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Dk & Df ALGEBRAIC MODEL v2.04

(NOTE: only change from v2.03 is correction of Cu conductivity used in surface roughness [Hurray model] calculation)

Let’s take a look…

WHAT YOU WILL SEE

Unchanged from v2.01

NOTE: The Change Parameter window is a visual basic macro. If you save the file to your computer, be sure to select the Maco-Enabled file type.

LOOK & FEEL

Quick preview of select frequencies

Version 2.01 LOSS SNAPSHOT: Loss at 5GHz: 23.79 dB Loss at 12.75GHz: 56.54 dB Loss at 14GHz: 61.66 dB

CHANGE PARAMETERS

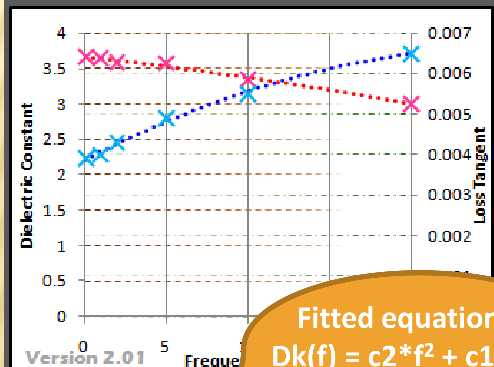
Click to change the input parameters

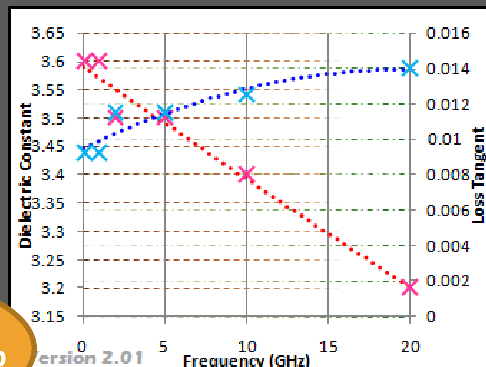
Backplane/Trace Material		
Length (inch)		20
Trace Width (mil)		8
Cu Thickness (mil)		0.6
Diel. Thickness (mil)		16
Freq	Dk	Df
1.00E+08	3.67	0.0039
1.00E+09	3.65	0.004
2.00E+09	3.59	0.0043
5.00E+09	3.576	0.0049
1.00E+10	3.3494	0.0055
2.00E+10	3	0.0065
Low Roughness	20	6.0E-07

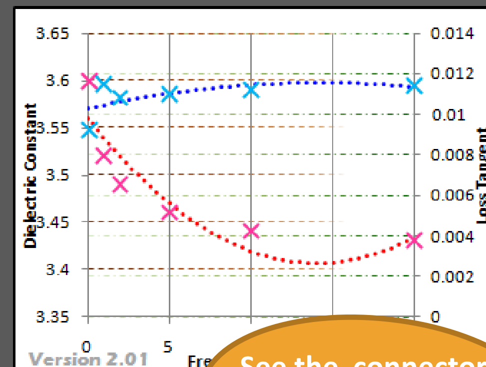
Linecard A Material		
Length (inch)		2
Trace Width (mil)		6
Cu Thickness (mil)		0.6
Diel. Thickness (mil)		12.9
Freq	Dk	Df
1.00E+08	3.6	0.0092
1.00E+09	3.6	0.0092
2.00E+09	3.5	0.0115
5.00E+09	3.5	0.0115
1.00E+10	3.4	0.0125
2.00E+10	3.2	0.014
High Roughness	65	6.0E-07

Linecard B Material		
Length (inch)		18
Trace Width (mil)		5
Cu Thickness (mil)		0.6
Diel. Thickness (mil)		11.1
Freq	Dk	Df
1.00E+08	3.6	0.0092
1.00E+09	3.52	0.0115
2.00E+09	3.49	0.0108
5.00E+09	3.46	0.011
1.00E+10	3.44	0.0112
2.00E+10	3.43	0.0114
High Roughness	65	6.0E-07

2 conne... each - 0.616dB loss at 5G - 1.123dB loss at 12.75G - 1.21dB loss at 14G







Fitted equation(s): Dk(f) = c2*f² + c1*f + b

See the connector & channel loss at specific frequencies

DK fit to Second Order Equation		
	b	3.672198659
Df fit to Second Order Equation	c2	-3.82611E-24
	c1	2.075E-13
	b	0.003870352

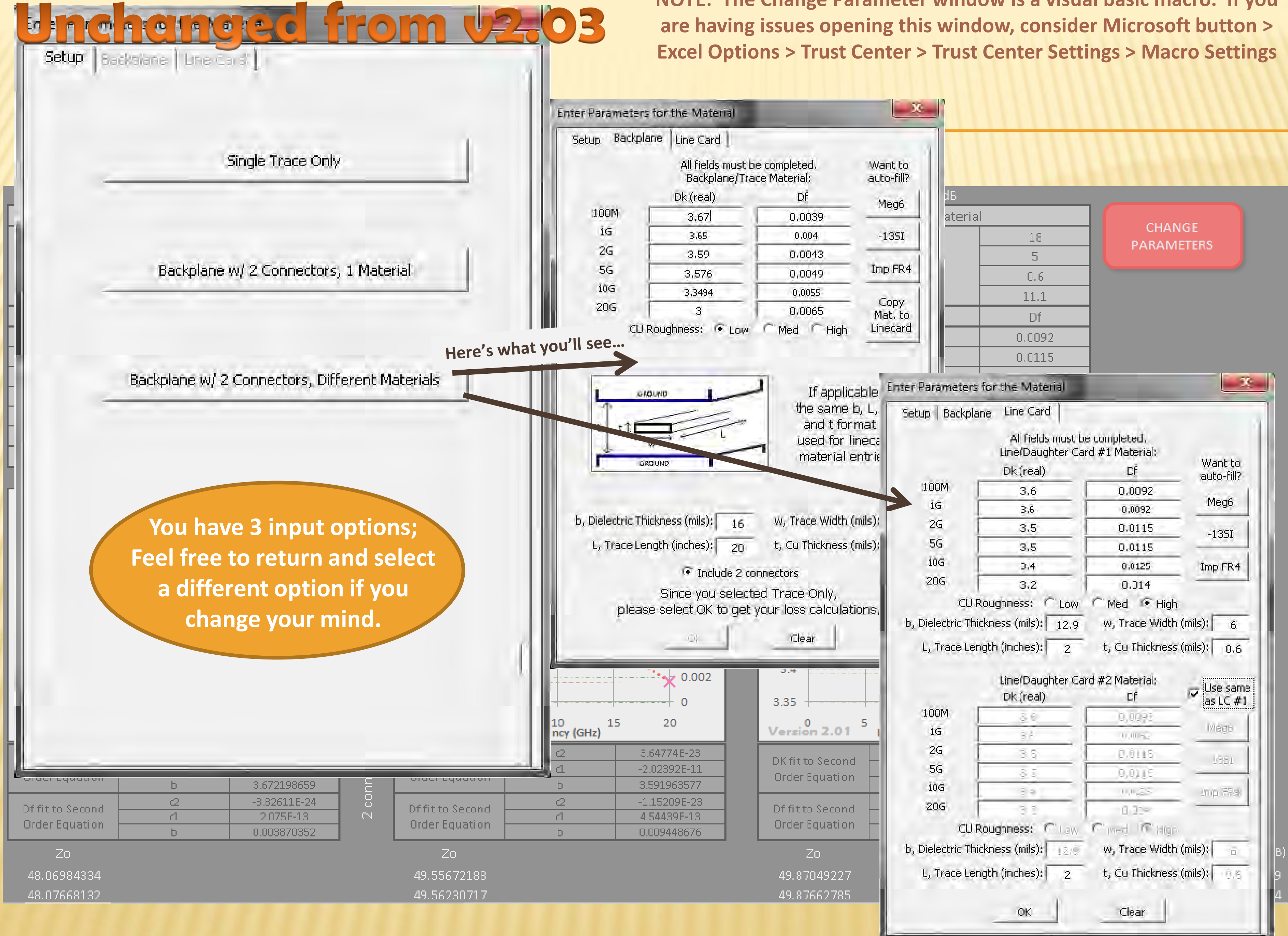
DK fit to Second Order Equation	c2	3.64774E-23
	c1	-2.02392E-11
	b	3.591963577
Df fit to Second Order Equation	c2	-1.15209E-23
	c1	4.54439E-13
	b	0.009448676

DK fit to Second Order Equation		
Df fit to Second Order Equation		
	c1	
	b	0.01025...

Zo	Zo	Zo	...n. Loss	A_{total} (dB)
48.06984334	49.55672188	49.87049227	0.0283	0.8099
48.07668132	49.56230717	49.87662785	0.0630	0.8454

Unchanged from v2.03

NOTE: The Change Parameter window is a visual basic macro. If you are having issues opening this window, consider Microsoft button > Excel Options > Trust Center > Trust Center Settings > Macro Settings

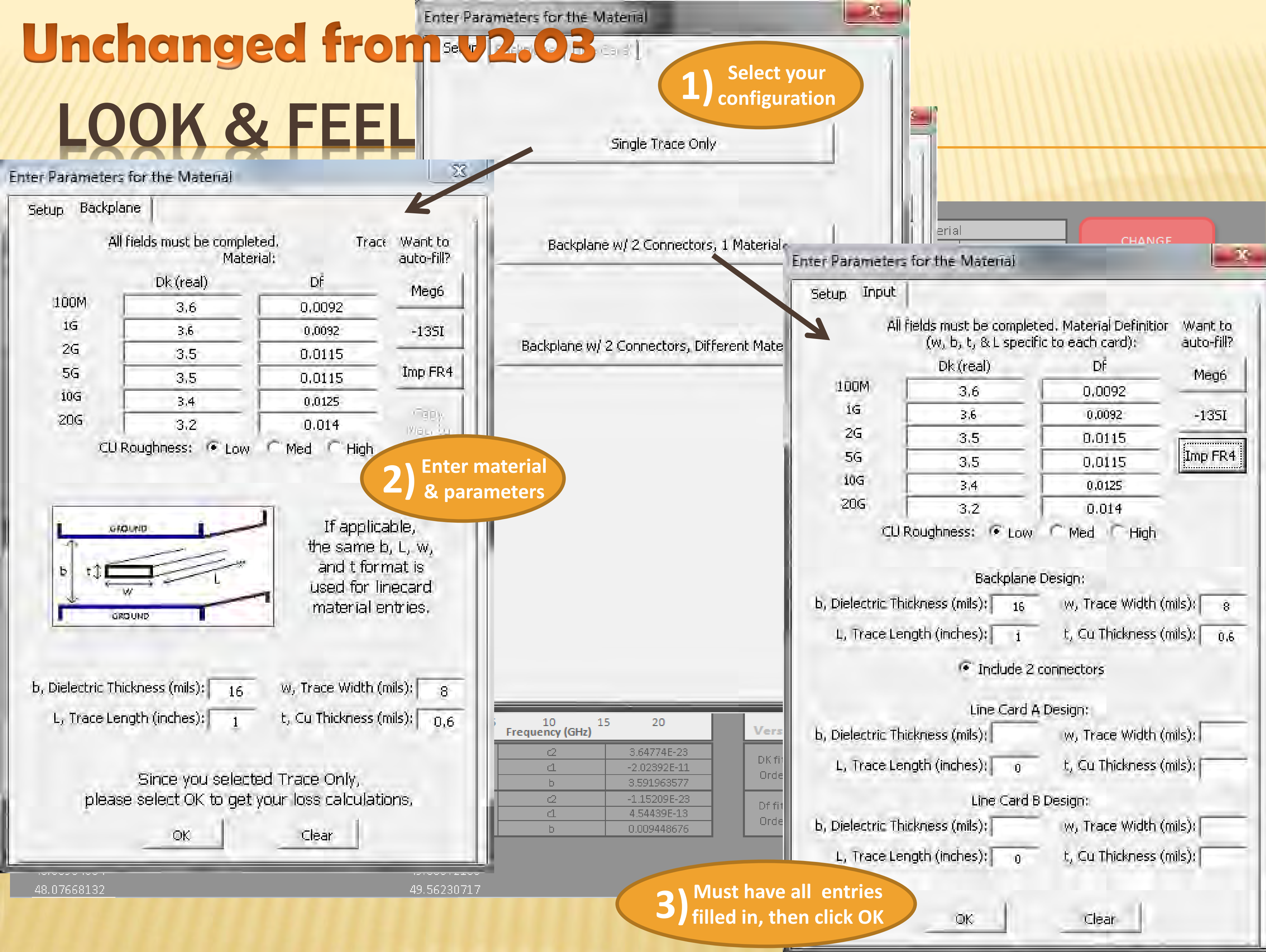


Here's what you'll see…

You have 3 input options; Feel free to return and select a different option if you change your mind.

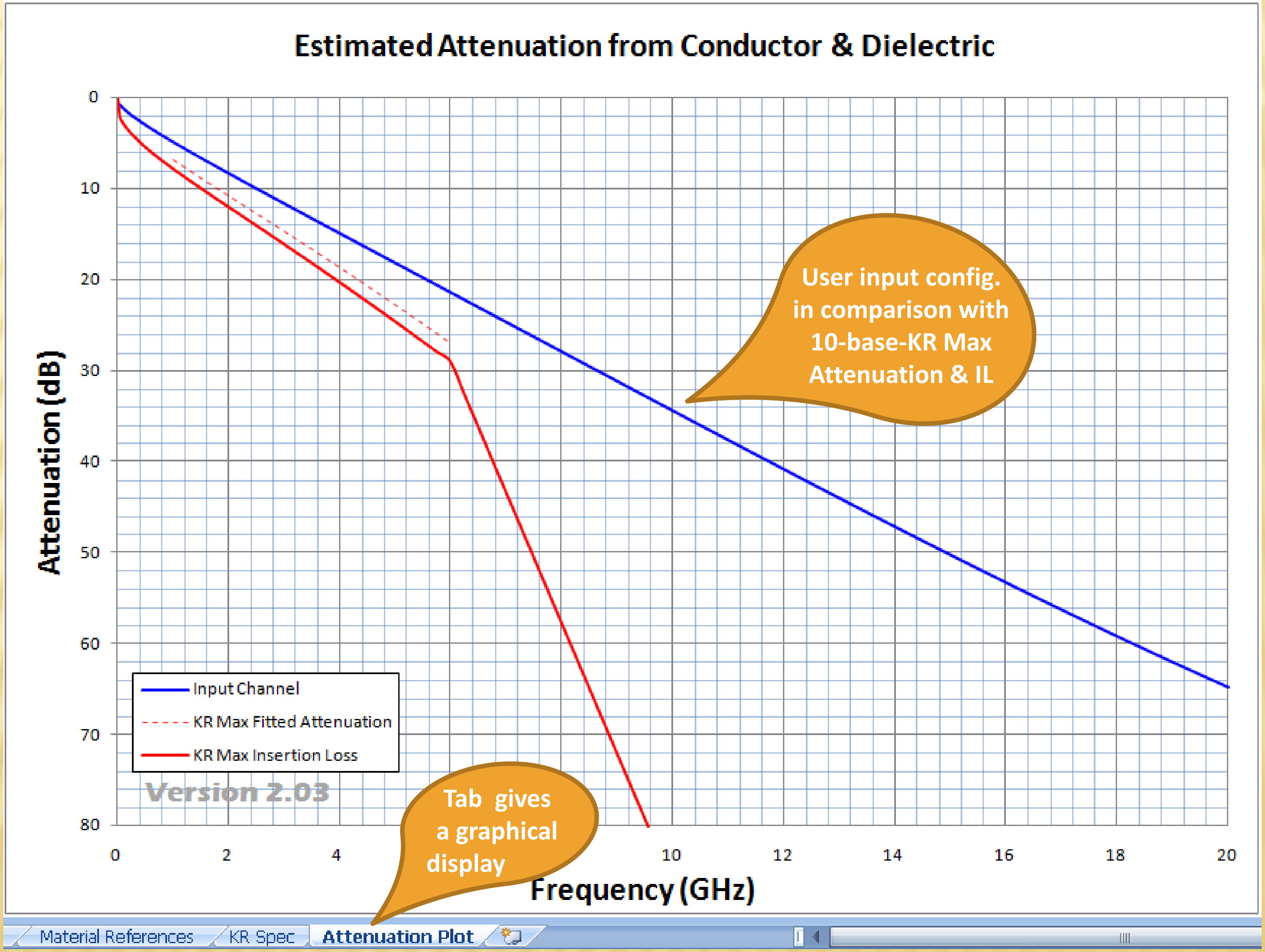
Unchanged from v2.03

LOOK & FEEL



Unchanged from v2.03

LOOK & FEEL



Behind the Scenes…

EQUATIONS AND REFERENCES OF MODEL

FREQUENCY DEPENDENCE

Unchanged from v2.01

- 6 input frequency points for Dk and Df
- Fit Dk and Df to second order equations
  - Coefficients shown on sheet
  - Graphical representation shown on sheet
- Note that frequency dependence fit is only approximated to 20G, therefore, loss approximations should only be considered to 20G
- Z_0 is calculated with Dk (or ε_r) at a given frequency; a similar technique is used in loss calculations

DK & DF SECOND ORDER EQUATIONS

Unchanged from v2.01

- Second order approximation is created using the LINEST function. This function essentially fits a 2nd order polynomial to the 6 frequency points given; resulting in

$$D_k = c_2*f^2 + c_1*f + b$$

- Function as implemented in the spreadsheet:

LINEST(C8:C13,B8:B13^[1,2])

Y Values
(Dk entered points)

X Values
(Freq. associated
w/ entered points)

Exponents of X;
Creating a second
order equation.

- See Excel HELP for more details on LINEST function. Methodology verified against “add trend line” within plot.

CHARACTERISTIC IMPEDANCE[2, EQU 4-5]

$$Z_0 = \frac{94.15}{\sqrt{\varepsilon_r}\left(\dfrac{\frac{w}{b}}{\frac{1-t}{b}} + \dfrac{c_f'}{0.0885\varepsilon_r}\right)} ohms$$

$$c_f' = \frac{0.0885\varepsilon_r}{\pi}\left\{\frac{2}{1-\frac{t}{b}}\log_e\left(\frac{1}{1-\frac{t}{b}}+1\right) - \left(\frac{1}{1-\frac{t}{b}}-1\right)\log_e\left(\frac{1}{\left(1-\frac{t}{b}\right)^2}-1\right)\right\}$$

ε_r = relative dielectric constant (at a given frequency)

b = plate spacing (mil)

w = trace width (mil)

t = trace thickness (mil)

c_f' = fringing capacitance (μf/cm) * assuming semi - infinite plate between two infinite ground planes, but good approximation for w/(b - t) ≥ 0.35

Unchanged from v1.01

ATTENUATION IN LOSSY LINES

Unchanged from v1.01

- Attenuation per length[1, EQN 9-54]:

$$\alpha_n = \sqrt{\frac{1}{2}\left[\sqrt{\left(R_L^2 + \omega^2 L_L^2\right)\left(G_L^2 + \omega^2 C_L^2\right)} - \omega^2 L_L C_L + R_L G_L\right]} \text{ nepers/length}$$

- Using a low-loss approximation[1, EQN 9-55]: (surface roughness ignored)

$$\alpha_n = \frac{1}{2}\left(\frac{R_L}{Z_0} + G_L Z_0\right) \text{ nepers/length}$$

- But we don't typically discuss in nepers…[1, EQN 9-57]

$$10^{\frac{\alpha_{dB}}{20}} = e^{\alpha_n} \qquad \therefore \alpha_{dB} = 20\log_{10} e \times \alpha_n$$

$$\text{for ease of notation: } Y_{n\to dB} = 20\log_{10} e$$

CONDUCTOR LOSS (per inch)

Unchanged from v2.01

$$\alpha_{cond} = \left(\mathrm{Y}_{\mathrm{n}\rightarrow\mathrm{dB}}\right)\times\frac{\mathrm{R}_{\mathrm{L}}}{\mathrm{Z}_{0}}$$

- α_{cond} = attenuation of amplitude due to conductor loss, in dB/length [1, EQN 9-59]
- $\mathrm{Y}_{\mathrm{n}\rightarrow\mathrm{dB}}$ = converstion from nepers to dB
- R_{L} = resistance per length of conductor
- Z_{0} = characteristic impedance

- Skin effect, ground resistance, and stripline effect are accounted for in resistance[3, EQNs 4.3a-4.10]:
  - R of signal trace & return path (w/skin effect)

$$R_{signalCu\,skin\,effect} = \frac{\sqrt{\pi\mu\rho f}}{w} \qquad R_{groundCu\,skin\,effect} = \frac{\sqrt{\pi\mu\rho f}}{6H}$$

  - AC surface resistance for microstrip (or 1 side of a stripline trace)

$$R_{ac\,microstrip} = R_{signal} + R_{ground}$$

Unchanged from v2.02

CONDUCTOR LOSS (per inch)

- Stripline approximation assumes parallel resistance of top and bottom microstrip approximations

$$R_L = \frac{\left[\sqrt{\pi\mu\rho f} * \left(\frac{1}{w} + \frac{1}{6H}\right)\right]^2}{2 * \left[\sqrt{\pi\mu\rho f} * \left(\frac{1}{w} + \frac{1}{6H}\right)\right]}$$

$$\begin{cases} R_L = \text{stripline surface resistance } (\Omega/\text{inch}) \\ w = \text{width of trace (inch)} \\ H = \text{height dielectric from ground to signal (inch)} \\ \mu = \text{permeability of Cu} \approx 4\pi \times 10^{-7} \times 0.999994 \ \frac{\text{H}}{\text{m}} \\ \rho = \text{resistivity of Cu} = \frac{1}{\sigma} = \frac{1}{5.96 \times 10^{7}} \ \Omega\,\text{m} \\ f = \text{frequency (hertz)} \end{cases}$$

- Conductor loss per inch as entered in the model:

$$\alpha_{cond} = \frac{1}{2} \times \left(20 \log_{10} e\right) \times \left(\frac{\left\lfloor \sqrt{\pi\mu\rho f} * \left(\frac{1}{w} + \frac{1}{6H}\right) \right\rfloor}{2} \right) \times \frac{1}{Z_0}$$

DIELECTRIC LOSS (per inch)

$$\alpha_{diel} = \left(\mathrm{Y}_{\mathrm{n}\to\mathrm{dB}}\right)\times \mathrm{G}_L Z_0 \quad \begin{cases} \alpha_{\mathrm{cond}} = \text{attenuation of amplitude due to dielectric loss, in dB/length}^{[1,\text{ EQN 9-60}]} \\ \mathrm{Y}_{\mathrm{n}\to\mathrm{dB}} = \text{converstion from nepers to dB} \\ \mathrm{G}_{\mathrm{L}} = \text{shunt conductance per length from dielectric} \\ \mathrm{Z}_0 = \text{characteristic impedance} \end{cases}$$

Unchanged from v1.01

- As developed by Bogatin…

$$G_L = \omega \tan(\delta) C_L$$

$$Z_0 = \frac{\sqrt{\varepsilon_r}}{c C_L}$$

$$\begin{cases} G_L \text{ equation}^{[1,\text{ EQN 9-19, EQN 9-60}]} \\ Z_0 \text{ equation}^{[1,\text{ EQN 9-67}]} \text{ is used to cancel the capacitance value,} \\ \qquad \text{the } \mathrm{Z}_0 \text{ value for a given frequency is NOT used} \\ c = \text{speed of light m/s} \quad \therefore \text{converstion m} \to \text{in. is needed} \end{cases}$$

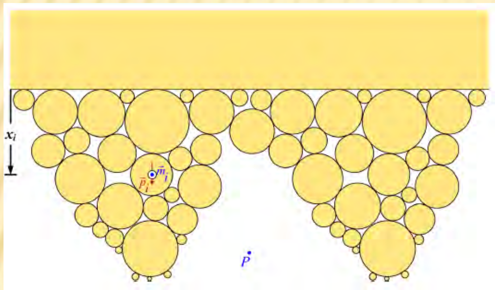
- Dielectric loss per inch as entered in the model:

$$\alpha_{diel} = \frac{1}{2}\times\left(20\log_{10} e\right)\times\left(2\pi f \times D_f\right)\times\frac{\sqrt{\varepsilon_r}}{299792458 * 39.37}$$

v2.04: Correction made to σ (was 5.69e7 instead of 5.96e7) affecting δ which, in turn, impacted $k_{snowball}$, and thus total loss.

SURFACE ROUGHNESS (multiplier)

- Through the snowball method (Huray Model[4, CHAP 6]), surface roughness is approximated as a collection of smaller spheres. *Note image shows non-uniform "snowballs"… model approximates using uniform spheres.



- Applied to trace: $\alpha_{total} = \alpha_{diel} + k_{snowball}\alpha_{cond(smooth)}$
- Surface roughness multiplier as entered in the model:

$$k_{snowball} \approx 1 + \frac{3}{2}\sum_{i=1}^{j}\frac{\left(\dfrac{N_i 4\pi a_i^2}{A_{flat}}\right)}{\left(1 + \dfrac{\delta}{a_i} + \dfrac{\delta^2}{2a_i^2}\right)}$$

$$\begin{cases} a_i = \text{radius of spheres (m)} \\ N_i = \text{number of snowballs of size } a_i \text{ per } A_{flat} \\ A_{flat} = \text{total area containing stacked snowballs} \\ \delta = \text{skin depth (m)} \quad \text{.... recall: } \delta = \dfrac{1}{\sqrt{\pi\mu\sigma f}} \end{cases}$$

CONNECTOR LOSS & CHANNEL LOSS

- Attempting to base on 25G technology connectors
- Used connector models from multiple vendors to draw this max* connector loss… used in model:

$$IL_{conn} = 9*10^{-6}*\sqrt{f} - 1.2*10^{-12}*f + 1.6*10^{-21}f^{2}$$

* Max loss when ignoring majority of ILD. Idea was to create equation that production connectors can beat. Note that this creates additional error in comparing model to measured, however, model should error in pessimistic direction. Connector implementation likely to be changed in future versions.

- Equation gives loss of: 0.6164dB @5G; 1.133dB @12.89G; 1.21dB @14G

- OVERALL CHANNEL LOSS EQUATION: (simple enough, right?)

$$A_{total} = a_{LCA\_total}*L_{LCA} + IL_{conn} + a_{BP\_total}*L_{BP} + IL_{conn} + L_{LCB\_total}*L_{LCB}$$

Unchanged from v2.01

REFERENCES

Unchanged from v2.01

[1] E. Bogatin. *Signal Integrity – Simplified*. Pearson Education, Inc., 2004. ISBN 0-13-066946-6.

[2] S. B. Cohn. "Problems in Strop Transmission Lines." *IRE Trans. Microwave Theory and Techniques*, Vol. MTT-3, March, 1955, pp 119-126.

[3] S. H. Hall, G.W.Hall, J. A. McCall. High-Speed Digital System Design: A Handbook of Interconnect Theory and Design Practices. John Wiley & Sons, Inc., 2000. ISBN-10 0471360902.

[4] P. G. Huray. The Foundations of Signal Integrity. John Wiley & Sons, Inc., 2010. ISBN-978-0-470-34360-9

TRACKING THE CHANGES

Version	Change
1.01	9/26/2011 – Initial release – second order Dk & Df approximation, track user input channel along with Meg-6 & Improved FR-4 for given length/width/thickness, 3 materials compared to KR limit line.
2.01	12/15/2011 – surface resistance updated to include return path resistance and stripline approximation, Huray model for surface roughness added, “worst-case” connector added, partitioning option added (backplane w/2 daughter cards), KR limit comparison made to attenuation max (instead of IL)
2.02 (a)	1/9/2012 – correction of error found in final multiplication/addition (A_{total})
2.03	2/1/2012 – correction of error found in surface roughness multiplier ($K_{snowball}$) for line cards (matched equation given in the explanation slides), GUI clarified for “Backplane w/ 2 connectors, same material”.