Analytical Model for 100 Gb/s Discrete Multi-Tone Modulation

40Gb/s and 100Gb/s Fiber Optic Task Force
IEEE 802.3bm Interim Session
Phoenix, Arizona
23-24 January 2013

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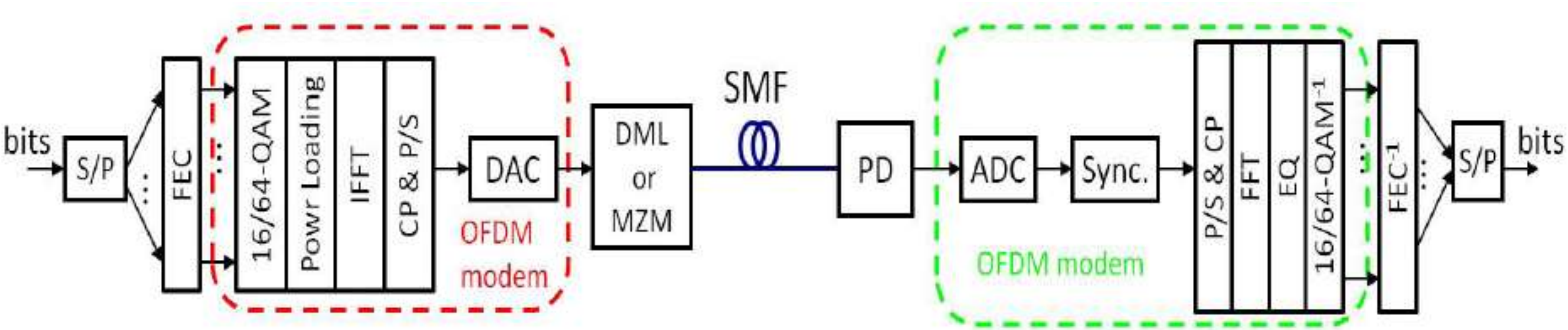
Outline

- Objectives
- DMT Analytical Model
- Impact of Thermal and Shot Noise
- Impact of Clipping Nonlinearity
- Impact of RIN
- Comparison of 100Gb/s DMT with PAM-M
- Conclusion

Objectives

- Continue higher order modulation analysis development in nicholl_01b_0312, ghiasi_01a_0912, and lyubomirsky_01a_1112_optx
- Gain physical insights from analytical models
- Compare performance of 802.3bm SMF PMD alternatives

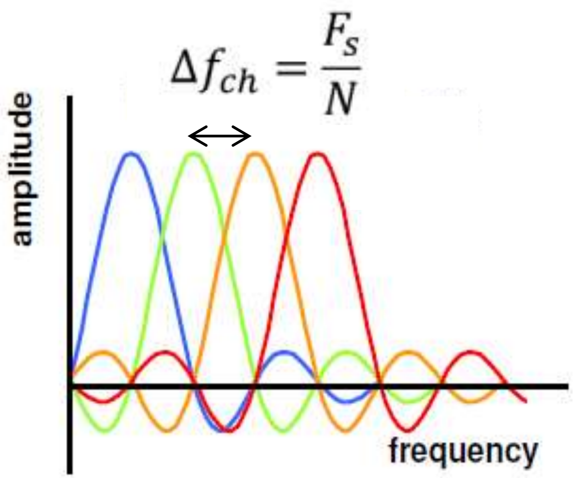
100 Gb/s DMT System Architecture



FEC: forward error correction **P/S:** parallel to serial conversion **CP:** cyclic prefix **Sync.:** Synchronization **PD:** photo-detector
(I)FFT: (inverse) fast Fourier transform **DAC/ADC:** digital to analog/analog to digital conversion

Source: J. Wei, et. al., "Performance Studies of 100 Gigabit Ethernet Enabled by Advanced Modulation Formats," IEEE 802.3, Next Gen. 40Gb/s and 100Gb/s Opt. Ethernet Study Group, ingham_01_0512_optx, May, 2012

DMT Effective SNR Model



$$SNR_{eff} = \frac{P_{ch}}{N_0 \Delta f_{ch}} = \frac{N_{sc} P_{ch}}{N_0 N_{sc} \Delta f_{ch}} = \frac{P_{sig}}{N_0 \Delta f}$$

$P_{sig} = N_{sc} P_{ch}$ = total DMT signal power at Rx

N_0 = one-sided noise power spectral density

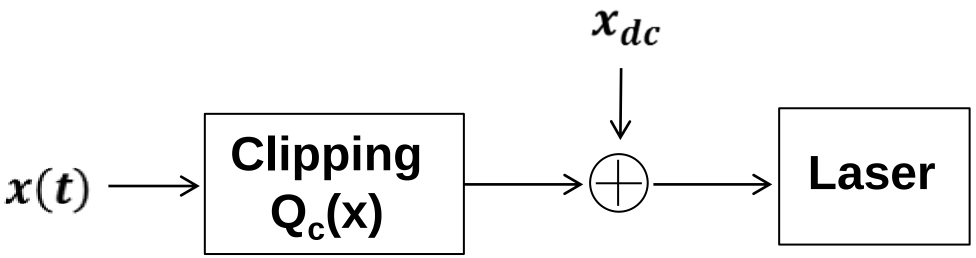
$\Delta f = \frac{N_{sc}}{N} F_s$ = effective noise bandwidth

N = FFT size

N_{sc} = number nonzero DMT subcarriers $< N/2$

F_s = DAC sampling rate

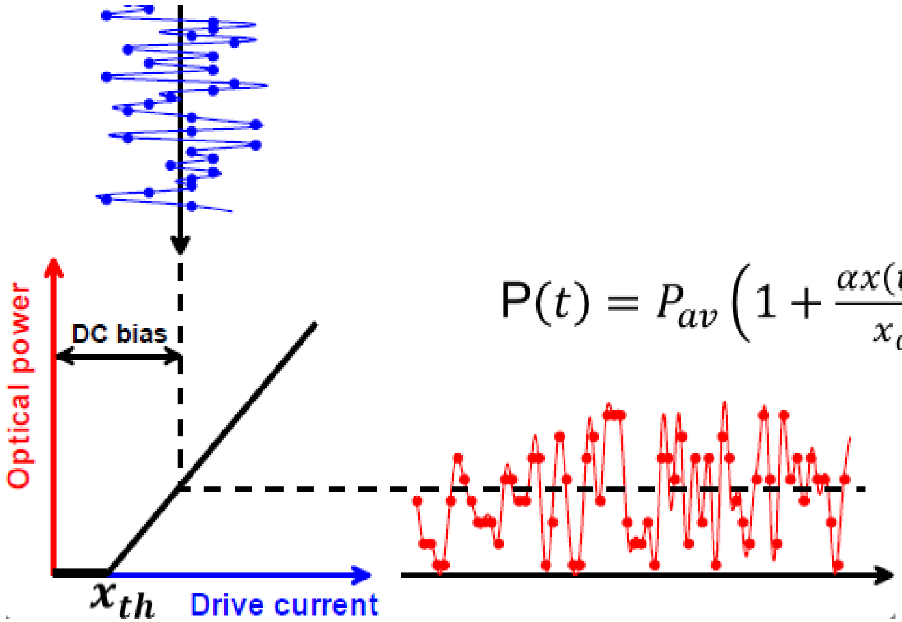
DMT Effective SNR Model (continued)



$$Q_c(x) = \begin{cases} x, & \text{if } |x| < x_0 \\ x_0 \cdot \text{sign}(x), & \text{if } |x| \geq x_0 \end{cases}$$

$$\text{Clipping Ratio} = R_{cl} = \frac{{x_0}^2}{\langle x^2 \rangle}$$

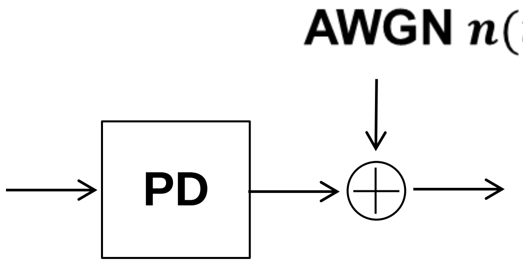
$$x_{drive}(t) = \alpha x(t) + n_{cl}(t) + x_{dc}$$



$$\text{P}(t) = P_{av}\left(1 + \frac{\alpha x(t) + n_{cl}(t)}{x_{dc} - x_{th}}\right)$$

DMT Effective SNR Model (continued)

$$\mathbf{P}(\boldsymbol{t}) = \boldsymbol{P}_{\boldsymbol{av}}\left(\mathbf{1} + \frac{\boldsymbol{\alpha x}(\boldsymbol{t}) + \boldsymbol{n}_{\boldsymbol{cl}}(\boldsymbol{t})}{\boldsymbol{x}_{\boldsymbol{dc}} - \boldsymbol{x}_{\boldsymbol{th}}}\right)$$



$$\boldsymbol{\rho P}_{\boldsymbol{av}}\left(\mathbf{1} + \frac{\boldsymbol{\alpha x}(\boldsymbol{t}) + \boldsymbol{n}_{\boldsymbol{cl}}(\boldsymbol{t})}{\boldsymbol{x}_{\boldsymbol{dc}} - \boldsymbol{x}_{\boldsymbol{th}}}\right) + \mathbf{n}(\mathbf{t})$$

$$P_{sig} = \left(\frac{\rho P_{av}}{x_{dc} - x_{th}}\right)^2 \alpha^2 \langle x^2 \rangle = (\rho P_{av})^2 \left(\frac{x_0}{x_{dc} - x_{th}}\right)^2 \frac{\alpha^2}{R_{cl}}$$

$$SNR_{eff} = \frac{P_{sig}}{P_{th} + P_{sh} + P_{rin} + P_{clip}}$$

$$P_{th} = {S_{th}}^2 \Delta f = Thermal\ Noise$$

$$P_{sh} = 2q\rho P_{av} \Delta f = Shot\ Noise$$

$$P_{rin} = 10^{RIN/10} (\rho P_{av})^2 \Delta f = RIN\ Noise$$

$$P_{clip} = Clipping\ Noise\ (see\ next\ slide)$$

DMT Effective SNR Model (continued)

$$\boldsymbol{x(t)} \longrightarrow \boxed{\textbf{Clipping } Q_c(x)} \longrightarrow \mathbf{y}(\boldsymbol{t}) = \boldsymbol{\alpha x(t)} + \boldsymbol{n_{cl}(t)}$$

$$Q_c(x) = \begin{cases} x, & \text{if } |x| < x_0 \\ x_0 \cdot \text{sign}(x), & \text{if } |x| \geq x_0 \end{cases}$$

$$R_{cl} = \frac{{x_0}^2}{\langle x^2 \rangle}$$

$$\langle x^2 \rangle = {\sigma_x}^2$$

Bussgang Theorem: Clipping noise $n_{cl}(t)$ uncorrelated with $x(t)$

$$\alpha = \frac{< x \cdot y >}{< x^2 >} = \frac{1}{\sqrt{2\pi}{\sigma_x}^3} \int_{-\infty}^{+\infty} x \cdot Q(x) \cdot \exp\left\{-\frac{x^2}{2{\sigma_x}^2}\right\} dx = 1 - \text{erfc}\left(\sqrt{R_{cl}/2}\right)$$

$$< y^2 > = \frac{1}{\sqrt{2\pi}\sigma_x} \int_{-\infty}^{+\infty} Q^2(x) \cdot \exp\left\{-\frac{x^2}{2{\sigma_x}^2}\right\} dx$$

$$< y^2 > = {\sigma_x}^2 \left(1 - \text{erfc}\left(\sqrt{R_{cl}/2}\right) + R_{cl}\,\text{erfc}\left(\sqrt{R_{cl}/2}\right) - \sqrt{2R_{cl}/\pi}\,\exp\{-R_{cl}/2\}\right)$$

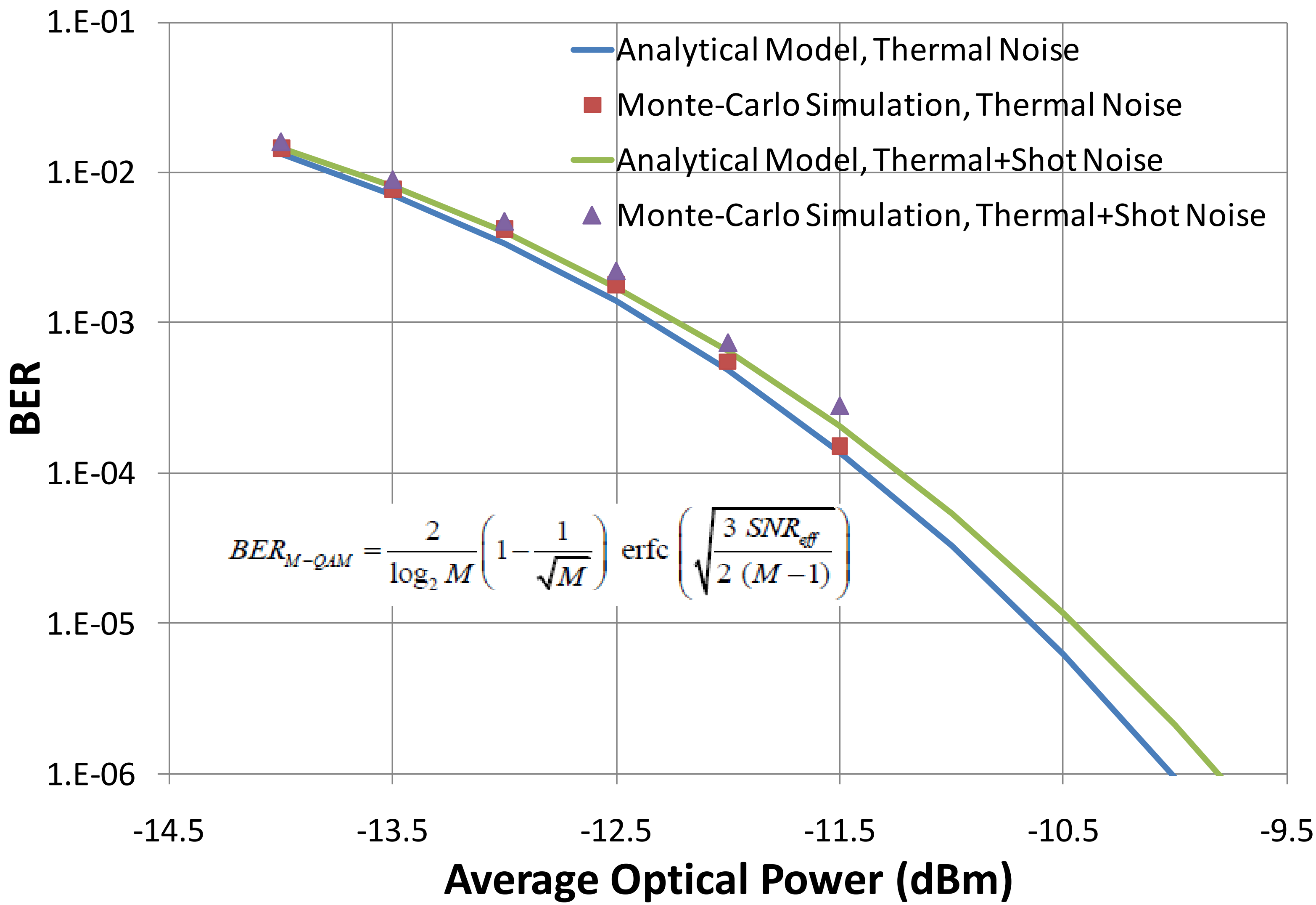
$$P_{clip} = \left(\frac{\rho P_{av}}{x_{dc} - x_{th}}\right)^2 \left(\langle y^2 \rangle - \alpha^2 {\sigma_x}^2\right)$$

Closed form formulas derived in E. Vanin, “Performance evaluation of intensity modulated optical OFDM system with digital baseband distortion,” Opt. Exp., Vol. 19, No. 5, pp. 4280-4293, 2011

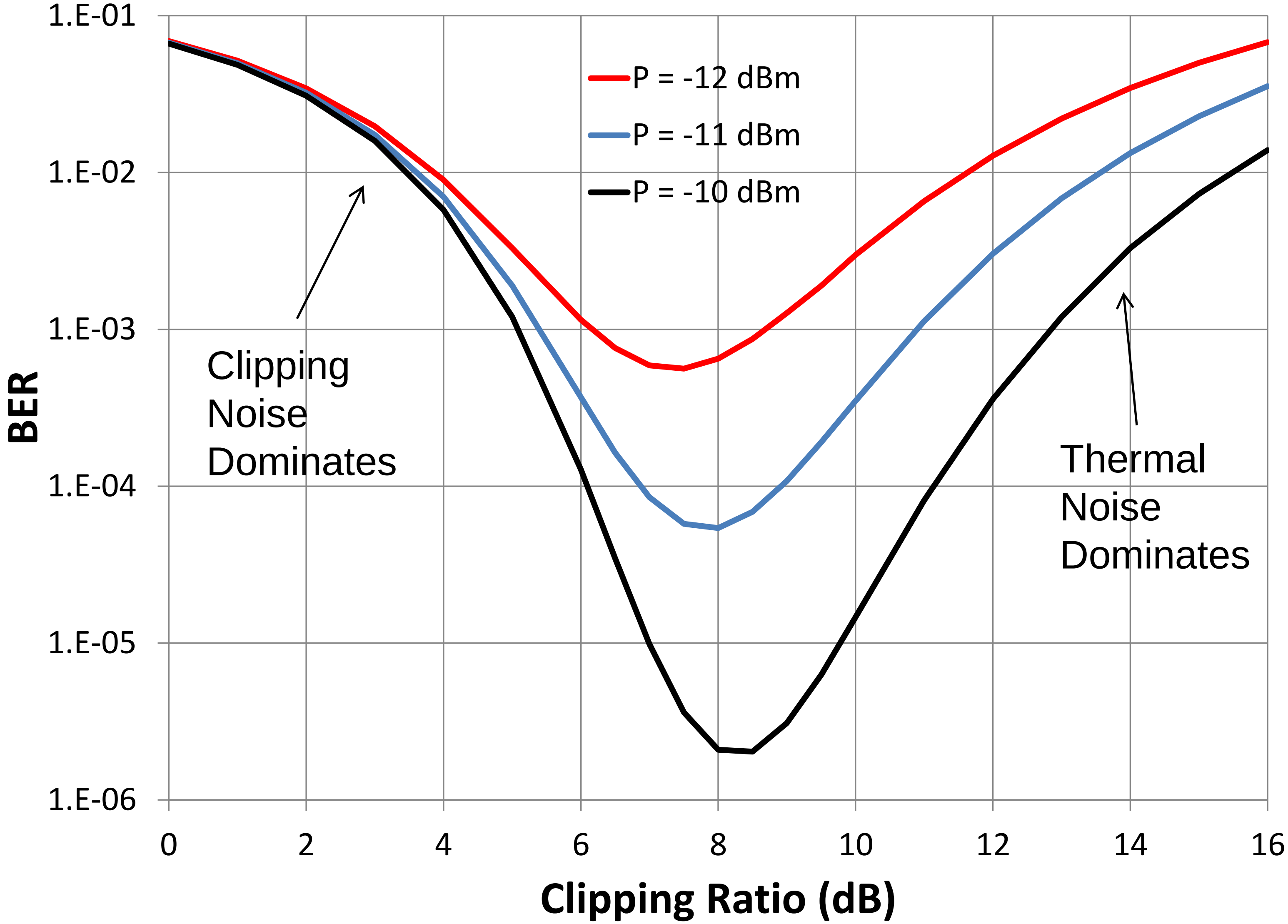
Monte-Carlo Simulation Parameters

Parameter	Value
Sampling rate, F_s	60 Gs/s
FFT size, N	128
Number of nonzero subcarriers, N_{sc}	55
High freq. subcarriers padded to zero	8
DC subcarriers padded to zero	1
Cyclic prefix, CP	4
Clipping ratio, R_{cl}	8 dB
QAM modulation order, M	16
Noise bandwidth, Δf	25.8 GHz
Thermal noise density, S_{th}	16 pA/sqrt(Hz)
Photodiode responsivity, ρ	0.8 A/W

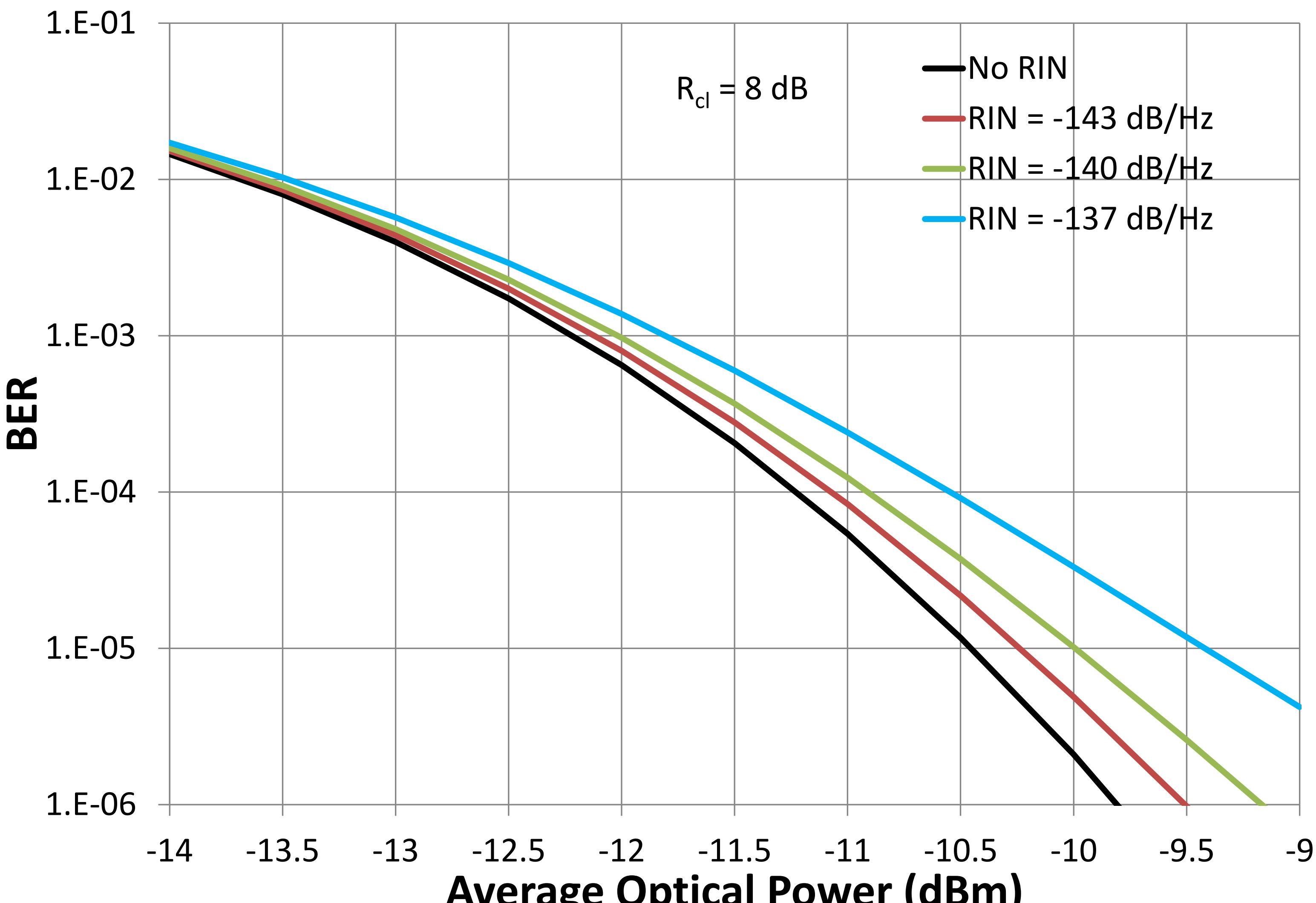
100 Gb/s DMT Analytical Model Results



Optimization of Clipping Ratio



Impact of RIN



Analytical Model for PAM-M

For optimum receiver thresholds, the symbol error probability is determined by an average over all the PAM eye Q-factors

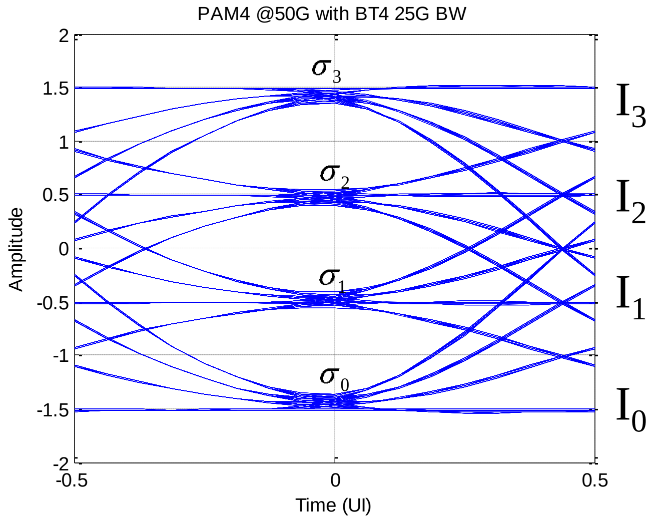
$$\mathrm{Q(k)} = \frac{I_{k+1} - I_k}{\sigma_{k+1} + \sigma_k}, \ \mathrm{k} = 0,\ldots,\mathrm{M-2}$$

$$I_{k+1} - I_k = \frac{2RP_{av}}{M-1}\frac{ER-1}{ER+1}$$

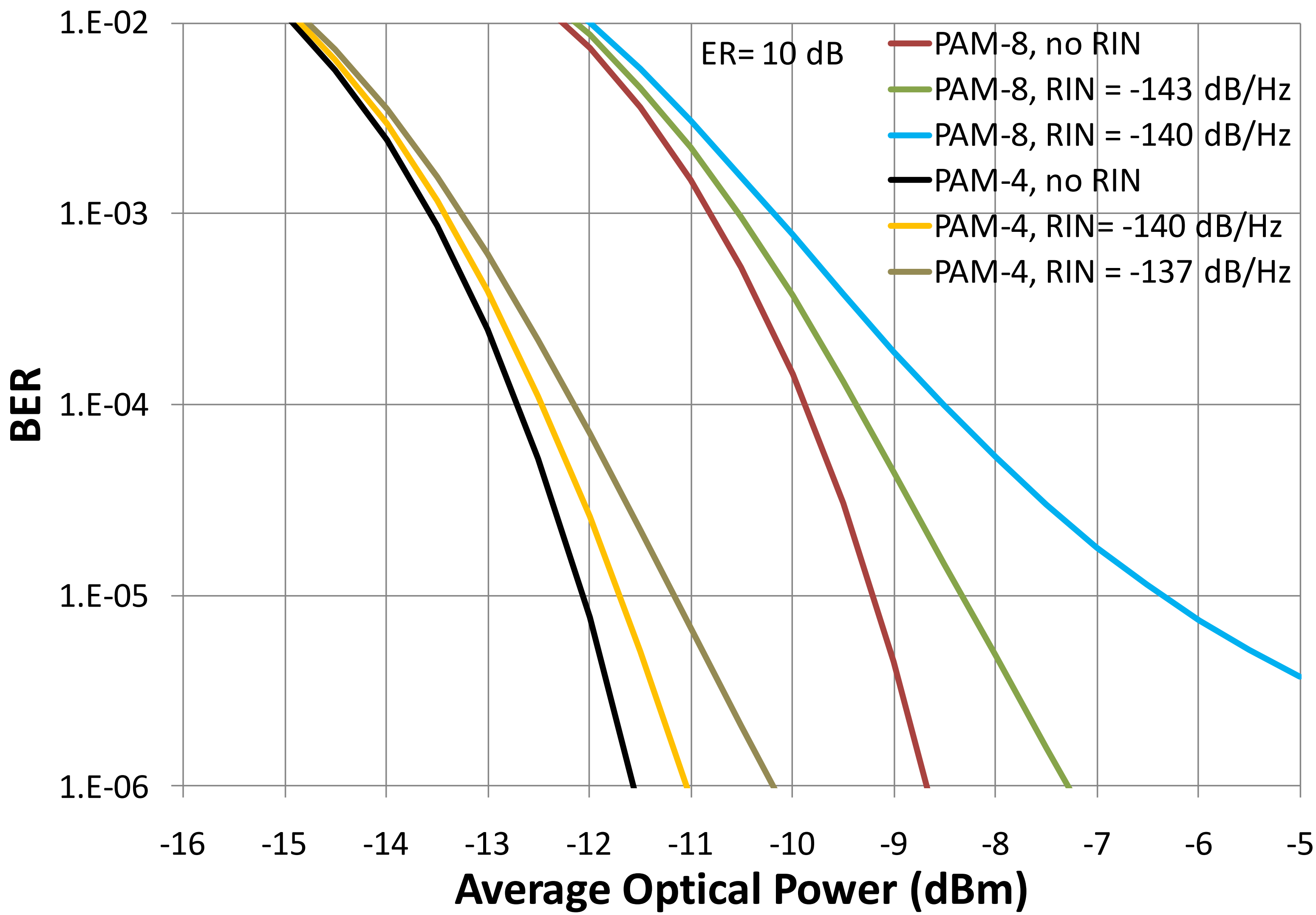
$$\sigma_k^2 = (S_{th}^2 + 2qI_k + 10^{\frac{RIN}{10}} I_k^2)\Delta \mathrm{f}$$

$$P(k) = erfc\left(\frac{Q(k)}{\sqrt{2}}\right)$$

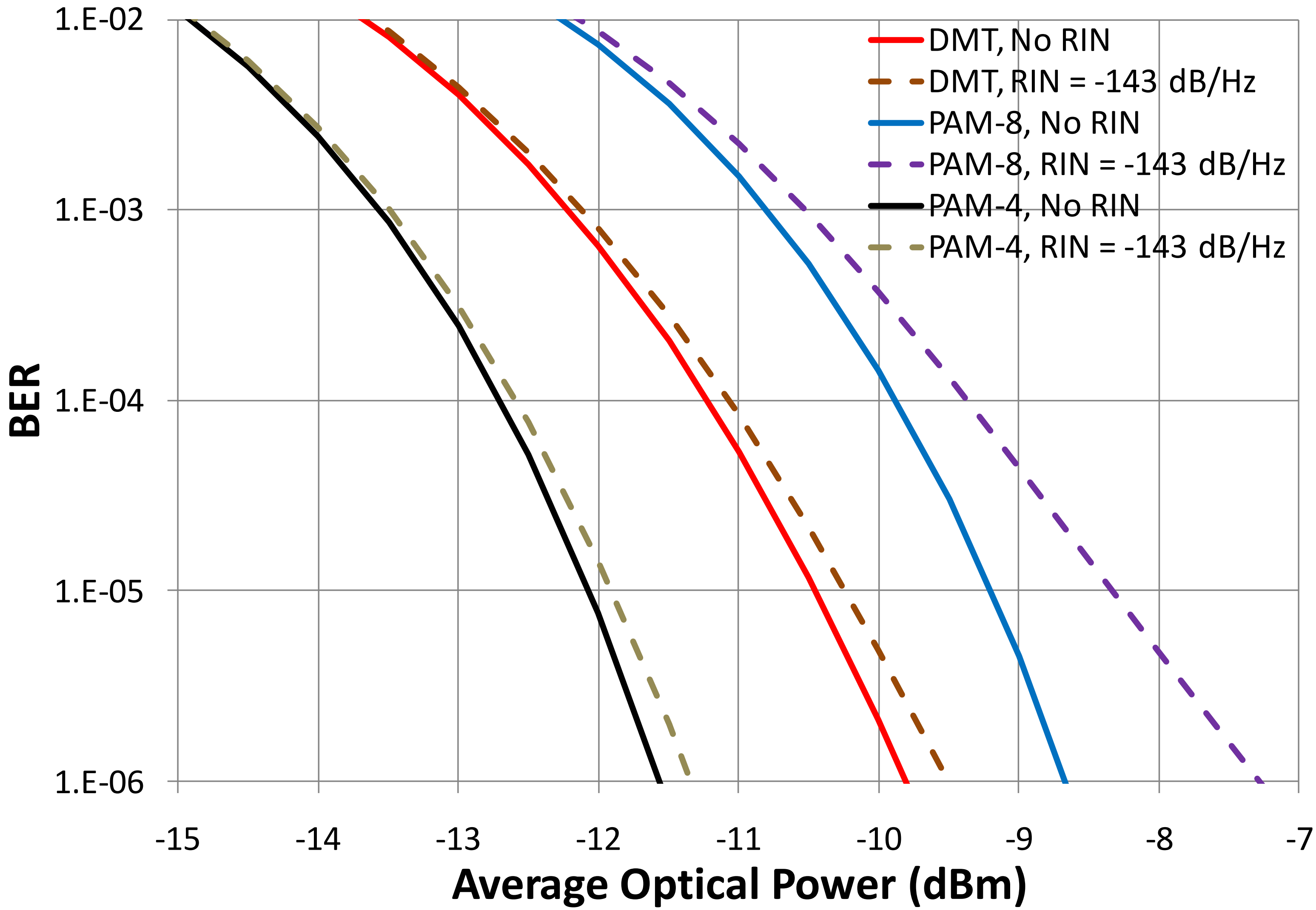
$$P_s = \frac{1}{M}\sum_{k=0}^{M-2} P(k)$$



100 Gb/s PAM-M Analytical Model Results



100 Gb/s DMT versus PAM-4 and PAM-8



Conclusions

- We presented an analytical model for DMT modulation based on an effective SNR approach

- The model provides a useful baseline for performance, as well as physical insight on the interplay of DMT system parameters