

EPOC FEC LDPC CODES FOR ACTIVE PLANT



Presenters: Rich Prodan and BZ Shen

CODES	Rate	Length
A	$R_A = 8/9$	16200
B	$R_B = 8/9$	16200
C	$R_C = 0.848$	5940
D	$R_D = 3/4$	1120
E	$R_E = 9/10$	14400
F	$R_F = 9/10$	10800
G	$R_G = 13/15$	5400
H	$R_H = 3/4$	960

DEPLOYMENT	Passive plant	Active plant
US, low band	F, G, H	B, C, D
DS, low band	E, (F), G, H	---
US, high band	E, (F), G, H	---
DS, high band	E, (F), G, H	A

REDUCED PROPOSED CODES FOR ACTIVE PLANT

CODES	Rate	Length
B	$R_B = 8/9$	16200
C	$R_C = 0.848$	5940
D	$R_D = 3/4$	1120
E	$R_E = 9/10$	14400
F	$R_F = 9/10$	10800
G	$R_G = 13/15$	5400
H	$R_H = 3/4$	960

DEPLOYMENT	Passive plant	Active plant
US, low band	F, G, H	B, C, D
DS, low band	E, (F), G, H	---
US, high band	E, (F), G, H	---
DS, high band	E, (F), G, H	B

This presentation will concentrate on:

CODES	Rate	Length
B	$R_B = 8/9$	16200
C	$R_C = 0.848$	5940
D	$R_D = 3/4$	1120

- **Proposed code matrices**
- **Performance evaluation criteria**
- **AWGN channel performance**
- **Performance under burst noise**
- **Conclusion**

- **Background on LDPC Code Algorithms**
 - LDPC codes in general
 - Iterative message passing decoding for LDPC codes
 - Possible encoding method

- **Definition of an (n, k) QC-LDPC code**

- A QC-LDPC parity-check matrix can be divided into blocks of L by L submatrices, where L represents the submatrix size or lifting factor.
- The parity-check matrix in compact circulant form is represented by an m by n block matrix:

$$H_{base} = \begin{bmatrix} H_{1,1} & H_{1,2} & H_{1,3} & \cdots & H_{1,n} \\ H_{2,1} & H_{2,2} & H_{2,3} & \cdots & H_{2,n} \\ H_{3,1} & H_{3,2} & H_{3,3} & \cdots & H_{3,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ H_{m,1} & H_{m,2} & H_{m,3} & \cdots & H_{m,n} \end{bmatrix}$$

- Each submatrix $H_{i,j}$ is a L by L all-zero submatrix or a cyclic right-shifted Identity submatrix.
- The parity-check matrix tables consist of entries $\{-1, 0, \dots, L-1\}$, where a '-1' value represents an all-zero submatrix, and the remaining values represent an identity submatrix which has been cyclically right-shifted by the specified value.
- The H_{base} matrix can be represented as $[H_1 | H_2]$ where H_2 represents the parity portion.
- The code rate is $(n-m)/n$ and a codeword length is $n L$ bits.

- The code will be used for both downstream and upstream

Long size code: Rate= 8/9 (16200, 14400) code, m=5 rows x n=45 columns, L=360

93	271	-1	83	26	208	245	200	-1	175	331	17	86	-1	337	-1	238	81	-1	307	-1	165	-1	47	76	73	150	349	139	331	118	345	27	294	-1	145	279	97	106	160	143	-1	-1	-1	-1
274	115	329	338	124	-1	293	-1	69	64	342	-1	88	139	-1	137	212	-1	157	195	357	81	194	1	159	56	72	126	277	156	32	111	175	-1	306	224	-1	206	-1	29	106	334	-1	-1	-1
134	355	175	24	253	242	-1	187	94	26	87	302	-1	191	323	22	-1	245	294	240	84	76	342	345	174	269	329	-1	214	-1	-1	-1	-1	218	104	40	197	73	229	63	-1	270	72	-1	-1
-1	-1	184	70	247	14	22	7	285	54	-1	352	26	108	10	298	123	139	117	-1	336	49	202	359	342	-1	224	106	-1	273	177	245	98	355	178	176	147	-1	280	-1	-1	-1	221	208	-1
253	273	90	-1	-1	151	311	320	339	-1	295	148	48	91	62	100	232	146	200	135	12	-1	179	-1	-1	232	-1	21	331	313	349	34	97	187	38	-1	235	52	170	58	-1	-1	-1	257	0

- **Number of equations: 1800**
- **Number of edges: 60840**
- **Density: 0.00209**
- **Number of nodes in the Tanner graph**
 - Bit nodes: 16200
 - Check nodes: 1800

- The code will be used for upstream only

Medium size code: Rate= 28/33(0.848) (5940, 5040) code, m=5 rows x n=33 columns, L=180

142	158	113	124	92	44	93	70	172	3	25	44	141	160	50	45	118	84	-1	64	66	97	1	115	8	108	-1	-1	22	-1	-1	-1	-1
54	172	145	28	55	19	159	22	96	12	85	-1	128	5	158	120	51	171	65	141	-1	42	83	7	-1	39	121	84	101	171	-1	-1	-1
63	11	112	114	61	123	72	55	114	20	53	114	42	33	4	66	163	50	46	17	175	-1	-1	-1	92	-1	41	138	-1	34	7	-1	-1
28	160	102	44	8	84	126	9	169	174	147	24	145	-1	26	-1	-1	-1	67	82	4	177	151	131	139	117	36	18	-1	-1	23	8	-1
52	159	75	74	46	71	42	11	108	153	-1	72	-1	163	-1	9	2	168	158	-1	1	49	89	63	179	10	75	161	-1	-1	-1	177	19

- Number of equations: 900
- Number of edges: 23580
- Density: 0.0044
- Number of nodes in the Tanner graph
 - Bit nodes: 5940
 - Check nodes: 900

- The code will be used for upstream only

Short size code: Rate= 3/4 (1120, 840) code, m=5 rows x n=20 columns, L=56

5	14	12	1	2	37	45	26	24	0	3	-1	34	7	46	10	-1	-1	-1	-1
0	35	1	26	0	10	16	16	34	4	2	23	0	51	-1	49	20	-1	-1	-1
12	28	22	46	3	16	51	2	25	29	19	18	52	-1	37	-1	34	39	-1	-1
0	51	16	31	13	39	27	33	8	27	53	13	-1	52	33	-1	-1	38	7	-1
36	6	3	51	4	19	4	45	48	9	-1	11	22	23	43	-1	-1	-1	14	1

- **Number of equations: 280**
- **Number of edges: 4424**
- **Density: 0.014**
- **Number of nodes in the Tanner graph**
 - Bit nodes: 1120
 - Check nodes: 280

▪ Evaluation parameters

- Constellation:
 - Downstream 256, 512, 1024, 2048, and 4096 QAM
 - Upstream 64,128,256,512 and 1024 QAM
- Two OFDM symbol durations
 - 20 μ s and 40 μ s
 - Cyclic prefix of 2.5 μ s
- Probability of error at threshold
 - BER=1e-8
 - WER=1e-6
- AWGN SNR threshold with and without burst error events
- Channel assumptions for burst error events
 - Downstream burst noise: 16 μ s at 20 dB SNR or 16 μ s at 5 dB SNR (two consecutive OFDM symbols)
 - Upstream burst noise: 1 μ s at 0 dB SNR (1 OFDM symbol) or 10 μ s at 10 dB SNR (two consecutive OFDM symbols)

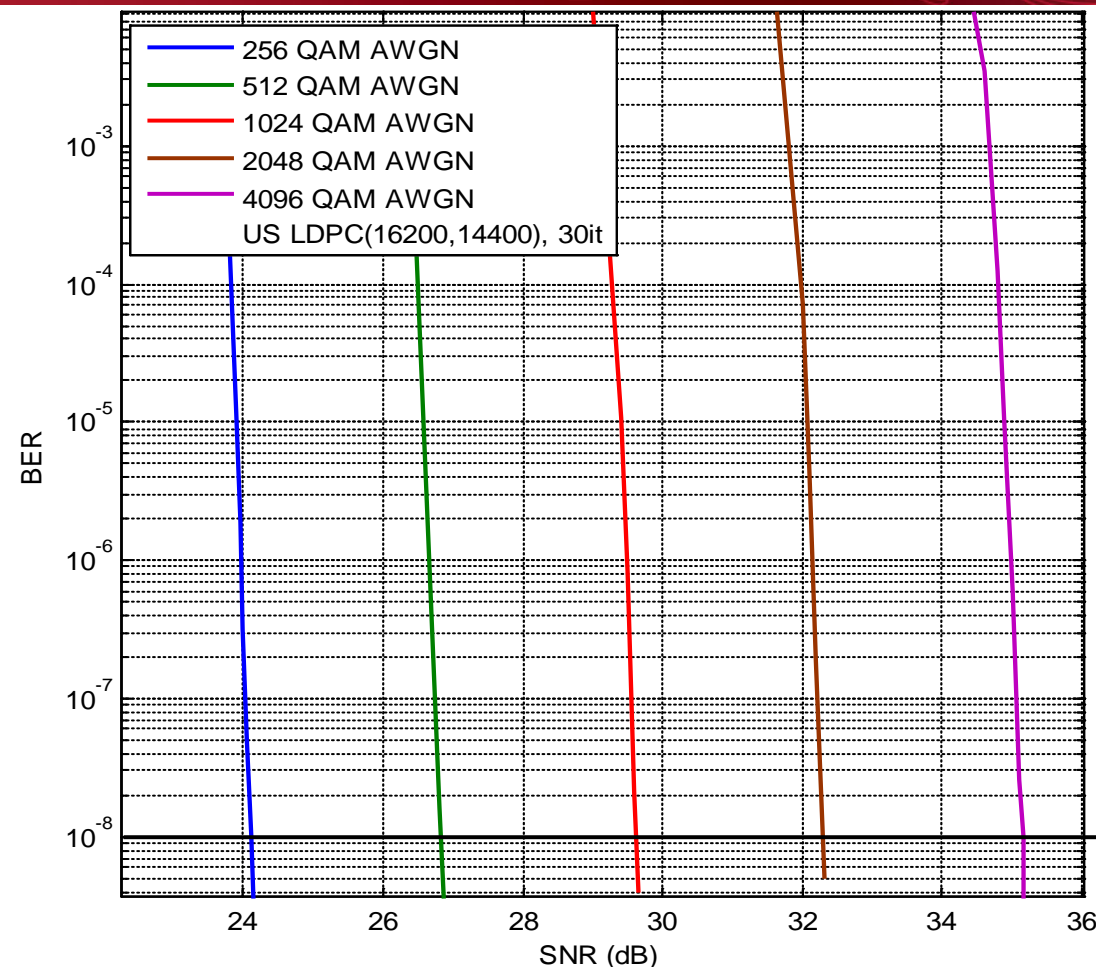
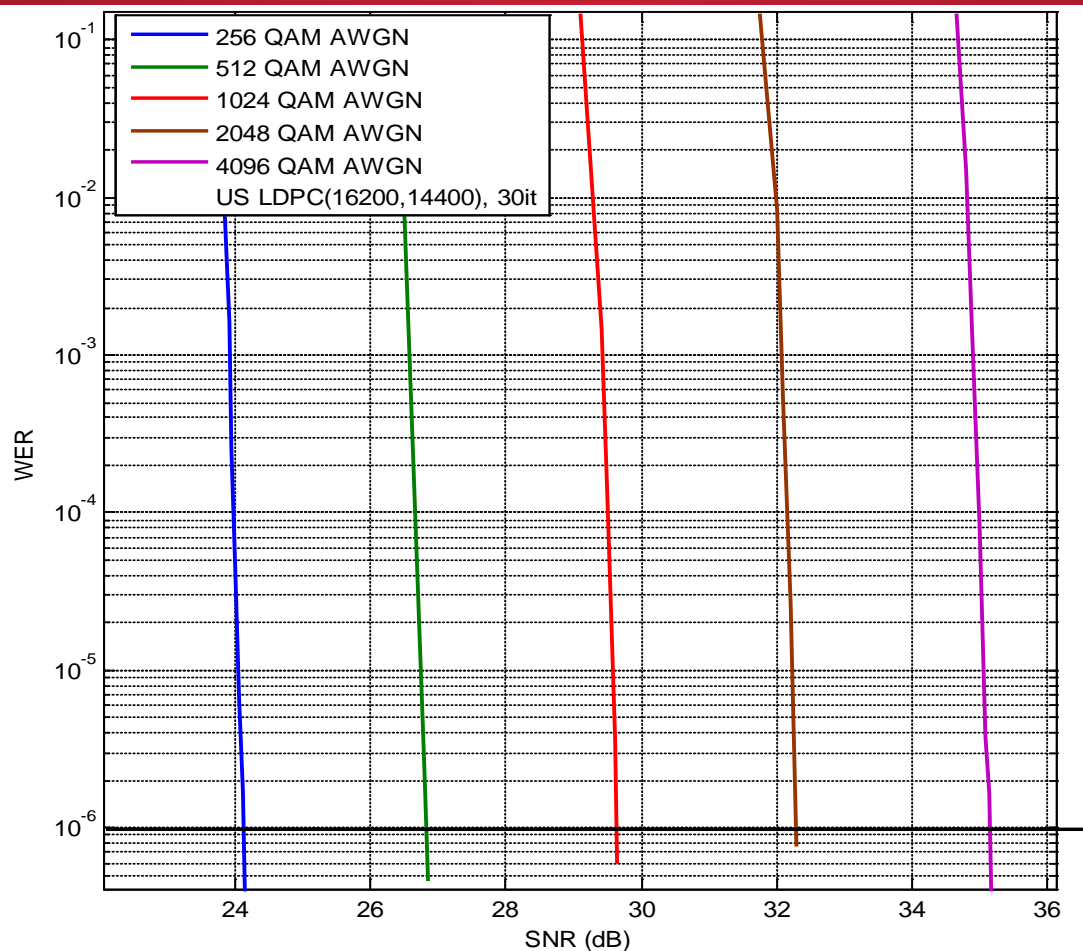
▪ Decoding procedure

- Iterative sum-product message passing algorithm
- Floating point
- Maximum of 15 and 30 flooding iterations

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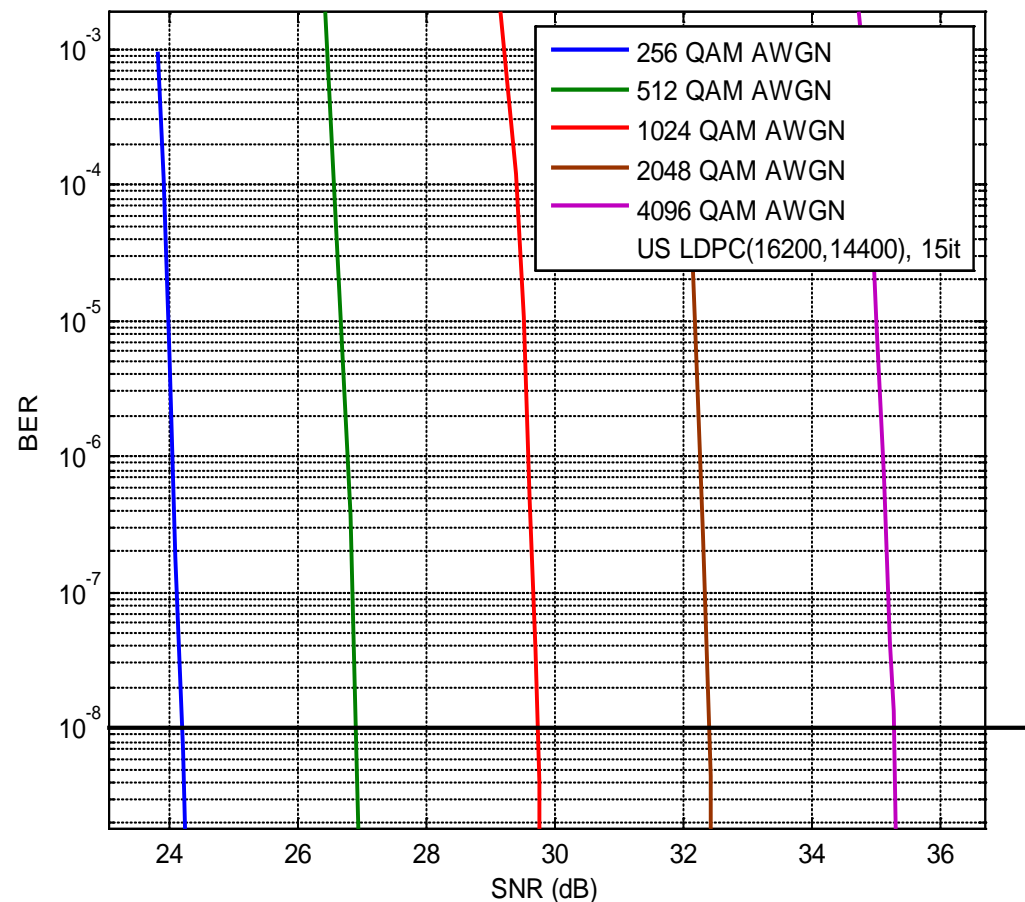
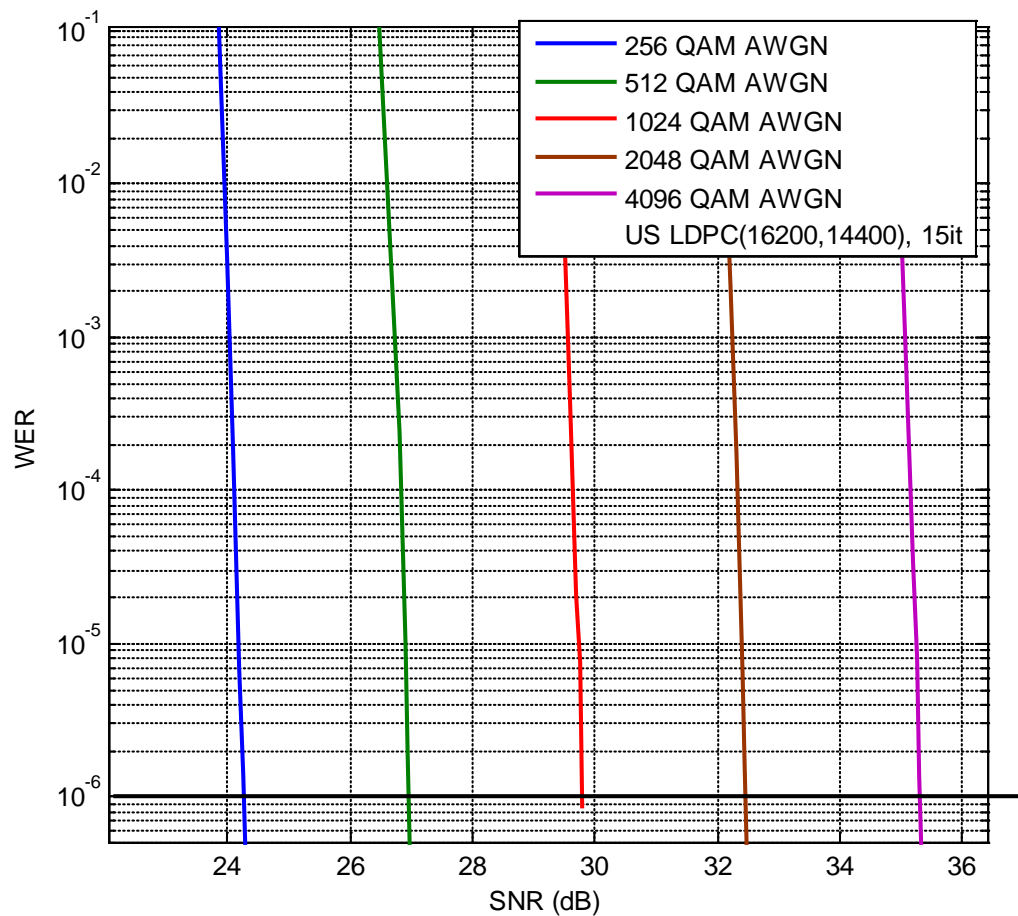
AWGN PERFORMANCE

ON AWGN CHANNEL (LONG SIZE CODE) (MAX. 30 ITERATIONS)



Upstream (16200, 14400) code (30 iterations)		256QAM	512QAM	1024QAM	2048QAM	4096QAM
	SNR@WER=1e-6		24.11dB	26.83dB	29.64dB	32.29dB
SNR@BER=1e-8		24.1dB	26.82dB	29.62dB	32.28dB	35.15dB

ON AWGN CHANNEL (LONG SIZE CODE) (MAX. 15 ITERATIONS)



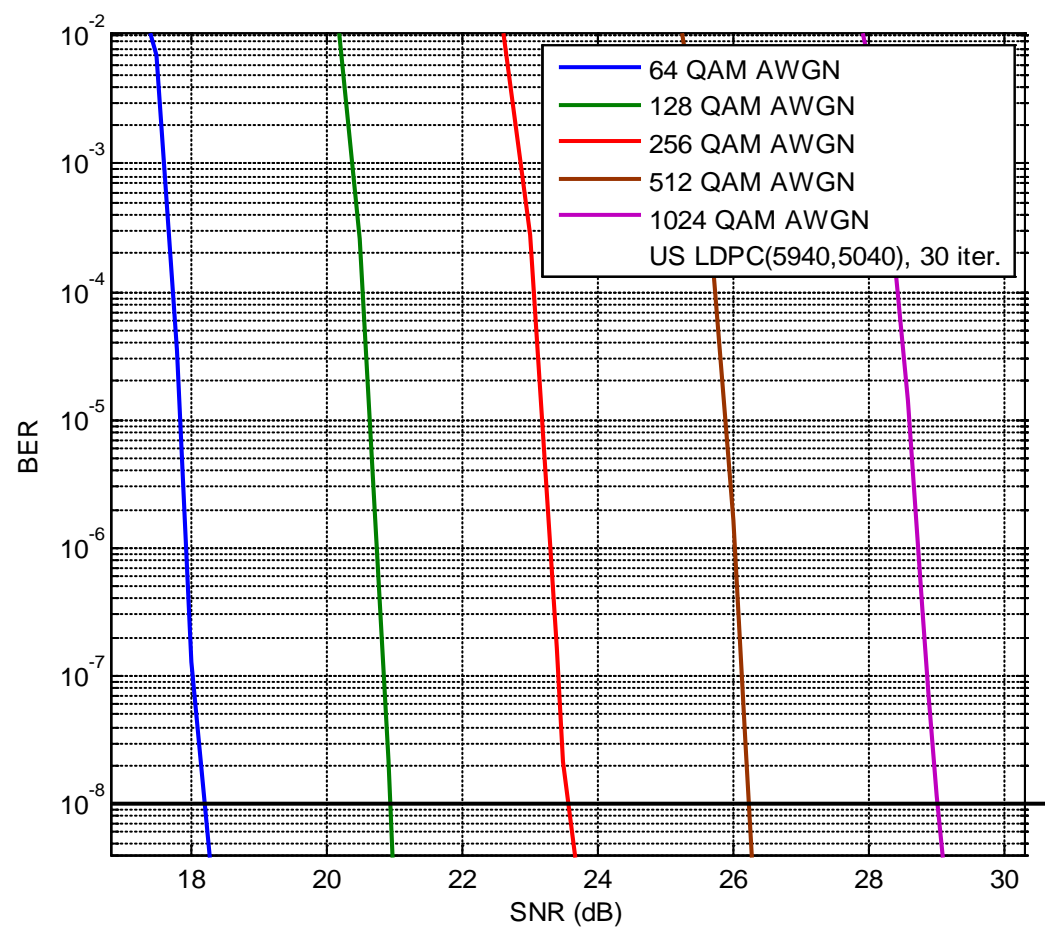
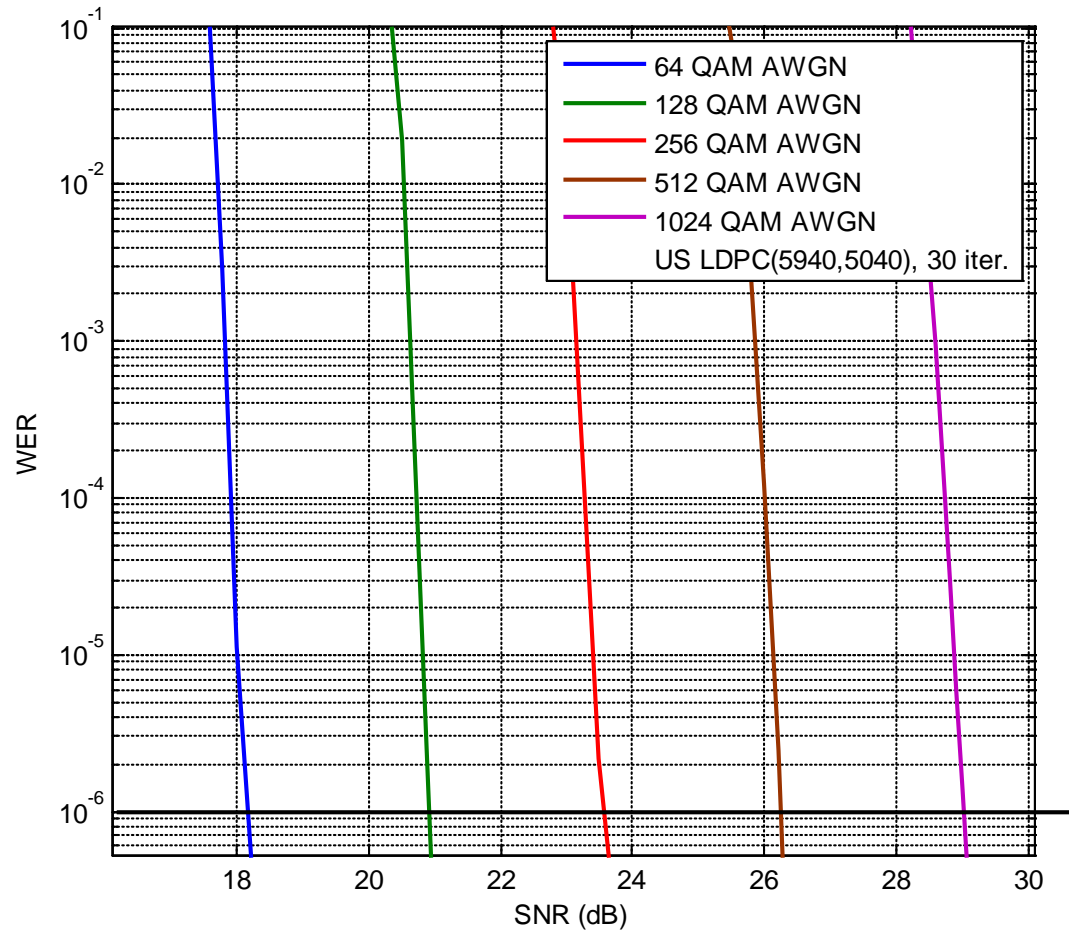
Upstream (16200, 14400) code (15 iterations)		256QAM	512QAM	1024QAM	2048QAM	4096QAM
	SNR @WER=1e-6	24.26dB	26.97dB	29.8dB	32.46dB	35.31dB
	SNR @BER=1e-8	24.19dB	26.89dB	29.73dB	32.38dB	35.26dB

ON AWGN CHANNEL (LONG SIZE CODE) DIFFERENCE BETWEEN 15 AND 30 ITERATIONS

Downstream code: Max. 30 iterations vs. Max. 15 iterations

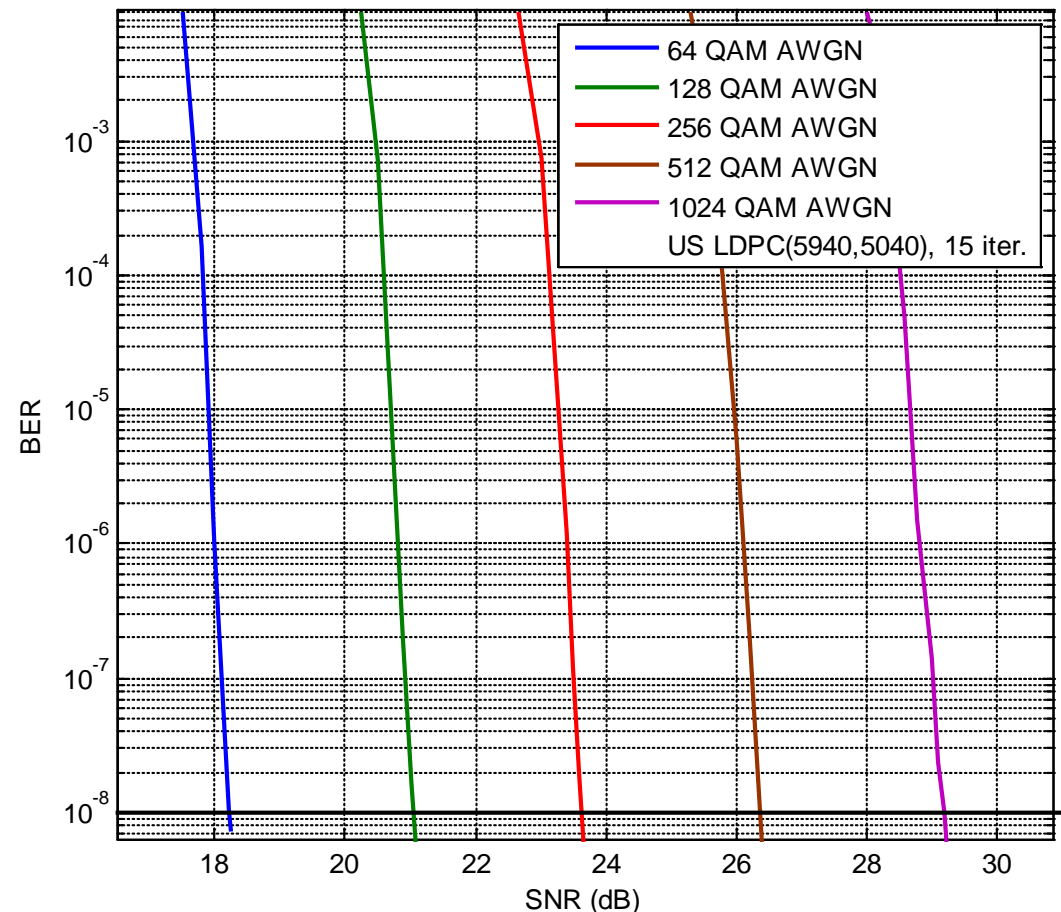
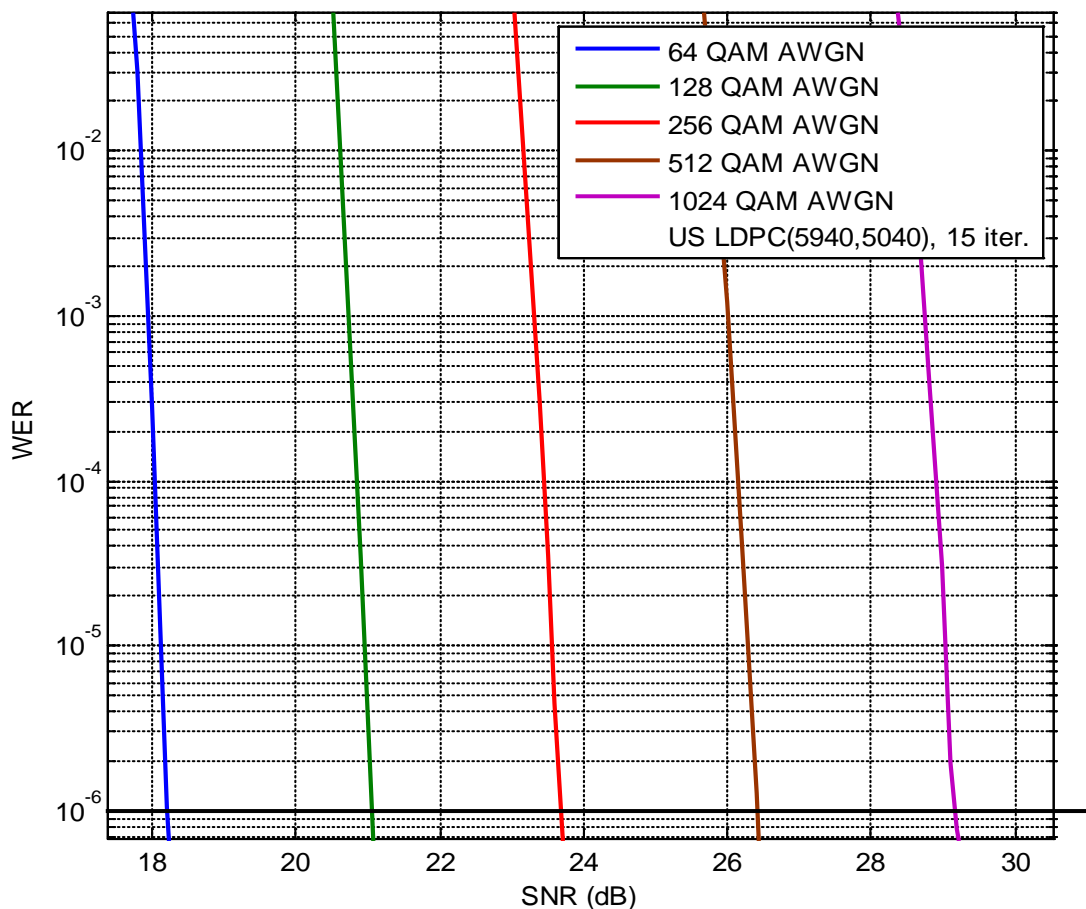
		256QAM	512QAM	1024QAM	2048QAM	4096QAM
SNR@WER=1e-6	Max.15 iterations	24.26dB	26.97dB	29.8dB	32.46dB	35.31dB
	Max.30 iterations	24.11dB	26.83dB	29.64dB	32.29dB	35.16dB
	Difference	0.15	0.14	0.16	0.17	0.15
SNR@BER=1e-8	Max. 15 iterations	24.19dB	26.89dB	29.73dB	32.38dB	35.26dB
	Max. 30 iterations	24.1dB	26.82dB	29.62dB	32.28dB	35.15dB
	Difference	0.09	0.07	0.11	0.1	0.11

ON AWGN CHANNEL (UPSTREAM MEDIUM SIZE CODE) (MAX. 30 ITERATIONS)



	64QAM	128QAM	256QAM	512QAM	1024QAM
SNR@WER=1e-6	18.15dB	20.93dB	23.59dB	26.26dB	29dB
SNR@BER=1e-8	18.17dB	20.93dB	23.6dB	26.25dB	28.99dB

ON AWGN CHANNEL (UPSTREAM MEDIUM SIZE CODE) (MAX. 15 ITERATIONS)

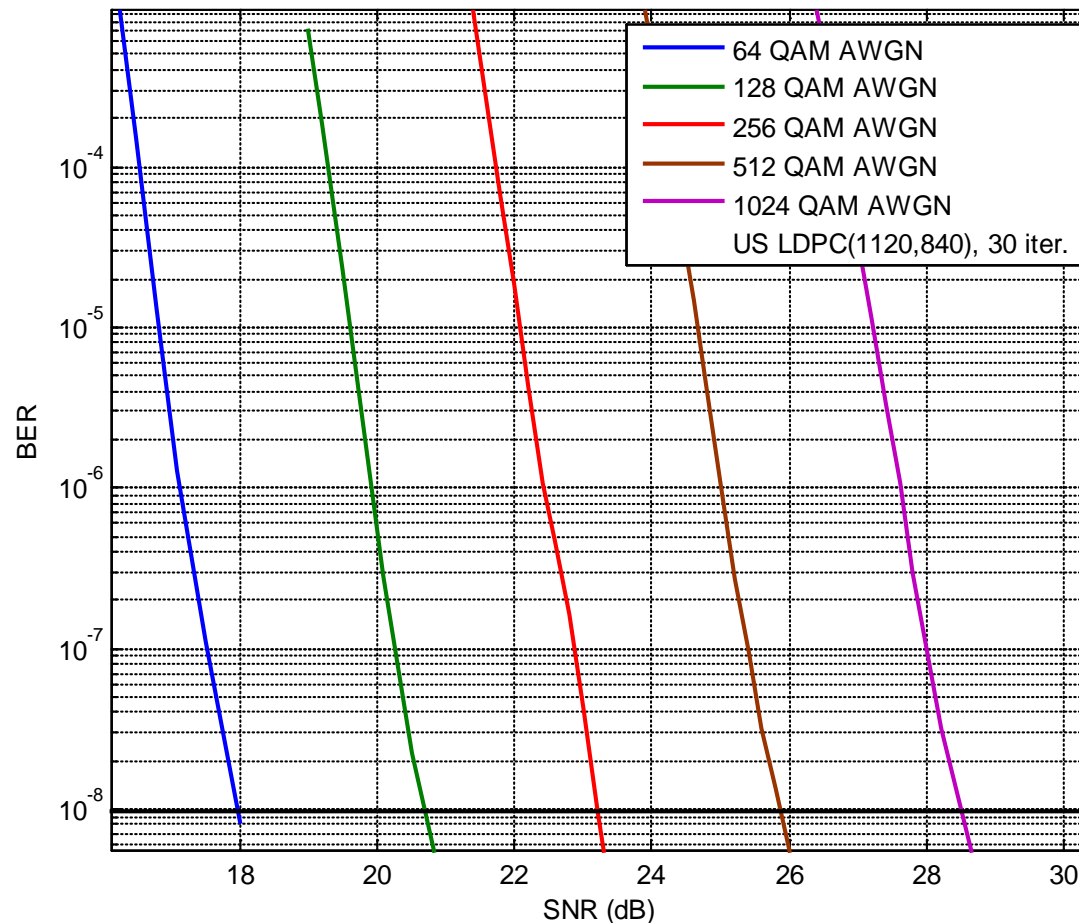
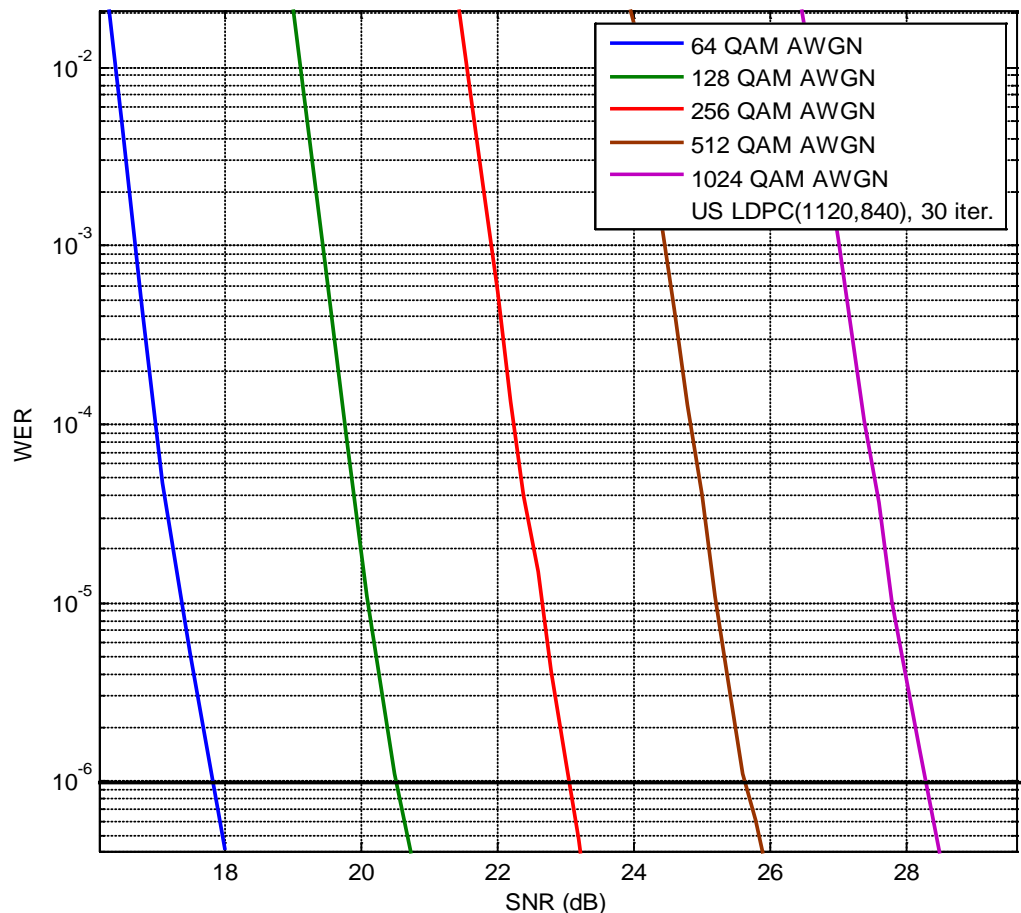


	64QAM	128QAM	256QAM	512QAM	1024QAM
SNR@WER=1e-6	18.2dB	21.05dB	23.69dB	26.4dB	29.18dB
SNR@BER=1e-8	18.2dB	21.03dB	23.69dB	26.38dB	29.16dB

DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (UPSTREAM MEDIUM SIZE CODE)

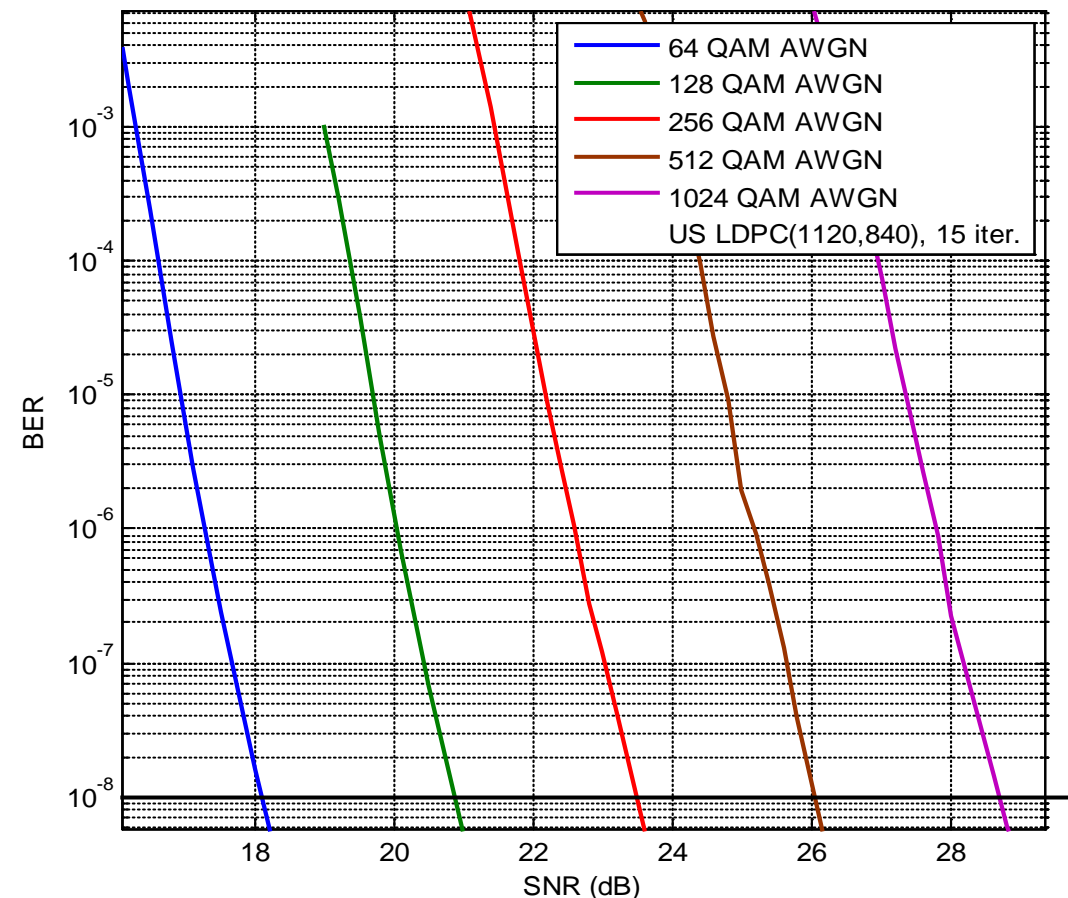
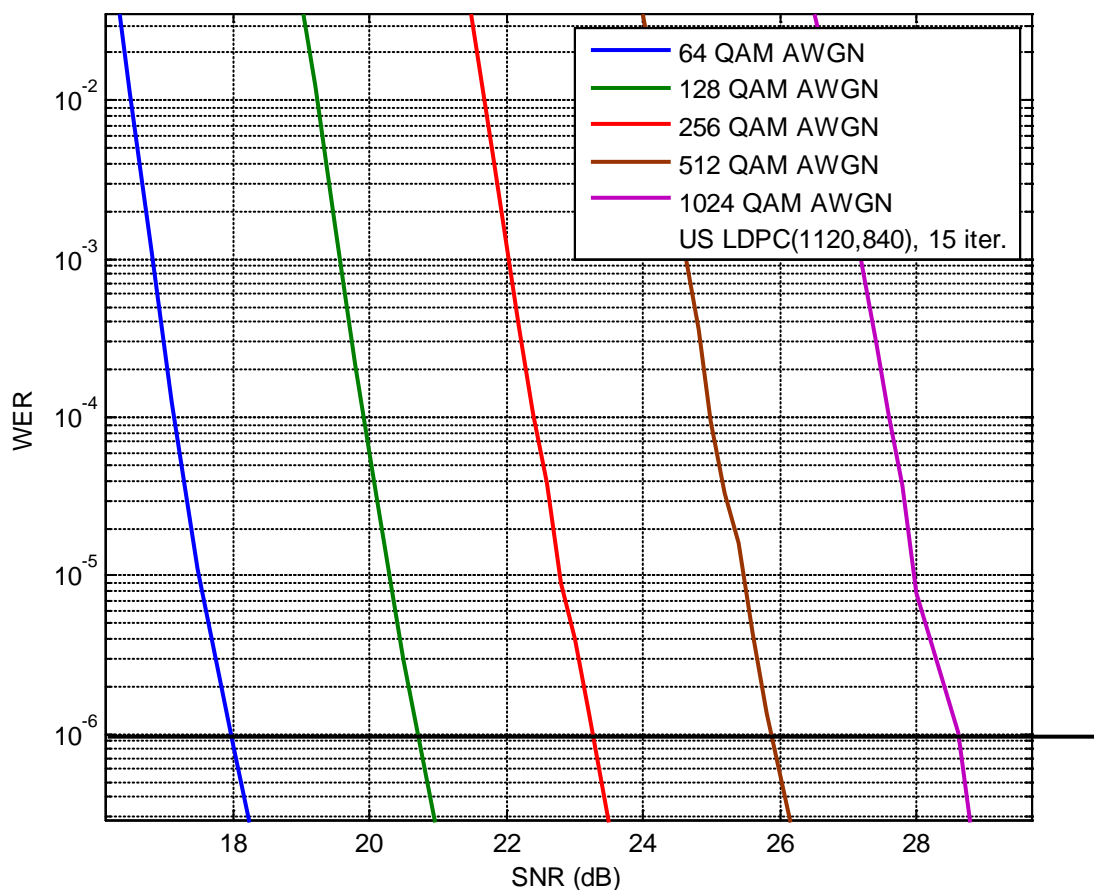
		64QAM	128QAM	256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	18.2dB	21.05dB	23.69dB	26.4dB	29.18dB
	Max. 30 iterations	18.15dB	20.93dB	23.59dB	26.26dB	29dB
	Difference	0.05	0.12	0.1	0.14	0.18
SNR@BER=1e-8	Max.15 iterations	18.2dB	21.03dB	23.69dB	26.38dB	29.16dB
	Max. 30 iterations	18.17dB	20.93dB	23.6dB	26.25dB	28.99dB
	Difference	0.03	0.1	0.09	0.13	0.17

ON AWGN CHANNEL (UPSTREAM SHORT SIZE CODE) (MAX. 30 ITERATIONS)



	64QAM	128QAM	256QAM	512QAM	1024QAM
SNR@WER=1e-6	17.83dB	20.52dB	23.08dB	25.69dB	28.3dB
SNR@BER=1e-8	17.96dB	20.69dB	23.21dB	26.01dB	28.59dB

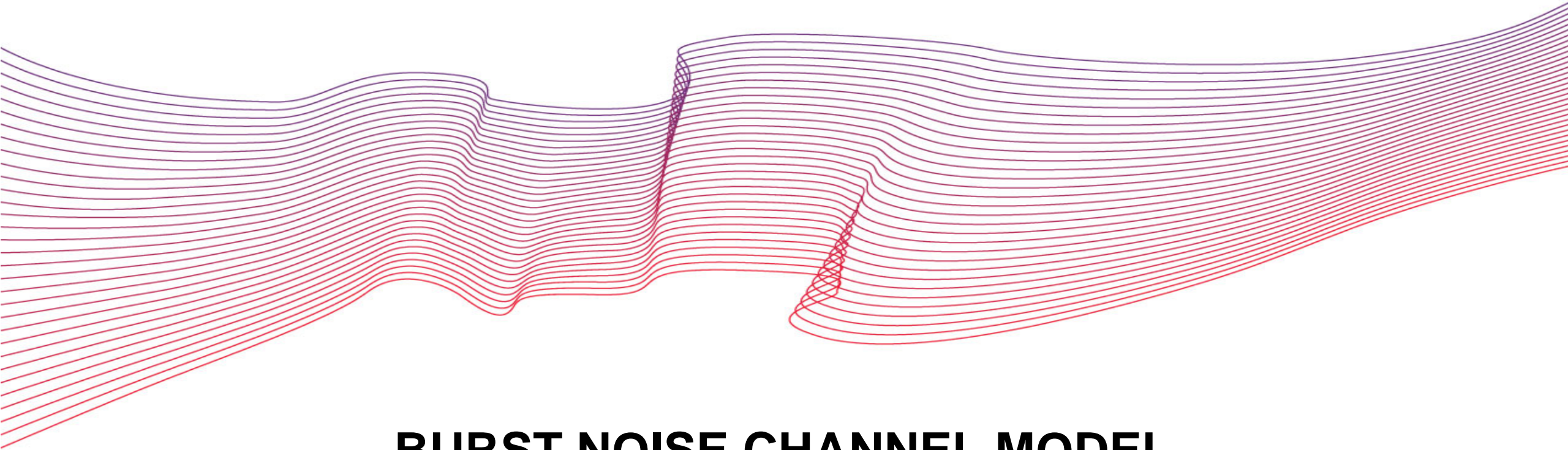
ON AWGN CHANNEL (UPSTREAM SHORT SIZE CODE) (MAX. 15 ITERATIONS)



	64QAM	128QAM	256QAM	512QAM	1024QAM
SNR@WER=1e-6	17.96dB	20.71dB	23.23dB	25.91dB	28.49dB
SNR@BER=1e-8	18.1dB	20.87dB	23.37dB	26.11dB	28.67dB

DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (UPSTREAM SHORT SIZE CODE)

		64QAM	128QAM	256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	17.96dB	20.71dB	23.23dB	25.91dB	28.49dB
	Max. 30 iterations	17.83dB	20.52dB	23.08dB	25.69dB	28.3dB
	Difference	0.13	0.19	0.15	0.22	0.19
SNR@BER=1e-8	Max.15 iterations	18.1dB	20.87dB	23.37dB	26.11dB	28.67dB
	Max. 30 iterations	17.96dB	20.69dB	23.21dB	26.01dB	28.59dB
	Difference	0.14	0.18	0.16	0.1	0.08



BURST NOISE CHANNEL MODEL

- **Two OFDM symbol durations**

- 20 μs
- 40 μs

- **Cyclic prefix**

- 2.5 μs

- **Channel assumption**

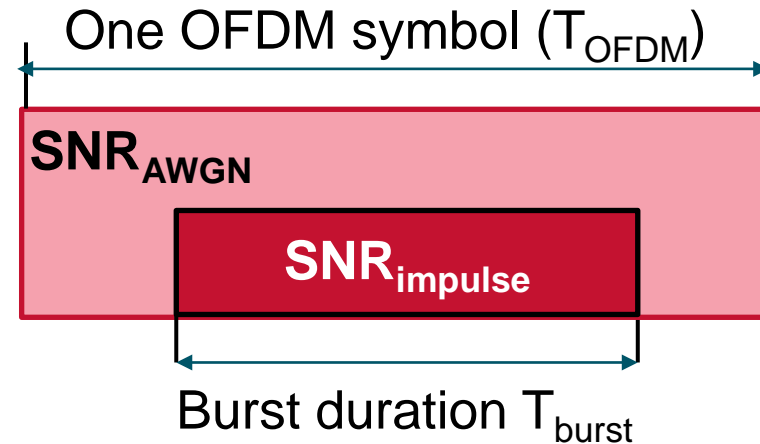
Burst noise	Duration	SNR
Downstream	16 μs	20dB
	16 μs	5dB
Upstream	10 μs	10dB
	1 μs	0dB

- **Interleaver latency threshold**

- 1 ms total PHY delay (upstream and downstream), suppose 400 μs each
 - 20 μs symbol: $400/22.5=17.7$ interleave depth
 - 40 μs symbol: $400/42.5=9.4$ interleave depth

- **Targeted probability of error**

- BER=1e-8
- WER=1e-6



- **Case I: the burst hits one OFDM symbol**

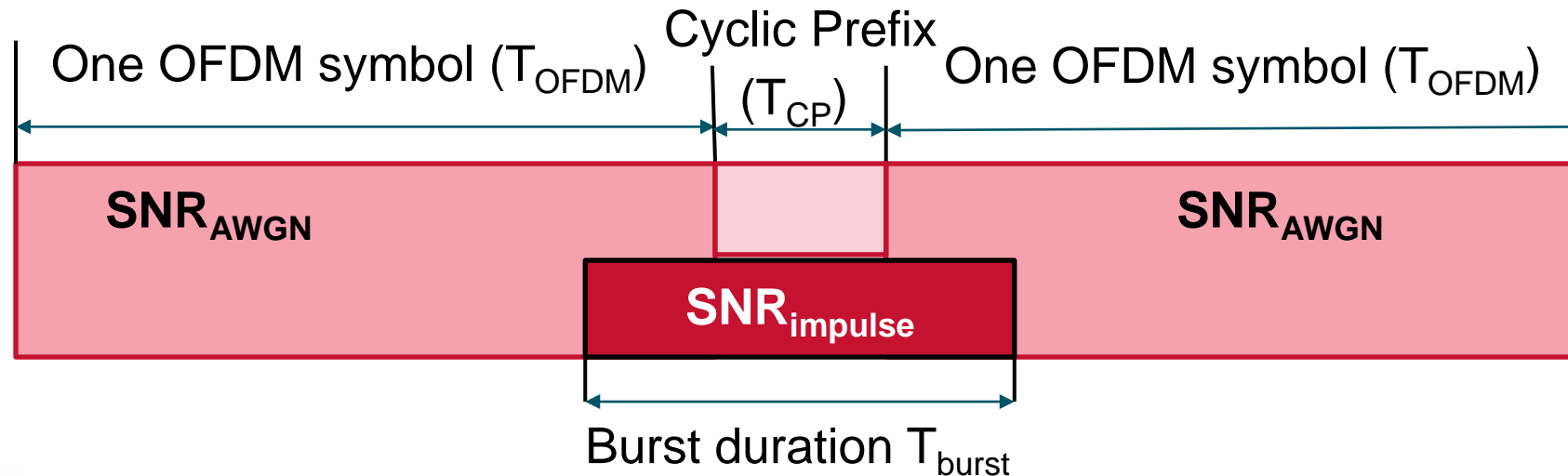
- SNR experienced by all sub-carriers in the OFDM symbol due to burst noise only is:

$$\text{SNR}_{\text{burst}} = \text{SNR}_{\text{impulse}} - 10 \log (T_{\text{burst}} / T_{\text{OFDM}})$$

T_{OFDM} :	OFDM symbol duration without cyclic prefix
T_{CP} :	duration of cyclic prefix
T_{burst} :	burst duration
$\text{SNR}_{\text{impulse}}$:	impulse SNR

- SNR experienced by all sub-carriers in the two OFDM symbols due to background noise only is:

$$\text{SNR}_{\text{background}} = \text{SNR}_{\text{AWGN}} - 10 \log (1 - [T_{\text{burst}} / T_{\text{OFDM}}])$$



- **Case II: the burst hits two consecutive OFDM symbols equally**

- SNR experienced by all sub-carriers in the two OFDM symbols due to burst noise only is:

$$\text{SNR}_{\text{burst}} = \text{SNR}_{\text{impulse}} - 10 \log (0.5 * (T_{\text{burst}} - T_{\text{CP}}) / T_{\text{OFDM}})$$

T_{OFDM} :	OFDM symbol duration without cyclic prefix
T_{CP} :	duration of cyclic prefix
T_{burst} :	burst duration
$\text{SNR}_{\text{impulse}}$:	impulse SNR

- SNR experienced by all sub-carriers in the two OFDM symbols due to background noise only is:

$$\text{SNR}_{\text{background}} = \text{SNR}_{\text{AWGN}} - 10 \log (1 - [0.5 * (T_{\text{burst}} - T_{\text{CP}}) / T_{\text{OFDM}}])$$

- **SNR on the burst noise impacted subcarrier in the presence of background AWGN is:**
 - $SNR_{\text{sub-carrier}} = -10 \text{ Log} (10^{[-SNR_{\text{burst}} / 10]} + 10^{[-SNR_{\text{background}} / 10]})$

$SNR_{\text{sub-carrier}}$: SNR experienced by all sub-carriers in the OFDM symbol

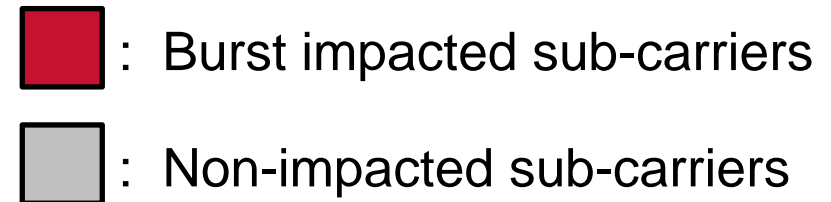
$SNR_{\text{background}}$: Background (thermal) Additive White Gaussian noise contribution

SNR_{burst} : Burst SNR contribution

- Our simulations show the minimum value for interleave depth N in order for the BER to reach $1e-8$ for a given worst case burst noise position at a specified duration and SNR and the associated minimum background AWGN SNR

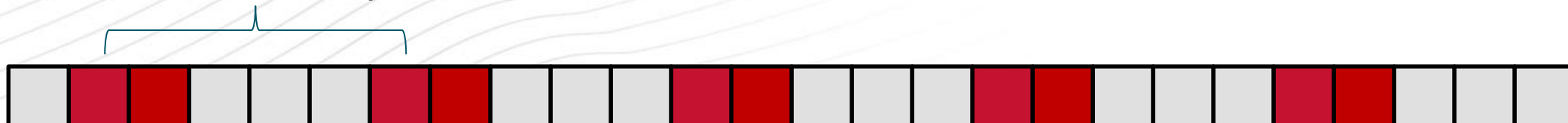
- Simulated cases**

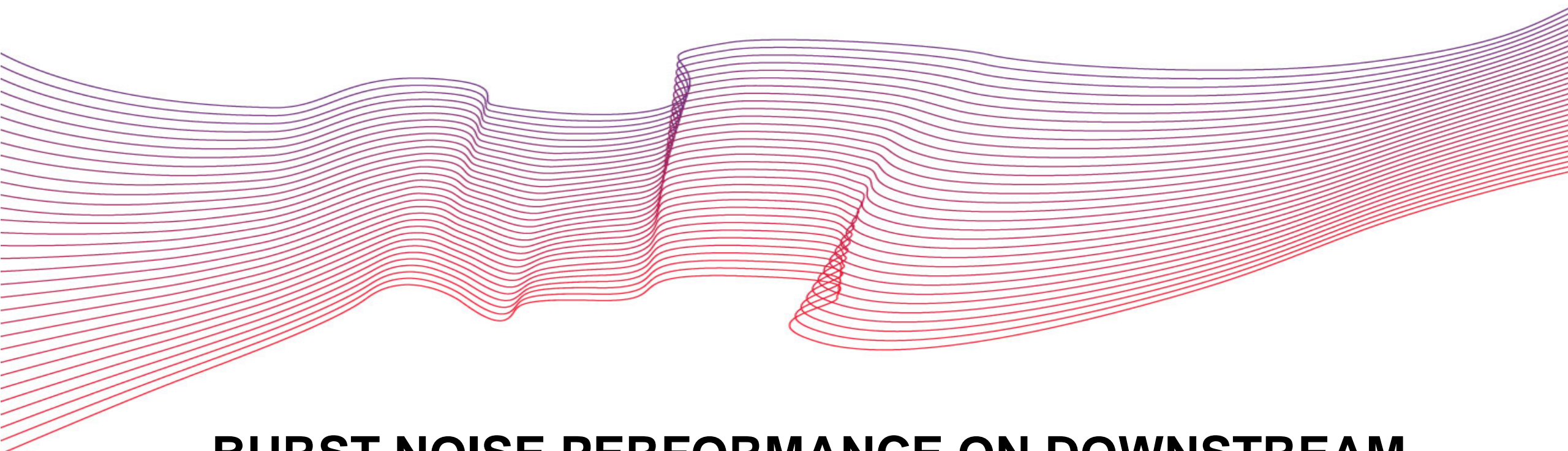
- Case 1: one OFDM symbol is impacted:
N sub-carriers apart



- Case 2 two consecutive OFDM symbols are impacted equally:

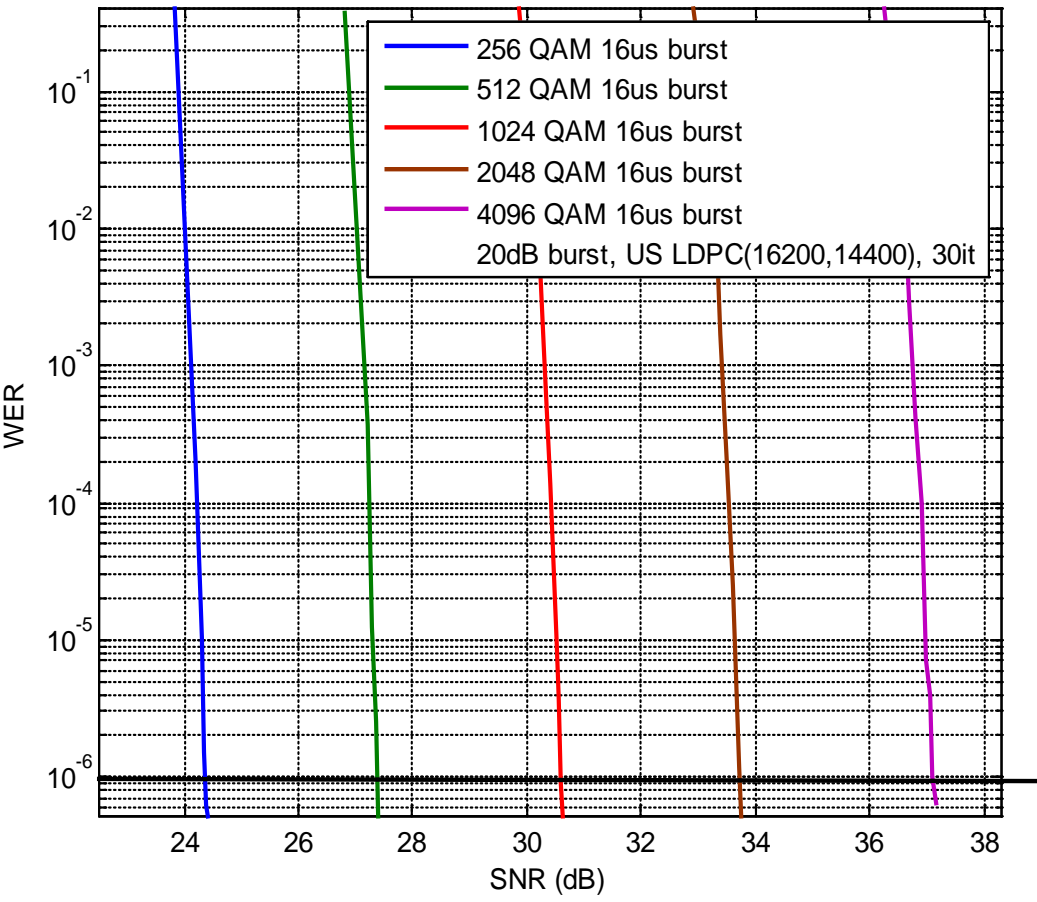
N sub-carriers apart



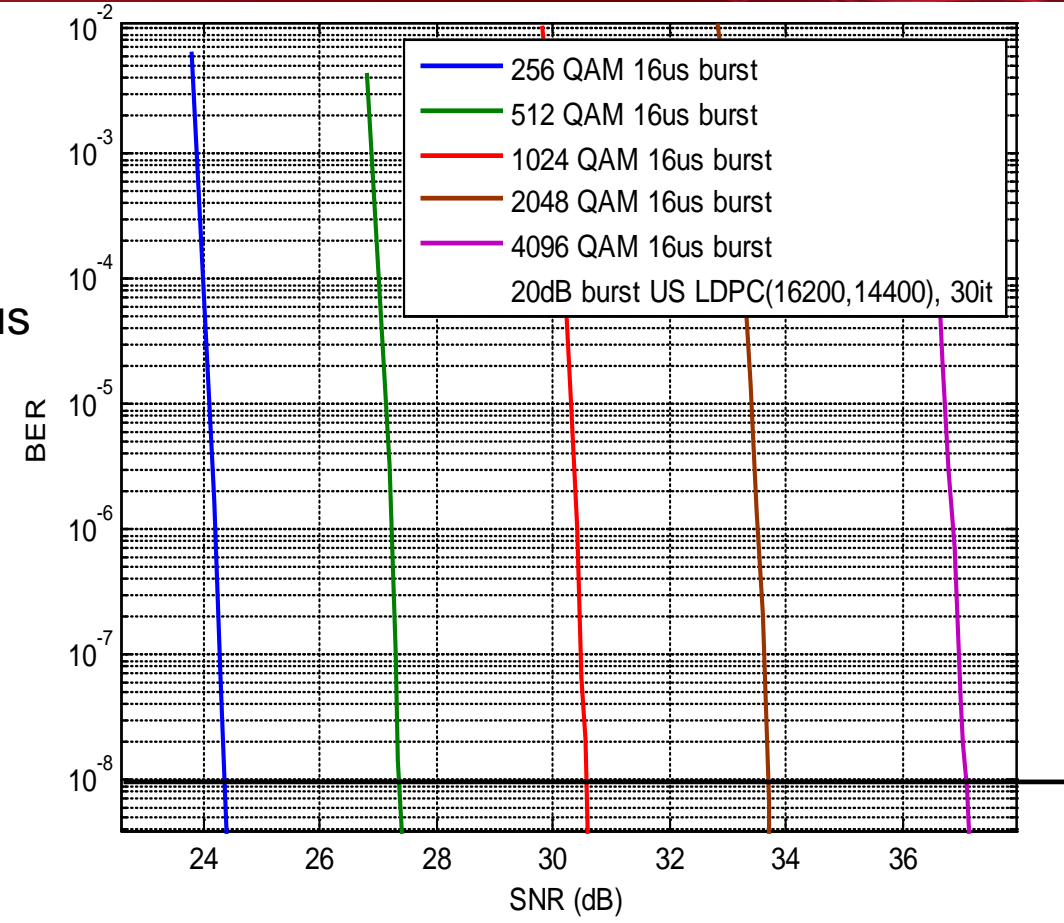


BURST NOISE PERFORMANCE ON DOWNSTREAM

20μs SYMBOLS ON 16μs 20dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS

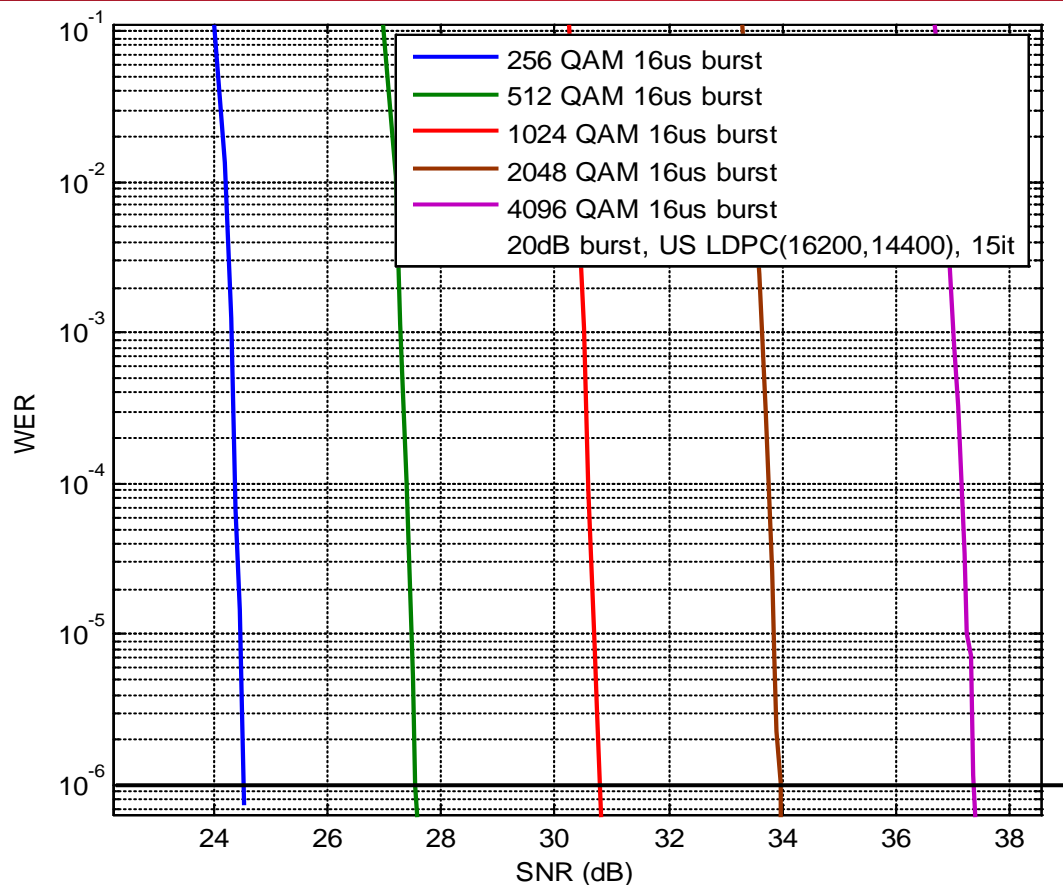


20μs symbol
(two affected)
Latency: 382.5μs
(depth:17)

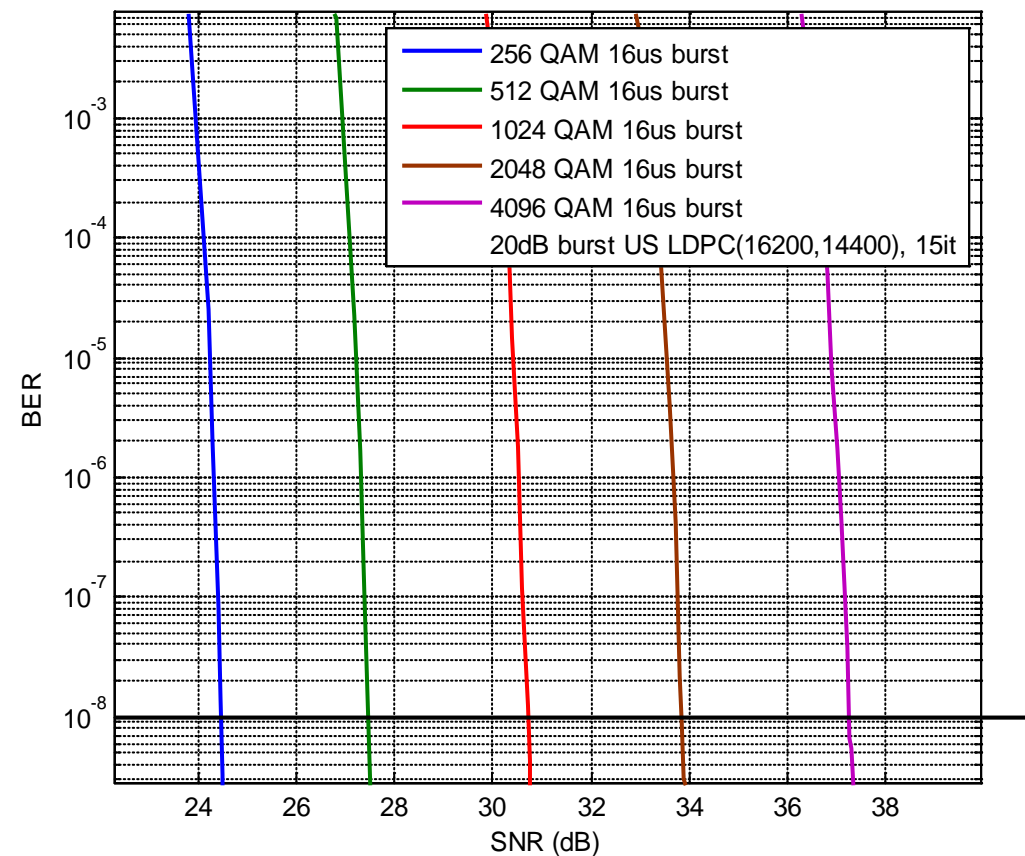


	256QAM	512QAM	1024QAM	2048QAM	4096QAM
SNR@WER=1e-6	24.38dB	27.38dB	30.59dB	33.71dB	37.10dB
SNR@BER=1e-8	24.36dB	27.36dB	30.58dB	33.69dB	37.09dB

20μs SYMBOLS ON 16μs 20dB BURST (LONG SIZE CODE) MAX 15 ITERATIONS



20μs symbol
(two affected)
Latency: 382.5μs
(depth:17)



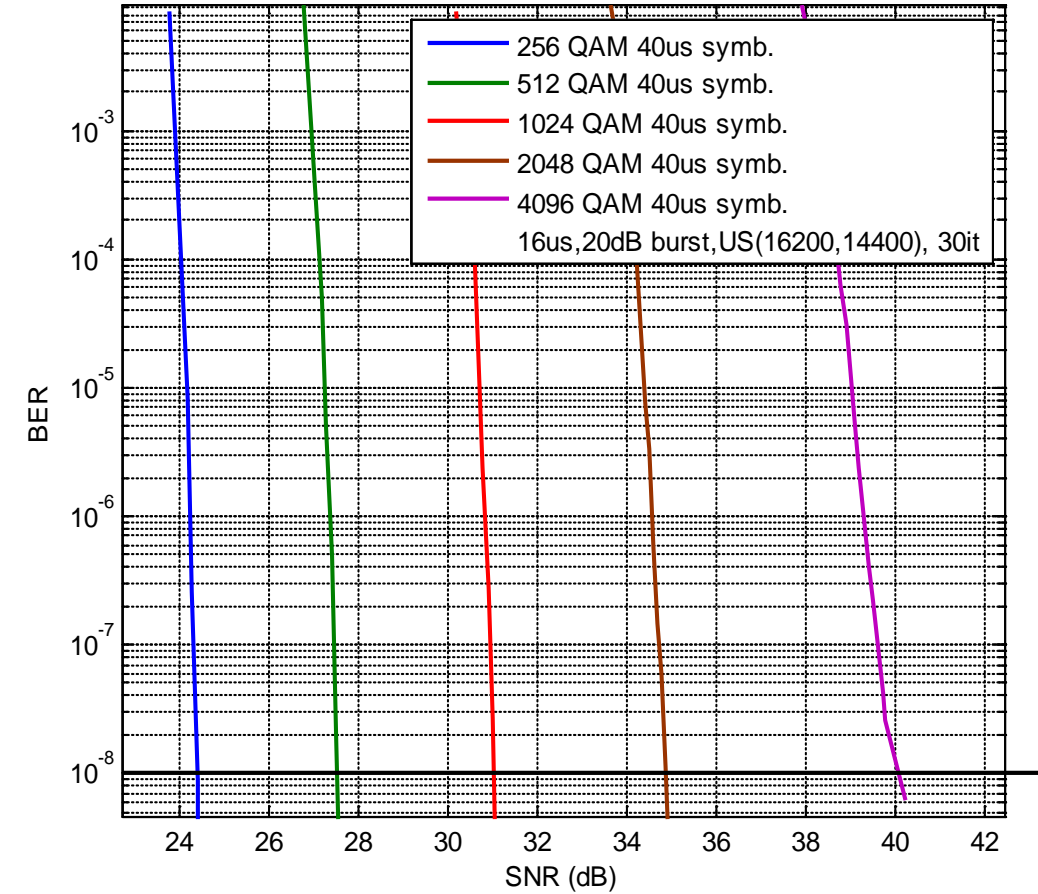
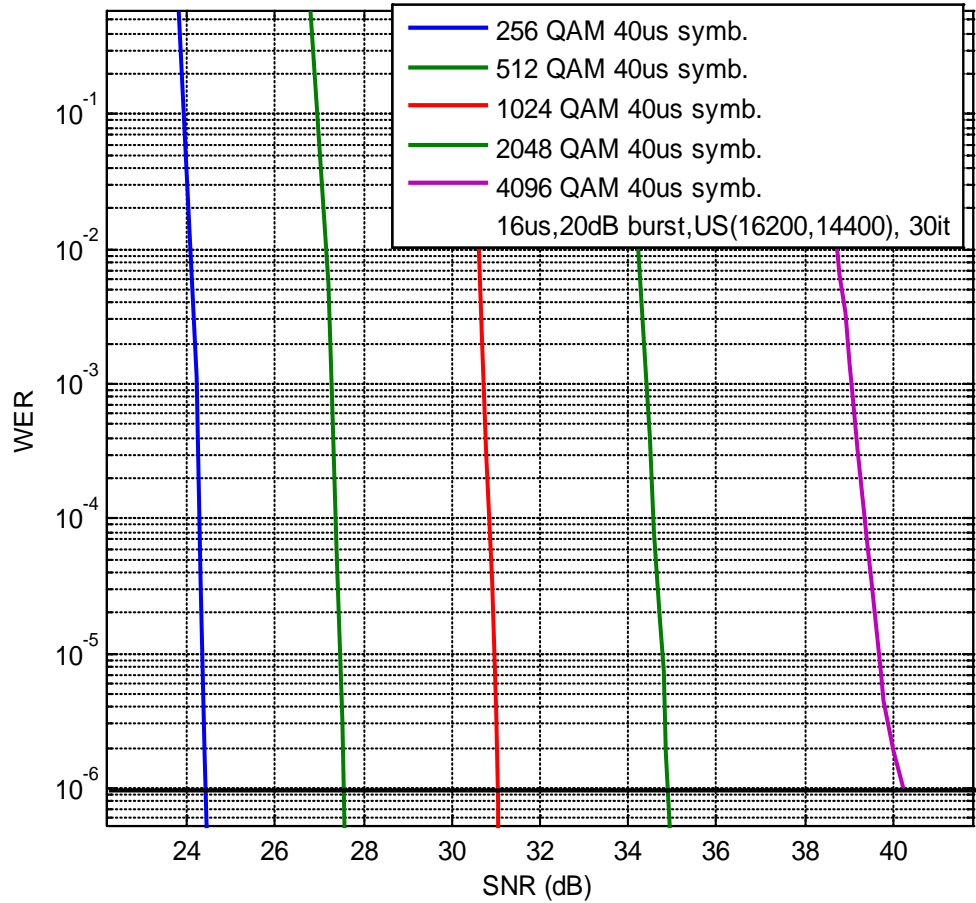
	256QAM	512QAM	1024QAM	2048QAM	4096QAM
SNR@WER=1e-6	24.54dB	27.55dB	30.78dB	33.95dB	37.36dB
SNR@BER=1e-8	24.26dB	27.46dB	30.71dB	33.84dB	37.24dB

20μs SYMBOLS ON 16μs 20dB BURST (LONG SIZE CODE)



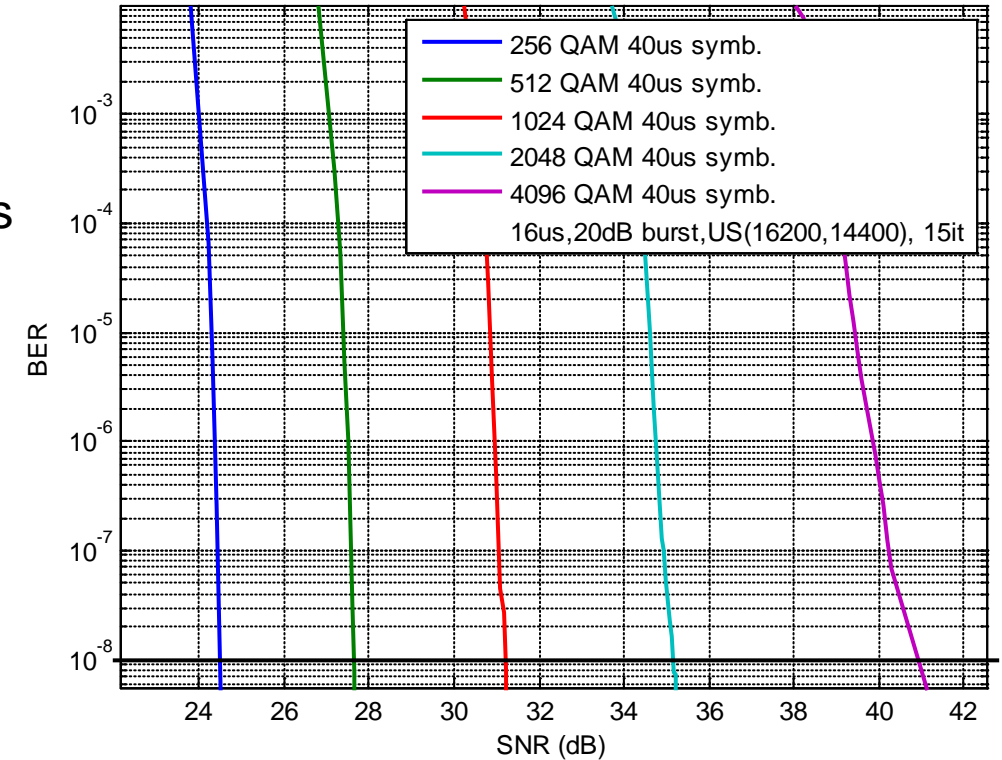
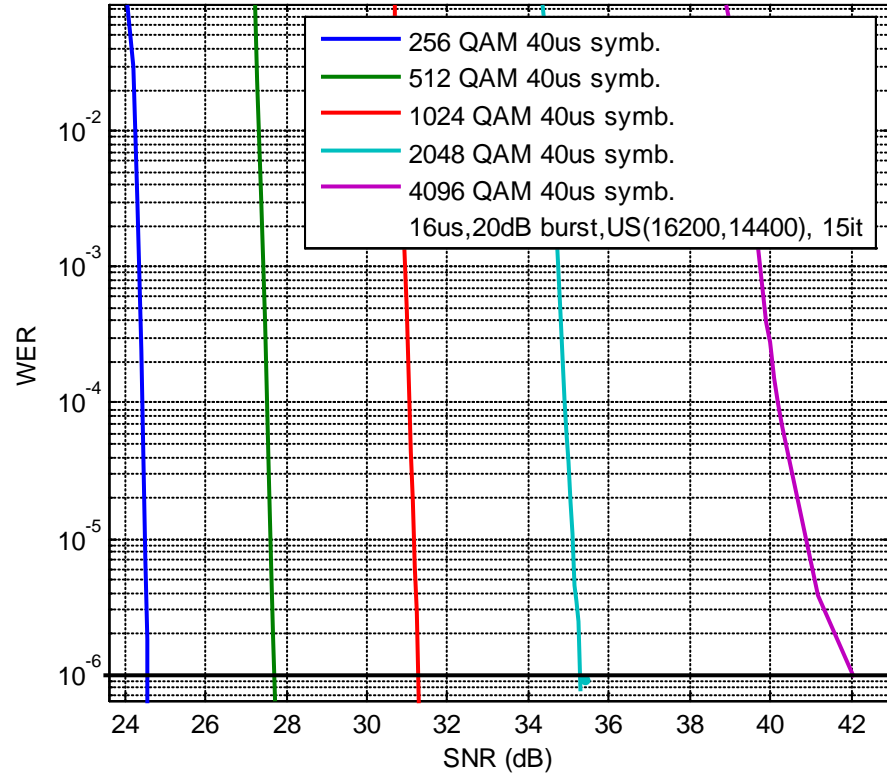
		64QAM	128QAM	256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	24.54dB	27.55dB	30.78dB	33.95dB	37.36dB
	Max. 30 iterations	24.38dB	27.38dB	30.59dB	33.71dB	37.10dB
	Difference	0.16	0.17	0.19	0.24	0.26
SNR@BER=1e-8	Max.15 iterations	24.26dB	27.46dB	30.71dB	33.84dB	37.24dB
	Max. 30 iterations	24.36dB	27.36dB	30.58dB	33.69dB	37.09dB
	Difference	0.1	0.1	0.13	0.15	0.15

40μs SYMBOLS ON 16μs 20dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS



	256QAM	512QAM	1024QAM	2048QAM	4096QAM
SNR@WER=1e-6	24.41dB	27.53dB	31.04dB	34.9dB	40.25dB
SNR@BER=1e-8	24.4dB	27.52dB	31.03dB	34.86dB	40.1dB

40μs SYMBOLS ON 16μs 20dB BURST (UPSTREAM LONG CODE) MAX 15 ITERATIONS



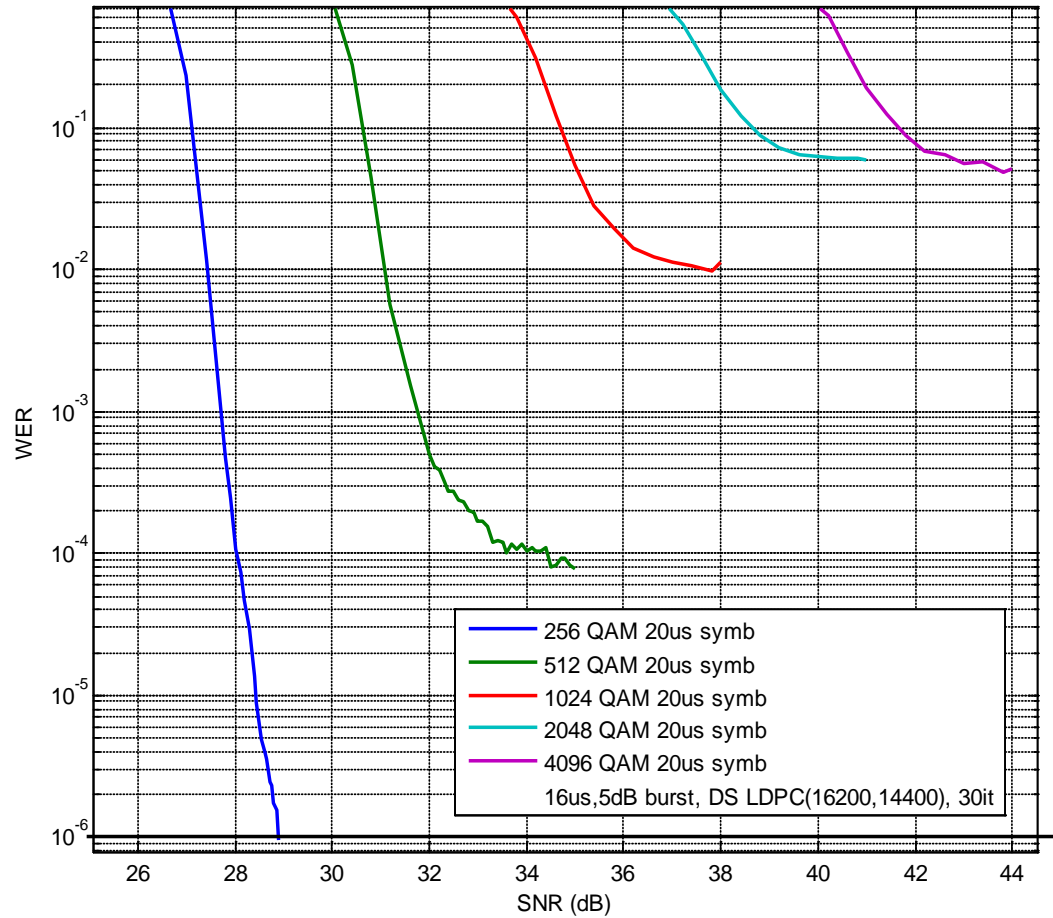
	256QAM	512QAM	1024QAM	2048QAM	4096QAM
SNR@WER=1e-6	24.57dB	27.73dB	31.28dB	35.29dB	42dB
SNR@BER=1e-8	24.48dB	27.64dB	31.19dB	35.13dB	40.9dB

40μs SYMBOLS ON 16μs 20dB BURST (UPSTREAM LONG CODE)

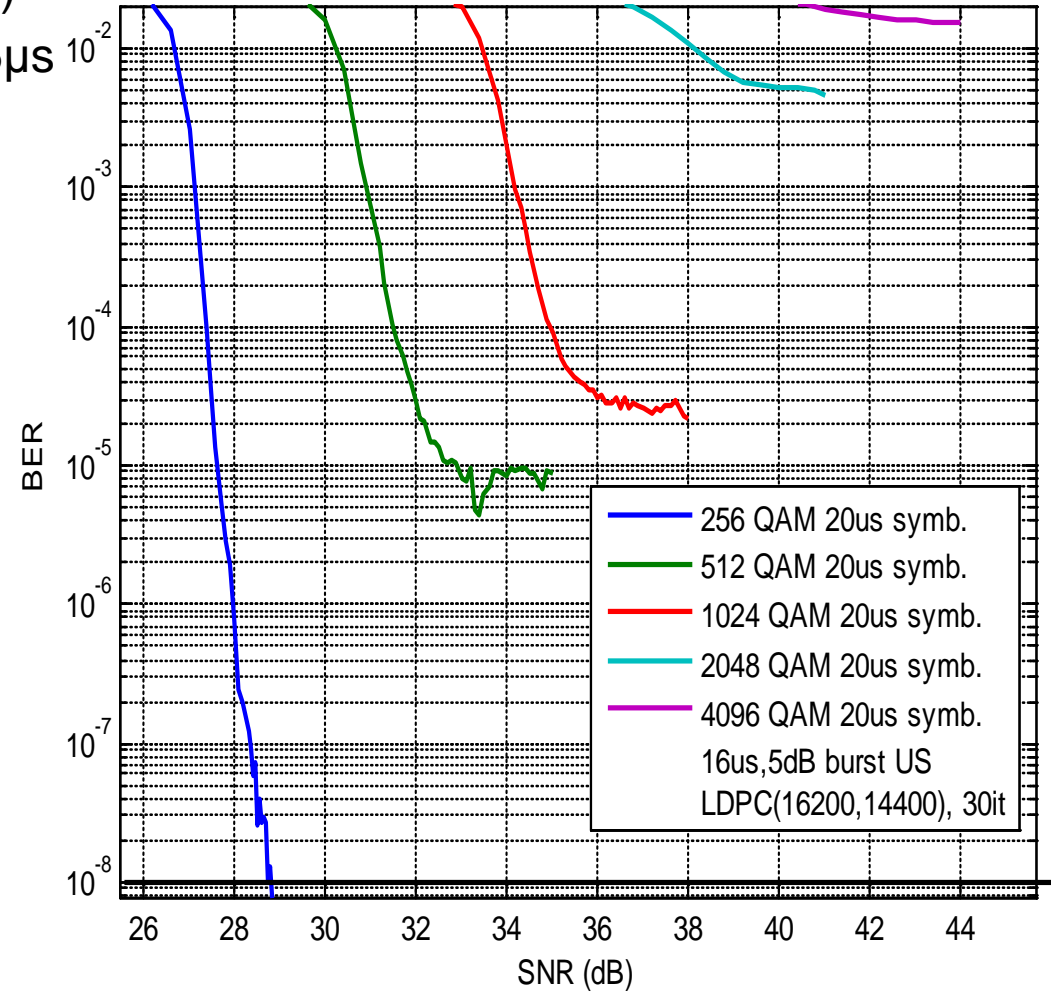


		64QAM	128QAM	256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	24.57dB	27.73dB	31.28dB	35.29dB	42dB
	Max. 30 iterations	24.41dB	27.53dB	31.04dB	34.9dB	40.25dB
	Difference	0.16	0.2	0.24	1.2	1.75
SNR@BER=1e-8	Max.15 iterations	24.48dB	27.64dB	31.19dB	35.13dB	40.9dB
	Max. 30 iterations	24.4dB	27.52dB	31.03dB	34.86dB	40.1dB
	Difference	0.04	0.12	0.16	0.27	0.8

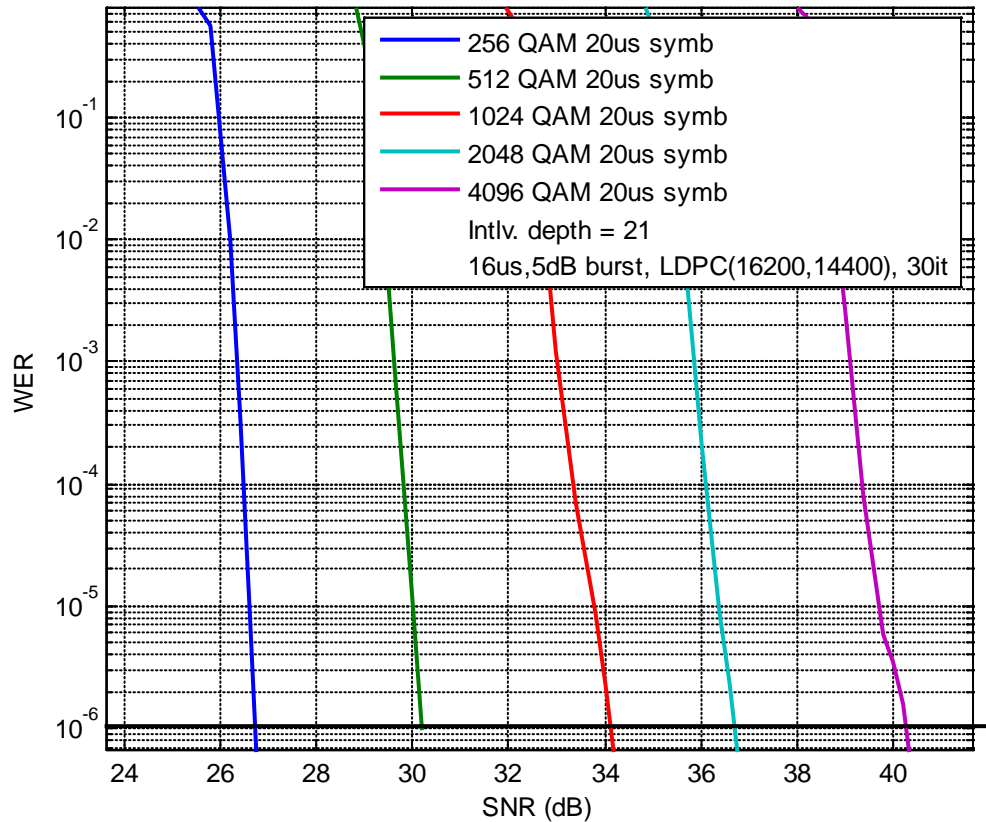
20 μ s SYMBOLS ON 16 μ s 5dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS



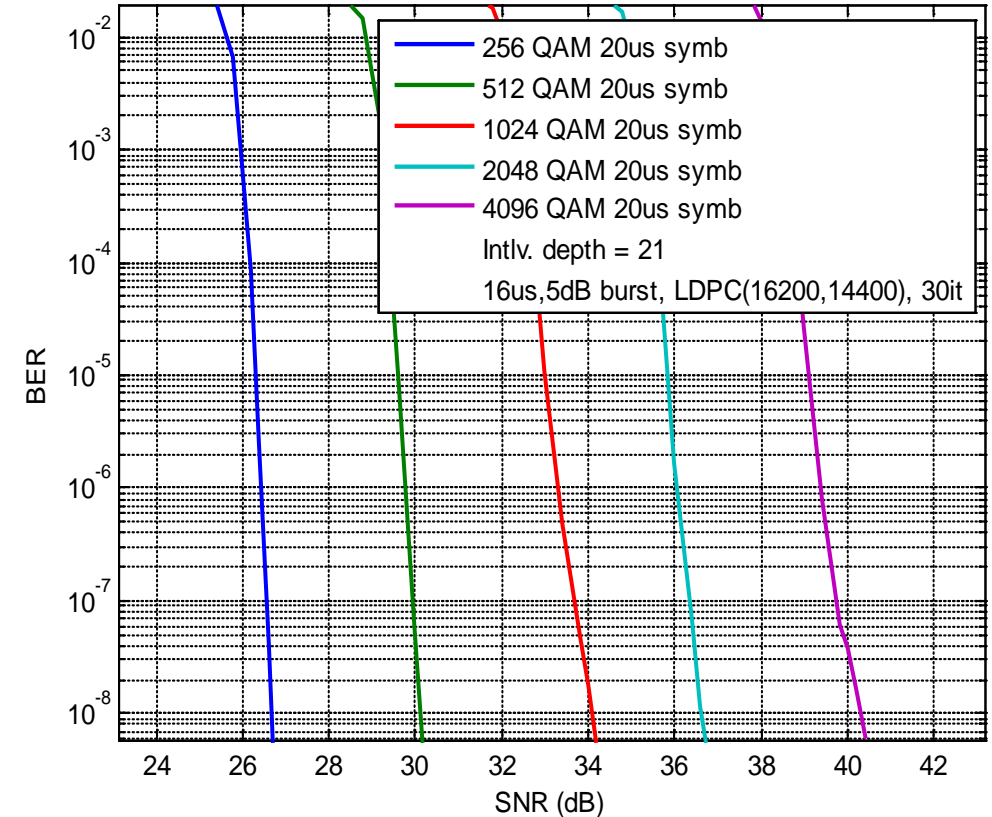
20 μ s symbol
(two affected)
Latency: 382.5 μ s
(depth:17)



20 μ s SYMBOLS ON 16 μ s 5dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS



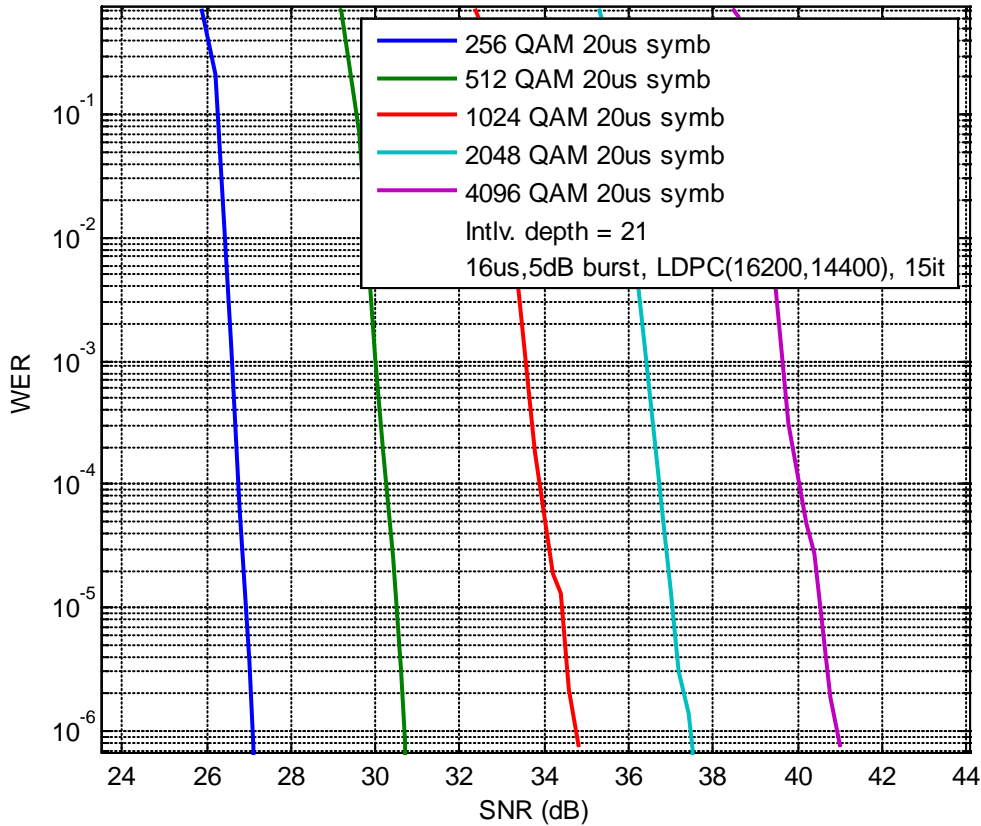
20 μ s symbol
(two affected)
Latency: 472.5 μ s
(depth: 21)



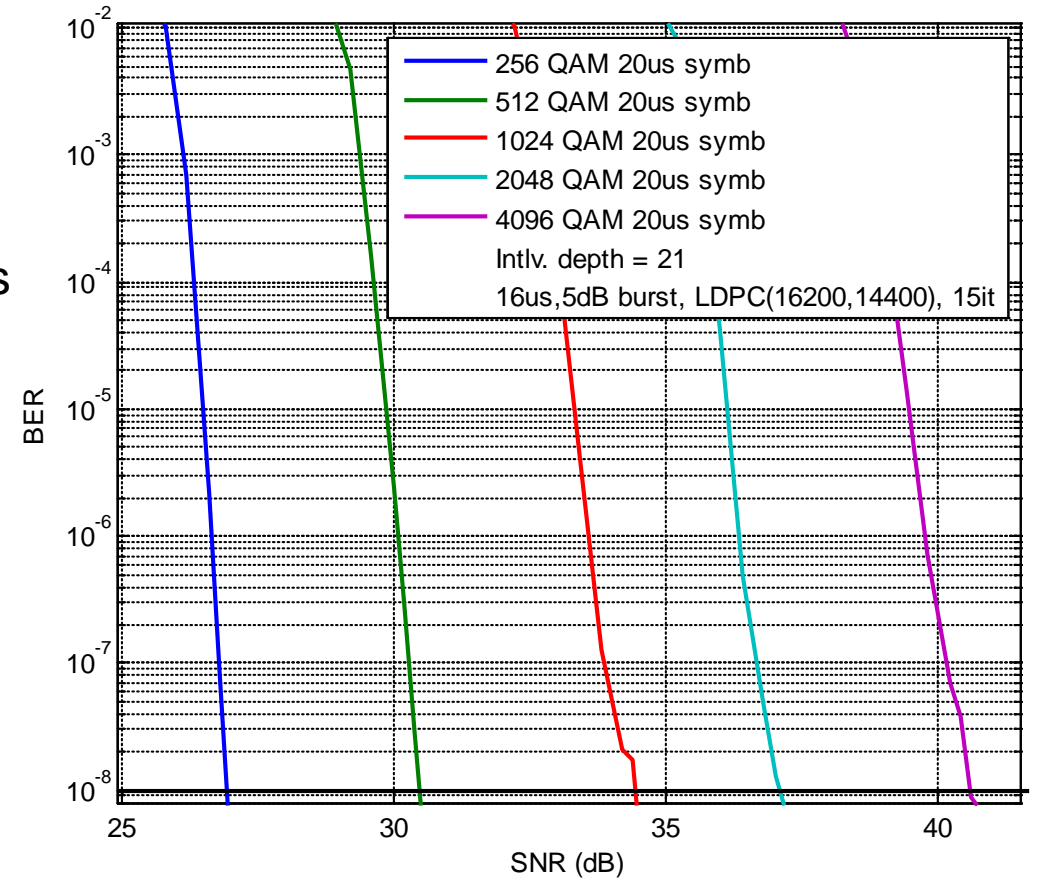
Remark: latency 450 μ s (depth 20) works for 30 iterations, but not for 15 iterations

	256QAM	512QAM	1024QAM	2048QAM	4096QAM
SNR@WER=1e-6	26.72dB	30.20dB	34.12dB	36.72dB	40.28dB
SNR@BER=1e-8	26.68dB	30.13dB	34.09dB	36.62dB	40.3dB

20 μ s SYMBOLS ON 16 μ s 5dB BURST (LONG SIZE CODE) MAX 15 ITERATIONS



20 μ s symbol
(two affected)
Latency: 472.5 μ s
(depth: 21)



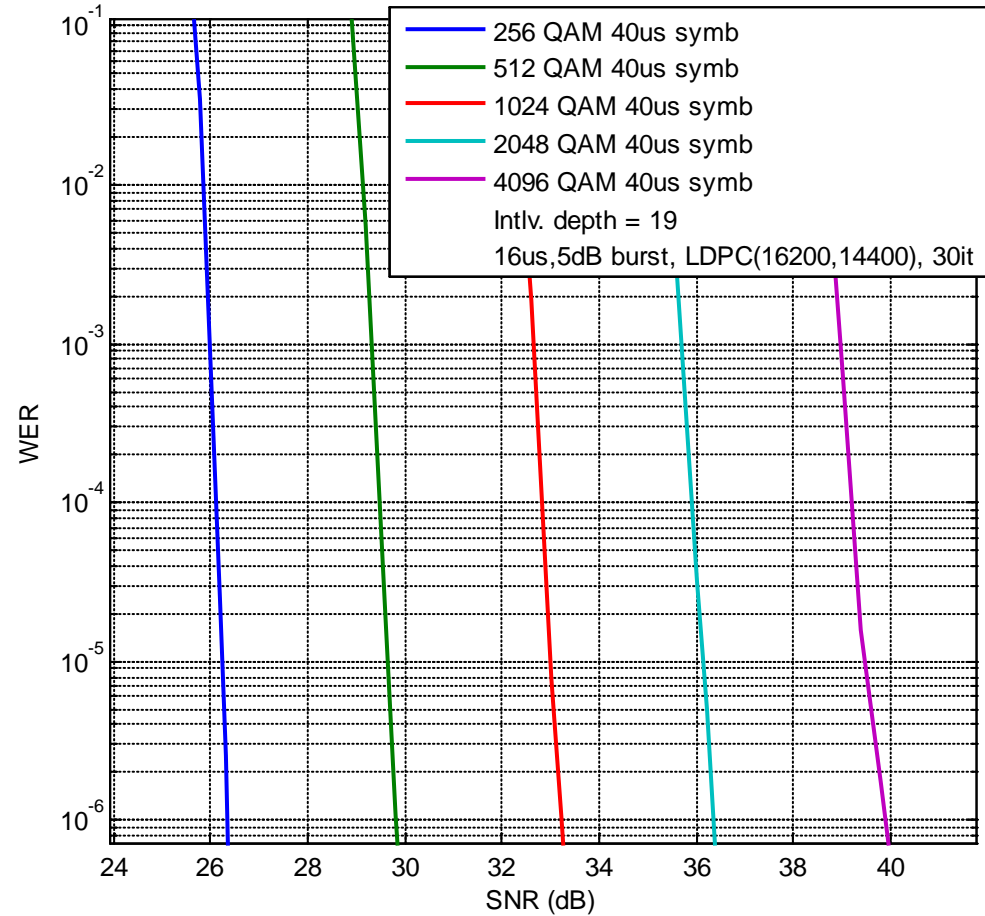
	256QAM	512QAM	1024QAM	2048QAM	4096QAM
SNR@WER=1e-6	27.08dB	30.68dB	34.74dB	37.45dB	40.94dB
SNR@BER=1e-8	26.94dB	30.49dB	34.45dB	37.07dB	40.58dB

20μs SYMBOLS ON 16μs 5dB BURST (LONG SIZE CODE)

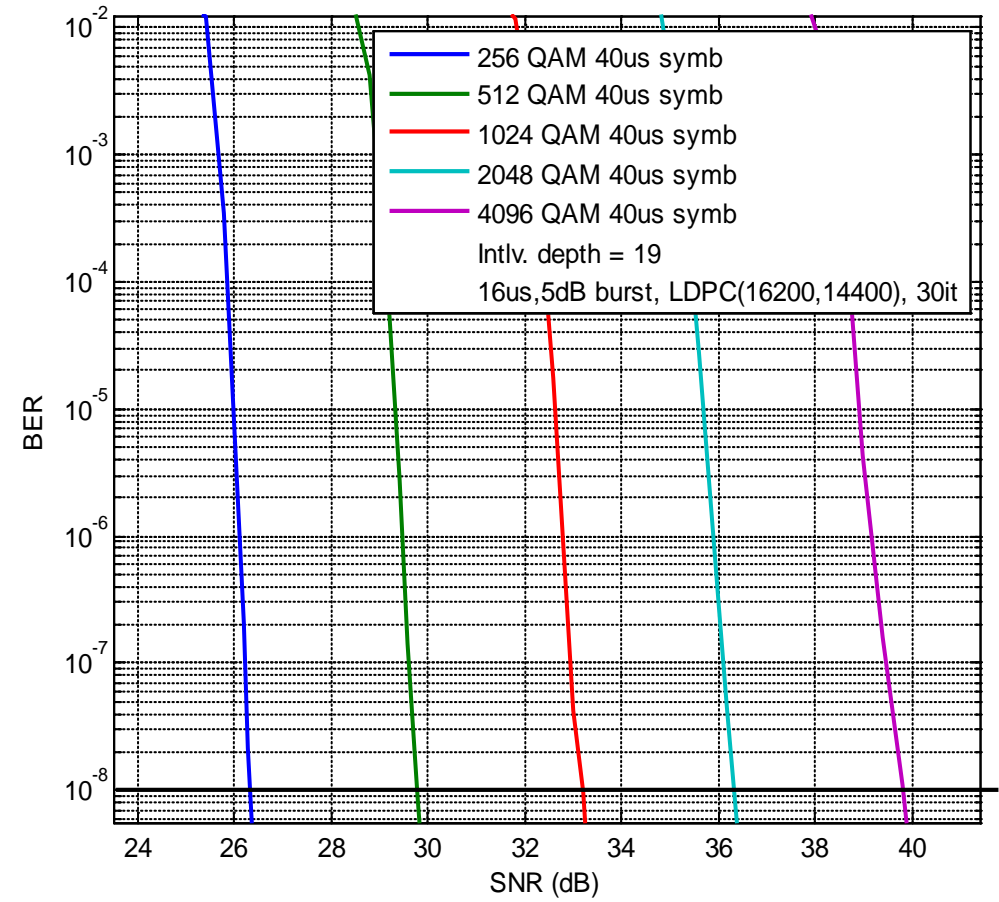


		64QAM	128QAM	256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	27.08dB	30.68dB	34.74dB	37.45dB	40.94dB
	Max. 30 iterations	26.72dB	30.20dB	34.12dB	36.72dB	40.28dB
	Difference	0.36	0.48	0.62	0.73	0.66
SNR@BER=1e-8	Max.15 iterations	26.94dB	30.49dB	34.45dB	37.07dB	40.58dB
	Max. 30 iterations	26.68dB	30.13dB	34.09dB	36.62dB	40.3dB
	Difference	0.26	0.36	0.36	0.35	0.128

40μs SYMBOLS ON 16μs 5dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS



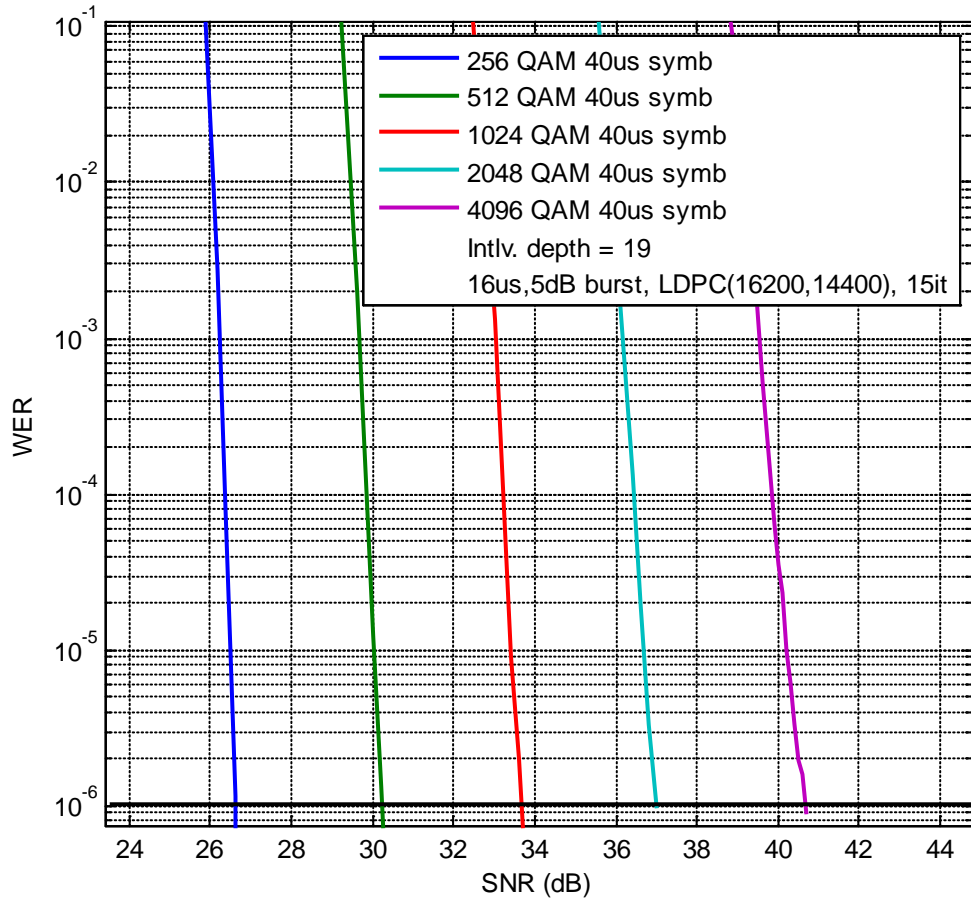
40μs symbol
(two affected)
Latency: 807.5.5μs
(depth: 19)



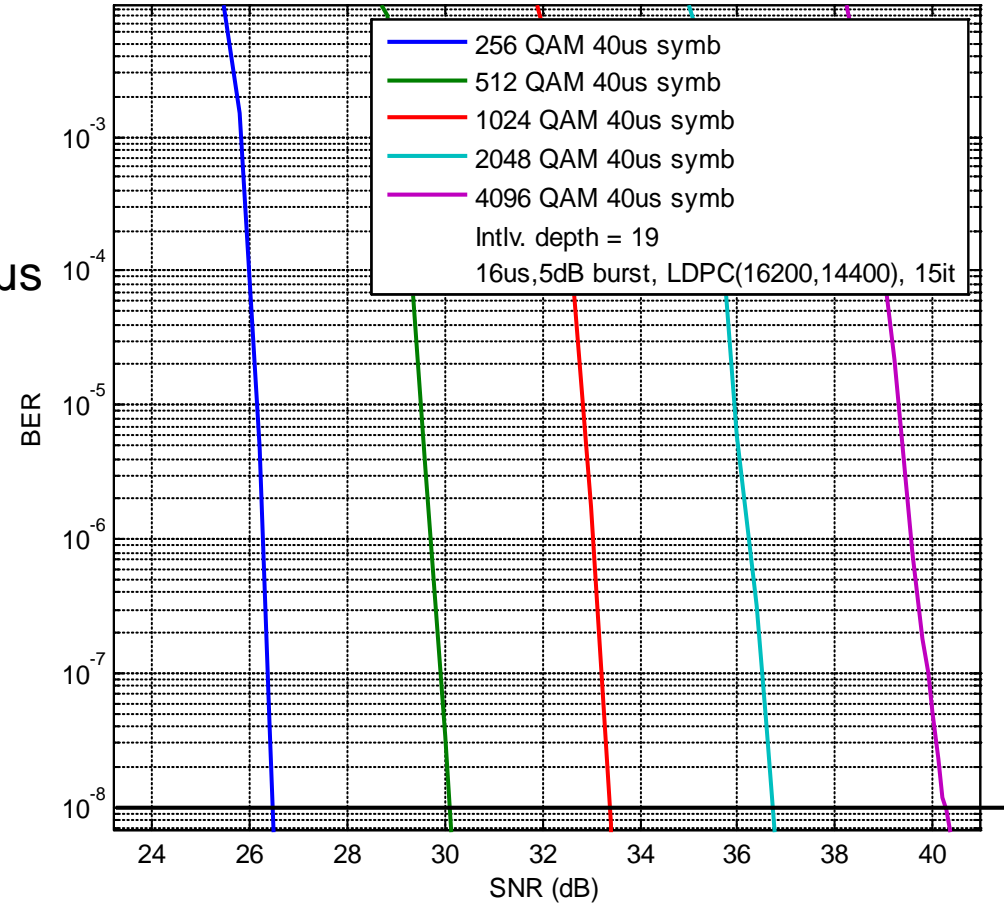
Remark: latency 765μs (depth 18) works for 30 iterations, but not for 15 iterations

	256QAM	512QAM	1024QAM	2048QAM	4096QAM
SNR@WER=1e-6	26.35dB	29.8dB	33.21dB	36.35dB	39.89dB
SNR@BER=1e-8	26,34dB	29.8dB	33.2dB	36.33dB	39.81dB

40μs SYMBOLS ON 16μs 5dB BURST (LONG SIZE CODE) MAX 15 ITERATIONS



40μs symbol
 (two affected)
 Latency: 807.5.5μs
 (depth: 19)



	256QAM	512QAM	1024QAM	2048QAM	4096QAM
SNR@WER=1e-6	26.61dB	30.22dB	33.68dB	36.99dB	40.68dB
SNR@BER=1e-8	26.47dB	30.09dB	33.38dB	36.73dB	40.28dB

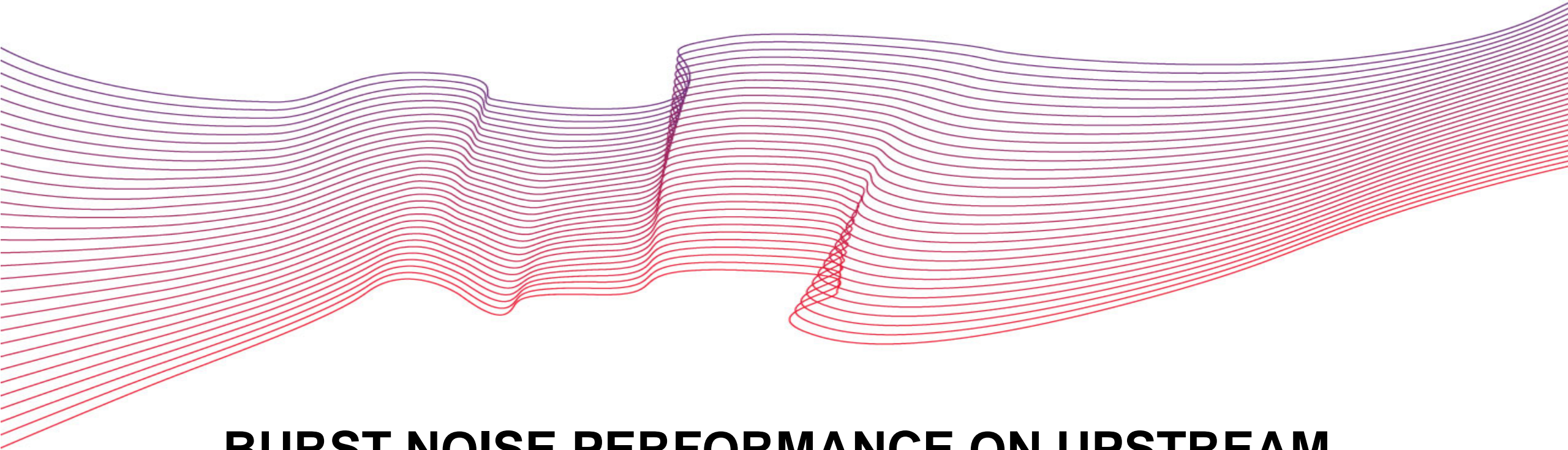
40μs SYMBOLS ON 16μs 5dB BURST (LONG SIZE CODE)

		64QAM	128QAM	256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	26.61dB	30.22dB	33.68dB	36.99dB	40.68dB
	Max. 30 iterations	26.35dB	29.8dB	33.21dB	36.35dB	39.89dB
	Difference	0.26	0.42	0.47	0.64	0.79
SNR@BER=1e-8	Max.15 iterations	26.47dB	30.09dB	33.38dB	36.73dB	40.28dB
	Max. 30 iterations	26,34dB	29.8dB	33.2dB	36.33dB	39.81dB
	Difference	0.13	0.29	0.18	0.4	0.47

MINIMUM INTERLEAVE DEPTH UNDER BURST NOISE (LONG SIZE CODE)

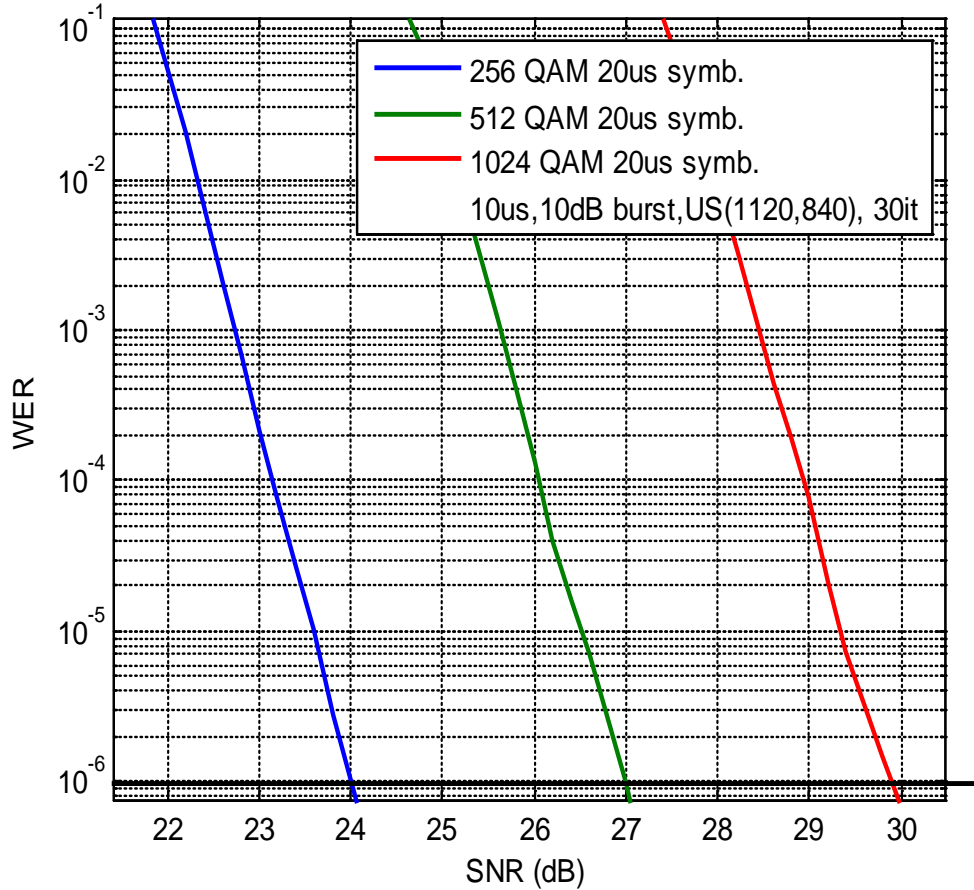
Achieving BER=1e-8/WER=1e-6 (4096-QAM)

	Burst duration	Burst SNR	Minimum Interleave depth
20μs symbol (two affected)	16μs	20dB	11 247.5μs
	16μs	5dB	21 472.5μs
40μs symbol (two affected)	16μs	20dB	8 340μs
	16μs	5dB	19 807.5μs

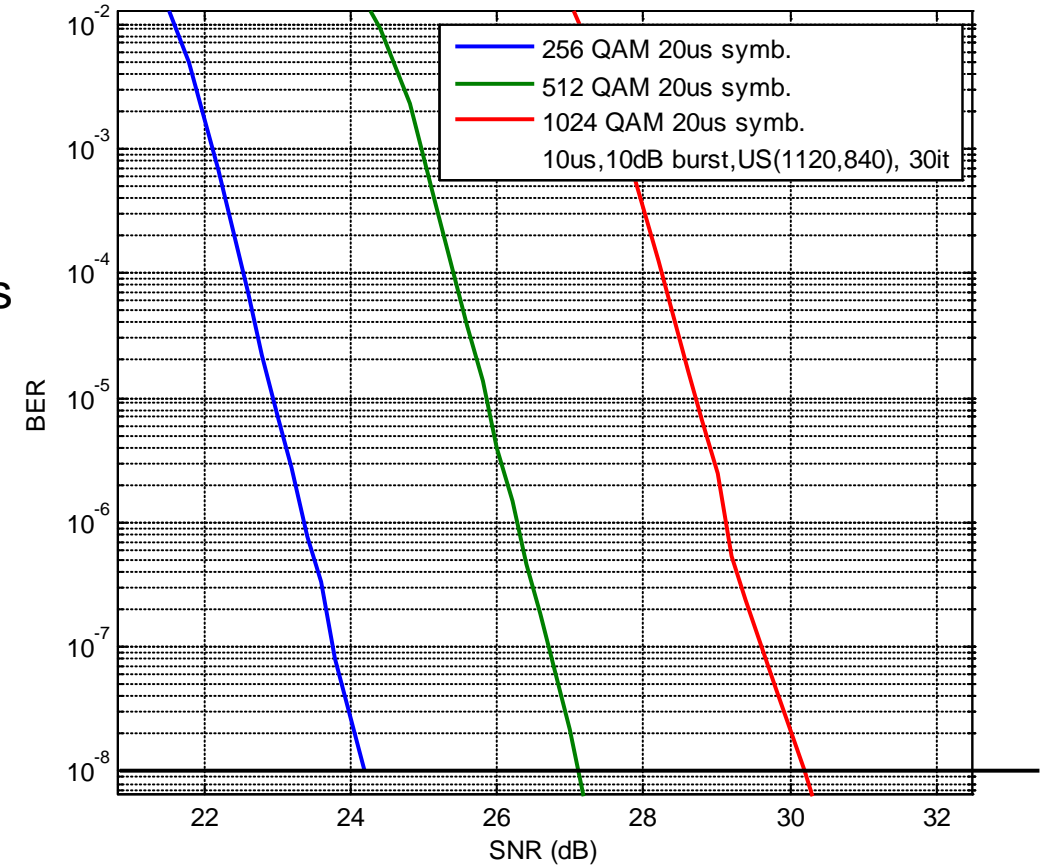


BURST NOISE PERFORMANCE ON UPSTREAM

20μs SYMBOLS ON 10μs 10dB BURST (SHORT SIZE CODE) MAX 30 ITERATIONS

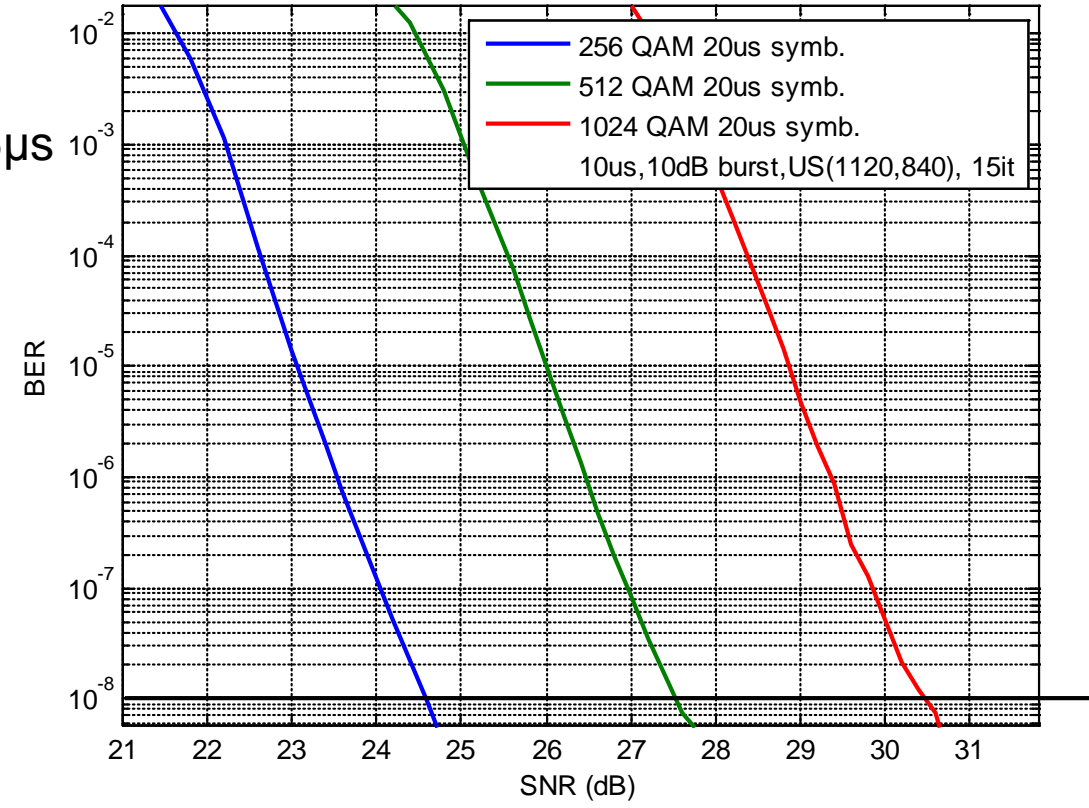
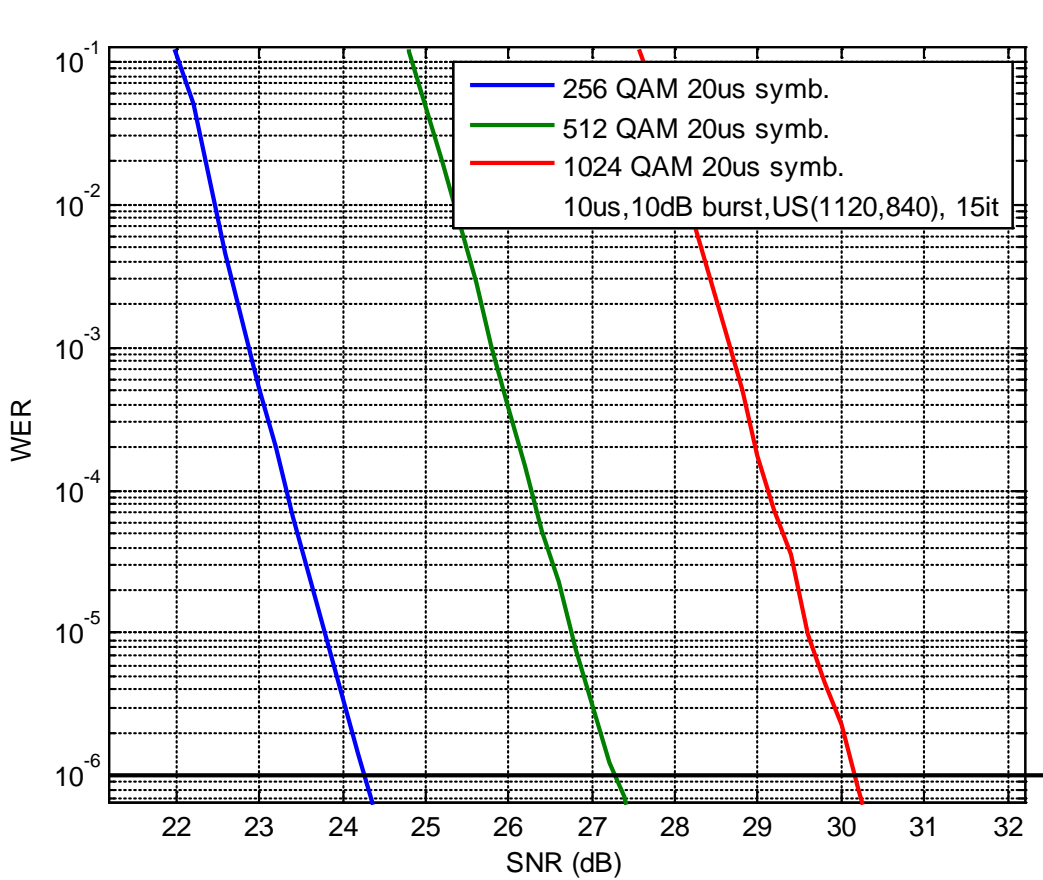


20μs symbol
(two affected)
Latency: 382.5μs
(depth:17)



	256QAM	512QAM	1024QAM
SNR@WER=1e-6	24.08dB	27.03dB	29.91dB
SNR@BER=1e-8	24.27dB	27.19dB	30.30dB

20μs SYMBOLS ON 10μs 10dB BURST (SHORT SIZE CODE) MAX 15 ITERATIONS



	256QAM	512QAM	1024QAM
SNR@WER=1e-6	24.27dB	27.28dB	30.16dB
SNR@BER=1e-8	24.58dB	27.55dB	30.47dB

DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (SHORT SIZE CODE)

20 μ s SYMBOLS ON 10 μ s 10dB BURST



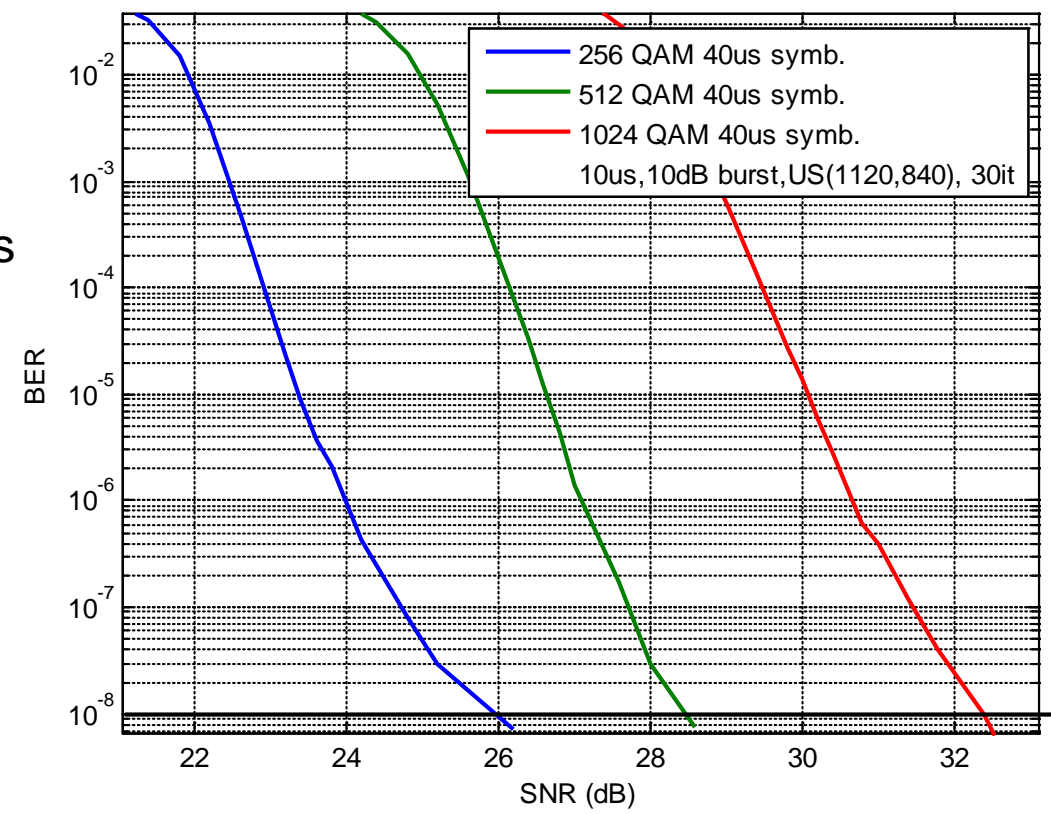
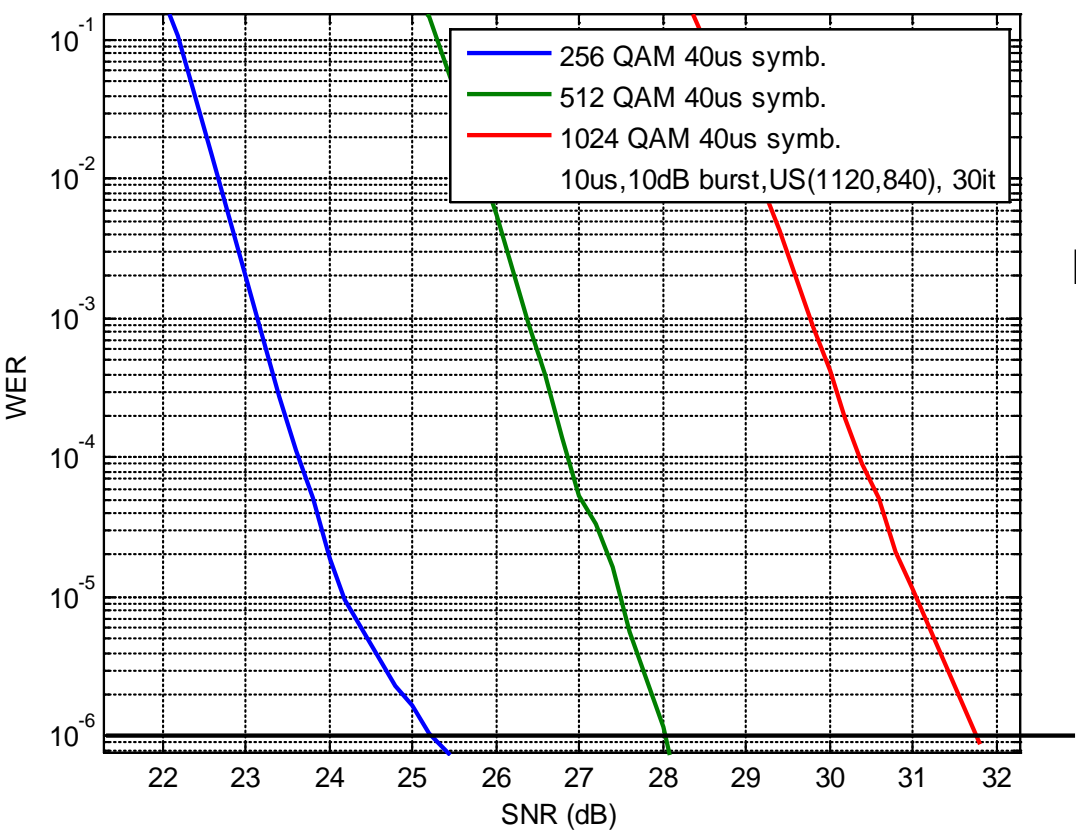
20 μ s symbol (two affected)

Latency: 382.5 μ s (depth:17)

Short size
(1120,840)
30 iterations
vs.
15 iterations

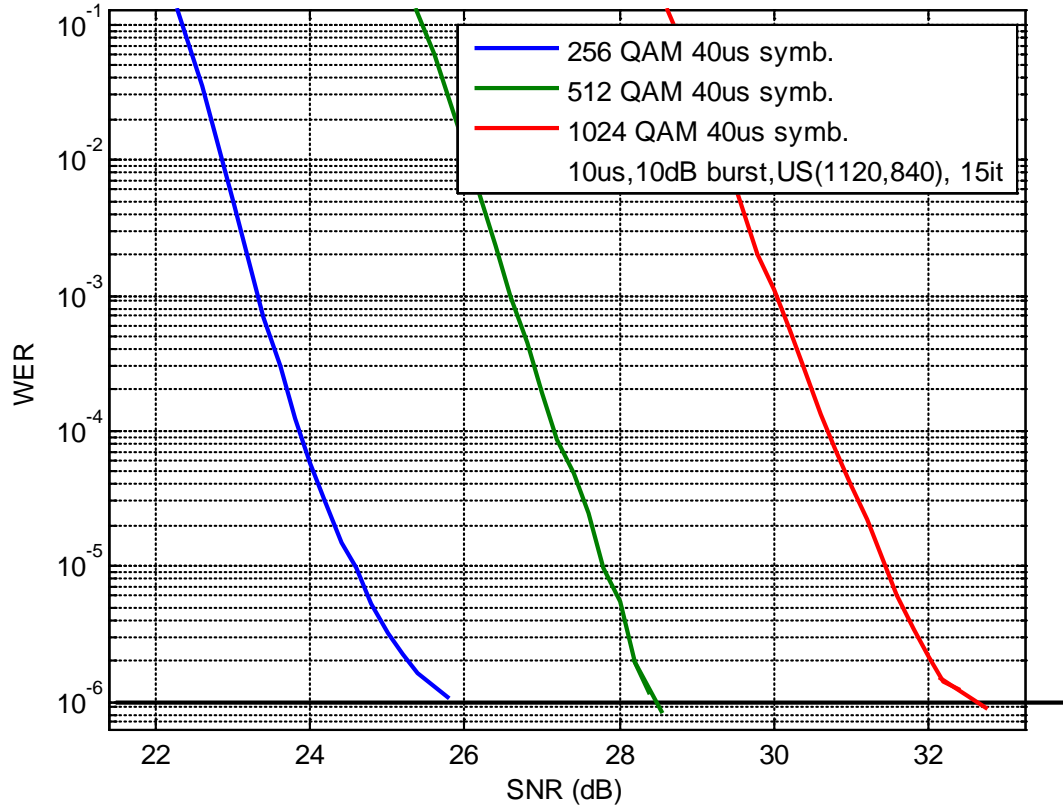
		256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	24.27dB	27.28dB	30.16dB
	Max.30 iterations	24.08dB	27.03dB	29.91dB
	Difference	0.19	0.25	0.25
SNR@BER=1e-8	Max. 15 iterations	24.58dB	27.55dB	30.47dB
	Max. 30 iterations	24.27dB	27.19dB	30.30dB
	Difference	0.31	0.36	0.17

40 μ s SYMBOLS ON 10 μ s 10dB BURST (SHORT SIZE CODE) MAX 30 ITERATIONS

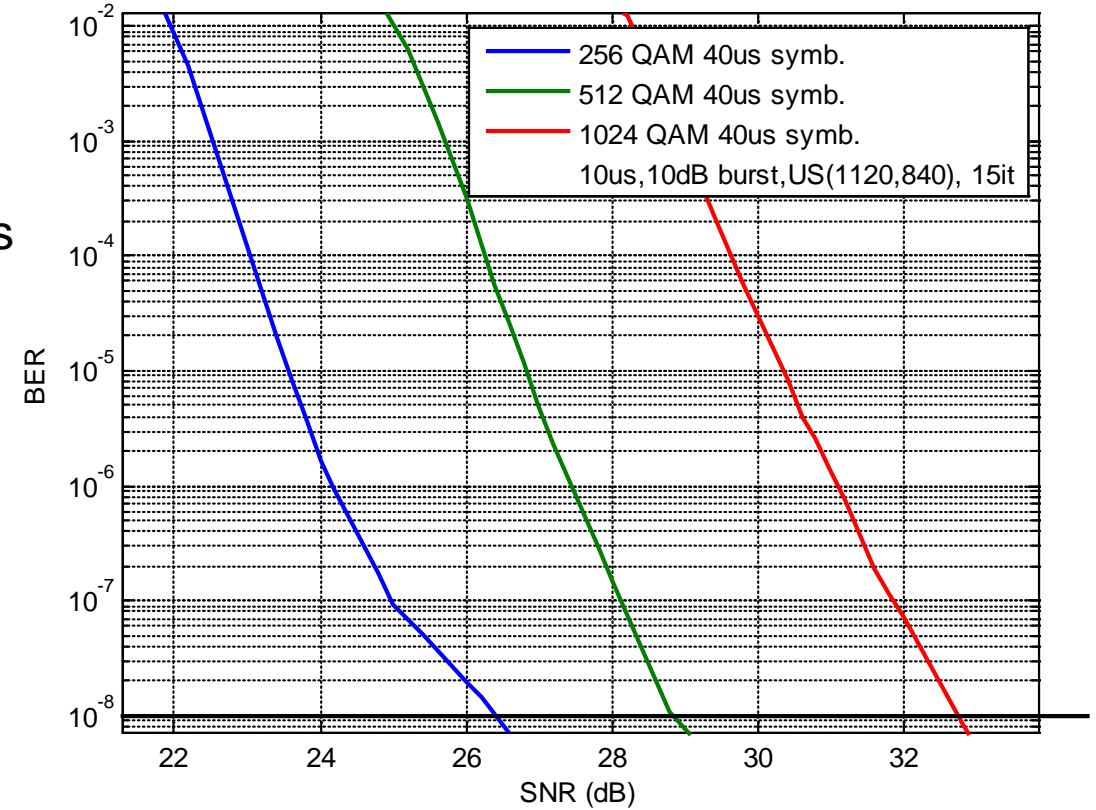


	256QAM	512QAM	1024QAM
SNR@WER=1e-6	25.25dB	28.03dB	31.8dB
SNR@BER=1e-8	25.97dB	28.52dB	32.42dB

40μs SYMBOLS ON 10μs 10dB BURST (SHORT SIZE CODE) MAX 15 ITERATIONS



40μs symbol
 (two affected)
 Latency: 340μs
 (depth:8)



	256QAM	512QAM	1024QAM
SNR@WER=1e-6	25.86dB	28.45dB	32.68dB
SNR@BER=1e-8	26.4dB	28.84dB	32.73dB

DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (SHORT SIZE CODE)

40 μ s SYMBOLS ON 10 μ s 10dB BURST

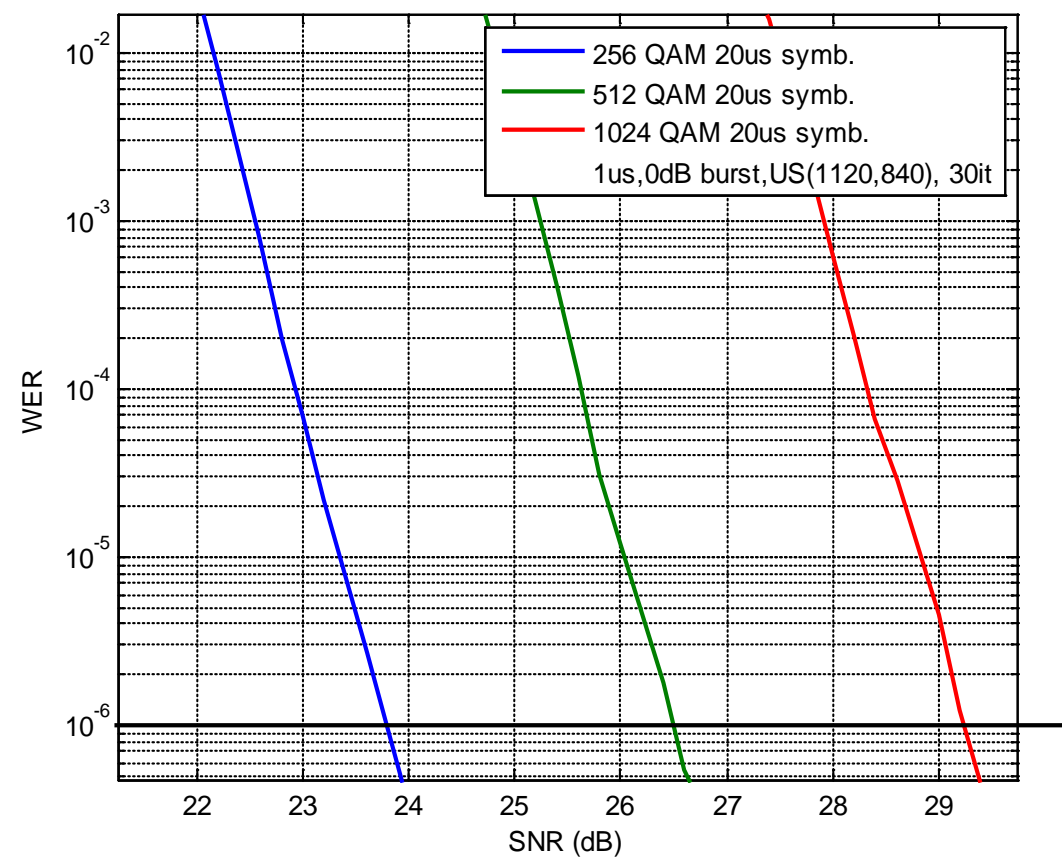
40 μ s symbol (two affected)

Latency: 340 μ s (depth:8)

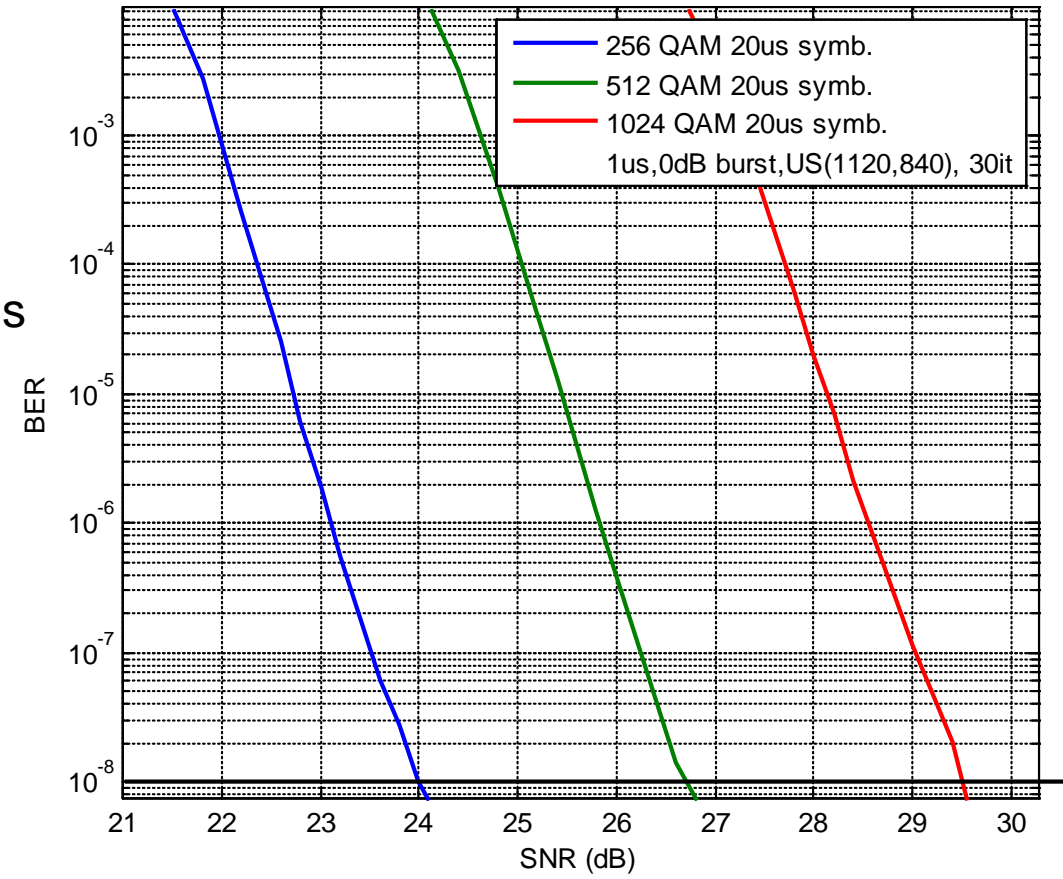
Short size
(1120,840)
30 iterations
vs.
15 iterations

		256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	25.86dB	28.45dB	32.68dB
	Max.30 iterations	25.25dB	28.03dB	31.8dB
	Difference	0.61	0.42	0.88
SNR@BER=1e-8	Max. 15 iterations	26.4dB	28.84dB	32.73dB
	Max. 30 iterations	25.97dB	28.52dB	32.42dB
	Difference	0.43	0.32	0.31

20 μ s SYMBOLS ON 1 μ s 0dB BURST (SHORT CODE) MAX 30 ITERATIONS

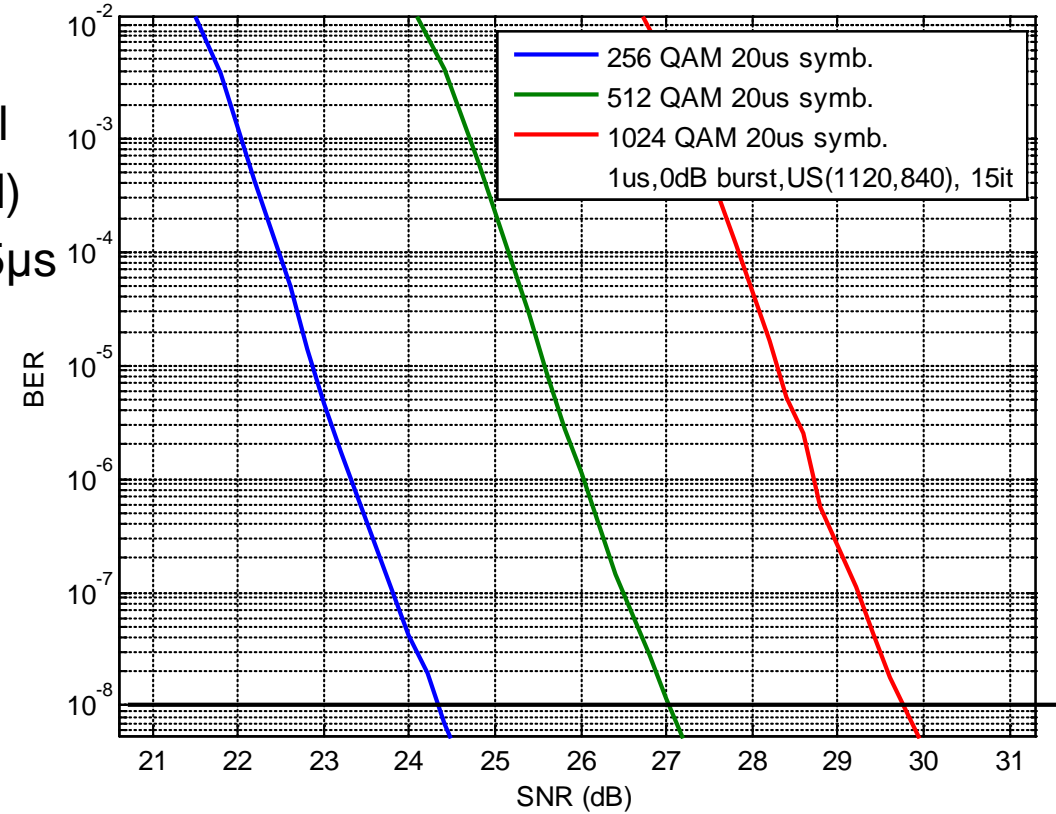
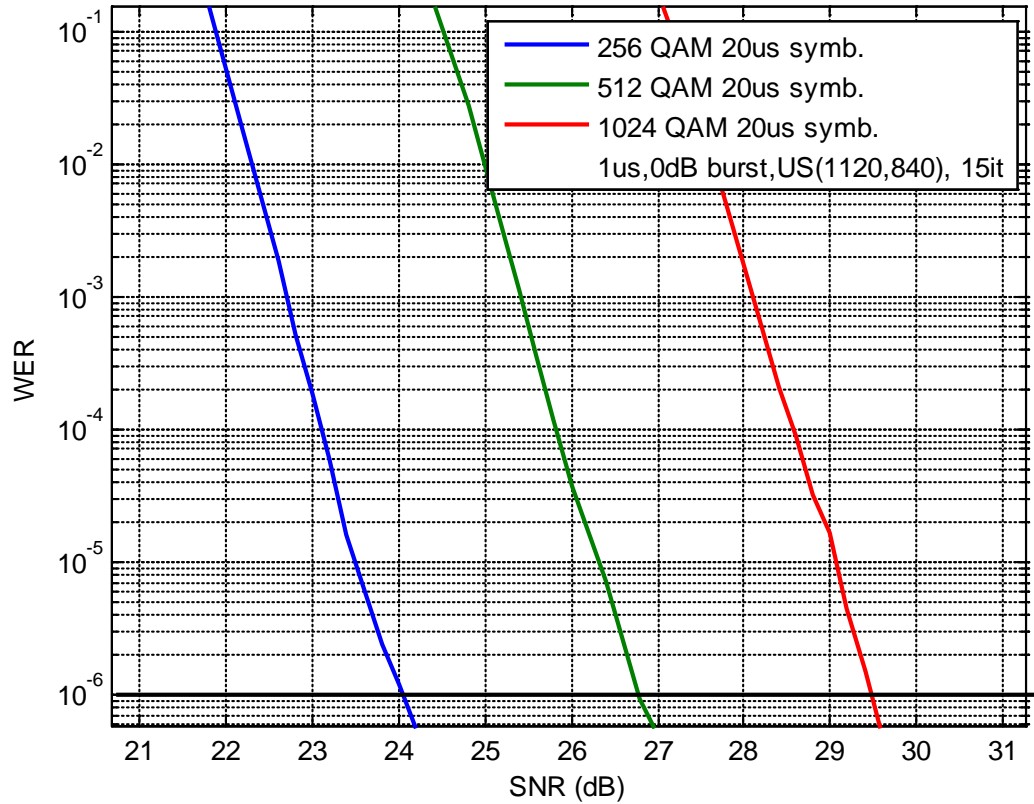


20 μ s symbol
(one affected)
Latency: 382.5 μ s
(depth:17)



	256QAM	512QAM	1024QAM
SNR@WER=1e-6	23.79dB	26.5dB	29.24dB
SNR@BER=1e-8	24.1dB	26.71dB	29.51dB

20 μ s SYMBOLS ON 1 μ s 0dB BURST (SHORT CODE) MAX 15 ITERATIONS



	256QAM	512QAM	1024QAM
SNR@WER=1e-6	24.05dB	26.78dB	29.48dB
SNR@BER=1e-8	24.33dB	27.04dB	29.77dB

DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (SHORT SIZE CODE)

20 μ s SYMBOLS ON 1 μ s 0dB BURST

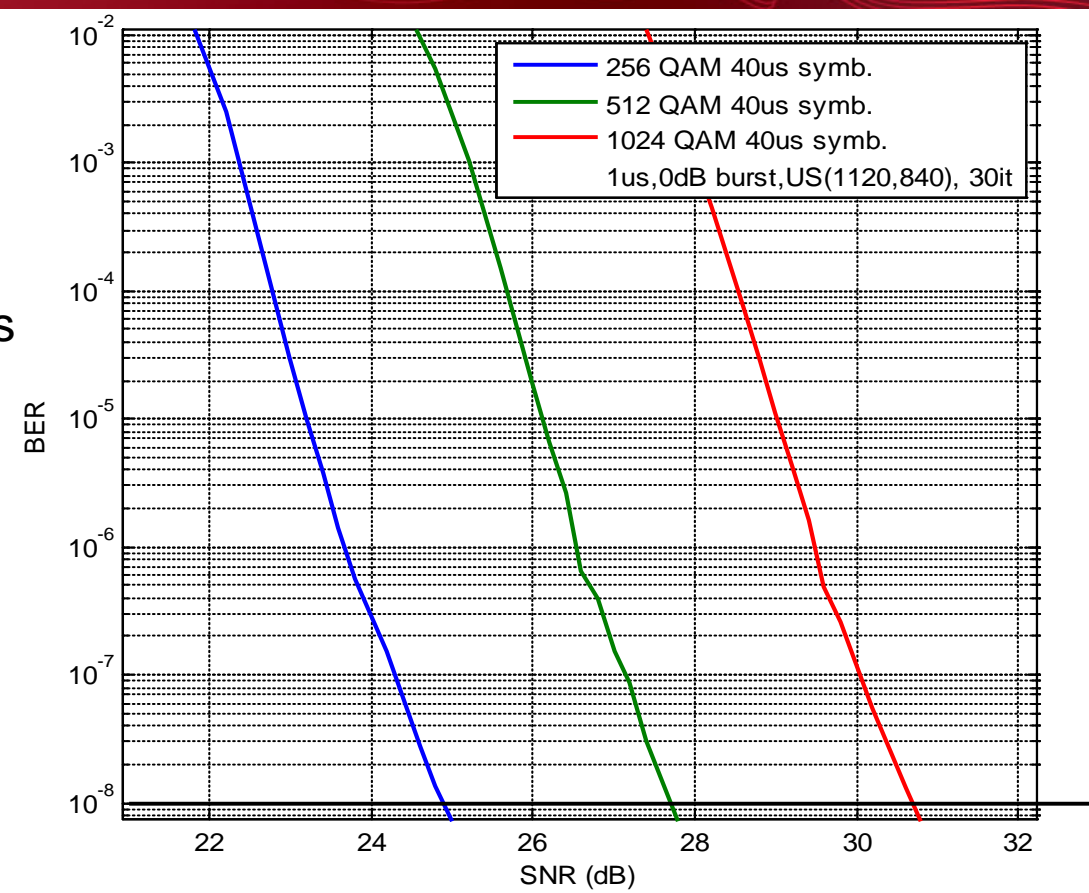
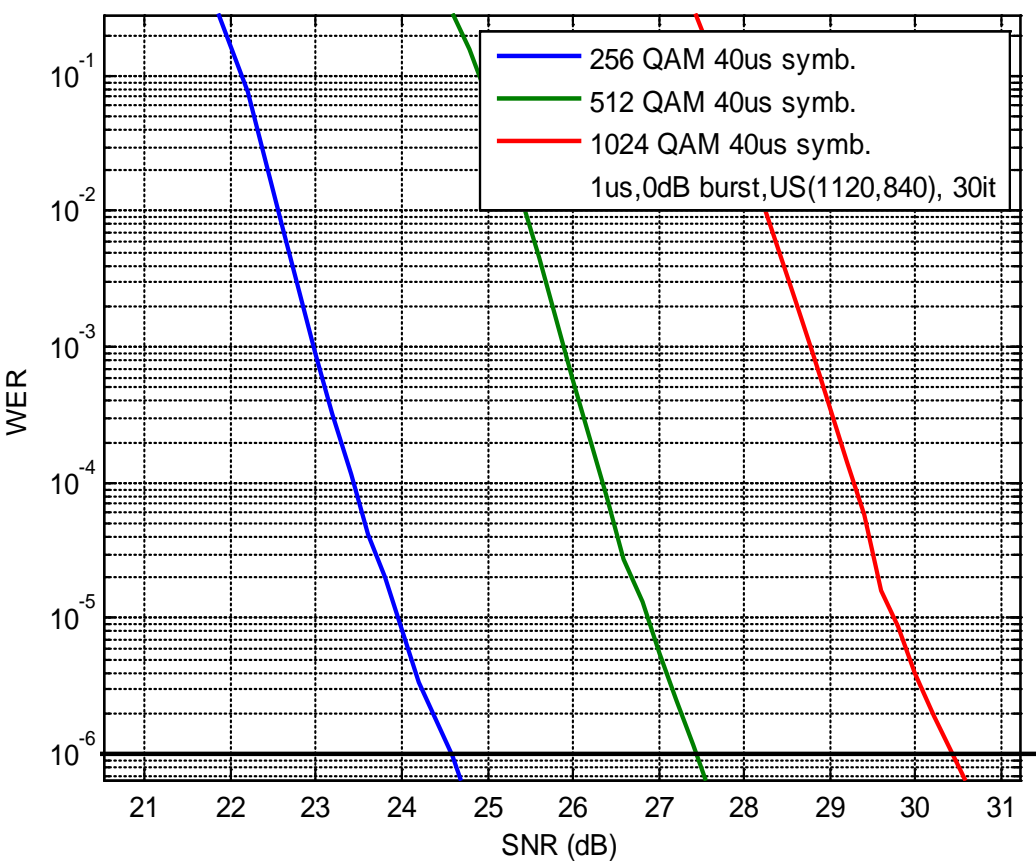
20 μ s symbol (two affected)

Latency: 382.5 μ s (depth:17)

Short size
(1120,840)
30 iterations
vs.
15 iterations

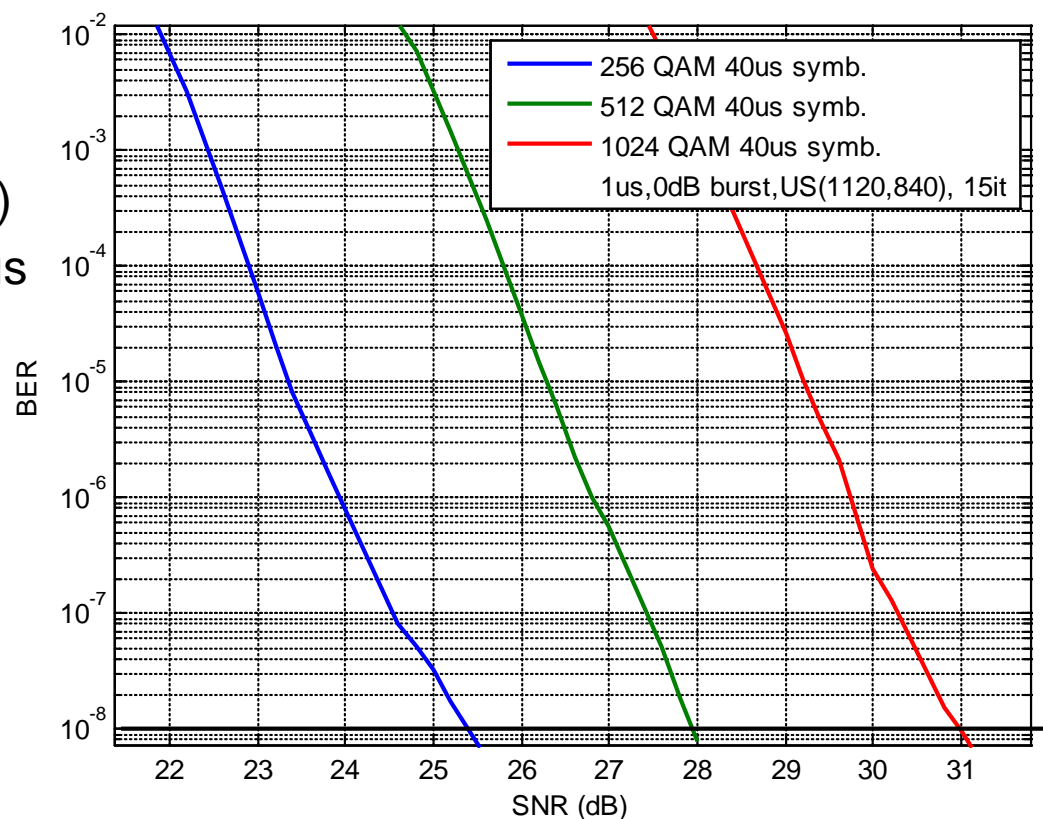
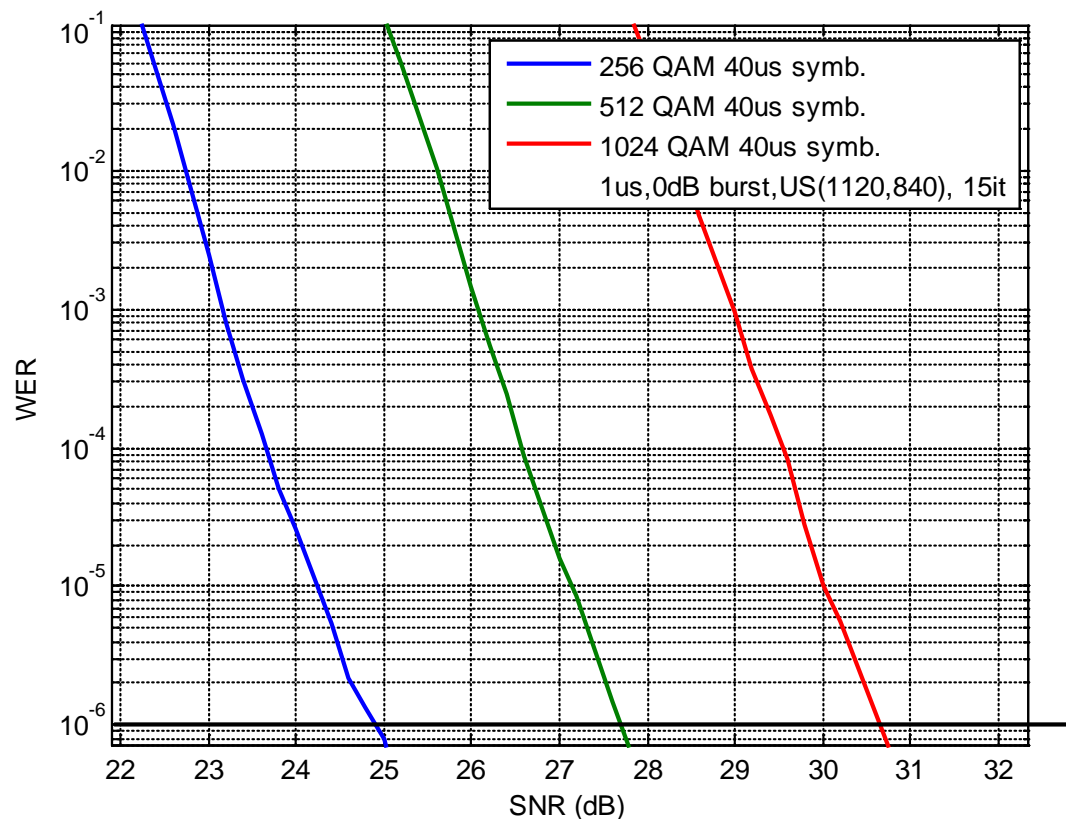
		256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	24.05dB	26.78dB	29.48dB
	Max.30 iterations	23.79dB	26.5dB	29.24dB
	Difference	0.26	0.28	0.24
SNR@BER=1e-8	Max. 15 iterations	24.33dB	27.04dB	29.77dB
	Max. 30 iterations	24.1dB	26.71dB	29.51dB
	Difference	0.23	0.33	0.26

40μs SYMBOLS ON 1μs 0dB BURST (SHORT CODE) MAX 30 ITERATIONS



	256QAM	512QAM	1024QAM
SNR@WER=1e-6	24.59dB	27.45dB	30.44dB
SNR@BER=1e-8	24.89dB	27.71dB	30.69dB

40μs SYMBOLS ON 1μs 0dB BURST (SHORT CODE) MAX 15 ITERATIONS



	256QAM	512QAM	1024QAM
SNR@WER=1e-6	24.91dB	27.7dB	30.7dB
SNR@BER=1e-8	25.4dB	27.94dB	31dB

DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (SHORT SIZE CODE)

40 μ s SYMBOLS ON 1 μ s 0dB BURST

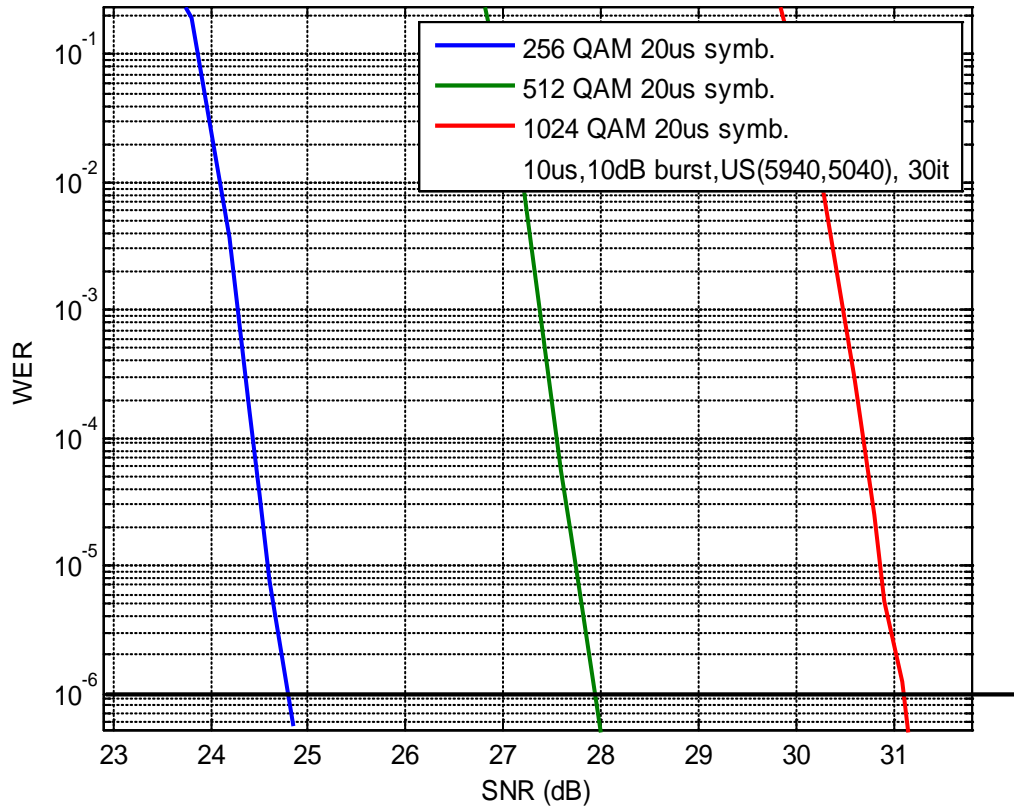
40 μ s symbol (two affected)

Latency: 340 μ s (depth:8)

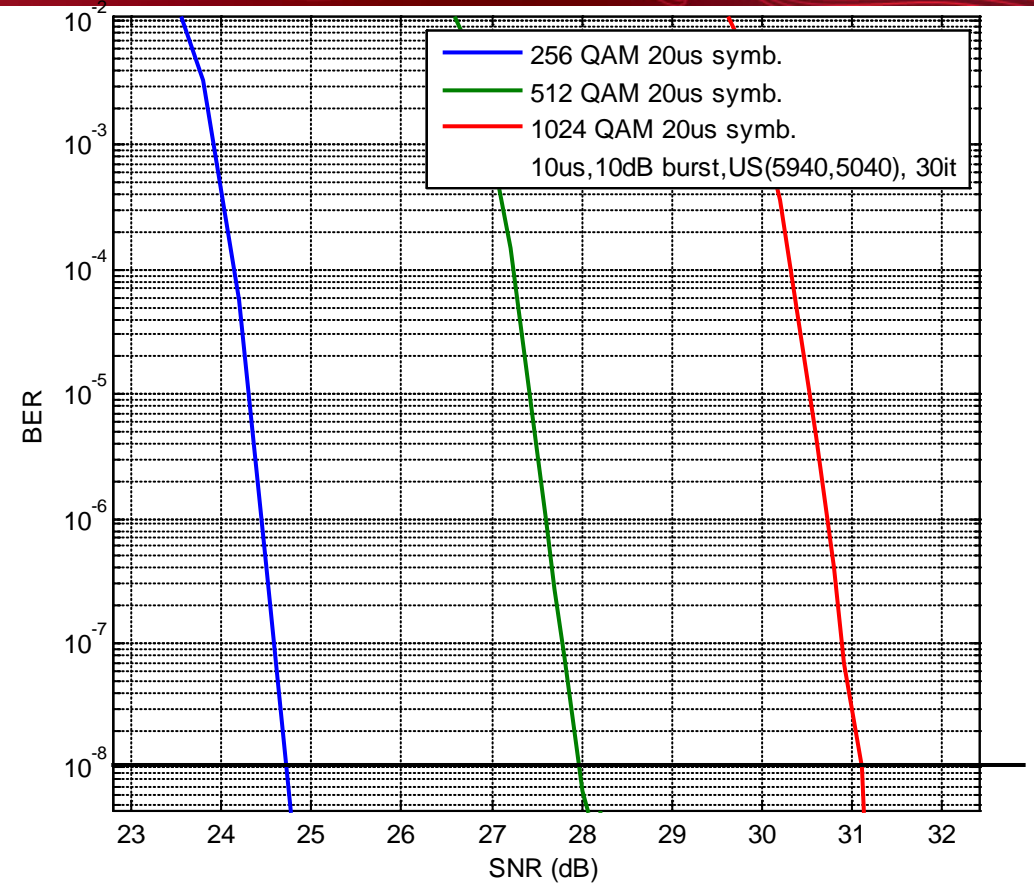
Short size
(1120,840)
30 iterations
vs.
15 iterations

		256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	24.91dB	27.7dB	30.7dB
	Max.30 iterations	24.59dB	27.45dB	30.44dB
	Difference	0.32	0.25	0.26
SNR@BER=1e-8	Max. 15 iterations	25.4dB	27.94dB	31dB
	Max. 30 iterations	24.89dB	27.71dB	30.69dB
	Difference	0.51	0.23	0.31

20μs SYMBOLS ON 10μs 10dB BURST (MEDIUM SIZE CODE) MAX 30 ITERATIONS

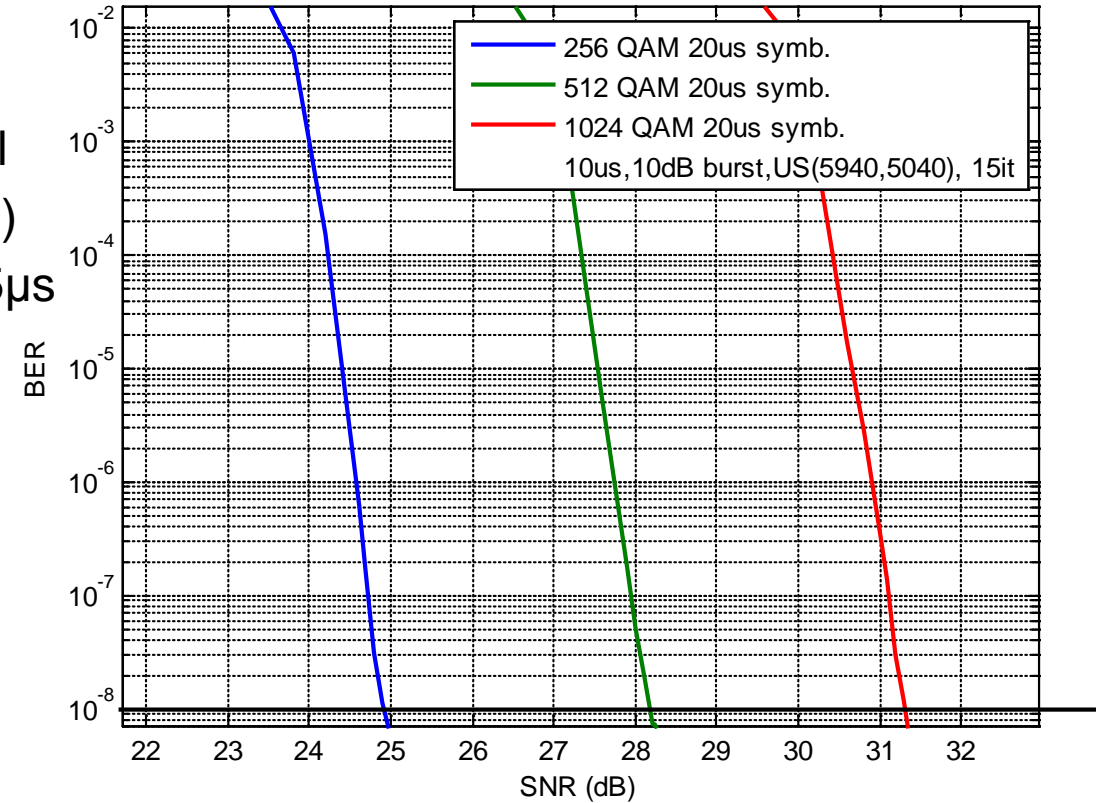
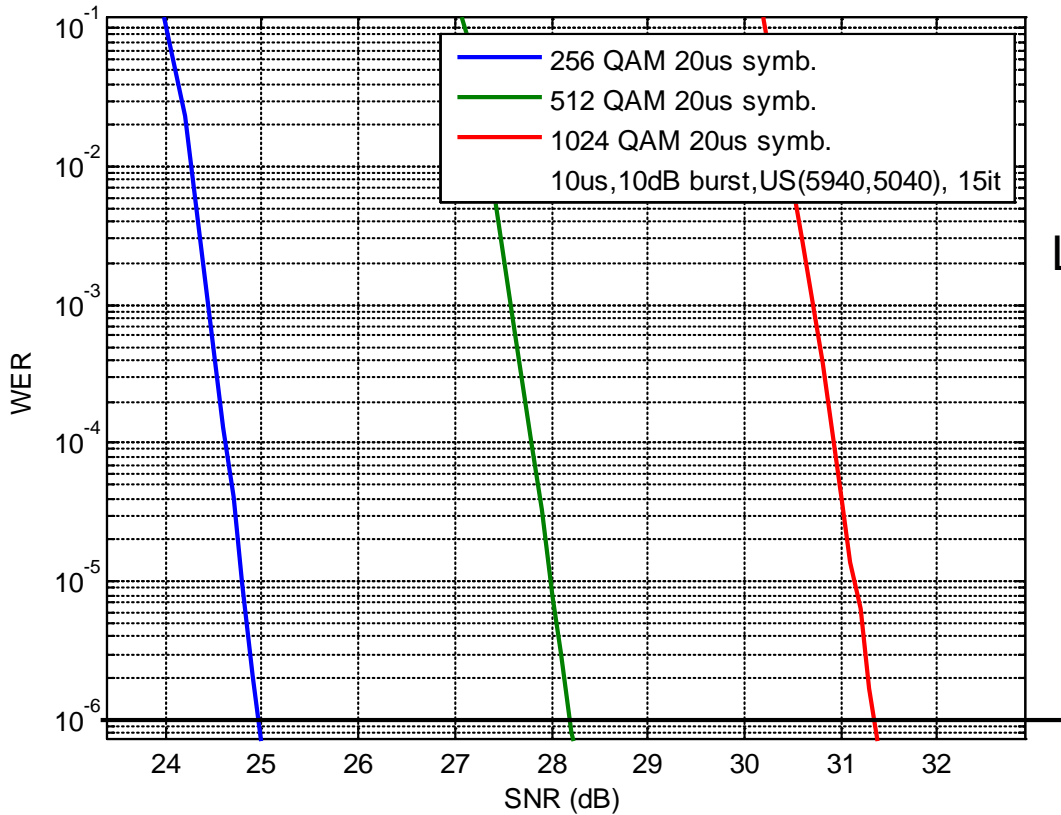


20μs symbol
(two affected)
Latency: 382.5μs
(depth:17)



	256QAM	512QAM	1024QAM
SNR@WER=1e-6	24.77dB	27.96dB	31.08dB
SNR@BER=1e-8	24.75dB	27.99dB	31.08dB

20μs SYMBOLS ON 10μs 10dB BURST (MEDIUM SIZE CODE) MAX 15 ITERATIONS



	256QAM	512QAM	1024QAM
SNR@WER=1e-6	24.96dB	28.19dB	31.35dB
SNR@BER=1e-8	24.92dB	28.17dB	31.32dB

DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (MEDIUM SIZE CODE)

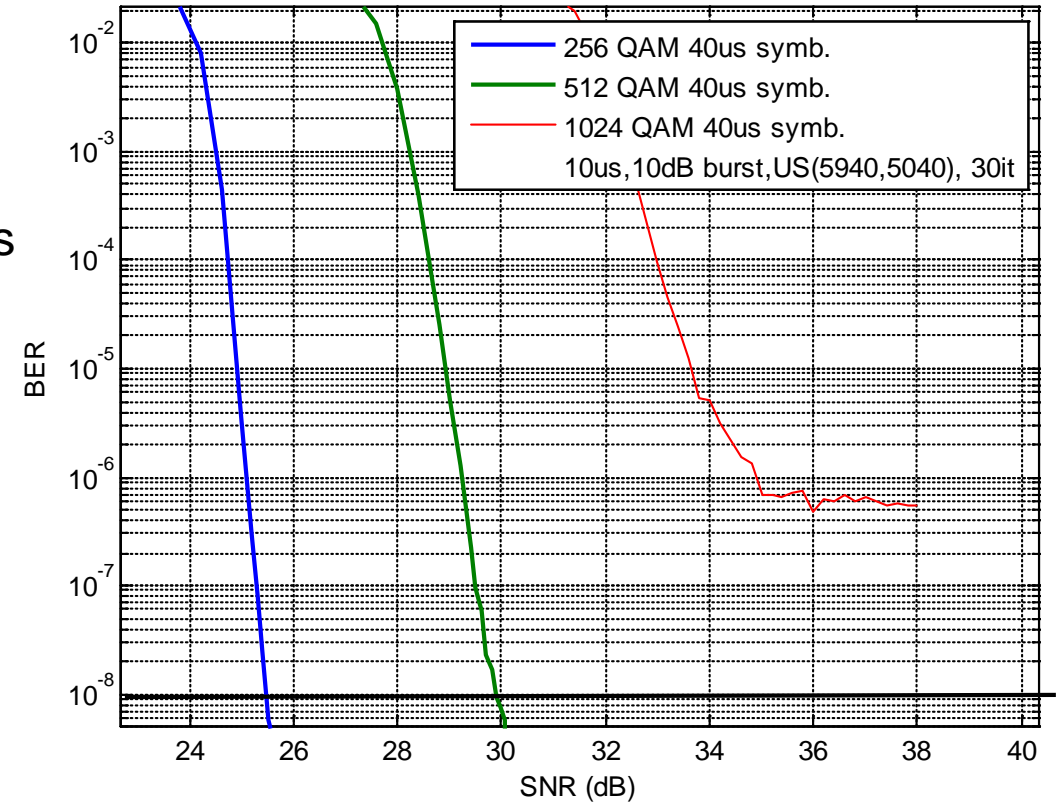
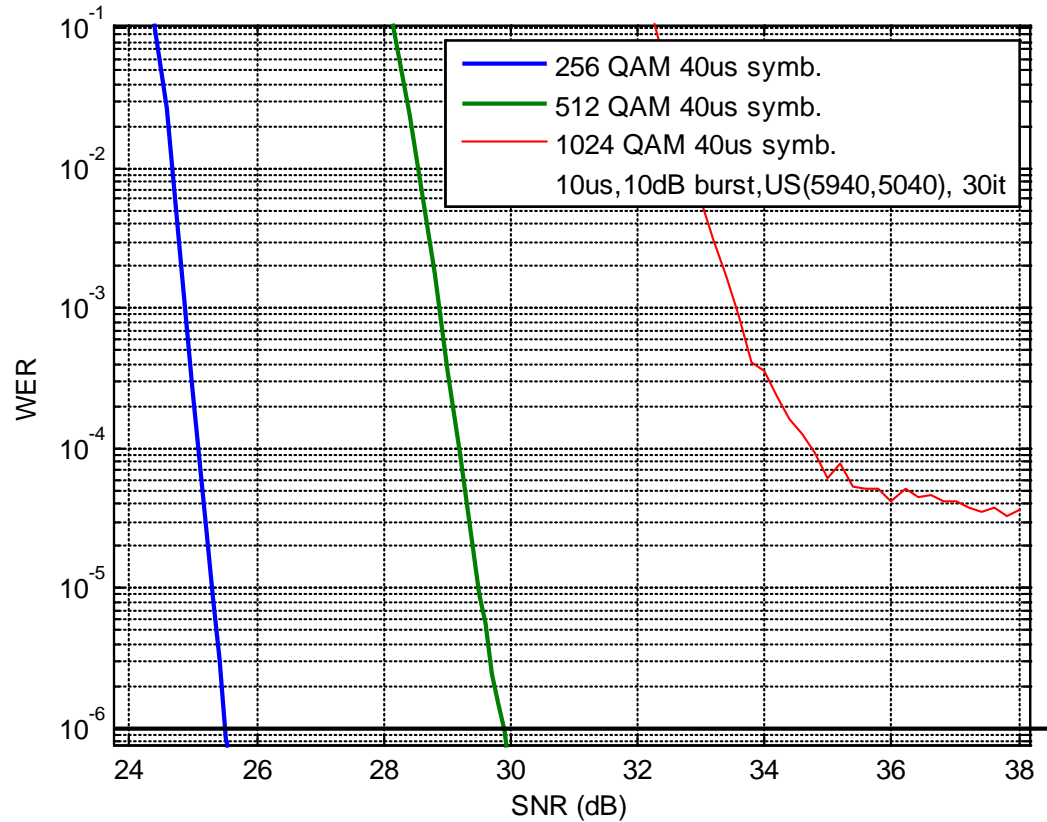
20 μ s SYMBOLS ON 10 μ s 10dB BURST

20 μ s symbol (two affected)
 Latency: 382.5 μ s (depth:17)

Medium size
 (5940,5040)
 30 iterations
 vs.
 15 iterations

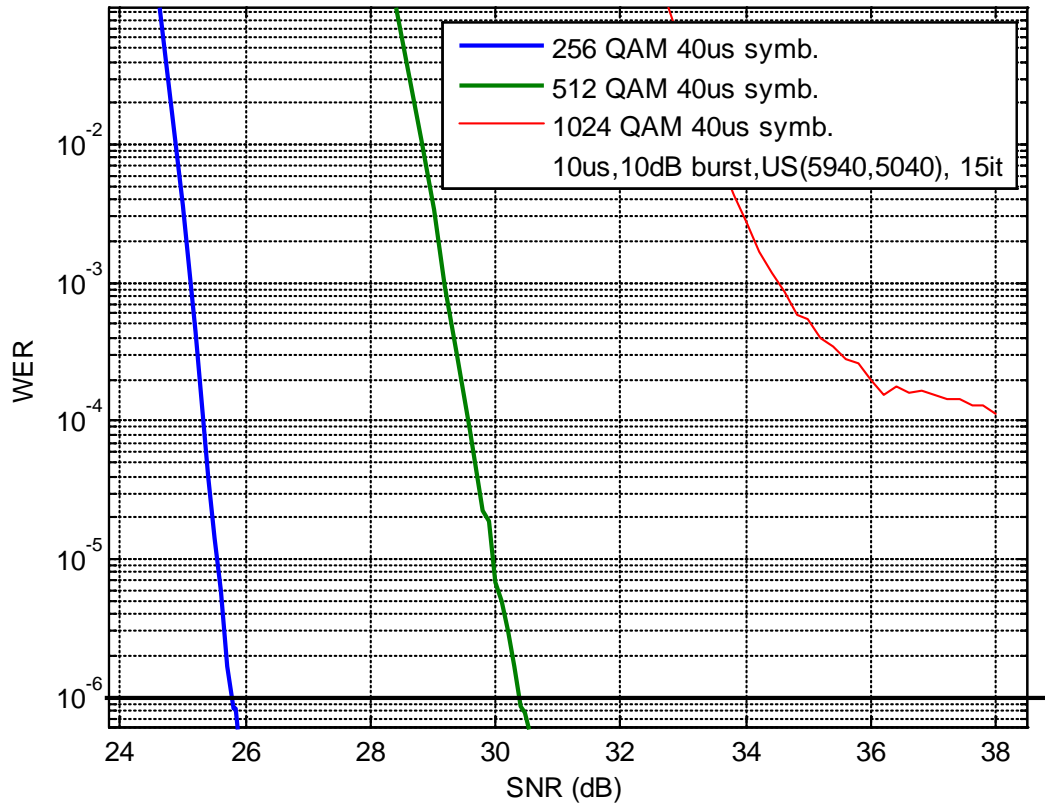
		256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	24.96dB	28.19dB	31.35dB
	Max.30 iterations	24.77dB	27.96dB	31.08dB
	Difference	0.19	0.23	0.23
SNR@BER=1e-8	Max. 15 iterations	24.92dB	28.17dB	31.32dB
	Max. 30 iterations	24.75dB	27.99dB	31.08dB
	Difference	0.17	0.18	0.24

40μs SYMBOLS ON 10μs 10dB BURST (MEDIUM SIZE CODE) MAX 30 ITERATIONS

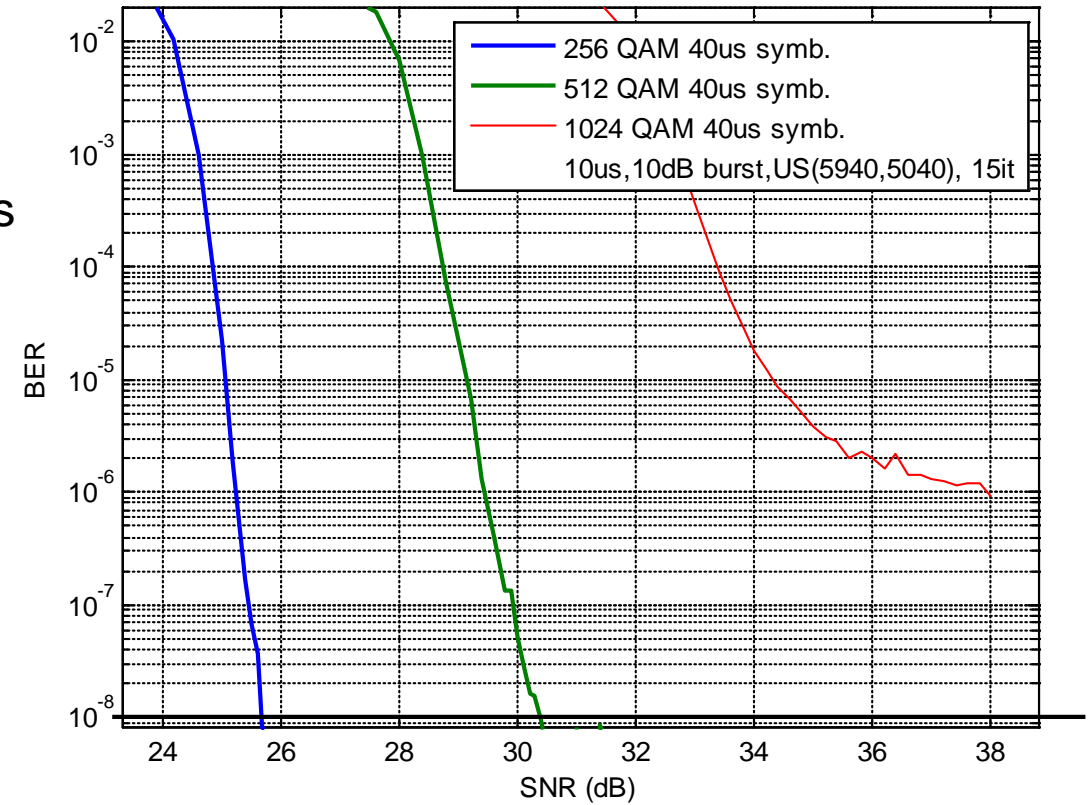


	256QAM	512QAM	1024QAM
SNR@WER=1e-6	25.49dB	29.8dB	
SNR@BER=1e-8	25.46dB	29.89dB	

40μs SYMBOLS ON 10μs 10dB BURST (MEDIUM SIZE CODE) MAX 15 ITERATIONS

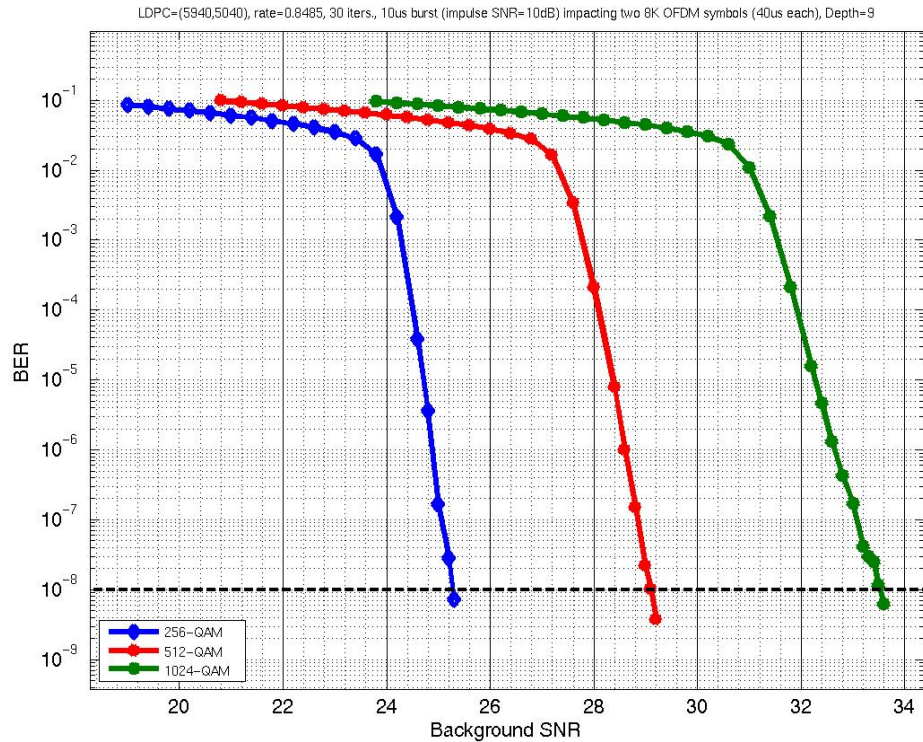


40μs symbol
(two affected)
Latency: 340μs
(depth:8)



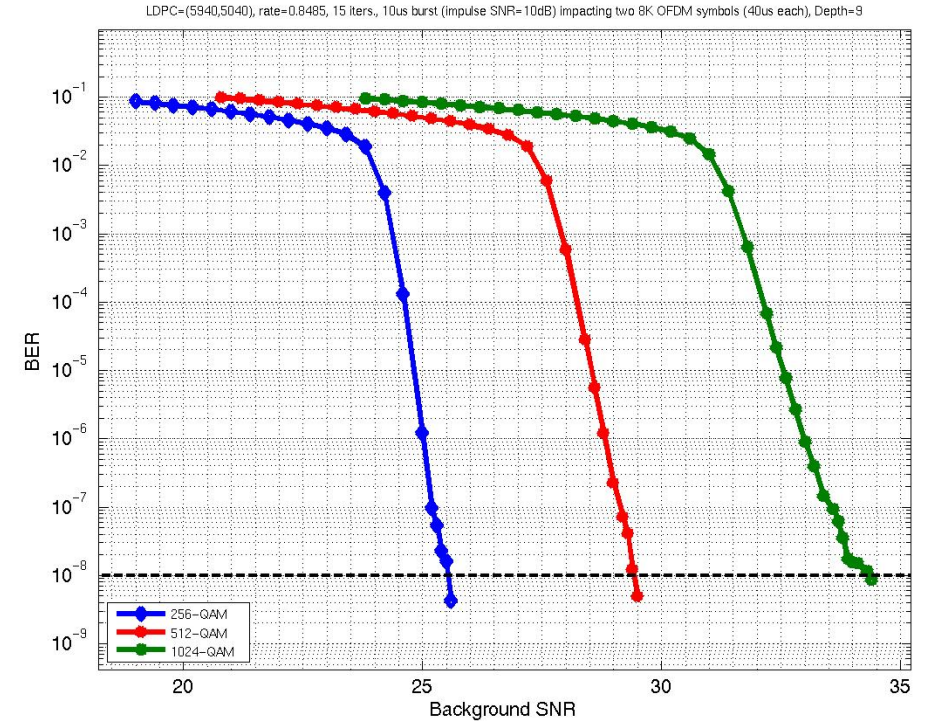
	256QAM	512QAM	1024QAM
SNR@WER=1e-6	25.77dB	30.38dB	
SNR@BER=1e-8	25.69dB	30.04dB	

40μs SYMBOLS ON 10μs 10dB BURST (MEDIUM SIZE CODE)



Max. 30 iterations

40μs symbol
(two affected)
Latency: 382.5μs
(depth:9)



Max. 15 iterations

		256QAM	512QAM	1024QAM
SNR@BER=1e-8	Max. 15 iterations	25.5dB	29.5dB	34.3dB
	Max. 30 iterations	25.3dB	29.1dB	33.5dB

DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (MEDIUM SIZE CODE)

40μs SYMBOLS ON 10μs 10dB BURST

40μs symbol (two affected) Latency: 340μs (depth:8)

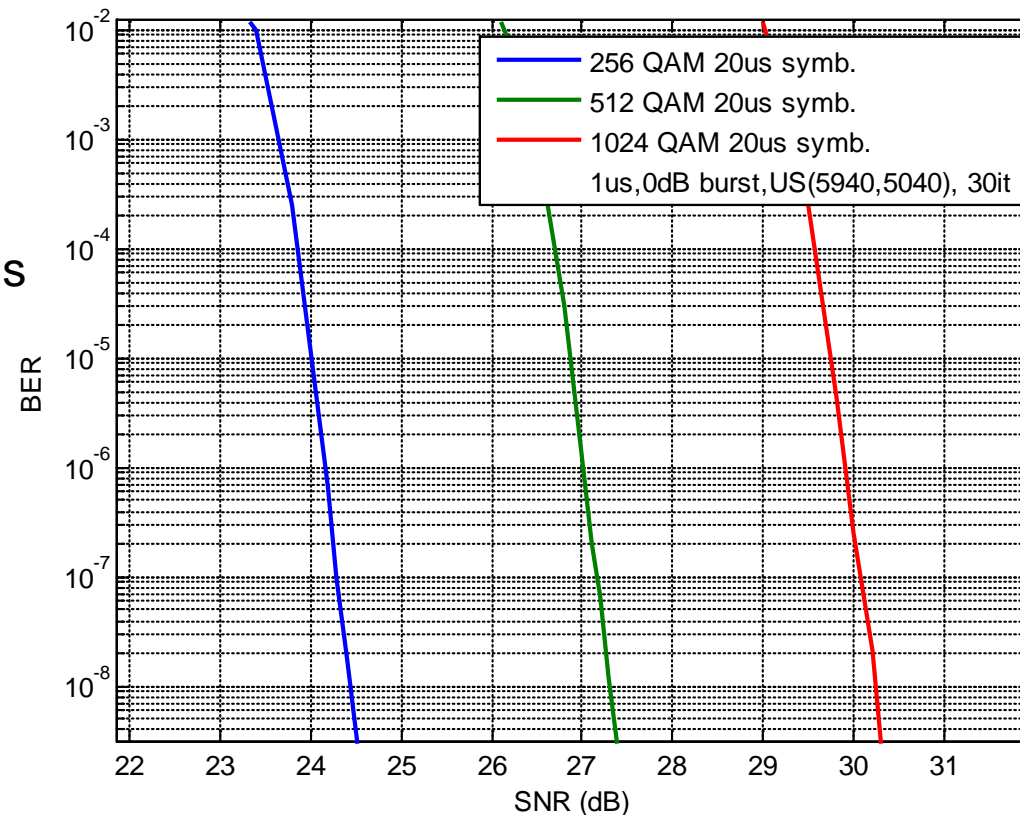
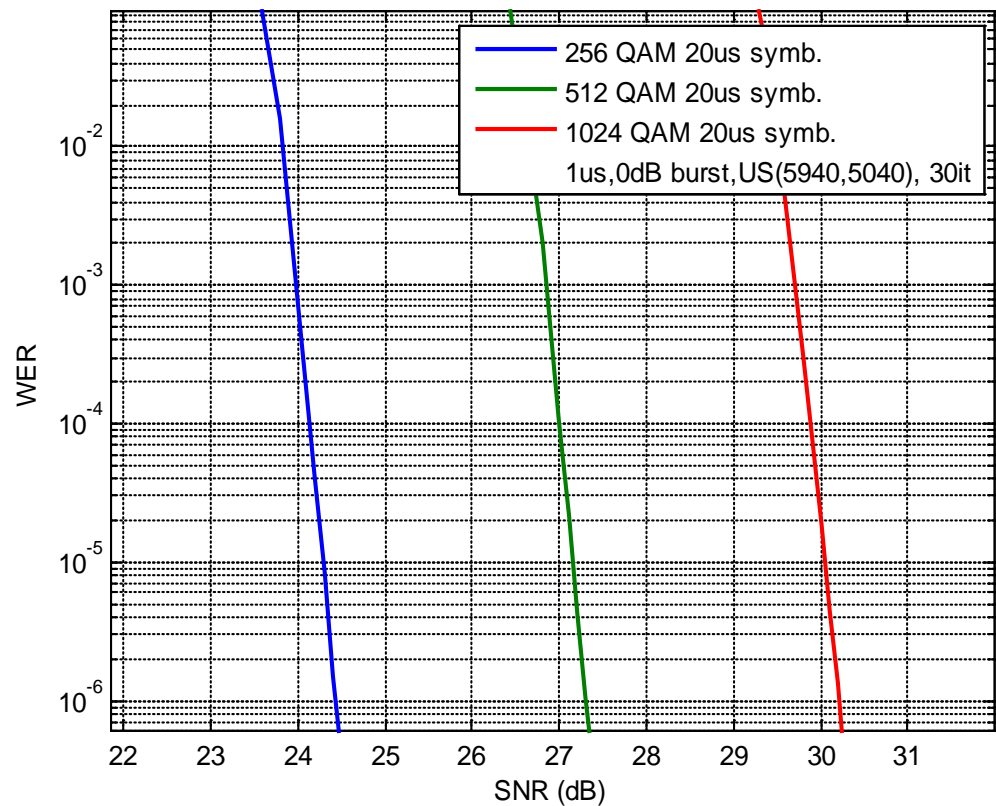
		256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	25.77dB	30.38dB	
	Max.30 iterations	25.49dB	29.8dB	
	Difference	0.28	0.58	
SNR@BER=1e-8	Max. 15 iterations	25.69dB	30.04dB	
	Max. 30 iterations	25.46dB	29.89dB	
	Difference	0.23	0.15	

Medium size
(5940,5040)
30 iterations
vs.
15 iterations

40μs symbol (two affected) Latency: 382.5μs (depth:9)

		256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations			
	Max.30 iterations			
	Difference			
SNR@BER=1e-8	Max. 15 iterations	25.5dB	29.5dB	34.3dB
	Max. 30 iterations	25.3dB	29.1dB	33.5dB
	Difference	0.2	0.4	0.8

20μs SYMBOLS ON 1μs 0dB BURST (MEDIUM CODE) MAX 30 ITERATIONS

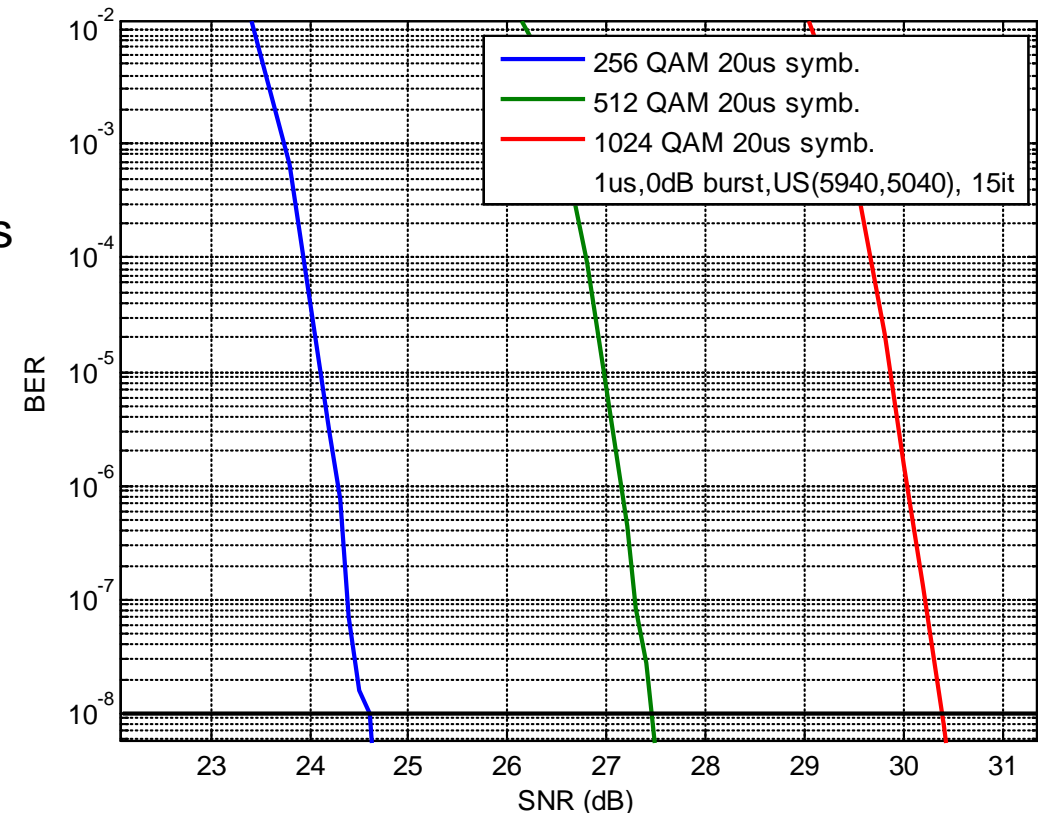
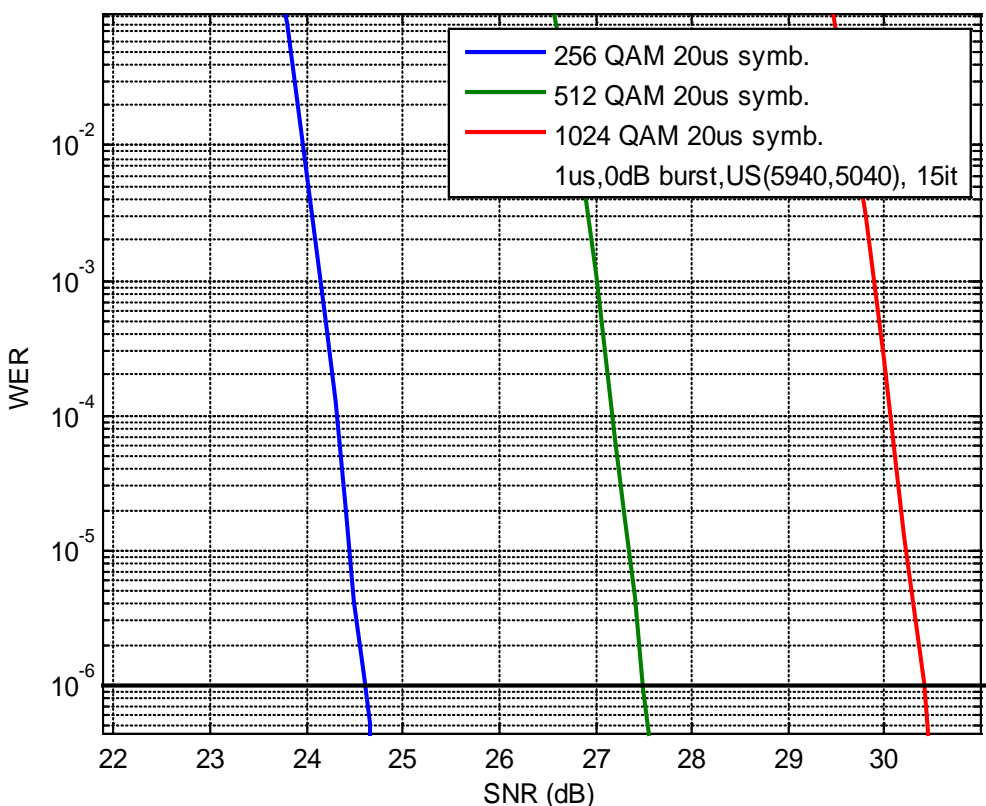


	256QAM	512QAM	1024QAM
SNR@WER=1e-6	24.43dB	27.30dB	30.22dB
SNR@BER=1e-8	24.44dB	27.31dB	30.22dB

20μs SYMBOLS ON 1μs 0dB BURST (MEDIUM CODE) MAX 15 ITERATIONS



20μs symbol
(one affected)
Latency: 382.5μs
(depth:17)



	256QAM	512QAM	1024QAM
SNR@WER=1e-6	24.61dB	27.5dB	30.4dB
SNR@BER=1e-8	24.6dB	27.45dB	30.38dB

DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (MEDIUM SIZE CODE)

20 μ s SYMBOLS ON 1 μ s 0dB BURST



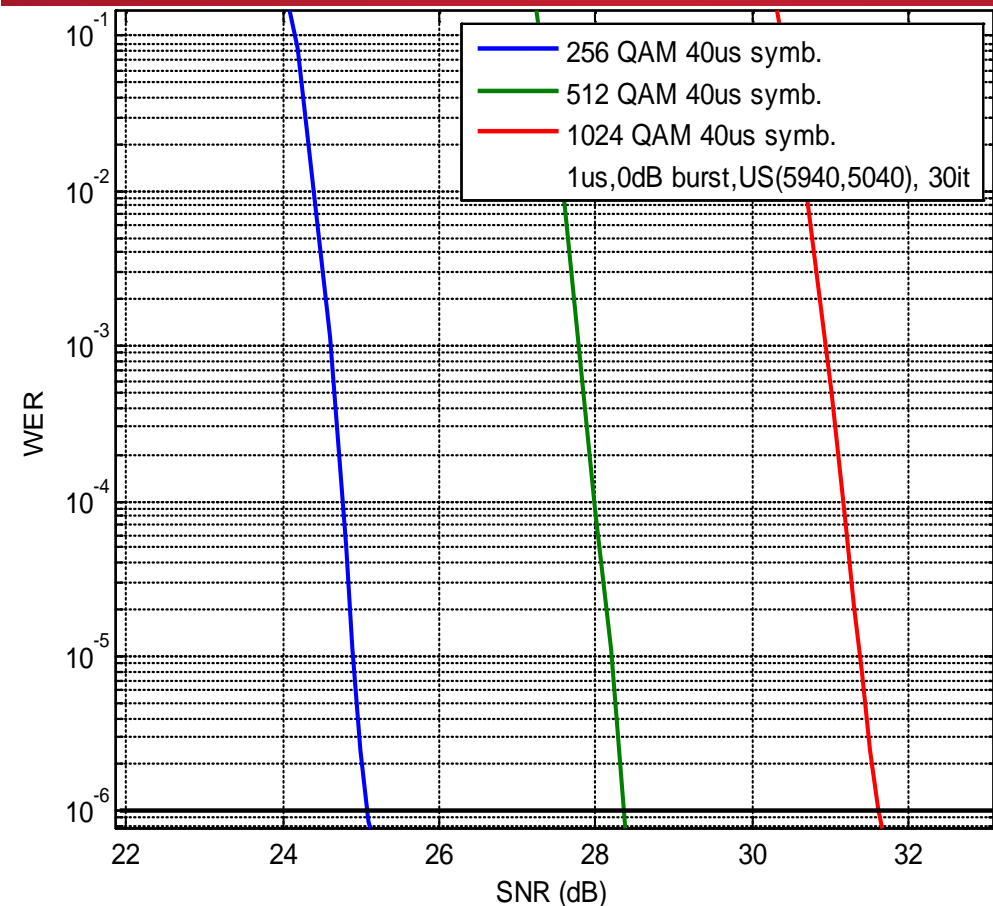
20 μ s symbol (one affected)

Latency: 382.5 μ s (depth:17)

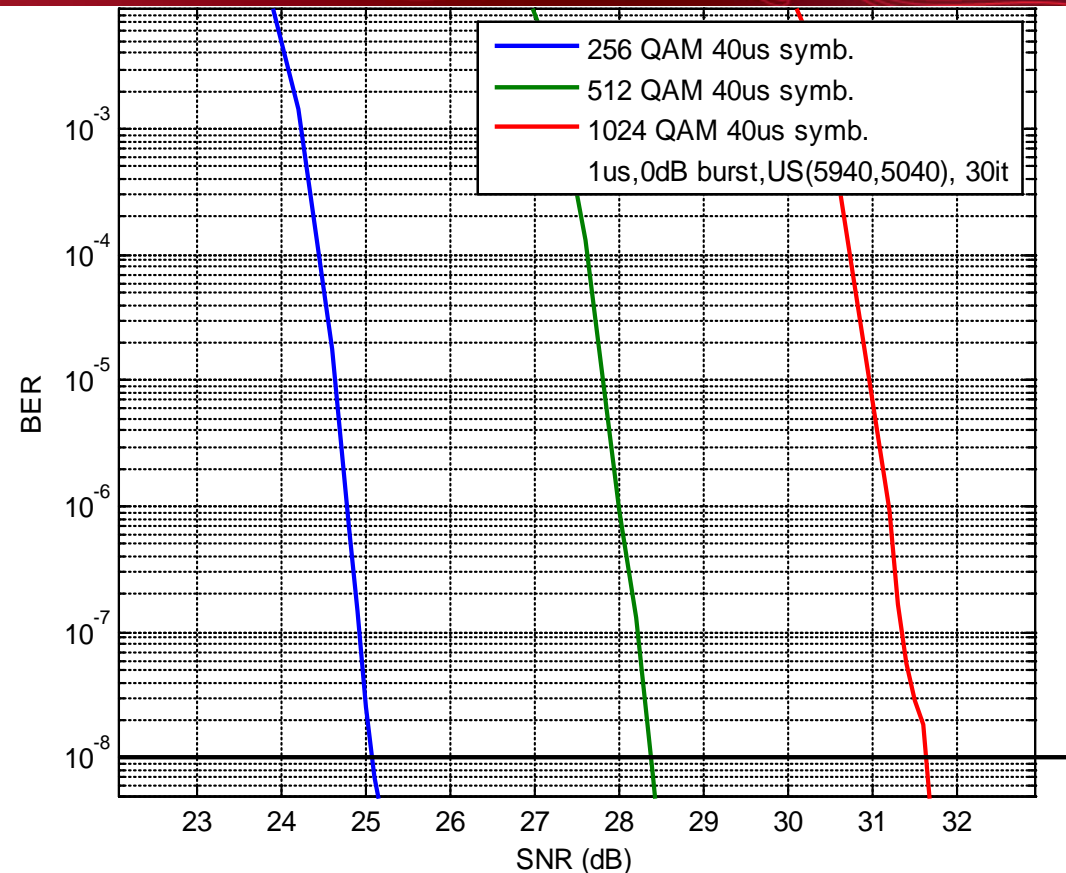
Medium size
(5940,5040)
30 iterations
vs.
15 iterations

		256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	24.61dB	27.5dB	30.4dB
	Max.30 iterations	24.43dB	27.3dB	30.22dB
	Difference	0.18	0.2	0.18
SNR@BER=1e-8	Max. 15 iterations	24.6dB	27.45dB	30.38dB
	Max. 30 iterations	24.44dB	27.31dB	30.22dB
	Difference	0.16	0.14	0.16

40μs SYMBOLS ON 1μs 0dB BURST (MEDIUM SIZE CODE) MAX 30 ITERATIONS

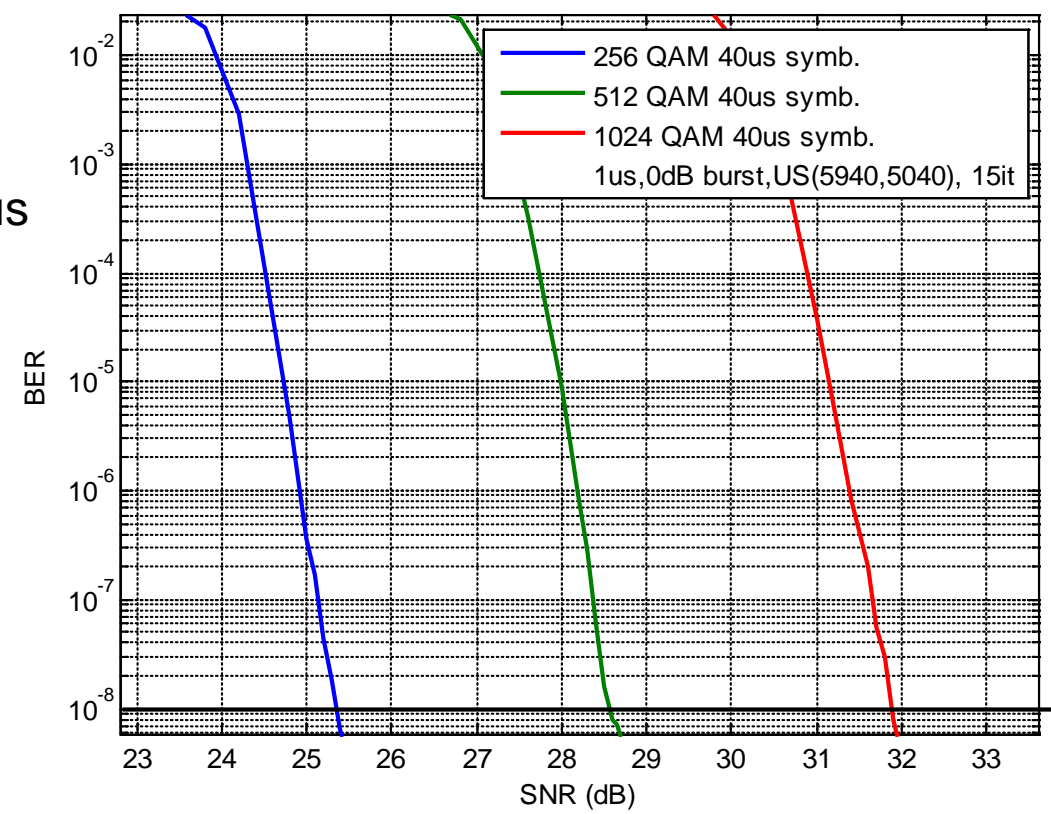
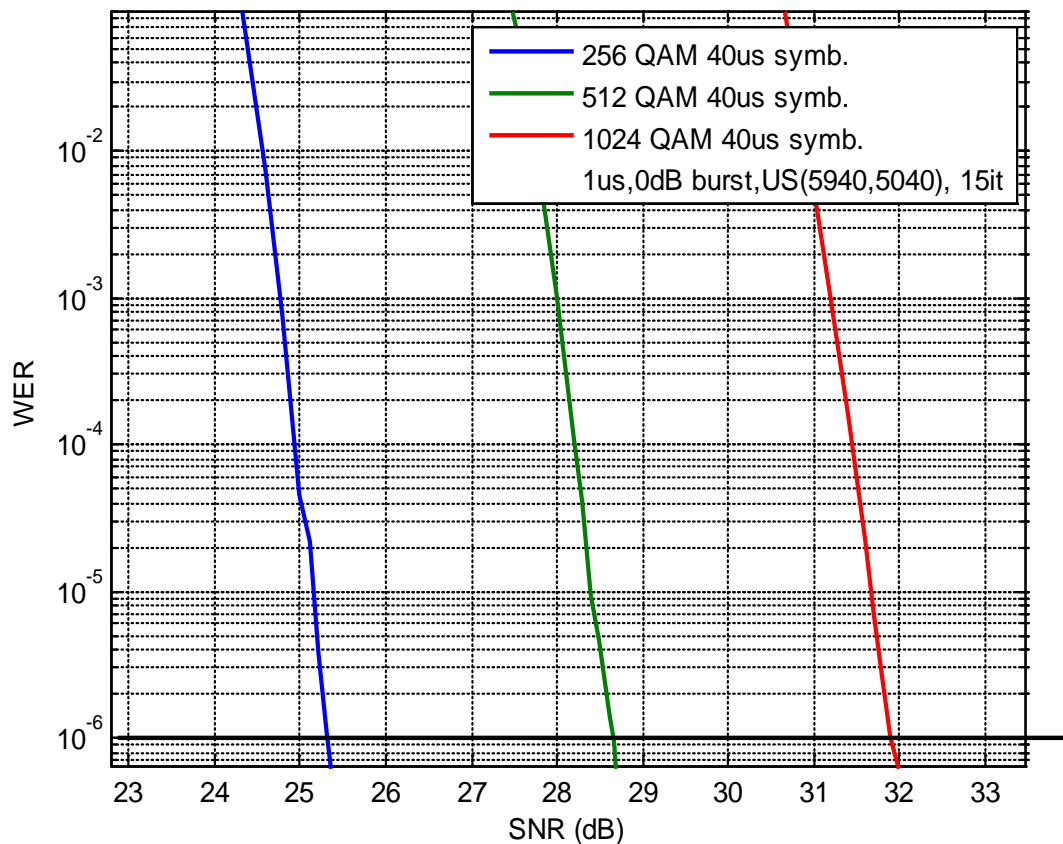


40μs symbol
(one affected)
Latency: 340μs
(depth:8)



	256QAM	512QAM	1024QAM
SNR@WER=1e-6	25.09dB	28.35dB	31.6dB
SNR@BER=1e-8	25.09dB	28.37dB	31.62dB

40μs SYMBOLS ON 1μs 0dB BURST (MEDIUM SIZE CODE) MAX 15 ITERATIONS



	256QAM	512QAM	1024QAM
SNR@WER=1e-6	25.31dB	28.65dB	31.9dB
SNR@BER=1e-8	25.29dB	28.57dB	31.89dB

DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (MEDIUM SIZE CODE)

40 μ s SYMBOLS ON 1 μ s 0dB BURST

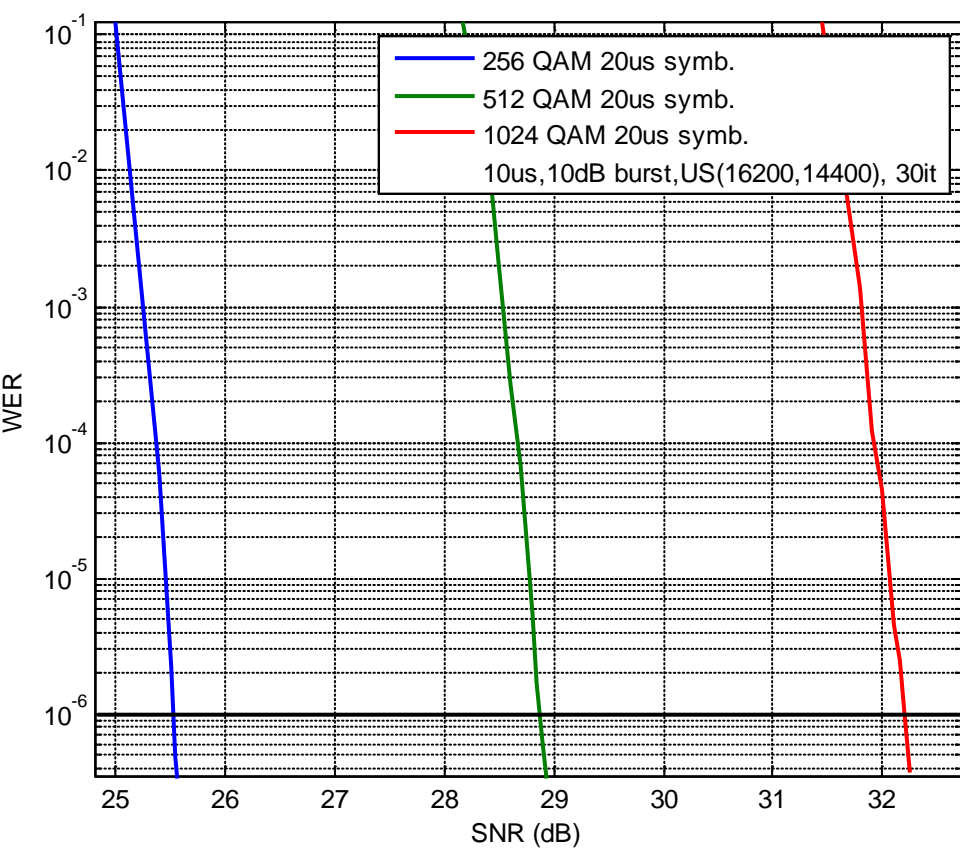


40 μ s symbol (one affected)
 Latency: 340 μ s (depth:8)

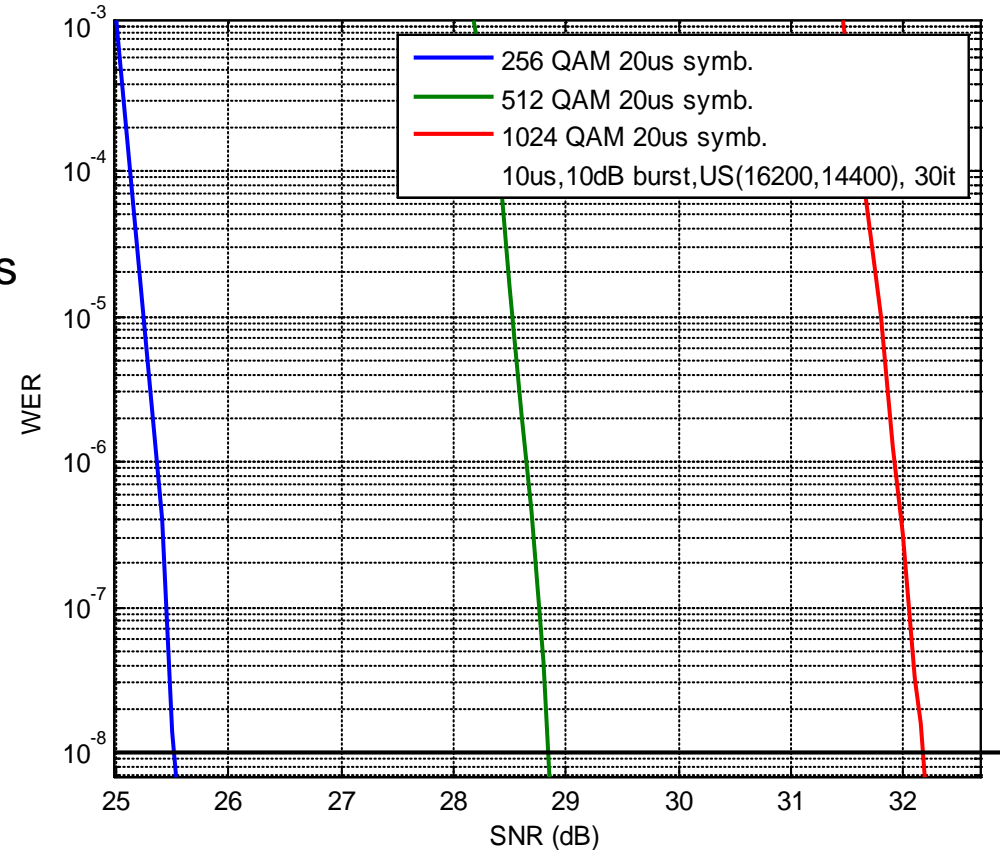
Medium size
 (5940,5040)
 30 iterations
 vs.
 15 iterations

		256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	25.31dB	28.65dB	31.9dB
	Max.30 iterations	25.09dB	28.35dB	31.6dB
	Difference	0.22	0.3	0.3
SNR@BER=1e-8	Max. 15 iterations	25.29dB	28.57dB	31.89dB
	Max. 30 iterations	25.09dB	28.37dB	31.62dB
	Difference	0.2	0.2	0.27

20μs SYMBOLS ON 10μs 10dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS

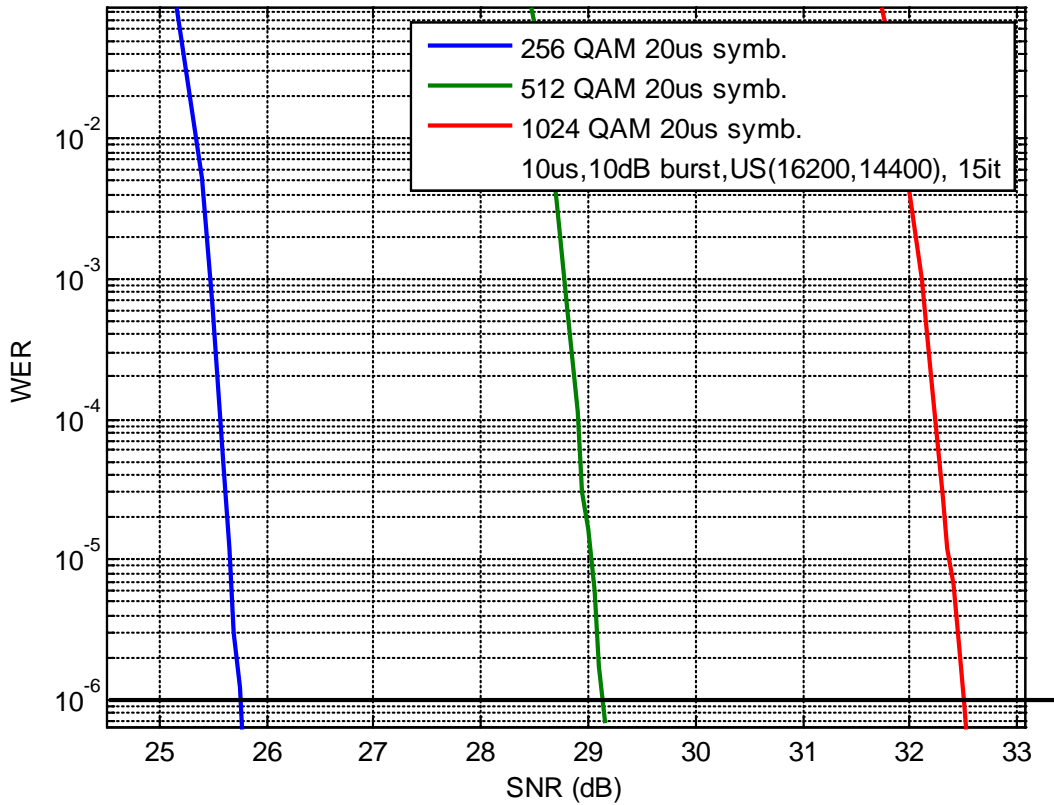


20μs symbol
(two affected)
Latency: 382.5μs
(depth:17)

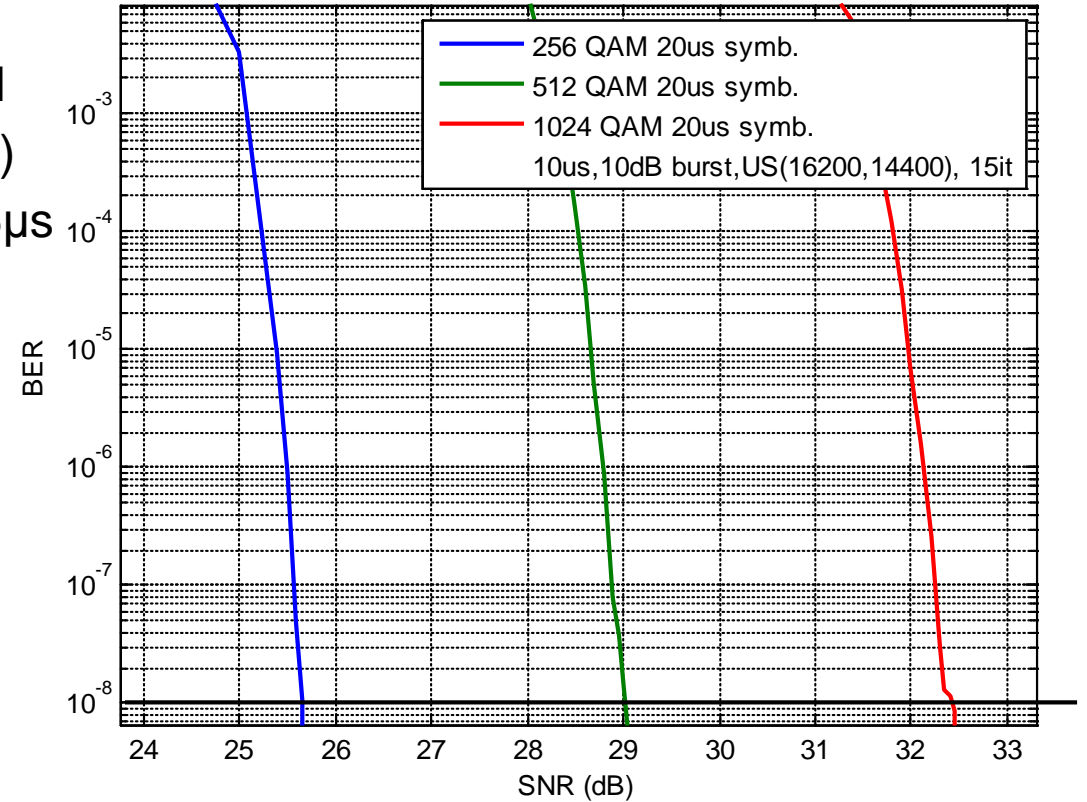


	256QAM	512QAM	1024QAM
SNR@WER=1e-6	25.53dB	28.88dB	32.2dB
SNR@BER=1e-8	25.52dB	28.86dB	32.17dB

20 μ s SYMBOLS ON 10 μ s 10dB BURST (LONG SIZE CODE) MAX 15 ITERATIONS



20 μ s symbol
(two affected)
Latency: 382.5 μ s
(depth:17)



	256QAM	512QAM	1024QAM
SNR@WER=1e-6	25.76dB	29.13dB	32.5dB
SNR@BER=1e-8	25.65dB	29.02dB	32.42dB

DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (LONG SIZE CODE)

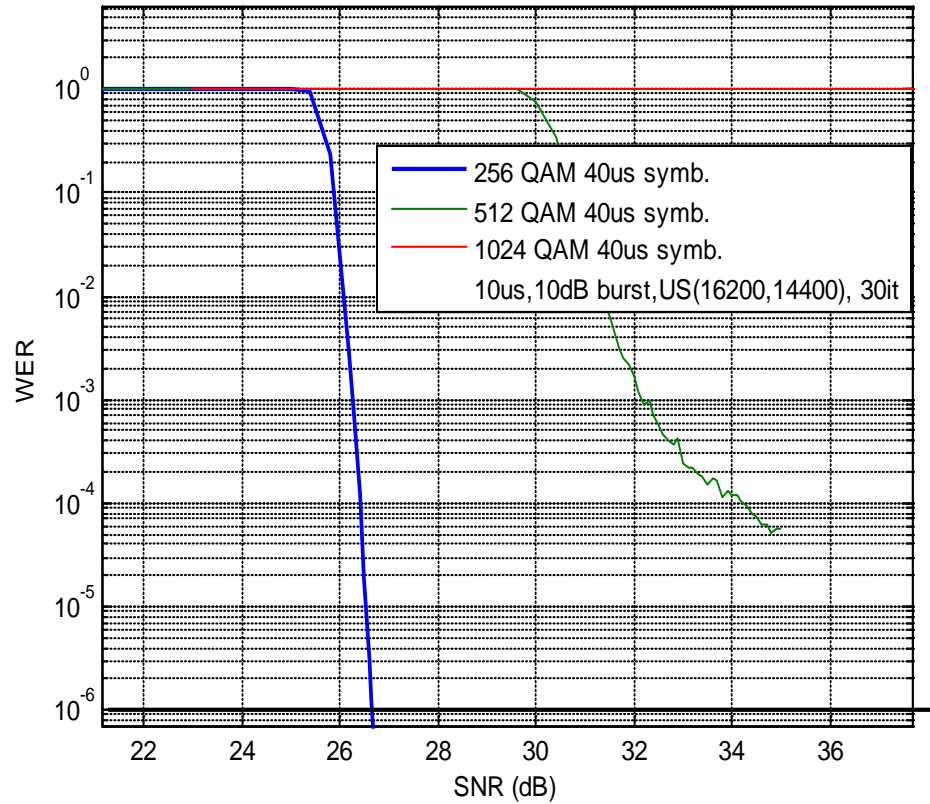
20 μ s SYMBOLS ON 10 μ s 10dB BURST

20 μ s symbol(two affected)
 Latency: 382.5 μ s (depth:17)

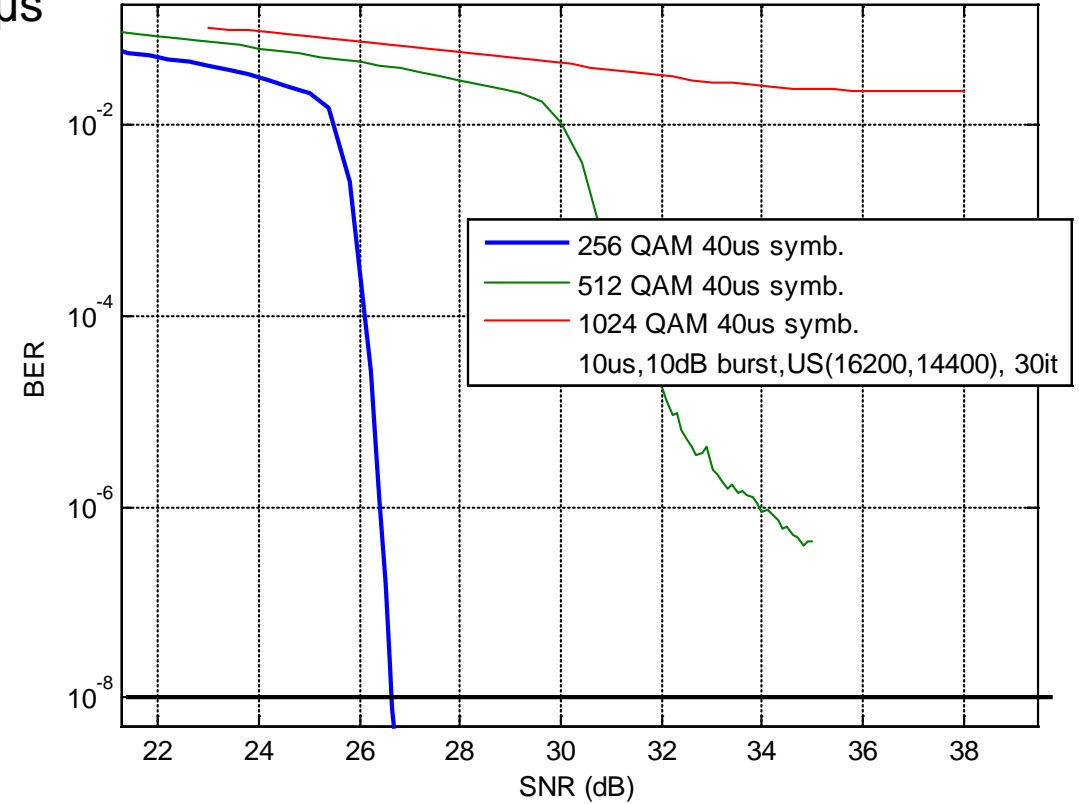
Long size
 (16200,14400)
 30 iterations
 vs.
 15 iterations

		256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	25.76dB	29.13dB	32.5dB
	Max.30 iterations	25.53dB	28.88dB	32.2dB
	Difference	0.23	0.25	0.3
SNR@BER=1e-8	Max. 15 iterations	25.53dB	28.88dB	32.2dB
	Max. 30 iterations	25.52dB	28.86dB	32.17dB
	Difference	0.01	0.02	0.03

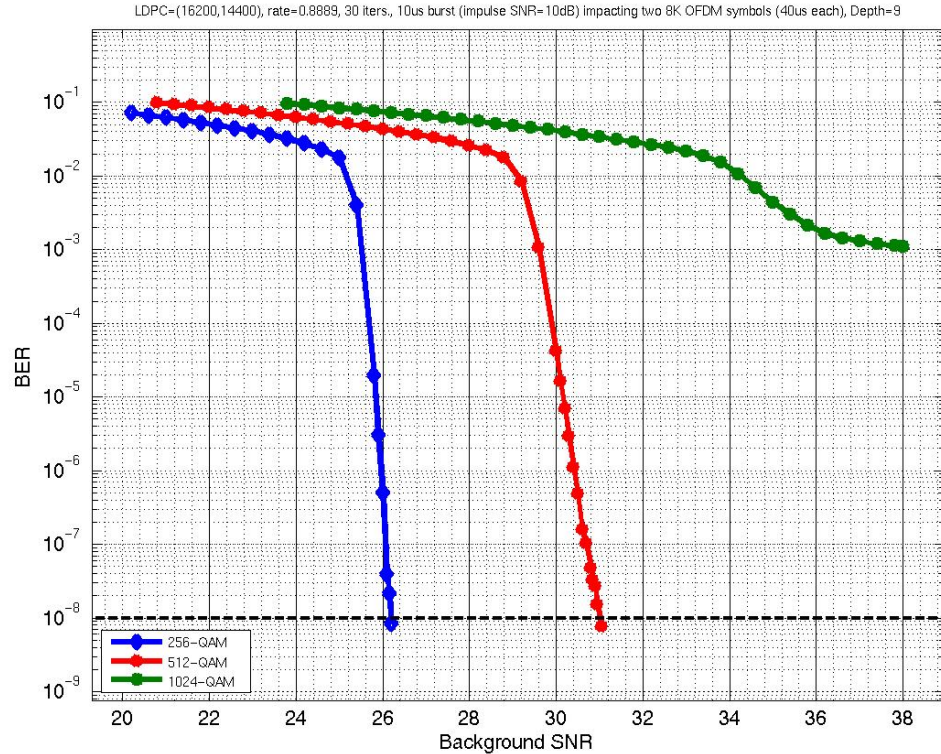
40 μ S SYMBOLS ON 10 μ s 10dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS



40 μ s symbol
(two affected)
Latency: 340 μ s
(depth: 8)

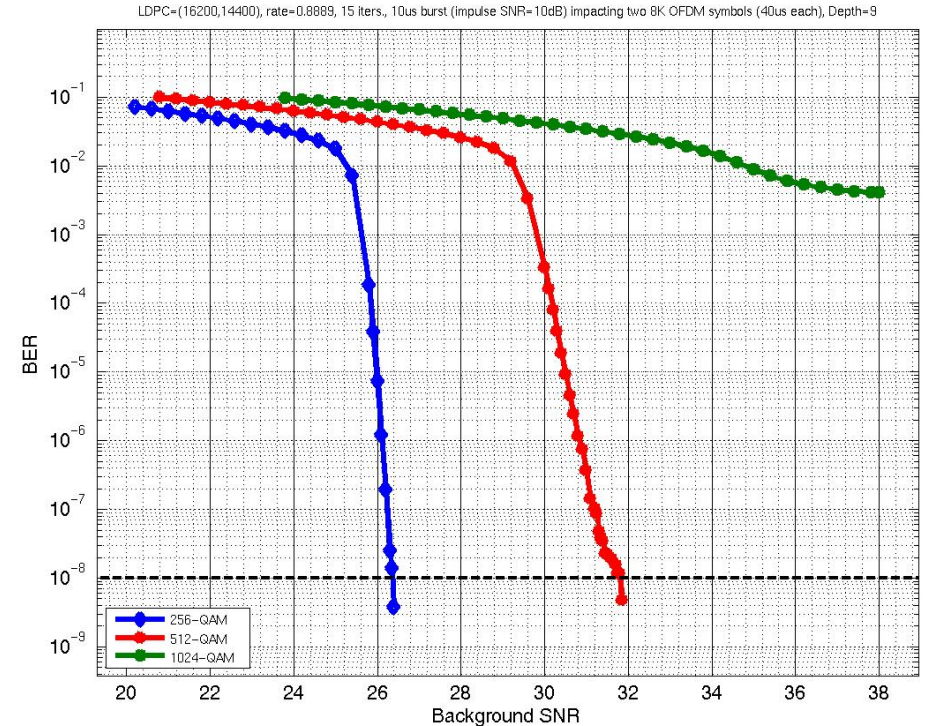


40μs SYMBOLS ON 10μs 10dB BURST (LONG SIZE CODE)



Max. 30 iterations

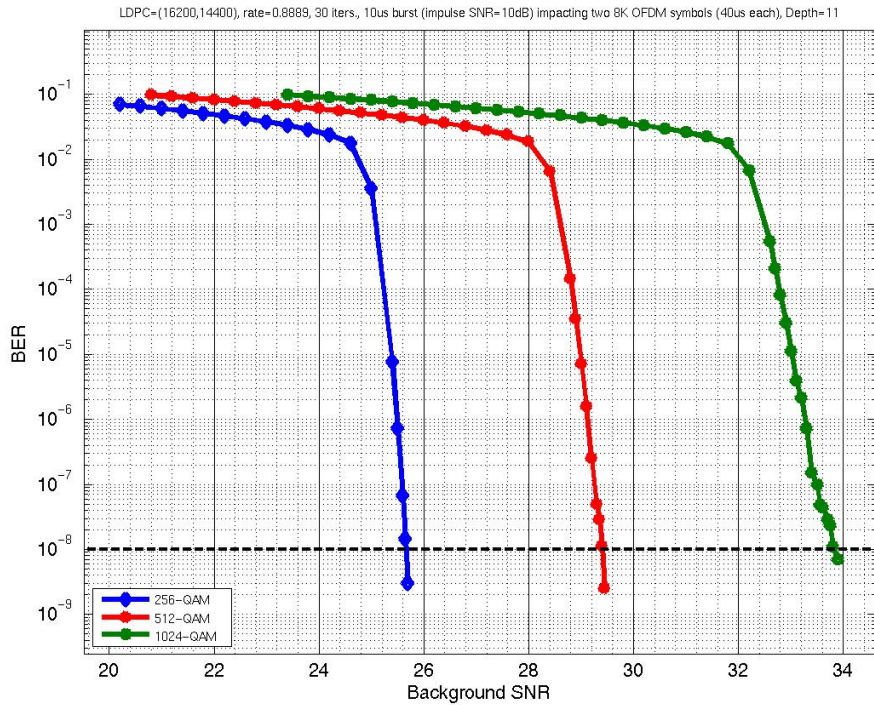
40μs symbol
(two affected)
Latency: 382.5μs
(depth:9)



Max. 15 iterations

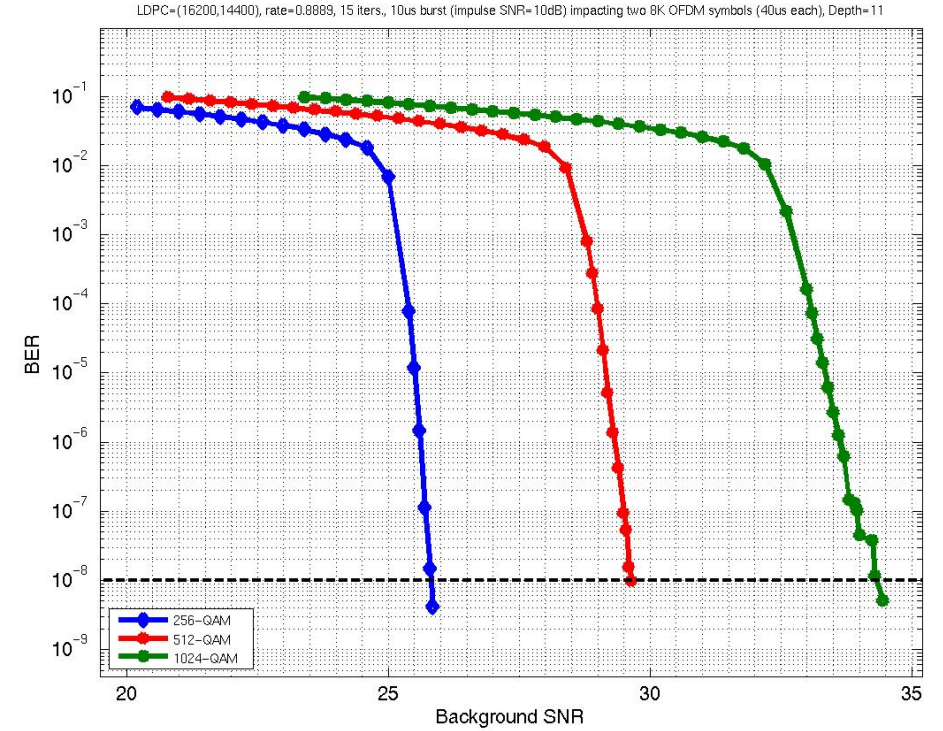
		256QAM	512QAM	1024QAM
SNR@BER=1e-8	Max. 15 iterations	26.5dB	31dB	
	Max. 30 iterations	26.4dB	31.8dB	

40μs SYMBOLS ON 10μs 10dB BURST (LONG SIZE CODE)



Max. 30 iterations

40μs symbol
(two affected)
Latency: 467.5μs
(depth:11)



Max. 15 iterations

		256QAM	512QAM	1024QAM
SNR@BER=1e-8	Max. 15 iterations	26.5dB	29.5dB	33.8dB
	Max. 30 iterations	25.6dB	29.6dB	34.3dB

DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (LONG SIZE CODE)

40 μ s SYMBOLS ON 10 μ s 10dB BURST

40 μ s symbol (two affected) Latency: 382.5 μ s (depth:9)

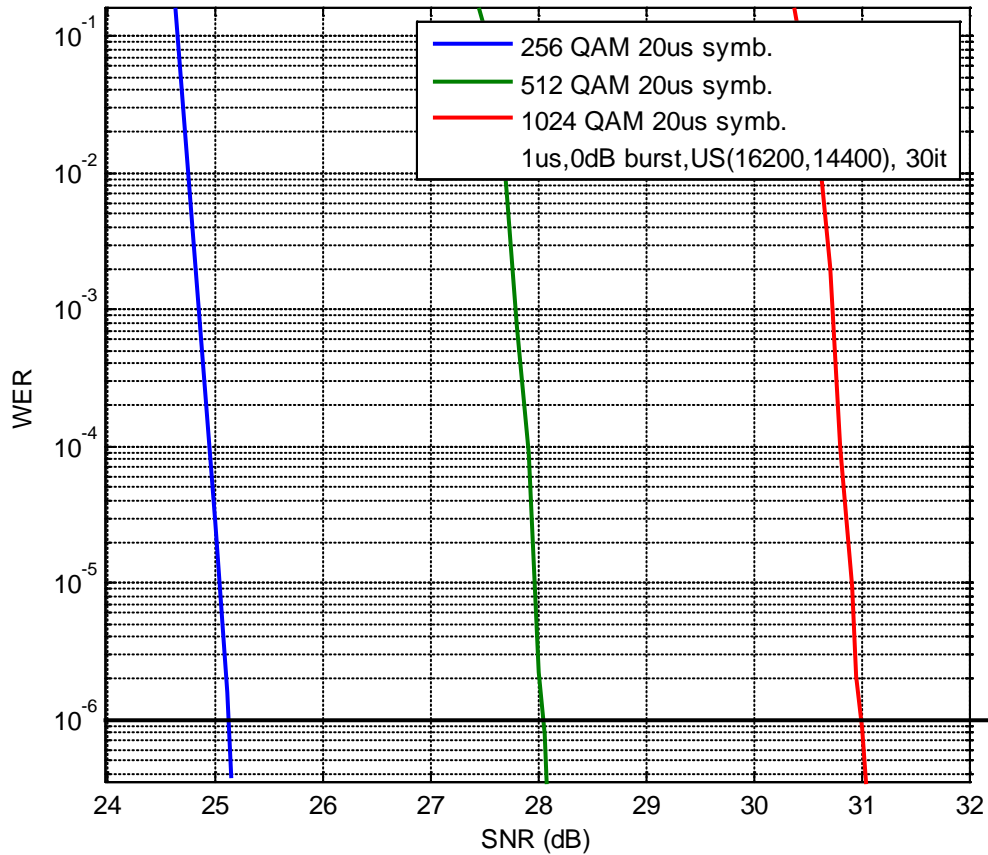
		256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations			
	Max.30 iterations			
	Difference			
SNR@BER=1e-8	Max. 15 iterations	26.5dB	31dB	
	Max. 30 iterations	26.4dB	31.8dB	
	Difference	0.1	0.8	

Long size
(16200,14400)
30 iterations
vs.
15 iterations

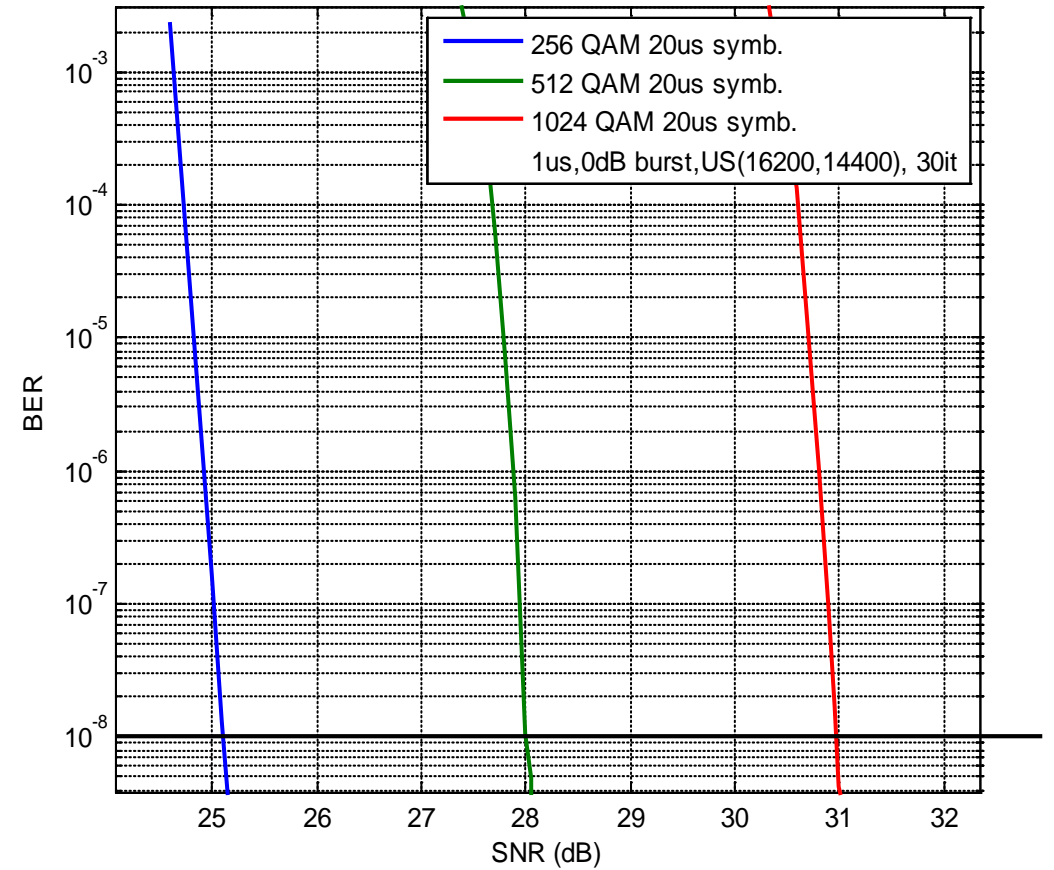
40 μ s symbol (two affected) Latency: 467.5 μ s (depth:11)

		256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations			
	Max.30 iterations			
	Difference			
SNR@BER=1e-8	Max. 15 iterations	26.5dB	29.5dB	33.8dB
	Max. 30 iterations	25.6dB	29.6dB	34.3dB
	Difference	0.9	0.1	0.5

20 μ s SYMBOLS ON 1 μ s 0dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS

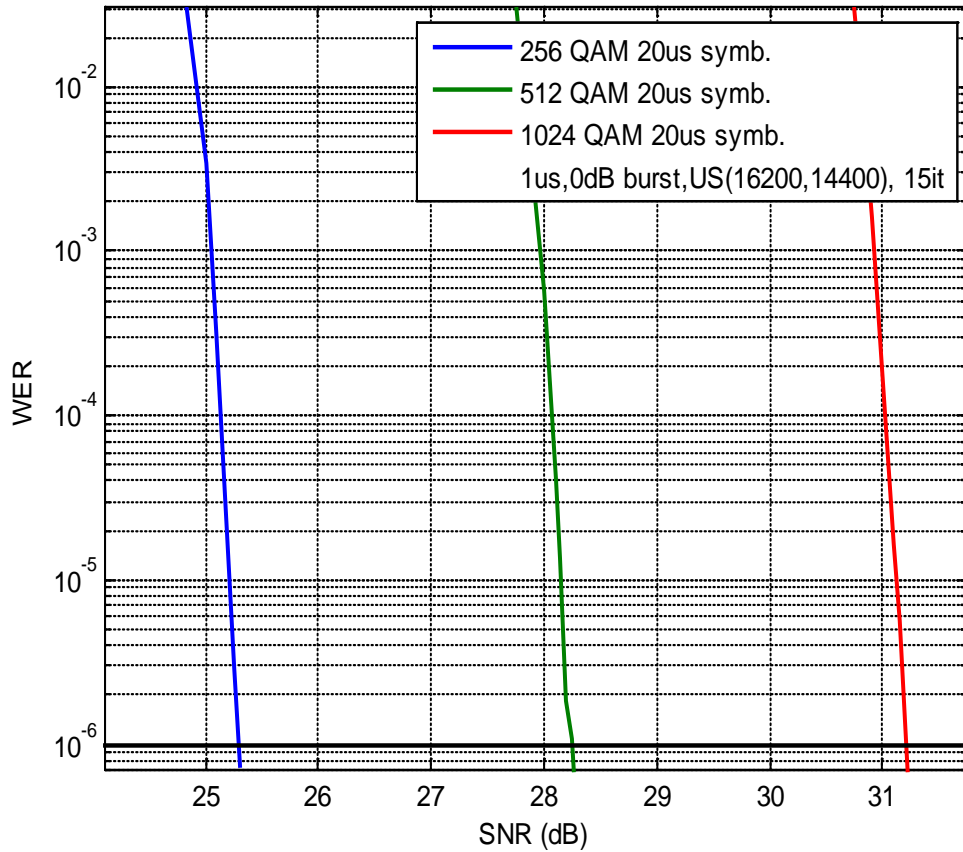


20 μ s symbol
(one affected)
Latency: 382.5 μ s
(depth:17)

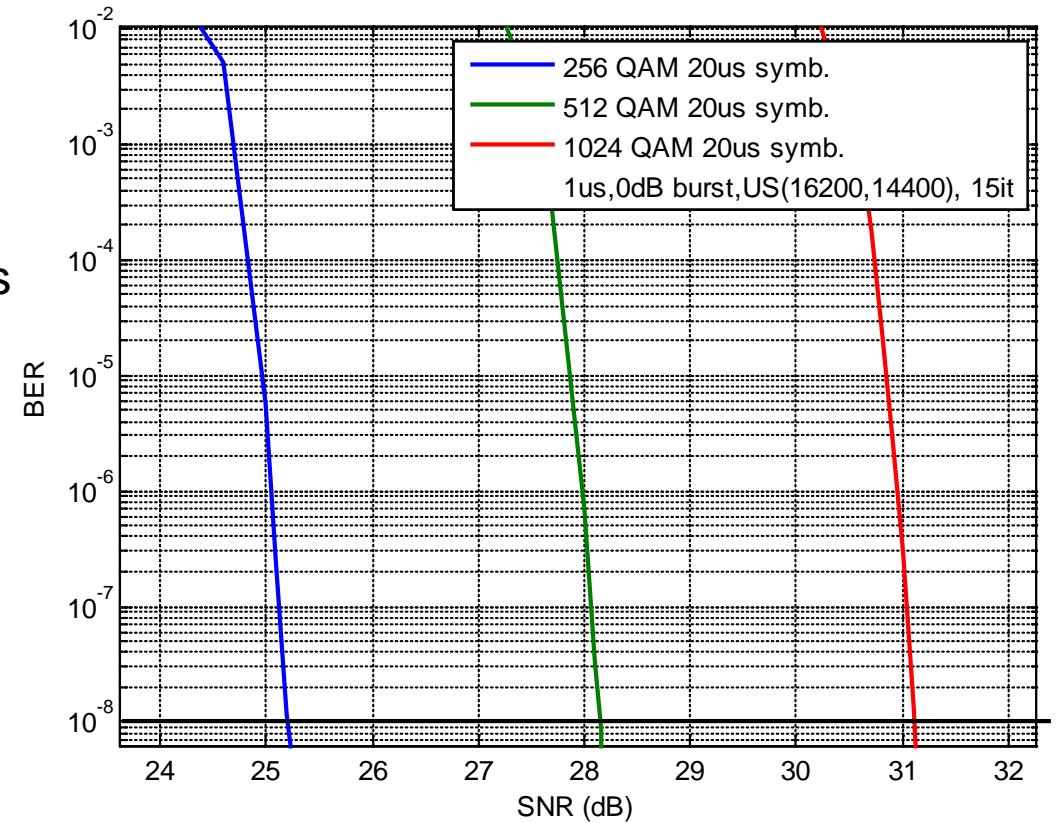


	256QAM	512QAM	1024QAM
SNR@WER=1e-6	25.11dB	28.04dB	30.99dB
SNR@BER=1e-8	25.1dB	28.01dB	30.97dB

20 μ s SYMBOLS ON 1 μ s 0dB BURST (LONG SIZE CODE) MAX 15 ITERATIONS



20 μ s symbol
(one affected)
Latency: 382.5 μ s
(depth:17)



	256QAM	512QAM	1024QAM
SNR@WER=1e-6	25.29dB	28.25dB	31.21dB
SNR@BER=1e-8	25.21dB	28.14dB	31.11dB

DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (LONG SIZE CODE)

20 μ s SYMBOLS ON 1 μ s 0dB BURST



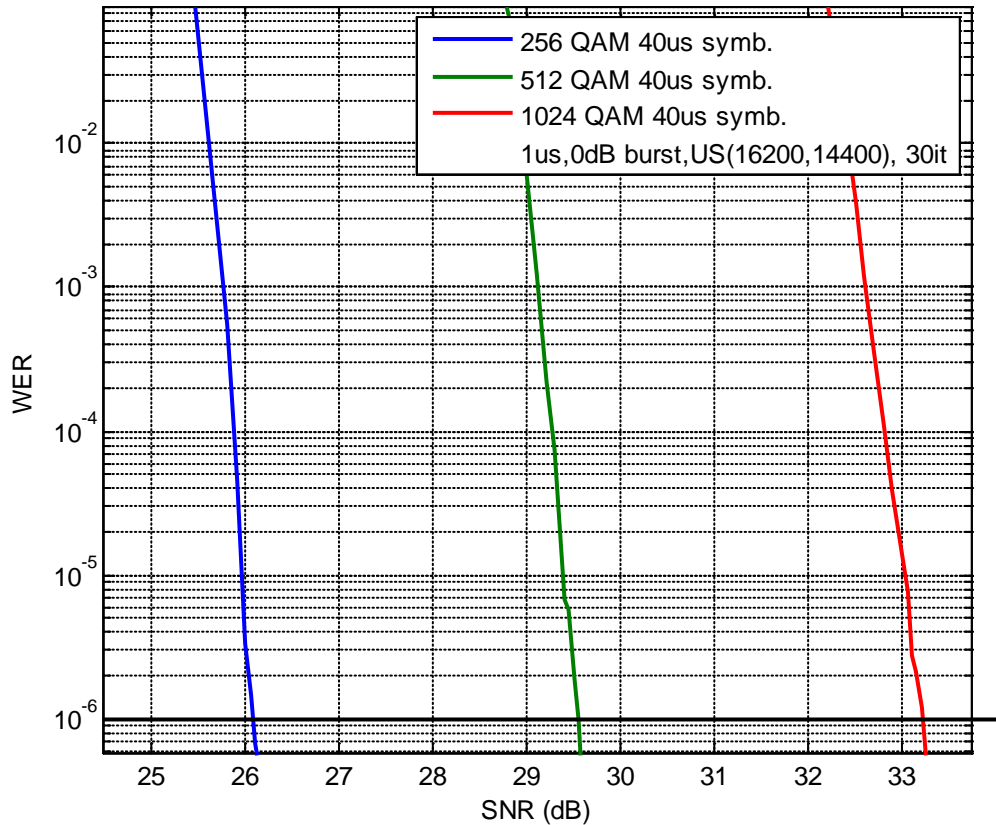
20 μ s symbol(two affected)

Latency: 382.5 μ s (depth:17)

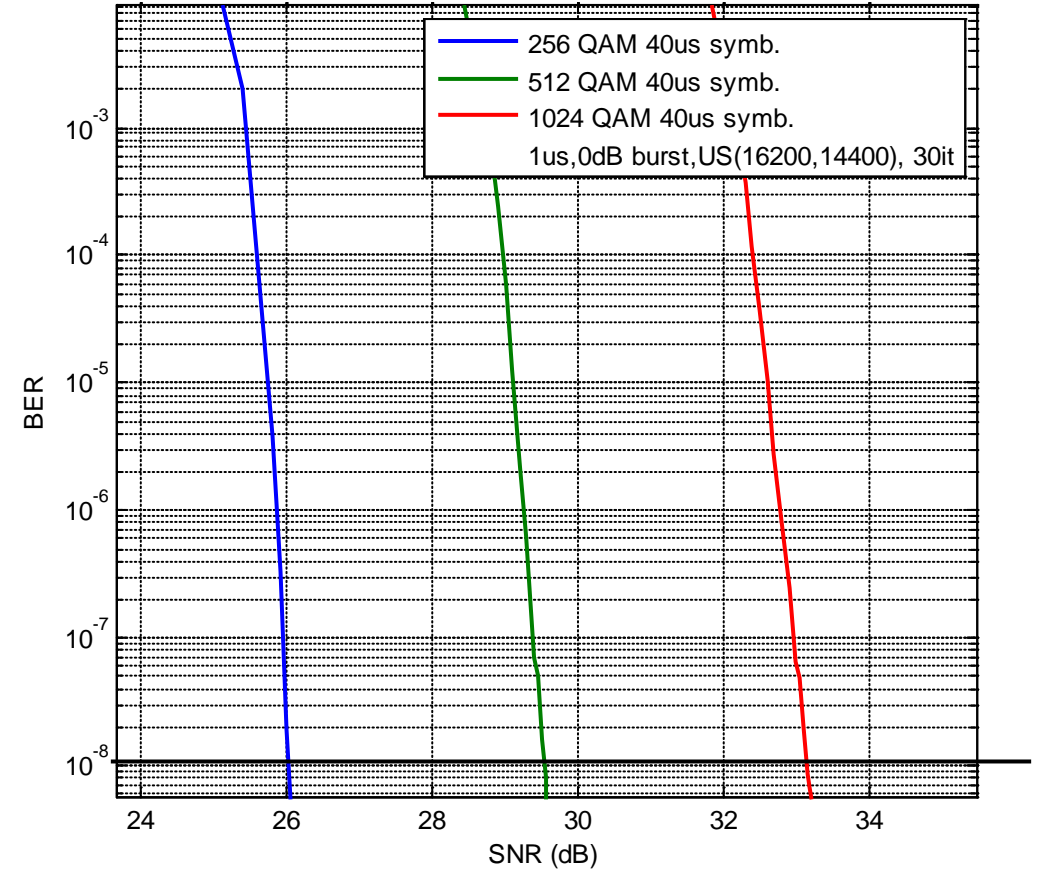
Long size
(16200,14400)
30 iterations
vs.
15 iterations

		256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	25.29dB	28.25dB	31.21dB
	Max.30 iterations	25.11dB	28.04dB	30.99dB
	Difference	0.18	0.21	0.22
SNR@BER=1e-8	Max. 15 iterations	25.21dB	28.14dB	31.11dB
	Max. 30 iterations	25.1dB	28.01dB	30.97dB
	Difference	0.2	0.13	0.14

40μs SYMBOLS ON 1μs 0dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS

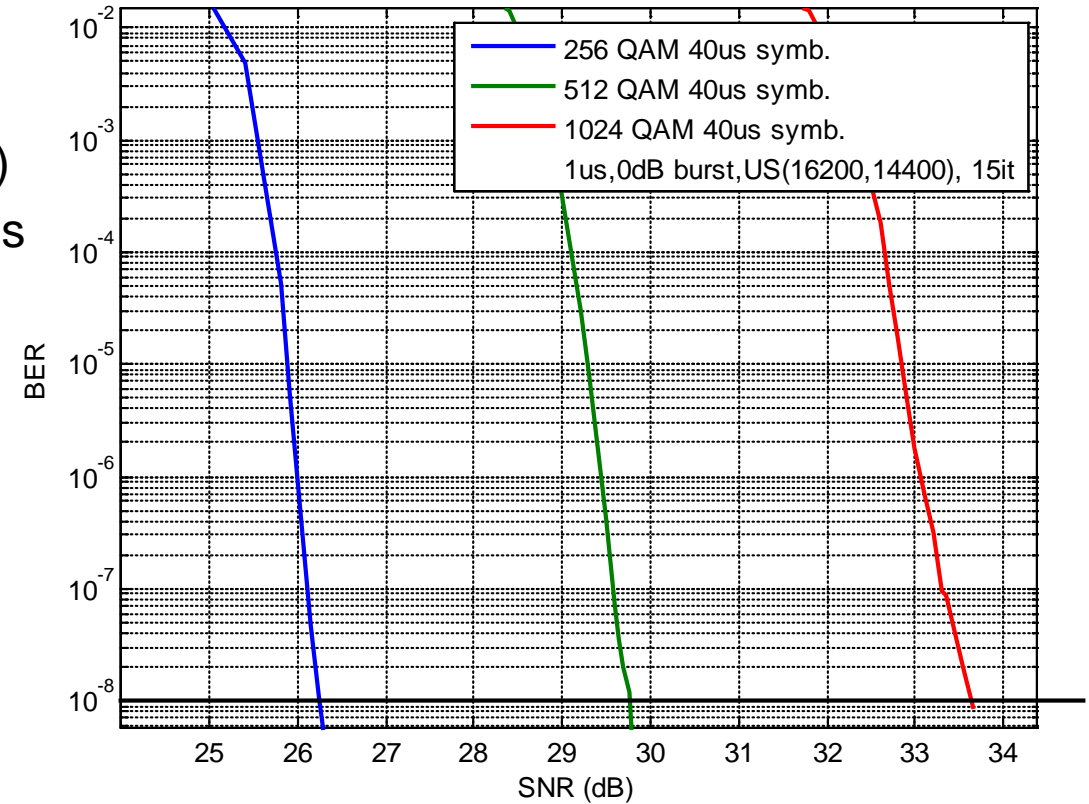
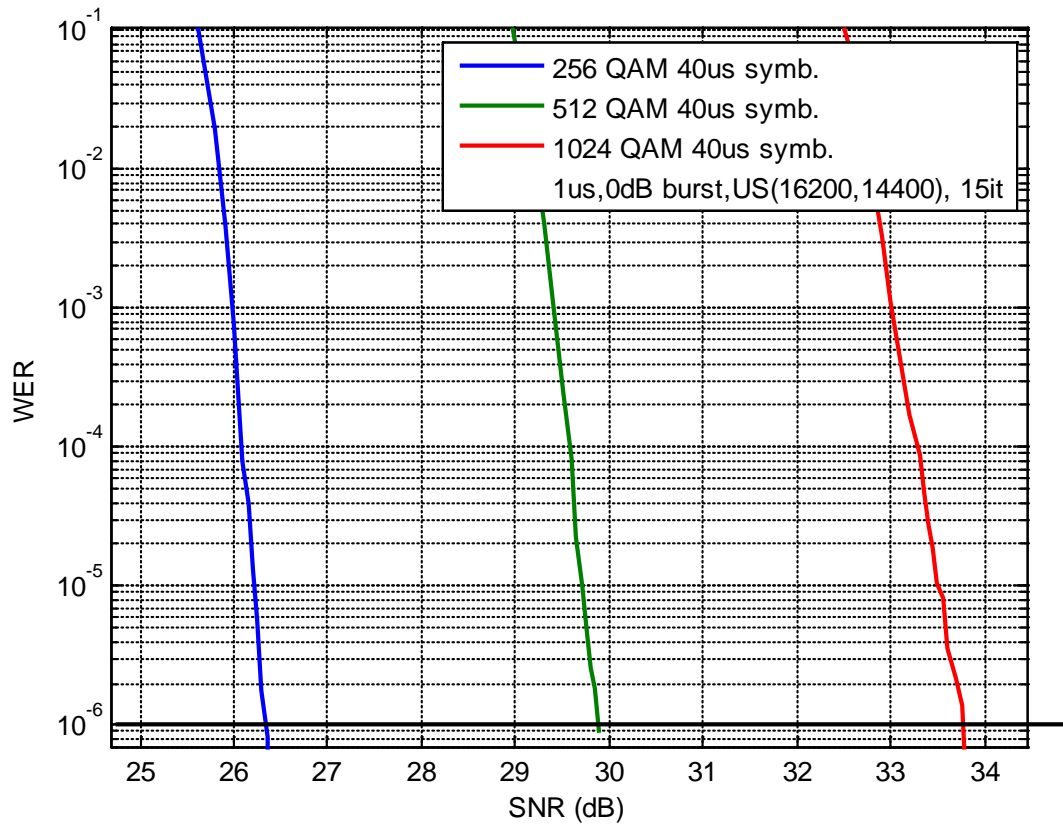


40μs symbol
(one affected)
Latency: 340μs
(depth:8)



	256QAM	512QAM	1024QAM
SNR@WER=1e-6	26.07dB	29.55dB	33.21dB
SNR@BER=1e-8	26.02dB	29.54dB	33.14dB

40μs SYMBOLS ON 1μs 0dB BURST (LONG SIZE CODE) MAX 15 ITERATIONS



	256QAM	512QAM	1024QAM
SNR@WER=1e-6	26.34dB	29.89dB	33.77dB
SNR@BER=1e-8	26.25dB	29.76dB	33.63dB

DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (LONG SIZE CODE)

40μs SYMBOLS ON 1μs 0dB BURST

40μs symbol (one affected)

Latency: 340μs (depth:8)

Long size
(16200,14400)
30 iterations
vs.
15 iterations

		256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	26.34dB	29.89dB	33.77dB
	Max.30 iterations	26.07dB	29.55dB	33.21dB
	Difference	0.27	0.34	0.56
SNR@BER=1e-8	Max. 15 iterations	26.25dB	29.76dB	33.63dB
	Max. 30 iterations	26.02dB	29.54dB	33.14dB
	Difference	0.23	0.22	0.49

MINIMUM INTERLEAVE DEPTH UNDER BURST NOISE (SHORT SIZE CODE)

Achieving BER=1e-8/WER=1e-6 (256QAM, 512QAM 1024-QAM)

	Burst duration	Burst SNR	Minimum Interleave depth
20 μ s symbol (two affected)	10 μ s	10dB	17 382.5 μ s
	1 μ s	0dB	17 382.5 μ s
40 μ s symbol (two affected)	10 μ s	10dB	8 340 μ s
	1 μ s	0dB	8 340 μ s

MINIMUM INTERLEAVE DEPTH UNDER BURST NOISE (MEDIUM SIZE CODE)

Achieving BER=1e-8/WER=1e-6
256 QAM, 512 QAM and 1024QAM

	Burst duration	Burst SNR	Minimum Interleave depth		
20µs symbol (two affected)	10µs	10dB	17 382.5µs		
	1µs	0dB	17 382.5µs		
40µs symbol (two affected)	10µs	10dB	8 340µs (256QAM)	8 340µs (256QAM)	9 382.5µs (1204QAM)
	1µs	0dB	8 340µs		

MINIMUM INTERLEAVE DEPTH UNDER BURST NOISE (LONG SIZE CODE)

Achieving BER=1e-8/WER=1e-6 (256QAM, 512QAM and 1024-QAM)

	Burst duration	Burst SNR	Minimum Interleave depth		
20µs symbol (two affected)	10µs	10dB	17 382.5µs		
	1µs	0dB	17 382.5µs		
40µs symbol (two affected)	10µs	10dB	8 340µs (256QAM)	9 382µs (512QAM)	11 467.5µs (1024QAM)
	1µs	0dB	8 340µs		

- **Three proposed LDPC codes for active plant have been presented**
 - Long (16200,14400) 89%, Medium (5940, 5040) 85%, and Short (1120, 840) 75% LDPC codes upstream
 - Long (16200,14400) 89% LDPC code downstream (same as upstream long code)
- **Performance evaluation criteria including modulation orders and channel model parameters have been discussed**
- **AWGN channel performance for all proposed codes has been presented**
- **Performance under burst noise conditions in AWGN has been presented**

Thank You



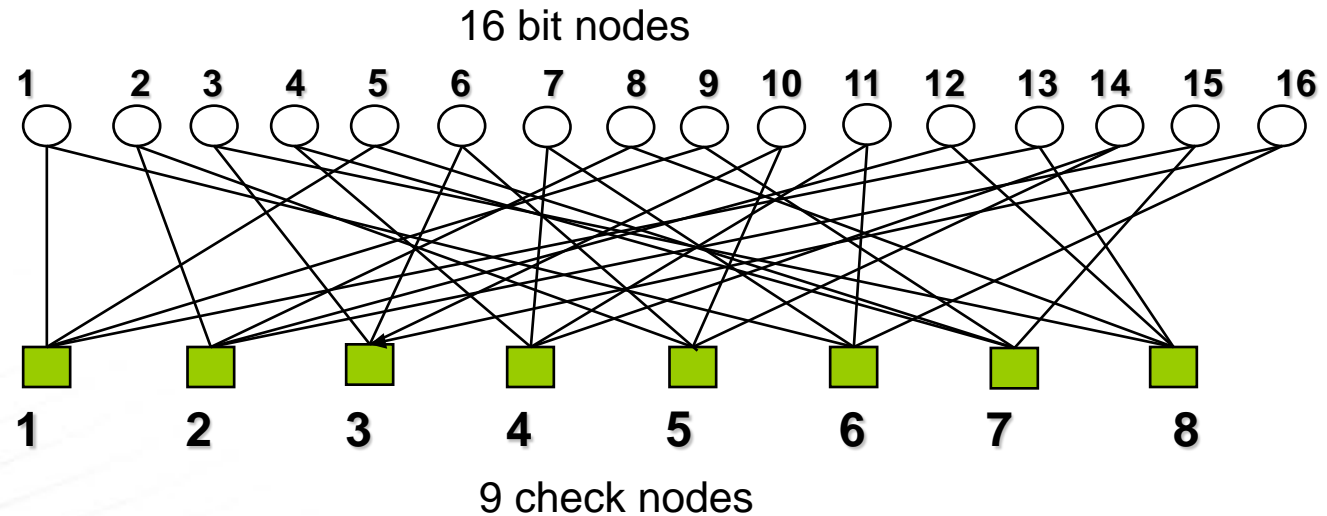
BACKGROUND ON LDPC CODES

- LDPC codes are specified by a (parity-check) matrix containing mostly 0's and relatively few 1's (R. G. Gallager 1960)
- A simple example

$$H = \begin{bmatrix} 1000 & 1000 & 1000 & 1000 \\ 0100 & 0001 & 0001 & 0010 \\ 0010 & 0100 & 0100 & 0001 \\ 0001 & 0010 & 0010 & 0100 \\ 0100 & 0100 & 0100 & 0100 \\ 1000 & 0010 & 0010 & 0001 \\ 0001 & 1000 & 1000 & 0010 \\ 0010 & 0001 & 0001 & 1000 \end{bmatrix}$$

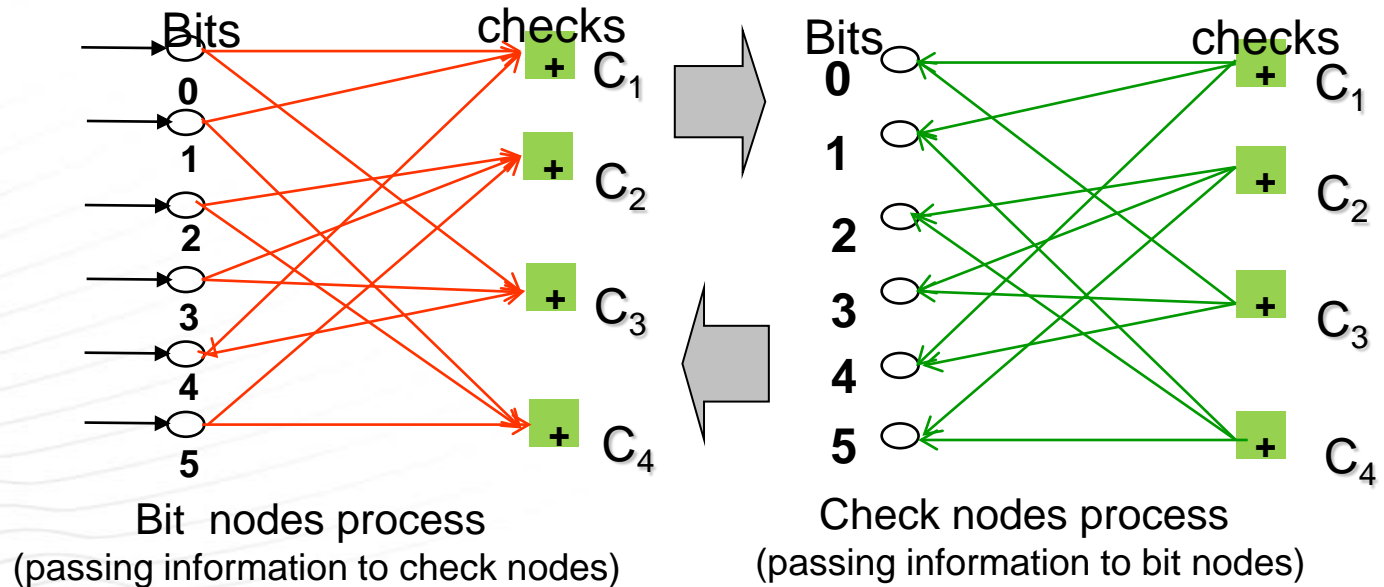
Parity check matrix of (16,9) regular LDPC code

32 1's , 128 0's
Density of 1's = 0.25



Tanner graphs

- **Belief propagation (AKA sum-product)** is a message passing algorithm for performing inference on Tanner graph.
 - It calculates the marginal distribution for each unobserved node, conditional on any observed nodes
 - First flooding iteration:
 - Observed nodes: bit nodes with known LLR value from the demodulation
 - Unobserved nodes: check nodes
 - Extrinsic LLR; calculated marginal distribution for updated check node
 - Second flooding iteration
 - Observed nodes: check nodes with extrinsic LLR
 - Unobserved node: bit nodes
 - Etc.



DECODING CAPABILITY ON (16,9) EXAMPLE CODE

tx: X=(0000000000000000)

SNR	-3dB	-1dB	0dB	0.5dB
Classical Bounded-distance decoding (hard)	Failed	Failed	Failed	Failed
Maximal likelihood decoding (MLD)(soft)	1010000010011 ($H\underline{x}^T=0$) but wrong codeword	corrected	corrected	corrected
Bit MLD (soft)	Failed ($H\underline{x}^T \neq 0$)	failed ($H\underline{x}^T \neq 0$)	corrected	corrected
Iterative message passing decoding (soft)	not converge	not converge	not converge	corrected (5 iterations)

QPSK on AWGN channel

- back-substitution algorithm

Denote the parity check matrix $H=[H_I, H_P]$, where

$$H_P = \begin{array}{|c|c|c|c|c|} \hline I(u_{0,0}) & 0 & 0 & 0 & 0 \\ \hline I(u_{1,0}) & I(u_{1,1}) & 0 & 0 & 0 \\ \hline 0 & I(u_{2,1}) & I(u_{2,2}) & 0 & 0 \\ \hline 0 & 0 & I(u_{3,2}) & I(u_{3,3}) & 0 \\ \hline 0 & 0 & 0 & I(u_{4,3}) & I(u_{4,4}) \\ \hline \end{array}$$

I : an L by L identity matrix ($L=360$ for long size code, $L=180$ for medium size code and $L=56$ for short size code)

$I(u)$: the matrix obtained by right cyclic shift I by u positions

$\mathbf{c}_I = (c_0, c_1, \dots, c_{k-1})$: $k=L(n-m)$ input (information) bits. .

$\mathbf{c}_P = (c_k, c_{k+1}, \dots, c_{Ln-1})$: Lm parity bits need to be computed by an encoder

Encoder will output $\mathbf{c}_P^T = \begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix}$, where $C_i = (c_{L(n-m+i)}, c_{L(n-m+i)+1}, \dots, c_{L(n-m+i)+L-1})^T$

Encoding procedure:

Input: $\mathbf{c}_I = (c_0, c_1, \dots, c_{k-1})$

Step 1: Compute $V_i = H_{I,i} \mathbf{c}_I^T \quad i = 0, \dots, 4$

Step 2: Back-substitution

$$\begin{aligned} C_0 &= V_0(L - u_{0,0}) \\ C_1 &= V_1(L - u_{1,1}) + C_0((L - u_{1,1} + u_{1,0}) \bmod L) \\ C_2 &= V_2(L - u_{2,2}) + C_1((L - u_{2,2} + u_{2,1}) \bmod L) \\ C_3 &= V_3(L - u_{3,3}) + C_2((L - u_{3,3} + u_{3,2}) \bmod L) \\ C_4 &= V_4(L - u_{4,4}) + C_3((L - u_{4,4} + u_{4,3}) \bmod L) \end{aligned}$$

A: a size L binary column vector.

A(u): a column vector obtained by up cyclic shifting V in u positions

Output: $\mathbf{c}_P^T = \begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix}$