OFDM TX Symbol Shaping
802.3bn

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Phoenix, January 2013
Recommendations

• TX window is specified as $N_t$ samples in taper region
  – No need for different set of Alpha for 4K and 8K FFT.
  – Avoid confusion for calculation of $N_t$ with variable Cyclic Prefix.
    \[
    \text{Alpha} = \frac{N_t}{N_{fft}}, \quad T_p = \frac{N_t}{204.8e6}
    \]

• $N_t = \{0, 32, 64, 128, 192, 256\}$
  – $T_p = \{0, 0.15625, 0.3125, 0.625, 0.9375, 1.25\}$ us
  – Alpha = \{0, 0.78125\%, 1.5625\%, 3.125\%, 4.6875\%, 6.25\% \} for 4K FFT
  – Alpha = \{0, 0.390625\%, 0.78125\%, 1.5625\%, 2.34375\%, 3.125\% \} for 8K FFT

• A postfix of $N_t$ samples is added, windowing is applied to cyclic prefix and postfix
  – Windowing is absorbed by CP.
  – Symbol time is independent of Window $N_t$.
  – Receiver sampling is independent of TX window.
  – TX window appears to RX as post-cursor multipath, affect only the following symbol, not the previous symbol.
Windowing Function
Raised-Cosine Window

• Raised-Cosine window in frequency domain (FD):

\[ P(f) = \left[ \frac{\sin(\pi f T)}{\pi f T} \left( \frac{\cos(\pi \alpha f T)}{1 - (2\alpha f T)^2} \right) \right], \quad 0 \leq \alpha \leq 1 \]

• Raised-Cosine window in time domain (TD):

\[ p(t) = \begin{cases} \frac{1}{T}, & 0 \leq |t| < \frac{T(1-\alpha)}{2} \\ \frac{1}{2T} \left\{ 1 + \cos \left[ \frac{\pi}{\alpha T} \left( |t| - \frac{T(1-\alpha)}{2} \right) \right] \right\}, & \frac{T(1-\alpha)}{2} \leq |t| \leq \frac{T(1+\alpha)}{2} \\ 0, & \text{otherwise} \end{cases} \]

\( \alpha = 0 \) is a rectangular window (no shaping)
Examples of Raised-Cosine Window

MATLAB code for TX window \( p \):  
\[
\begin{align*}
N_{\text{fft}} &= 4096; & \text{% FFT size} \\
CP &= 256; & \text{% Nb. samples in Cyclic Prefix} \\
\text{Alpha} &= 1/32; & \text{% RX Alpha} \\
N_t &= 2*\text{round}(N_{\text{fft}}*\text{Alpha}/2); & \text{% Nb. samples in taper region} \\
p &= 1/2*(1+\cos(\pi*[-N_t+1/2:N_t-1/2]/N_t)); & \text{% Raised-Cosine in TD} \\
p &= [p(1:N_t), \text{ones}(1,N_{\text{fft}}+CP-N_t), p(N_t+1:2*N_t)]; & \text{% Add ones in middle}
\end{align*}
\]

Taper Region weight for Alpha = 1/128 (32 points):  
\[
\begin{array}{cccccccc}
0.0006 & 0.0054 & 0.0150 & 0.0292 & 0.0480 & 0.0711 & 0.0984 & 0.1295 \\
0.1642 & 0.2022 & 0.2429 & 0.2862 & 0.3316 & 0.3785 & 0.4266 & 0.4755 \\
0.5245 & 0.5734 & 0.6215 & 0.6684 & 0.7138 & 0.7571 & 0.7978 & 0.8358 \\
0.8705 & 0.9016 & 0.9289 & 0.9520 & 0.9708 & 0.9850 & 0.9946 & 0.9994 \\
\end{array}
\]

The taper region should not change with different CP \( \rightarrow N_t = \text{Alpha} \times N_{\text{fft}} \)
OFDM TX Windowing, RX sampling offset unaffected by TX Windowing

$T_s$ is independent of RC Window Alpha
Leakage in In-Band and Adjacent SC-QAM
4K FFT, $N_t = 128$, CP = 1.25 us, 8 MHz Spectral Exclusion, SQRT-RC 12% 5.35 MHz Filter

Leakage = -52.04 dBc
4K & 8K FFT, CP = 1.25 us, 8 MHz Spectral Exclusion, SQRT-RC 12% 5.35 MHz Filter

OFDM Leakage in SC-QAM Channel vs Nb of Taper Samples

Δ = 2.5 dB
OFDM TX Power Spectral Density, CP = 1.25 us
Adjacent Channel Guard-Band, 6 MHz rectangular and SQRT-RC 12% 5.35 MHz Filter, CP = 1.25 us

4k FFT, OFDM Power Leakage vs $N_t$

Guard Band BW (MHz)

$N_t$ samples in taper region

-45 dBc, 5.35 MHz
-50 dBc, 5.35 MHz
-55 dBc, 5.35 MHz
-45 dBc, 6.0 MHz
-50 dBc, 6.0 MHz
-55 dBc, 6.0 MHz
In-Band Exclusion BW, 6 MHz rectangular and SQRT-RC 12% 5.35 MHz Filter, CP = 1.25 us

4k FFT, OFDM Power Leakage vs $N_t$

-45 dBc, 5.35 MHz
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-55 dBc, 5.35 MHz
-45 dBc, 6.0 MHz
-50 dBc, 6.0 MHz
-55 dBc, 6.0 MHz

Exclusion BW (MHz)

Nt samples in taper region
In-Band Exclusion BW, 8 MHz rectangular and SQRT-RC 15% 6.952 MHz Filter, CP = 1.25 us

4k FFT, OFDM Power Leakage vs $N_t$
OFDM TX Power Spectral Density,
CP = 1.25 us

8K FFT, Power Spectral Density at Band-Edge

Frequency Offset from Band-Edge (MHz)

Magnitude (dB)
Adjacent Channel Guard-Band, 6 MHz rectangular and SQRT-RC 12% 5.35 MHz Filter, CP = 1.25 us

8k FFT, OFDM Power Leakage vs $N_t$

-45 dBc, 5.35 MHz
-50 dBc, 5.35 MHz
-55 dBc, 5.35 MHz
-45 dBc, 6.0 MHz
-50 dBc, 6.0 MHz
-55 dBc, 6.0 MHz

Guard Band BW (MHz) vs $N_t$ samples in taper region
In-Band Exclusion BW, 6 MHz rectangular and SQRT-RC 12% 5.35 MHz Filter, CP = 1.25 us

8k FFT, OFDM Power Leakage vs $N_t$

-45 dBc, 5.35 MHz
-50 dBc, 5.35 MHz
-55 dBc, 5.35 MHz
-45 dBc, 6.0 MHz
-50 dBc, 6.0 MHz
-55 dBc, 6.0 MHz
In-Band Exclusion BW, 8 MHz rectangular and SQRT-RC 15% 6.952 MHz Filter, CP = 1.25 us

8k FFT, OFDM Power Leakage vs $N_t$

Exclusion BW (MHz)

$N_t$ samples in taper region

-45 dBc, 6.952 MHz
-50 dBc, 6.952 MHz
-55 dBc, 6.952 MHz
-45 dBc, 8.0 MHz
-50 dBc, 8.0 MHz
-55 dBc, 8.0 MHz
Thank You