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# **Dk & Df ALGEBRAIC MODEL v2.04**

(NOTE: only change from v2.03 is correction of Cu conductivity used in surface roughness [Hurray model] calculation)

Let's take a look...

**WHAT YOU WILL SEE**

# Unchanged from v2.01

## LOOK & FEEL

NOTE: The Change Parameter window is a visual basic macro.  
If you save the file to your computer, be sure to select the Maco-Enabled file type.

Quick preview of select frequencies

Version 2.01

LOSS SNAPSHOT:

Loss at 5GHz: 23.79 dB

Loss at 12.75GHz: 56.54 dB

Loss at 14GHz: 61.66 dB

Backplane/Trace Material

Length (inch)	20	
Trace Width (mil)	8	
Cu Thickness (mil)	0.6	
Diel. Thickness (mil)	16	
Freq	Dk	Df
1.00E+08	3.67	0.0039
1.00E+09	3.65	0.004
2.00E+09	3.59	0.0043
5.00E+09	3.576	0.0049
1.00E+10	3.3494	0.0055
2.00E+10	3	0.0065
Low Roughness	20	6.0E-07

Linecard A Material

Length (inch)	2	
Trace Width (mil)	6	
Cu Thickness (mil)	0.6	
Diel. Thickness (mil)	12.9	
Freq	Dk	Df
1.00E+08	3.6	0.0092
1.00E+09	3.6	0.0092
2.00E+09	3.5	0.0115
5.00E+09	3.5	0.0115
1.00E+10	3.4	0.0125
2.00E+10	3.2	0.014
High Roughness	65	6.0E-07

Linecard B Material

Length (inch)	18	
Trace Width (mil)	5	
Cu Thickness (mil)	0.6	
Diel. Thickness (mil)	11.1	
Freq	Dk	Df
1.00E+08	3.6	0.0092
1.00E+09	3.52	0.0115
2.00E+09	3.49	0.0108
5.00E+09	3.46	0.011
1.00E+10	3.44	0.0112
2.00E+10	3.43	0.0114
High Roughness	65	6.0E-07

CHANGE PARAMETERS

Click to change the input parameters

23.79 dB loss at 5G - 1.123dB loss at 12.75G - 1.21dB loss at 14G

each - 0.616dB loss at 5G - 1.123dB loss at 12.75G - 1.21dB loss at 14G

2 connector

Dielectric Constant

Loss Tangent

Frequency (GHz)

Fitted equation(s):  
 $Dk(f) = c2*f^2 + c1*f + b$

DK fit to Second Order Equation

Df fit to Second Order Equation

Zo

48.06984334

48.07668132

3.65

3.6

3.55

3.5

3.45

3.4

3.35

3.3

3.25

3.2

3.15

0.016

0.014

0.012

0.01

0.008

0.006

0.004

0.002

0

0

5

10

15

20

DK fit to Second Order Equation

Df fit to Second Order Equation

Zo

49.55672188

49.56230717

3.65

3.6

3.55

3.5

3.45

3.4

3.35

3.3

3.25

3.2

3.15

0.014

0.012

0.01

0.008

0.006

0.004

0.002

0

0

5

10

15

20

DK fit to Second Order Equation

Df fit to Second Order Equation

Zo

49.87049227

49.87662785

See the connector & channel loss at specific frequencies

Conn. Loss	A <sub>total</sub> (dB)
0.0283	0.8099
0.0630	0.8454



# Unchanged from v2.03

NOTE: The Change Parameter window is a visual basic macro. If you are having issues opening this window, consider Microsoft button > Excel Options > Trust Center > Trust Center Settings > Macro Settings

Setup | Backplane | Line Card

Single Trace Only

Backplane w/ 2 Connectors, 1 Material

Backplane w/ 2 Connectors, Different Materials

Enter Parameters for the Material

Setup | Backplane | Line Card

All fields must be completed. Backplane/Trace Material:

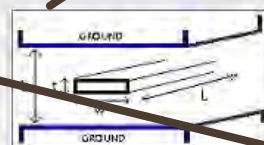
	Dk (real)	Df	Want to auto-fill?
100M	3.67	0.0039	Meg6
1G	3.65	0.004	-135I
2G	3.59	0.0043	Imp FR4
5G	3.576	0.0049	
10G	3.3494	0.0055	Copy Mat. to Linecard
20G	3	0.0065	

Cu Roughness: ☒ Low ☐ Med ☐ High

Change Parameters

Material	
	18
	5
	0.6
	11.1
	Df
	0.0092
	0.0115

Here's what you'll see...



If applicable the same b, L, and t format used for linecard material entries

b, Dielectric Thickness (mils): 16 w, Trace Width (mils):

L, Trace Length (inches): 20 t, Cu Thickness (mils):

☒ Include 2 connectors

Since you selected Trace Only, please select OK to get your loss calculations.

OK Clear

Enter Parameters for the Material

Setup | Backplane | Line Card

All fields must be completed. Line/Daughter Card #1 Material:

	Dk (real)	Df	Want to auto-fill?
100M	3.6	0.0092	Meg6
1G	3.6	0.0092	
2G	3.5	0.0115	-135I
5G	3.5	0.0115	
10G	3.4	0.0125	Imp FR4
20G	3.2	0.014	

Cu Roughness: ☐ Low ☐ Med ☒ High

b, Dielectric Thickness (mils): 12.9 w, Trace Width (mils): 6

L, Trace Length (inches): 2 t, Cu Thickness (mils): 0.6

Line/Daughter Card #2 Material:

	Dk (real)	Df	Use same as LC #1
100M	3.6	0.0092	<input checked="" type="checkbox"/>
1G	3.6	0.0092	
2G	3.5	0.0115	
5G	3.5	0.0115	
10G	3.4	0.0125	
20G	3.2	0.014	

Cu Roughness: ☐ Low ☐ Med ☒ High

b, Dielectric Thickness (mils): 12.9 w, Trace Width (mils): 6

L, Trace Length (inches): 2 t, Cu Thickness (mils): 0.6

OK Clear

You have 3 input options; Feel free to return and select a different option if you change your mind.

10 ncy (GHz)

2	3.64774E-23
Df fit to Second Order Equation	-3.82611E-24
b	3.591963577
Df fit to Second Order Equation	-1.15209E-23
d	4.54439E-13
b	0.009448676

Version 2.01

2	3.35
Df fit to Second Order Equation	0
Df fit to Second Order Equation	5

2 conn

Equation	b	3.672198659
Df fit to Second Order Equation	d	-3.82611E-24
	d	2.075E-13
	b	0.003870352

Zo

48.06984334

48.07668132

Equation	b	3.64774E-23
Df fit to Second Order Equation	d	-2.02392E-11
	b	3.591963577
Df fit to Second Order Equation	d	-1.15209E-23
	d	4.54439E-13
	b	0.009448676

Zo

49.55672188

49.56230717

Equation	b	3.35
Df fit to Second Order Equation	d	0
Df fit to Second Order Equation	d	5

Zo

49.87049227

49.87662785

# Unchanged from v2.03

## LOOK & FEEL

1) Select your configuration

2) Enter material & parameters

3) Must have all entries filled in, then click OK

Enter Parameters for the Material

Single Trace Only

Backplane w/ 2 Connectors, 1 Material

Backplane w/ 2 Connectors, Different Mate

Enter Parameters for the Material

Setup Backplane

All fields must be completed. Material:

	Dk (real)	Df	Meg6
100M	3.6	0.0092	Meg6
1G	3.6	0.0092	-135I
2G	3.5	0.0115	Imp FR4
5G	3.5	0.0115	
10G	3.4	0.0125	
20G	3.2	0.014	

CU Roughness: ☒ Low ☐ Med ☐ High

If applicable, the same b, L, w, and t format is used for linecard material entries.

b, Dielectric Thickness (mils): 16 w, Trace Width (mils): 8

L, Trace Length (inches): 1 t, Cu Thickness (mils): 0.6

Since you selected Trace Only, please select OK to get your loss calculations.

OK Clear

Enter Parameters for the Material

Setup Input

All fields must be completed. Material Definition (w, b, t, & L specific to each card):

	Dk (real)	Df	Meg6
100M	3.6	0.0092	Meg6
1G	3.6	0.0092	-135I
2G	3.5	0.0115	Imp FR4
5G	3.5	0.0115	
10G	3.4	0.0125	
20G	3.2	0.014	

CU Roughness: ☒ Low ☐ Med ☐ High

Backplane Design:

b, Dielectric Thickness (mils): 16 w, Trace Width (mils): 8

L, Trace Length (inches): 1 t, Cu Thickness (mils): 0.6

☒ Include 2 connectors

Line Card A Design:

b, Dielectric Thickness (mils): w, Trace Width (mils):

L, Trace Length (inches): 0 t, Cu Thickness (mils):

Line Card B Design:

b, Dielectric Thickness (mils): w, Trace Width (mils):

L, Trace Length (inches): 0 t, Cu Thickness (mils):

OK Clear

Frequency (GHz)

10	15	20
$\alpha_2$	3.64774E-23	
$\alpha_1$	-2.02392E-11	
b	3.591963577	
$\alpha_2$	-1.15209E-23	
$\alpha_1$	4.54439E-13	
b	0.009448676	

Vers

DK fil

Orde

Df fil

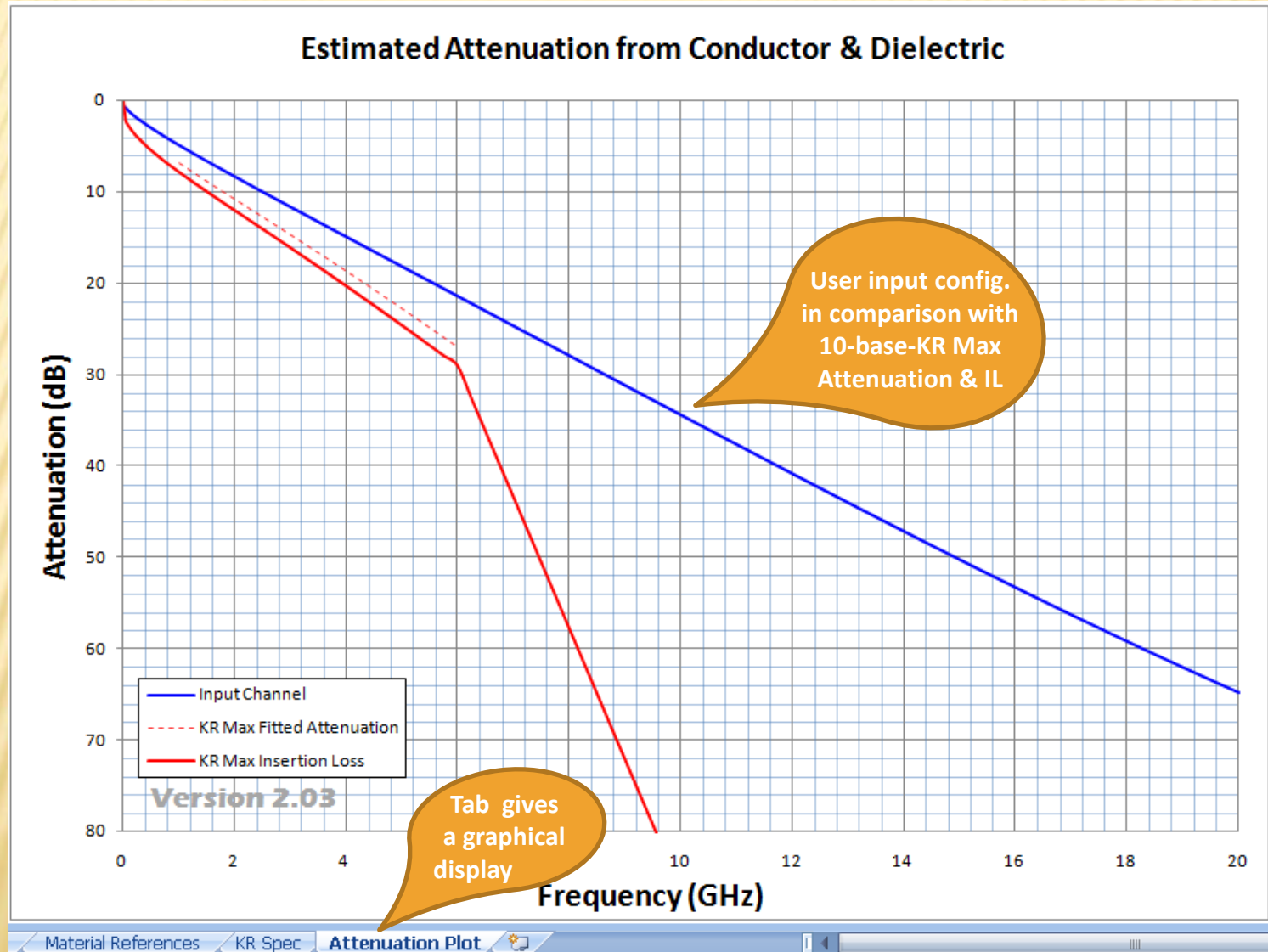
Orde

48.07668132

49.56230717

# Unchanged from v2.03

## LOOK & FEEL





Behind the Scenes...

# **EQUATIONS AND REFERENCES OF MODEL**

# FREQUENCY DEPENDENCE

Unchanged  
from v2.01

- ✖ 6 input frequency points for  $D_k$  and  $D_f$
- ✖ Fit  $D_k$  and  $D_f$  to second order equations
  - + Coefficients shown on sheet
  - + Graphical representation shown on sheet
- ✖ Note that frequency dependence fit is only approximated to 20G, therefore, loss approximations should only be considered to 20G
- ✖  $Z_0$  is calculated with  $D_k$  (or  $\epsilon_r$ ) at a given frequency; a similar technique is used in loss calculations



# DK & DF SECOND ORDER EQUATIONS

- ✖ Second order approximation is created using the LINEST function. This function essentially fits a 2<sup>nd</sup> order polynomial to the 6 frequency points given; resulting in

$$D_k = c_2 * f^2 + c_1 * f + b$$

- ✖ Function as implemented in the spreadsheet:

LINEST(C8:C13,B8:B13^[1,2])

Y Values  
(Dk entered points)

X Values  
(Freq. associated  
w/ entered points)

Exponents of X;  
Creating a second  
order equation.

- ✖ See Excel HELP for more details on LINEST function.  
Methodology verified against “add trend line” within plot.

# CHARACTERISTIC IMPEDANCE<sup>[2, EQU 4-5]</sup>

$$Z_0 = \frac{94.15}{\sqrt{\epsilon_r} \left( \frac{\frac{w}{b}}{1-t} + \frac{c'_f}{0.0885\epsilon_r} \right)} \text{ ohms}$$

$$c'_f = \frac{0.0885\epsilon_r}{\pi} \left\{ \frac{2}{1-\frac{t}{b}} \log_e \left( \frac{1}{1-\frac{t}{b}} + 1 \right) - \left( \frac{1}{1-\frac{t}{b}} - 1 \right) \log_e \left( \frac{1}{\left(1-\frac{t}{b}\right)^2} - 1 \right) \right\}$$

$\epsilon_r$  = relative dielectric constant (at a given frequency)

$b$  = platespacing (mil)

$w$  = trace width (mil)

$t$  = trace thickness (mil)

$c'_f$  = fringing capacitance ( $\mu\text{f}/\text{cm}$ ) \* assuming semi - infinite plate between two infinite ground planes, but good approximation for  $w/(b - t) \geq 0.35$

Unchanged from v1.01



# ATTENUATION IN LOSSY LINES

- ✗ Attenuation per length<sup>[1, EQN 9-54]</sup>:

$$\alpha_n = \sqrt{\frac{1}{2} \left[ \sqrt{(R_L^2 + \omega^2 L_L^2)(G_L^2 + \omega^2 C_L^2)} - \omega^2 L_L C_L + R_L G_L \right]} \text{ nepers/length}$$

- ✗ Using a low-loss approximation<sup>[1, EQN 9-55]</sup>: (surface roughness ignored)

$$\alpha_n = \frac{1}{2} \left( \frac{R_L}{Z_0} + G_L Z_0 \right) \text{ nepers/length}$$

- ✗ But we don't typically discuss in nepers...<sup>[1, EQN 9-57]</sup>

$$10^{\frac{\alpha_{dB}}{20}} = e^{\alpha_n} \quad \therefore \alpha_{dB} = 20 \log_{10} e \times \alpha_n$$

for ease of notation:  $Y_{n \rightarrow dB} = 20 \log_{10} e$

Unchanged  
from v1.01



## CONDUCTOR LOSS (per inch)

from v2.01

$$\alpha_{cond} = (Y_{n \rightarrow dB}) \times \frac{R_L}{Z_0} \quad \left\{ \begin{array}{l} \alpha_{cond} = \text{attenuation of amplitude due to conductor loss, in dB/length}^{[1, \text{EQN 9-59}] \\ Y_{n \rightarrow dB} = \text{conversion from nepers to dB} \\ R_L = \text{resistance per length of conductor} \\ Z_0 = \text{characteristic impedance} \end{array} \right.$$

- ✖ Skin effect, ground resistance, and stripline effect are accounted for in resistance<sup>[3, EQNs 4.3a-4.10]</sup>:

- R of signal trace & return path (w/skin effect)

$$R_{signalCu \text{ skin effect}} = \frac{\sqrt{\pi \mu \rho f}}{w} \quad R_{groundCu \text{ skin effect}} = \frac{\sqrt{\pi \mu \rho f}}{6H}$$

- AC surface resistance for microstrip (or 1 side of a stripline trace)

$$R_{ac \text{ microstrip}} = R_{signal} + R_{ground}$$

## CONDUCTOR LOSS (per inch) from v2.02

- Stripline approximation assumes parallel resistance of top and bottom microstrip approximations

$$R_L = \frac{\left[ \sqrt{\pi \mu \rho f} * \left( \frac{1}{w} + \frac{1}{6H} \right) \right]^2}{2 * \left[ \sqrt{\pi \mu \rho f} * \left( \frac{1}{w} + \frac{1}{6H} \right) \right]}$$

$$\left\{ \begin{array}{l} R_L = \text{stripline surface resistance } (\Omega/\text{inch}) \\ w = \text{width of trace (inch)} \\ H = \text{height dielectric from ground to signal (inch)} \\ \mu = \text{permeability of Cu} \approx 4\pi \times 10^{-7} \times 0.999994 \frac{\text{H}}{\text{m}} \\ \rho = \text{resistivity of Cu} = \frac{1}{\sigma} = \frac{1}{5.96 \times 10^7} \Omega \text{ m} \\ f = \text{frequency (hertz)} \end{array} \right.$$

- ✖ Conductor loss per inch as entered in the model:

$$\alpha_{cond} = \frac{1}{2} \times (20 \log_{10} e) \times \left( \frac{\left[ \sqrt{\pi \mu \rho f} * \left( \frac{1}{w} + \frac{1}{6H} \right) \right]}{2} \right) \times \frac{1}{Z_0}$$



# DIELECTRIC LOSS (per inch)

$$\alpha_{diel} = (Y_{n \rightarrow dB}) \times G_L Z_0 \quad \left\{ \begin{array}{l} \alpha_{cond} = \text{attenuation of amplitude due to dielectric loss, in dB/length}^{[1, EQN 9-60]} \\ Y_{n \rightarrow dB} = \text{conversion from nepers to dB} \\ G_L = \text{shunt conductance per length from dielectric} \\ Z_0 = \text{characteristic impedance} \end{array} \right.$$

Unchanged  
from v1.01

✖ As developed by Bogatin...

$$\begin{array}{l} G_L = \omega \tan(\delta) C_L \\ Z_0 = \frac{\sqrt{\epsilon_r}}{c C_L} \end{array} \quad \left\{ \begin{array}{l} G_L \text{ equation}^{[1, EQN 9-19, EQN 9-60]} \\ Z_0 \text{ equation}^{[1, EQN 9-67]} \text{ is used to cancel the capacitance value,} \\ \text{the } Z_0 \text{ value for a given frequency is NOT used} \\ c = \text{speed of light m/s} \quad \therefore \text{conversion m} \rightarrow \text{in. is needed} \end{array} \right.$$

✖ Dielectric loss per inch as entered in the model:

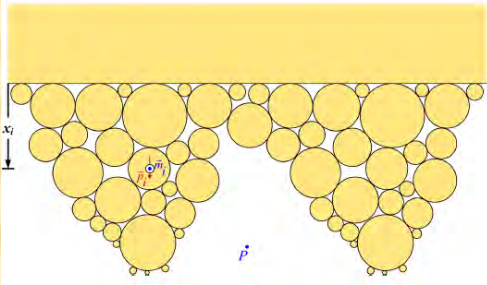
$$\alpha_{diel} = \frac{1}{2} \times (20 \log_{10} e) \times (2\pi f \times D_f) \times \frac{\sqrt{\epsilon_r}}{299792458 * 39.37}$$



**v2.04: Correction made to  $\sigma$  (was 5.69e7 instead of 5.96e7) affecting  $\delta$  which, in turn, impacted  $k_{\text{snowball}}$ , and thus total loss.**

## SURFACE ROUGHNESS (multiplier)

- ✗ Through the snowball method (Huray Model<sup>[4, CHAP 6]</sup>), surface roughness is approximated as a collection of smaller spheres. \*Note image shows non-uniform “snowballs”... model approximates using uniform spheres.



- ✗ Applied to trace:  $\alpha_{\text{total}} = \alpha_{\text{diel}} + k_{\text{snowball}} \alpha_{\text{cond(smooth)}}$
- ✗ Surface roughness multiplier as entered in the model:

$$k_{\text{snowball}} \approx 1 + \frac{3}{2} \sum_{i=1}^j \frac{\left( \frac{N_i 4\pi a_i^2}{A_{\text{flat}}} \right)}{\left( 1 + \frac{\delta}{a_i} + \frac{\delta^2}{2a_i^2} \right)}$$

$$\left\{ \begin{array}{l} a_i = \text{radius of spheres (m)} \\ N_i = \text{number of snowballs of size } a_i \text{ per } A_{\text{flat}} \\ A_{\text{flat}} = \text{total area containing stacked snowballs} \\ \delta = \text{skin depth (m)} \quad \dots \text{recall: } \delta = \frac{1}{\sqrt{\pi \mu \sigma f}} \end{array} \right.$$

# CONNECTOR LOSS & CHANNEL LOSS

- ✗ Attempting to base on 25G technology connectors
- ✗ Used connector models from multiple vendors to draw this max\* connector loss... used in model:

$$IL_{conn} = 9 * 10^{-6} * \sqrt{f} - 1.2 * 10^{-12} * f + 1.6 * 10^{-21} f^2$$

- \* Max loss when ignoring majority of ILD. Idea was to create equation that production connectors can beat. Note that this creates additional error in comparing model to measured, however, model should error in pessimistic direction. Connector implementation likely to be changed in future versions.
- ✗ Equation gives loss of: 0.6164dB @5G; 1.133dB @12.89G; 1.21dB @14G
- ✗ OVERALL CHANNEL LOSS EQUATION: (simple enough, right?)

$$A_{total} = a_{LCA\_total} * L_{LCA} + IL_{conn} + a_{BP\_total} * L_{BP} + IL_{conn} + L_{LCB\_total} * L_{LCB}$$



# REFERENCES

Unchanged  
from v2.01

- [1] E. Bogatin. *Signal Integrity – Simplified*. Pearson Education, Inc., 2004. ISBN 0-13-066946-6.
- [2] S. B. Cohn. “Problems in Strop Transmission Lines.” *IRE Trans. Microwave Theory and Techniques*, Vol. MTT-3, March, 1955, pp 119-126.
- [3] S. H. Hall, G.W.Hall, J. A. McCall. *High-Speed Digital System Design: A Handbook of Interconnect Theory and Design Practices*. John Wiley & Sons, Inc., 2000. ISBN-10 0471360902.
- [4] P. G. Huray. *The Foundations of Signal Integrity*. John Wiley & Sons, Inc., 2010. ISBN-978-0-470-34360-9



# TRACKING THE CHANGES

Version	Change
1.01	9/26/2011 – Initial release – second order Dk & Df approximation, track user input channel along with Meg-6 & Improved FR-4 for given length/width/thickness, 3 materials compared to KR limit line.
2.01	12/15/2011 – surface resistance updated to include return path resistance and stripline approximation, Huray model for surface roughness added, “worst-case” connector added, partitioning option added (backplane w/2 daughter cards), KR limit comparison made to attenuation max (instead of IL)
2.02 (a)	1/9/2012 – correction of error found in final multiplication/addition ( $A_{total}$ )
2.03	2/1/2012 – correction of error found in surface roughness multiplier ( $K_{snowball}$ ) for line cards (matched equation given in the explanation slides), GUI clarified for “Backplane w/ 2 connectors, same material”.