Improved MPI "Upper Bound" Analysis

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Outline

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- Setup
- Defining MPI as optical vertical eye closure penalty
- Improved Upper Bound Method
- Statistical upper bound method
- Summary

Objective

- In this presentation we derive statistical upper bound to measure the MPI impact on various PAM modulation schemes

- We preserve the conservative nature of penalty analysis.

- We show that the return loss spec of -35dB is a good choice to keep MPI impact to small values

Definitions

Signal representation in the Optical Field:

We use the same nomenclature as bhatt_01_0512 starting from page 18 for “Upper-bound based analysis”. Since this presentation is trying to deal with “Upper-Bound” methods, assuming the Laser source is perfectly “coherent” is the right pessimistic assumption. The coherent signal in the optical field is represented as complex function

$$e(t)=A_{i(t)}e^{jwt}$$

A_i is the optical amplitude of the signal with $i(t) \in [1,m]$.
m is the modulation order. (ie 16 for PAM-16)
A_1 is the lowest transmit level and A_m is the highest transmit level.

Photo-Detector /TIA function maps the “optical power” to “electrical current”

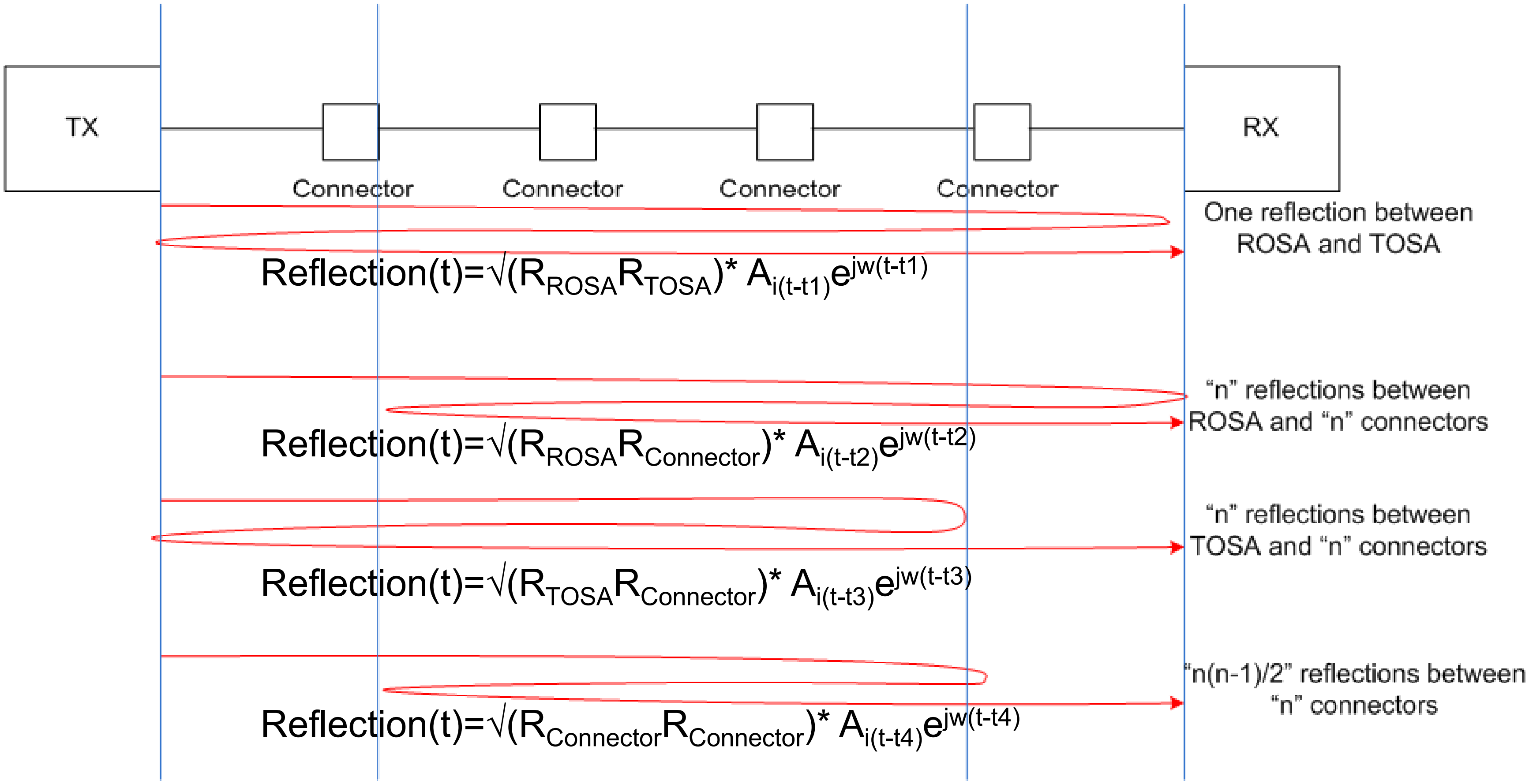
$$I(t) \propto |e(t)|^2 = A^2_{i(t)}$$

The return-loss factor RL of a connection, specifies the amount of optical power that is reflected by the connection

$$Reflection(t)=\sqrt{R}^* A_{i(t)}e^{jwt}$$

Setup

How many reflections are there for "n" connector link?



Modeling MPI as Vertical Eye reduction penalty

MPI Penalty

- In the following analysis we make some pessimistic assumptions and then calculate the MPI as an amplitude dependent closure of the signal eye for PAM modulati**on.**

- Define Optical MPI penalty in dB as

$$10\mathrm{Log}10\ \frac{\mathrm{Signal\_Eye\_Height\_Without\_MPI}}{\mathrm{Reduced\_Signal\_Eye\_Height\_due\_to\_MPI}}$$

Improved Upper bound method

Similar to bhatt_01_0512 starting from page 18 for “Upper-bound based analysis”

Page 18 Pessimistic assumptions:

- **Pessimistic Assumption 1**: Fiber insertion loss is zero
- **Pessimistic Assumption 2**: Connector insertion loss is zero
- **Pessimistic Assumption 3**: All interfering optical signals are perfectly aligned in polarization

Additional Pessimistic assumptions

Pessimistic Assumption 4: Laser spectral width is zero (perfect coherent laser)
Pessimistic Assumption 5: All interfering optical signals are perfectively constructively added

$$e(t)=A_{i(t)}e^{jwt}+e^{j(wt+\theta)}\sum A_{k(t)}\sqrt{}(R_{k1}R_{k2})$$

A_i is the amplitude of the received signal in optical field. $i(t) \in [1,m]$ and m is modulation order
$A_{k(t)}$ are the interfering signals. The relative phase of all interfering signals to received signals is θ.
R_1 and R_2 are the two reflection points return-loss

Improved Upper bound method

Pessimistic Assumption 6: Assume all interfering signals are at their maximum

$$e(t)=A_{i(t)}e^{jwt}+e^{j(wt+\theta)}\sum A_m\sqrt{(R_{k1}R_{k2})}$$

Which is

$$e(t)=A_{i(t)}e^{jwt}+e^{j(wt+\theta)}\,A_m\sum\sqrt{(R_{k1}R_{k2})}$$

Replace the sum $\sum\sqrt{(R_{k1}R_{k2})}$ with the variable S calculated as

$$\mathbf{S=n(n-1)R_{Conn}/2+n\sqrt{(R_{Conn}R_{TOSA})}+n\sqrt{(R_{Conn}R_{ROSA})}+\sqrt{(R_{TOSA}R_{ROSA})}}$$

$$e(t)=A_{i(t)}e^{jwt}+e^{j(wt+\theta)}\,A_m\,S$$

The receive current is proportional to

$$I(t)\propto|e(t)|^2=A_{i(t)}^2+2A_{i(t)}A_mS\cos(\theta)+S^2A_m^2$$

$$I(t)\propto I_{i(t)}+2S\cos(\theta)\sqrt{(I_{i(t)}I_m)}+S^2I_m$$

And finally the MPI Eye Closure is

$$\mathbf{MPI\_Eye\_Closure=2S\cos(\theta)\sqrt{(I_{i(t)}I_m)}+S^2I_m}$$

Improved Upper bound method

•The worst case MPI happens for all levels from 2 to m-1 when Cos θ is +1 or -1. For level 1 the worst case is when Cos θ is +1 and for level m the worst case is when Cos θ is -1.

•Since the MPI is amplitude dependent, the penalty for level 1 is pretty small.

Pessimistic assumption 7: Cos θ=-1 and the term $S^2 I_m$ is so small that it can be ignored in practical cases.

$$S=n(n-1)R_{Conn}/2+n\sqrt{(R_{Conn}R_{TOSA})}+n\sqrt{(R_{Conn}R_{ROSA})}+\sqrt{(R_{TOSA}R_{ROSA})}$$

$$MPI\_Eye\_Closure=-2S\sqrt{(I_{i(t)}I_m)}$$

Improved Upper bound method

•Bhatt_01_0512 further assumed the received signal Ii is at maximum I_m and calculated the MPI penalty as

$$S=n(n-1)R_{Conn}/2+n\sqrt{(R_{Conn}R_{TOSA})}+n\sqrt{(R_{Conn}R_{ROSA})}+\sqrt{(R_{TOSA}R_{ROSA})}$$

$$MPI\_Eye\_Closure=-2S\sqrt{(I_m I_m)}$$

•For example setting $R_{TOSA}=R_{ROSA}=R_{Conn}=R$ and ER=6dB, the following MPI table is produced

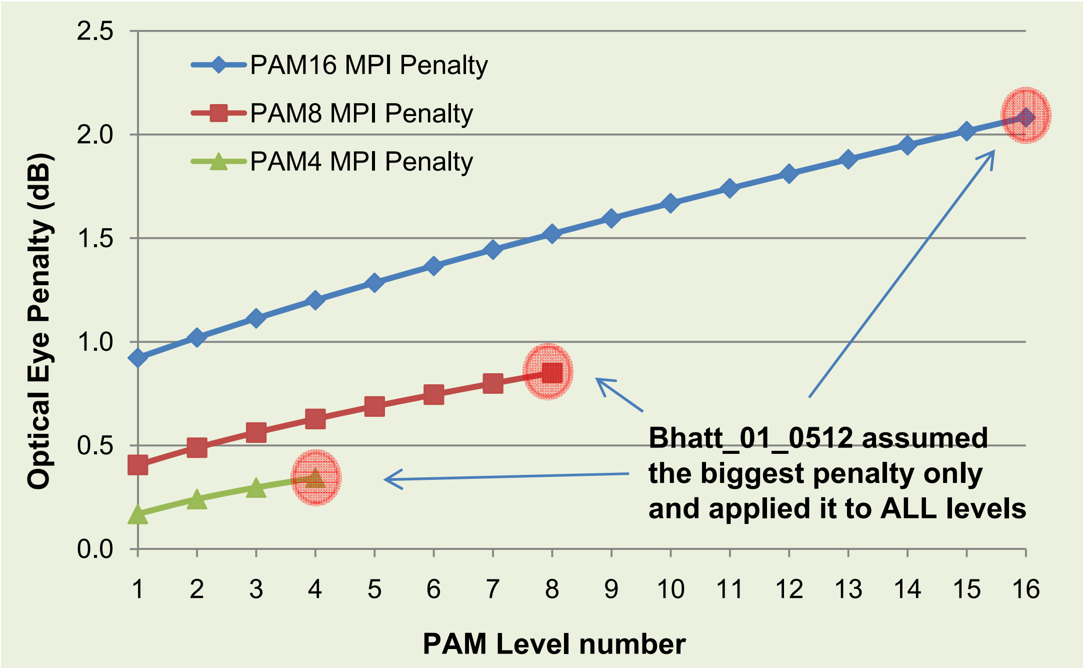
MPI Penalty upper bound in dB, for the worst case combination of all reflecting terms at the highest PAM amplitude, zero error rate, Extinction Ratio E = 4 (6 dB), dual-trunk cabling, 4 connectors (six reflectance points, N = 15), m = number of PAM amplitude levels. Results do not include the effect of backscatter.

R (dB)	PAM-2	PAM-4	PAM-8	PAM-16
-26	0.97	4.01	--	--
-30	0.36	1.19	3.57	--
-35	0.11	0.34	0.85	2.07
-40	0.03	0.11	0.25	0.56

Assuming $I_{i(t)}=I_m$ and then applying the flat penalty to all levels is too pessimistic. Lets model the amplitude dependent MPI.

Improved Upper bound method

Using $R_{ROSA}=R_{TOSA}=R_{Conn}$=-35dB ,4 connectors and 6dB ER, below graph shows level-dependent MPI penalty



R (dB)	PAM-2	PAM-4	PAM-8	PAM-16
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Statistical Upper Bound method

Lets make the “Upper Bound” Analysis more statistical by taking into account the fact that interfering signals have random optical amplitude rather than always staying at maximum. Let’s call this “Statistical Upper Bound method”

Statistical Upper Bound method

Refine the following pessimistic assumption
Pessimistic Assumption 6: Assume all interfering signals are at their maximum

We are going to assume the sum of all interfering sources are going to look more like "Normal" distribution $\chi(\mu_m,\sigma_m^2)$

•This assumption is a good estimate if there are more connectors (ie 4). Keep in mind that the more connectors, the more the MPI impact so 4 connector is a reasonable assumption.

• It is a good estimate if the Connector RL, TOSA RL and ROSA RL are close to each other.

We call this type of analysis "Statistical Upper Bound" not to be confused with Minneapolis time domain simulations.

The following table shows PAM16,8 and 4 mean and variance with ER=6dB assuming a peak vale of 1.

Modulation	Mean μ_m	Variance σ_m^2
PAM16	7.76E-1	2.29E-2
PAM8	7.74E-1	2.62E-2
PAM4	7.69E-1	4.03E-2

Statistical Upper Bound method

Start from original equation

$$e(t)=A_{i(t)}e^{jwt}+e^{j(wt+\theta)}\sum A_{k(t)}\sqrt{(R_{k1}R_{k2})}$$

Replace the sum $\sum A_{k(t)} \sqrt{(R_{k1}R_{k2})}$ with the variable $\chi(\mu_{scale}\ \mu_m, \sigma^2_{scale}\ \sigma_m^2)$ which takes into account proper scaling of μ_m and σ_m^2 as function of $R_{k1}R_{k2}$.

$$\mu_{scale} = n(n-1)R_{Conn}/2+n\sqrt{(R_{Conn}R_{TOSA})} + n\sqrt{(R_{Conn}R_{ROSA})}+\sqrt{(R_{TOSA}R_{ROSA})}$$
$$\sigma^2_{scale} = n(n-1)R_{Conn}^2/2+nR_{Conn}R_{TOSA}+nR_{Conn}R_{ROSA}+R_{TOSA}R_{ROSA}$$

$$e(t)=A_{i(t)}e^{jwt}+e^{j(wt+\theta)}\ \chi(\mu_{scale}\ \mu_m, \sigma^2_{scale}\ \sigma_m^2)$$

The receive current is proportional to

$$I(t) \propto |e(t)|^2 = A_{i(t)}^2+2A_{i(t)}\ \chi(\mu_{scale}\ \mu_m, \sigma^2_{scale}\ \sigma_m^2)\ \mathrm{Cos}(\theta)+ \chi(\mu_{scale}\ \mu_m, \sigma^2_{scale}\ \sigma_m^2)^2$$

It is safe to assume $\chi(\mu_{scale}\ \mu_m, \sigma^2_{scale}\ \sigma_m^2)^2$ is very small and can be omitted.

$$I(t) \propto |e(t)|^2 = A_{i(t)}^2+2A_{i(t)}\ \chi(\mu_{scale}\ \mu_m, \sigma^2_{scale}\ \sigma_m^2)\ \mathrm{Cos}(\theta)$$
$$I(t) \propto I_{i(t)}+2\ \chi(\mu_{scale}\ \mu_m, \sigma^2_{scale}\ \sigma_m^2)\ \mathrm{Cos}(\theta)\sqrt{I_{i(t)}}$$

And finally the MPI noise is

$$\mathrm{MPI\_Eye\_Closure\_Mean}=2\ \mu_{scale}\ \mu_m\ \mathrm{Cos}(\theta)\sqrt{I_{i(t)}}$$
$$\mathrm{MPI\_Eye\_additive\_noise}=2\ \chi(0, \sigma^2_{scale}\ \sigma_m^2)\ \mathrm{Cos}(\theta)\sqrt{I_{i(t)}}$$

Statistical Upper Bound method

•The worst case MPI happens for all levels from 2 to m-1 when Cos θ is +1 or -1. For level 1 the worst case is when Cos θ is +1 and for level m the worst case is when Cos θ is -1.

•Since the MPI is amplitude dependent, the penalty for level 1 is pretty small.

Pessimistic assumption 7: Cos θ=-1

$$\mu_{scale} = n(n-1)R_{Conn}/2 + n\sqrt{(R_{Conn}R_{TOSA})} + n\sqrt{(R_{Conn}R_{ROSA})} + \sqrt{(R_{TOSA}R_{ROSA})}$$

$$\sigma^2{}_{scale} = n(n-1)R_{Conn}{}^2/2 + nR_{Conn}R_{TOSA} + nR_{Conn}R_{ROSA} + R_{TOSA}R_{ROSA}$$

$$MPI\_Eye\_Closure\_Mean = -2\ \mu_{scale}\ \mu_m\ \sqrt{I_{i(t)}}$$

$$MPI\_additive\_noise = 2\ \chi(0,\ \sigma^2{}_{scale}\ \sigma_m{}^2)\ \sqrt{I_{i(t)}}$$

Statistical Upper Bound method

It is reasonable to practically ignore the MPI additive noise component since it is very small. For example, assuming $R_{ROSA}=R_{TOSA}=R_{Conn}=-35dB$ and ER=6dB and assuming $I_{i(t)}=I_m$, below table shows MPI noise component for different PAM modulation schemes.

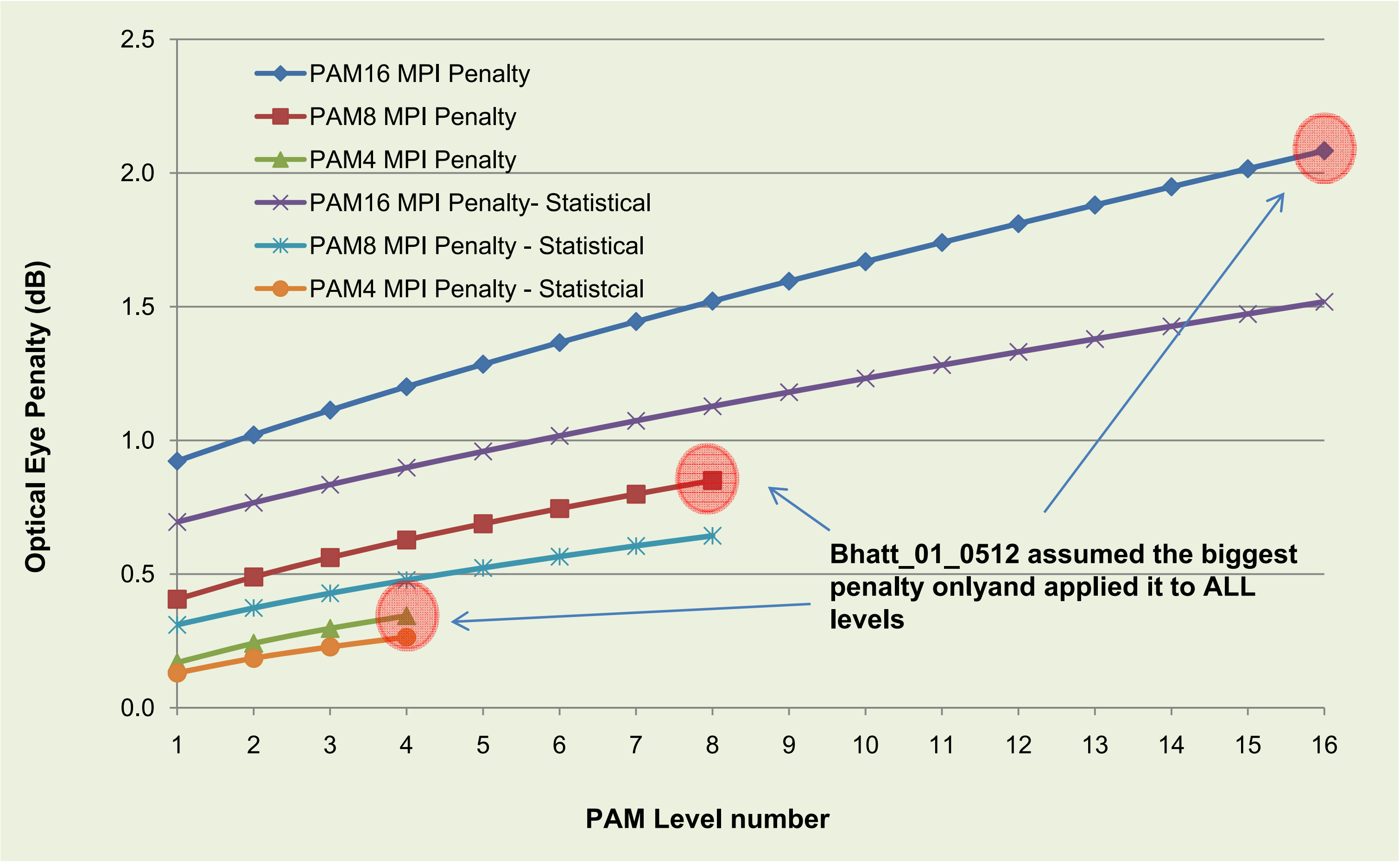
PAM modulation order	MPI noise limited RX SNR
PAM-16	36.6dB
PAM-8	43.2dB
PAM-4	50.6dB

$$\mu_{scale} = n(n-1)R_{Conn}/2 + n\sqrt{(R_{Conn}R_{TOSA})} + n\sqrt{(R_{Conn}R_{ROSA})} + \sqrt{(R_{TOSA}R_{ROSA})}$$

$$MPI\_Eye\_Closure\_Mean = -2\, \mu_{scale}\, \mu_m \sqrt{I_{i(t)}}$$

Statistical Upper Bound method

Using $R_{ROSA}=R_{TOSA}=R_{Conn}$=-35dB ,4 connectors and 6dB ER, below graph shows level-dependent MPI penalty



Statistical Upper Bound method

Using $R_{ROSA}=R_{TOSA}=R_{Conn}=-35dB$,4 connectors and 6dB ER, the table below shows the **maximum** statistical upper bound MPI penalty compared to Bhatt_01_0512 Upper Bound method MPI penalty

PAM order	Bhatt_01_0512 Upper Bound MPI penalty	Max Statistical Upper Bound MPI penalty	Delta for the upper Eye
PAM-16	2.07dB	1.52dB	0.55dB
PAM-8	0.85dB	0.64dB	0.19dB
PAM-4	0.34dB	0.26	0.08dB

The statistical upper bound method does result in some saving. However, the bigger part of the saving comes from correct modeling of amplitude dependent MPI as shown earlier in this presentation.

Statistical Upper Bound method

To understand the impact of the MPI on the slicer SNR, lets use the following method

- Similar to RIN and Shot noise, MPI Penalty is amplitude dependent. The higher the amplitude, the more MPI penalty is.

- There are other noise components such as TX electrical noise, TIA thermal noise and RX electrical noise that are not amplitude dependent

- The combination of the above noise sources make the segment SNR to look very close to a linear ramp going from value *a* to *b* starting from lowest eye to the highest eye. Assuming 1.5dB SNR change from *a* to *b*, is a good typical assumption for demonstration purposes

- Because the segment SNR varies as a function of segment number, each segment will make a different contribution to Symbol-Error-Rate (SER)

- The overall SER of the system is the average of all the segment SERs.

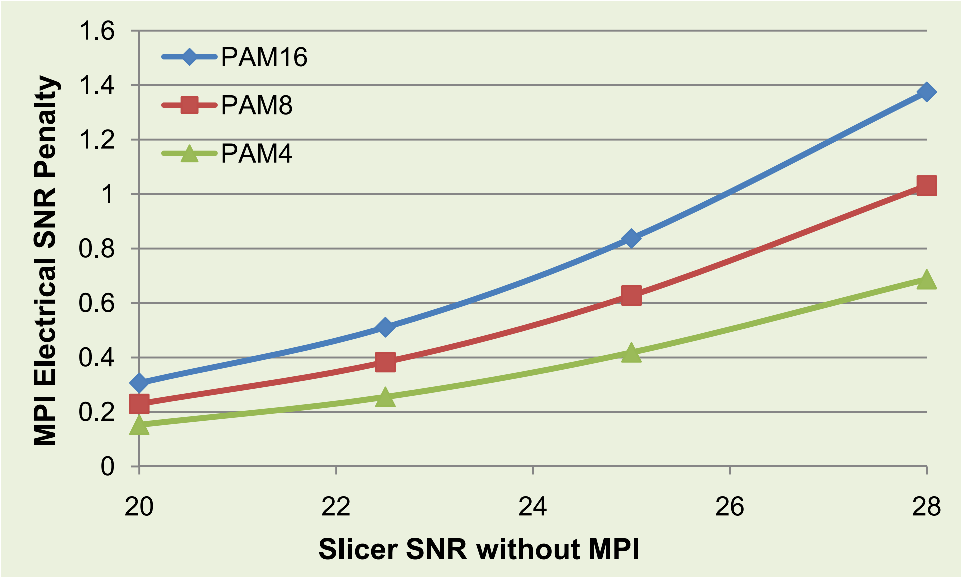
- The average SER can then be converted back to the “Effective SNR” for the modulation order under investigation.

- We then calculate the Effective SNR with and without MPI and define MPI Electrical SNR penalty as

“MPI Electrical SNR Penalty”=“Effective Slicer SNR without MPI”-”Effective Slicer SNR with MPI”

Statistical Upper Bound method

Using R_{ROSA}=R_{TOSA}=R_{Conn}=-35dB ,4 connectors and 6dB ER, SNR ramp with 1.5dB delta, below graph shows MPI Electrical SNR Penalty for different PAM options at different "Effective SNR without MPI"



- As shown in the above graph, if the Slicer SNR was 20dB before MPI, with MPI if will be reduced by **0.15dB to 0.3dB** as a function of PAM modulation order. This is a manageable penalty.

- For slicer SNR of 28dB the PAM16 MPI Electrical SNR penalty is **1.4dB.** (for PAM16 to work with bj FEC, Slicer SNR should be actually bigger than 31dB)

Summary

• We started from Bhatt_01_0512 upper bound analysis method for MPI and refined it by introducing an improved statistical method, while preserving the conservative approach.

• We recommend to adopt the ISO 11801 return-loss spec to -35dB as stated in Bhatt_01_0512.

• We showed that with -35dB RL, for a reasonable slicer SNR assumption of around 20dB, the MPI impact is small.

Potential Future Work

• In this presentation we assumed a perfect Coherent Laser with zero line-width

• We then assumed all interfering noise sources are perfectly aligned in phase

• It is worth investigating if substantial improvement can be made by modeling the above two pessimistic assumptions more accurately