1	1	0
18	1.89	1.97	0	1	13	72	1.97	2.05	1	1	25
19	1.89	1.97	1	-1	0	73	2.05	1.89	0	-1	0
20	1.89	1.97	1	-1	19	74	2.05	1.89	0	-1	25
21	1.89	1.97	1	0	0	75	2.05	1.89	0	0	0
22	1.89	1.97	1	0	25	76	2.05	1.89	0	0	13
23	1.89	1.97	1	1	0	77	2.05	1.89	0	1	0
24	1.89	1.97	1	1	13	78	2.05	1.89	0	1	19
25	1.89	2.05	0	-1	0	79	2.05	1.89	1	-1	0
26	1.89	2.05	0	-1	27	80	2.05	1.89	1	-1	25
27	1.89	2.05	0	0	0	81	2.05	1.89	1	0	0
28	1.89	2.05	0	0	15	82	2.05	1.89	1	0	13
29	1.89	2.05	0	1	0	83	2.05	1.89	1	1	0
30	1.89	2.05	0	1	21	84	2.05	1.89	1	1	19
31	1.89	2.05	1	-1	0	85	2.05	1.97	0	-1	0
32	1.89	2.05	1	-1	27	86	2.05	1.97	0	-1	15
33	1.89	2.05	1	0	0	87	2.05	1.97	0	0	0
34	1.89	2.05	1	0	15	88	2.05	1.97	0	0	21
35	1.89	2.05	1	1	0	89	2.05	1.97	0	1	0
36	1.89	2.05	1	1	21	90	2.05	1.97	0	1	27
37	1.97	1.89	0	-1	0	91	2.05	1.97	1	-1	0
38	1.97	1.89	0	-1	21	92	2.05	1.97	1	-1	15
39	1.97	1.89	0	0	0	93	2.05	1.97	1	0	0
40	1.97	1.89	0	0	27	94	2.05	1.97	1	0	21
41	1.97	1.89	0	1	0	95	2.05	1.97	1	1	0
42	1.97	1.89	0	1	15	96	2.05	1.97	1	1	27
43	1.97	1.89	1	-1	0	97	2.05	2.05	0	-1	0
44	1.97	1.89	1	-1	21	98	2.05	2.05	0	-1	17
45	1.97	1.89	1	0	0	99	2.05	2.05	0	0	0
46	1.97	1.89	1	0	27	100	2.05	2.05	0	0	23
47	1.97	1.89	1	1	0	101	2.05	2.05	0	1	0
48	1.97	1.89	1	1	15	102	2.05	2.05	0	1	11
49	1.97	1.97	0	-1	0	103	2.05	2.05	1	-1	0
50	1.97	1.97	0	-1	23	104	2.05	2.05	1	-1	17
51	1.97	1.97	0	0	0	105	2.05	2.05	1	0	0
52	1.97	1.97	0	0	11	106	2.05	2.05	1	0	23
53	1.97	1.97	0	1	0	107	2.05	2.05	1	1	0
54	1.97	1.97	0	1	17	108	2.05	2.05	1	1	11