

# EPOC FEC LDPC CODES FOR ACTIVE PLANT



Presenters: Rich Prodan and BZ Shen

<b>CODES</b>	<b>Rate</b>	<b>Length</b>
A	$R_A = 8/9$	16200
B	$R_B = 8/9$	16200
C	$R_C = 0.848$	5940
D	$R_D = 3/4$	1120
E	$R_E = 9/10$	14400
F	$R_F = 9/10$	10800
G	$R_G = 13/15$	5400
H	$R_H = 3/4$	960

<b>DEPLOYMENT</b>	<b>Passive plant</b>	<b>Active plant</b>
US, low band	F, G, H	B, C, D
DS, low band	E, (F), G, H	---
US, high band	E, (F), G, H	---
DS, high band	E, (F), G, H	A

# REDUCED PROPOSED CODES FOR ACTIVE PLANT

<b>CODES</b>	<b>Rate</b>	<b>Length</b>
B	$R_B = 8/9$	16200
C	$R_C = 0.848$	5940
D	$R_D = 3/4$	1120
E	$R_E = 9/10$	14400
F	$R_F = 9/10$	10800
G	$R_G = 13/15$	5400
H	$R_H = 3/4$	960

<b>DEPLOYMENT</b>	<b>Passive plant</b>	<b>Active plant</b>
US, low band	F, G, H	B, C, D
DS, low band	E, (F), G, H	---
US, high band	E, (F), G, H	---
DS, high band	E, (F), G, H	B

This presentation will concentrate on:

<b>CODES</b>	<b>Rate</b>	<b>Length</b>
B	$R_B = 8/9$	16200
C	$R_C = 0.848$	5940
D	$R_D = 3/4$	1120

- **Proposed code matrices**
- **Performance evaluation criteria**
- **AWGN channel performance**
- **Performance under burst noise**
- **Conclusion**
  
- **Background on LDPC Code Algorithms**
  - LDPC codes in general
  - Iterative message passing decoding for LDPC codes
  - Possible encoding method

- **Definition of an (n, k) QC-LDPC code**

- A QC-LDPC parity-check matrix can be divided into blocks of L by L submatrices, where L represents the submatrix size or lifting factor.
- The parity-check matrix in compact circulant form is represented by an m by n block matrix:

$$H_{base} = \begin{bmatrix} H_{1,1} & H_{1,2} & H_{1,3} & \cdots & H_{1,n} \\ H_{2,1} & H_{2,2} & H_{2,3} & \cdots & H_{2,n} \\ H_{3,1} & H_{3,2} & H_{3,3} & \cdots & H_{3,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ H_{m,1} & H_{m,2} & H_{m,3} & \cdots & H_{m,n} \end{bmatrix}$$

- Each submatrix  $H_{i,j}$  is a L by L all-zero submatrix or a cyclic right-shifted Identity submatrix.
- The parity-check matrix tables consist of entries  $\{-1, 0, \dots, L-1\}$ , where a '-1' value represents an all-zero submatrix, and the remaining values represent an identity submatrix which has been cyclically right-shifted by the specified value.
- The  $H_{base}$  matrix can be represented as  $[H_1 | H_2]$  where  $H_2$  represents the parity portion.
- The code rate is  $(n-m)/n$  and a codeword length is  $n L$  bits.

- The code will be used for both downstream and upstream

Long size code: Rate= 8/9 (16200, 14400) code, m=5 rows x n=45 columns, L=360

93	271	-1	83	26	208	245	200	-1	175	331	17	86	-1	337	-1	238	81	-1	307	-1	165	-1	47	76	73	150	349	139	331	118	345	27	294	-1	145	279	97	106	160	143	-1	-1	-1	-1
274	115	329	338	124	-1	293	-1	69	64	342	-1	88	139	-1	137	212	-1	157	195	357	81	194	1	159	56	72	126	277	156	32	111	175	-1	306	224	-1	206	-1	29	106	334	-1	-1	-1
134	355	175	24	253	242	-1	187	94	26	87	302	-1	191	323	22	-1	245	294	240	84	76	342	345	174	269	329	-1	214	-1	-1	-1	-1	218	104	40	197	73	229	63	-1	270	72	-1	-1
-1	-1	184	70	247	14	22	7	285	54	-1	352	26	108	10	298	123	139	117	-1	336	49	202	359	342	-1	224	106	-1	273	177	245	98	355	178	176	147	-1	280	-1	-1	-1	221	208	-1
253	273	90	-1	-1	151	311	320	339	-1	295	148	48	91	62	100	232	146	200	135	12	-1	179	-1	-1	232	-1	21	331	313	349	34	97	187	38	-1	235	52	170	58	-1	-1	-1	257	0

- **Number of equations: 1800**
- **Number of edges: 60840**
- **Density: 0.00209**
- **Number of nodes in the Tanner graph**
  - Bit nodes: 16200
  - Check nodes: 1800

- The code will be used for upstream only

Medium size code: Rate= 28/33(0.848) (5940, 5040) code, m=5 rows x n=33 columns, L=180

142	158	113	124	92	44	93	70	172	3	25	44	141	160	50	45	118	84	-1	64	66	97	1	115	8	108	-1	-1	22	-1	-1	-1	-1
54	172	145	28	55	19	159	22	96	12	85	-1	128	5	158	120	51	171	65	141	-1	42	83	7	-1	39	121	84	101	171	-1	-1	-1
63	11	112	114	61	123	72	55	114	20	53	114	42	33	4	66	163	50	46	17	175	-1	-1	-1	92	-1	41	138	-1	34	74	-1	-1
28	160	102	44	8	84	126	9	169	174	147	24	145	-1	26	-1	-1	-1	67	82	4	177	151	131	139	117	36	18	-1	-1	23	8	-1
52	159	75	74	46	71	42	11	108	153	-1	72	-1	163	-1	9	2	168	158	-1	1	49	89	63	179	10	75	161	-1	-1	-1	177	19

- Number of equations: 900
- Number of edges: 23580
- Density: 0.0044
- Number of nodes in the Tanner graph
  - Bit nodes: 5940
  - Check nodes: 900

- The code will be used for upstream only

Short size code: Rate= 3/4 (1120, 840) code, m=5 rows x n=20 columns, L=56

5	14	12	1	2	37	45	26	24	0	3	-1	34	7	46	10	-1	-1	-1	-1
0	35	1	26	0	10	16	16	34	4	2	23	0	51	-1	49	20	-1	-1	-1
12	28	22	46	3	16	51	2	25	29	19	18	52	-1	37	-1	34	39	-1	-1
0	51	16	31	13	39	27	33	8	27	53	13	-1	52	33	-1	-1	38	7	-1
36	6	3	51	4	19	4	45	48	9	-1	11	22	23	43	-1	-1	-1	14	1

- **Number of equations: 280**
- **Number of edges: 4424**
- **Density: 0.014**
- **Number of nodes in the Tanner graph**
  - Bit nodes: 1120
  - Check nodes: 280



## ▪ Evaluation parameters

- Constellation:
  - Downstream 256, 512, 1024, 2048, and 4096 QAM
  - Upstream 64,128,256,512 and 1024 QAM
- Two OFDM symbol durations
  - 20  $\mu$ s and 40  $\mu$ s
  - Cyclic prefix of 2.5  $\mu$ s
- Probability of error at threshold
  - BER=1e-8
  - WER=1e-6
- AWGN SNR threshold with and without burst error events
- Channel assumptions for burst error events
  - Downstream burst noise: 16  $\mu$ s at 20 dB SNR or 16  $\mu$ s at 5 dB SNR (two consecutive OFDM symbols)
  - Upstream burst noise: 1  $\mu$ s at 0 dB SNR (1 OFDM symbol) or 10  $\mu$ s at 10 dB SNR (two consecutive OFDM symbols)

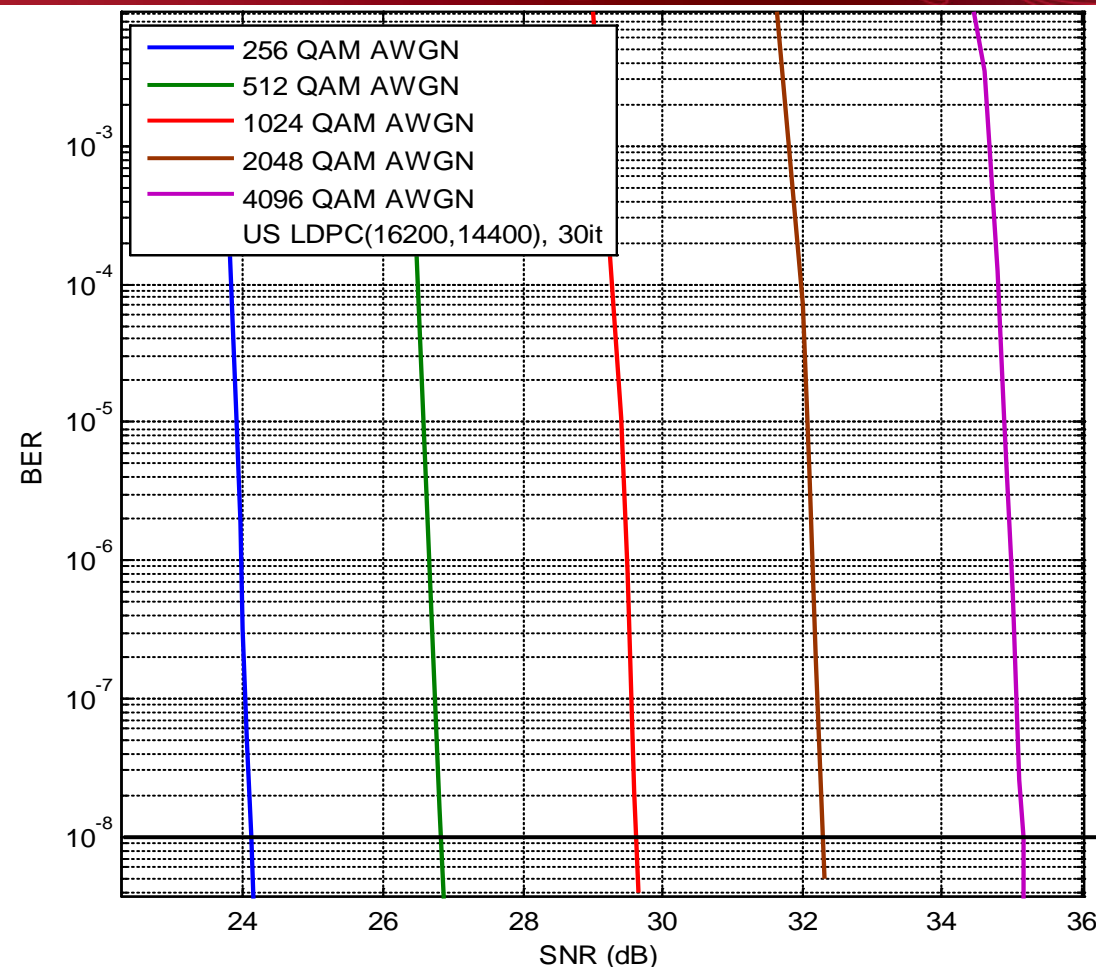
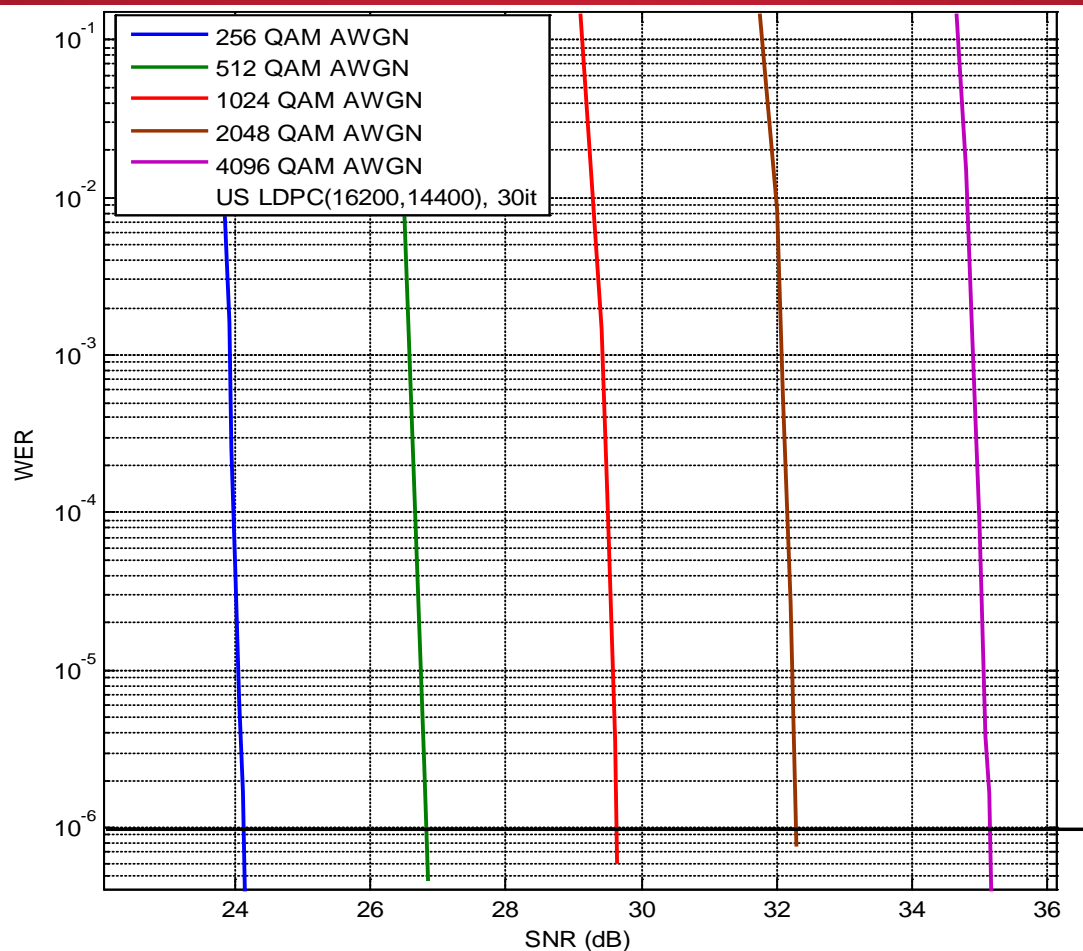
## ▪ Decoding procedure

- Iterative sum-product message passing algorithm
- Floating point
- Maximum of 15 and 30 flooding iterations

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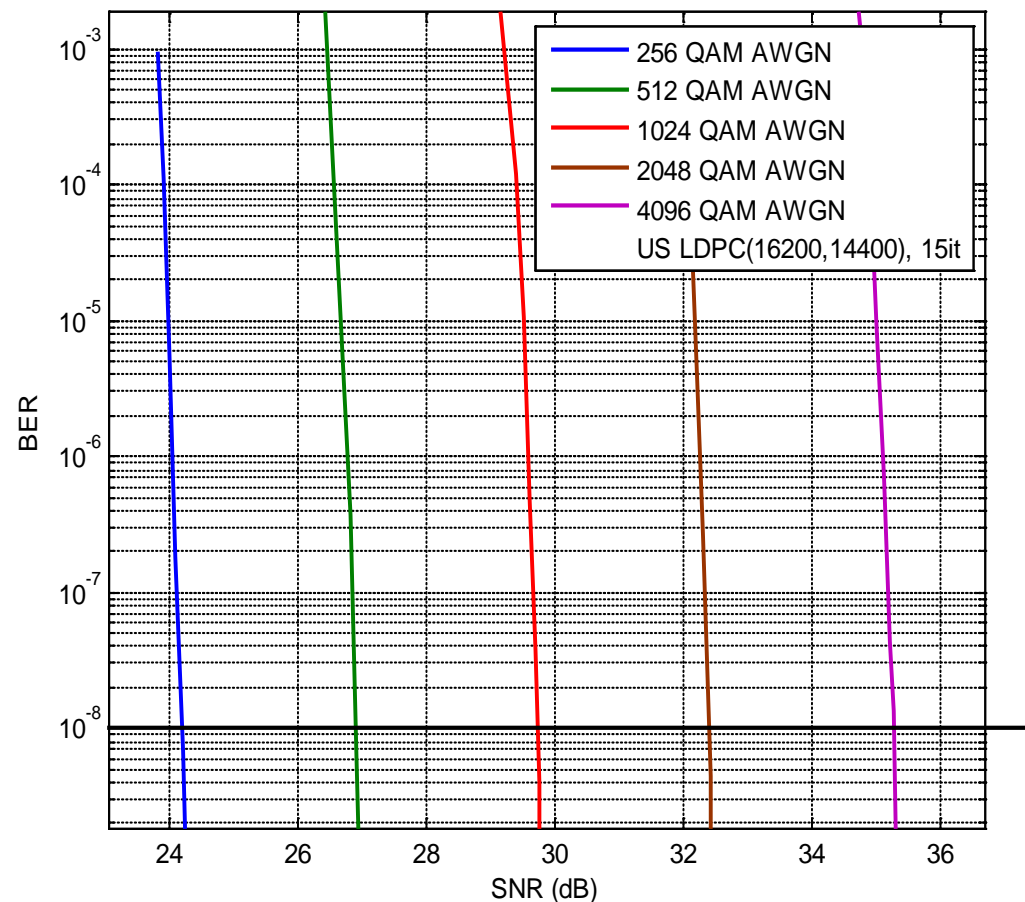
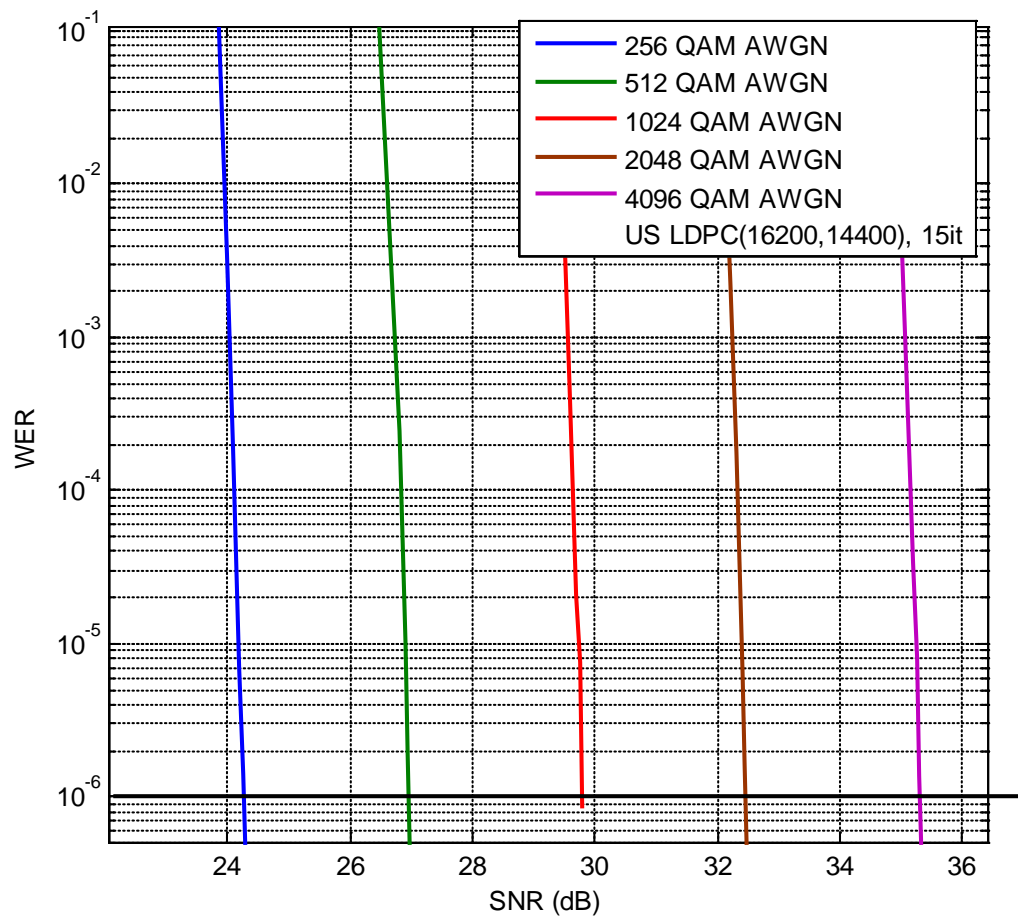
# AWGN PERFORMANCE

# ON AWGN CHANNEL (LONG SIZE CODE) (MAX. 30 ITERATIONS)



Upstream (16200, 14400) code (30 iterations)		256QAM	512QAM	1024QAM	2048QAM	4096QAM
	<b>SNR@WER=1e-6</b>		24.11dB	26.83dB	29.64dB	32.29dB
<b>SNR@BER=1e-8</b>		24.1dB	26.82dB	29.62dB	32.28dB	35.15dB

# ON AWGN CHANNEL (LONG SIZE CODE) (MAX. 15 ITERATIONS)



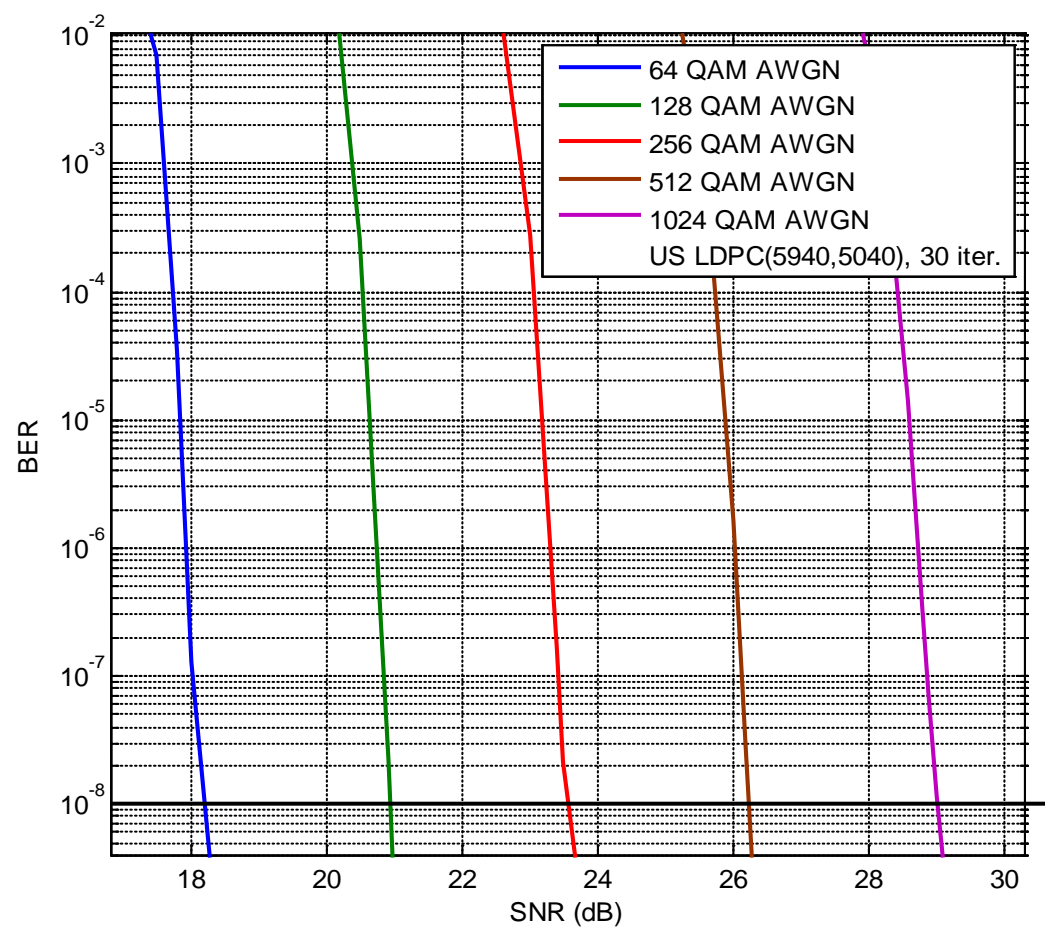
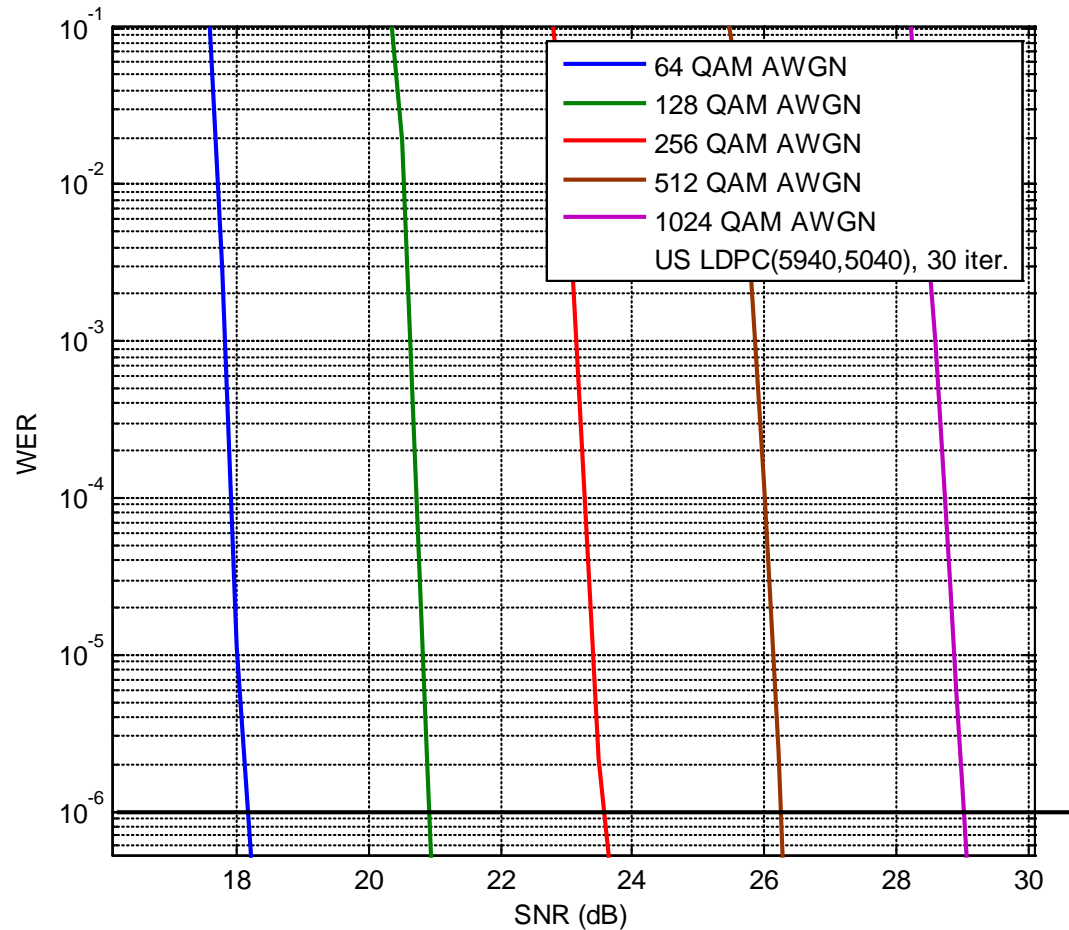
Upstream (16200, 14400) code (15 iterations)		256QAM	512QAM	1024QAM	2048QAM	4096QAM
	<b>SNR @WER=1e-6</b>	24.26dB	26.97dB	29.8dB	32.46dB	35.31dB
	<b>SNR @BER=1e-8</b>	24.19dB	26.89dB	29.73dB	32.38dB	35.26dB

# ON AWGN CHANNEL (LONG SIZE CODE) DIFFERENCE BETWEEN 15 AND 30 ITERATIONS

Downstream code: Max. 30 iterations vs. Max. 15 iterations

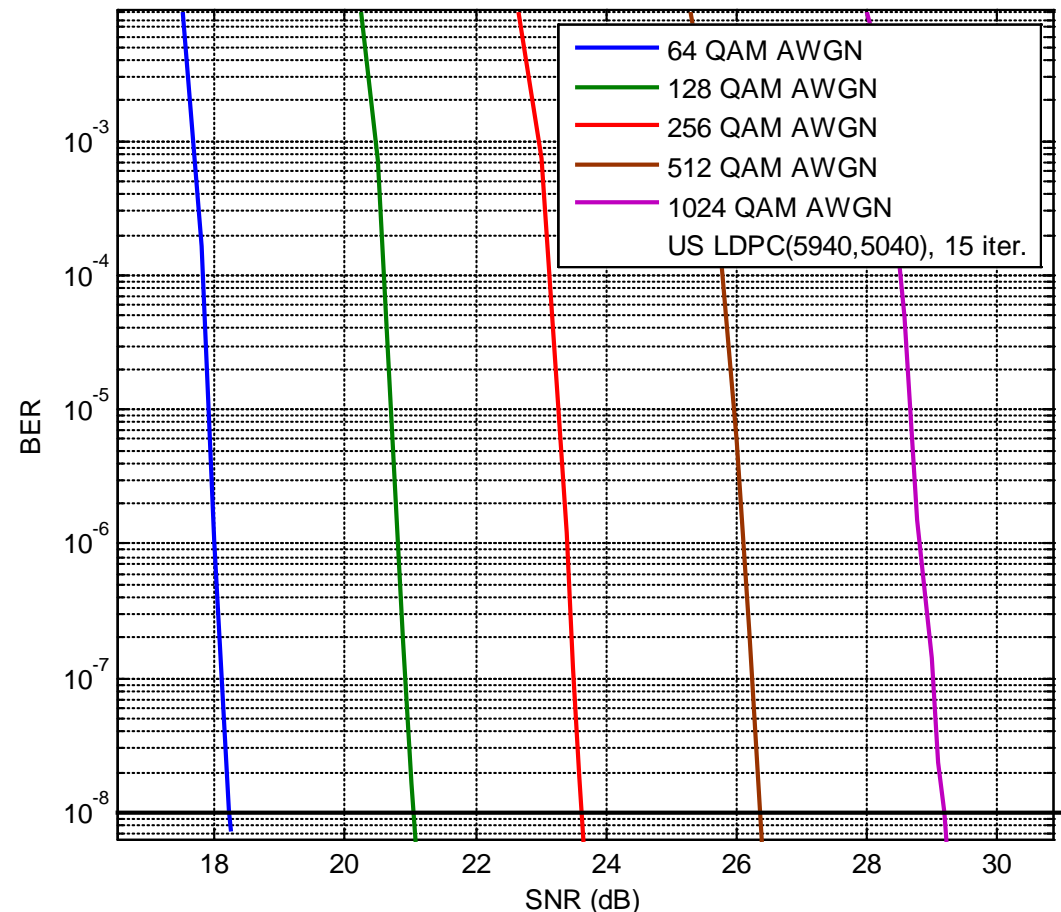
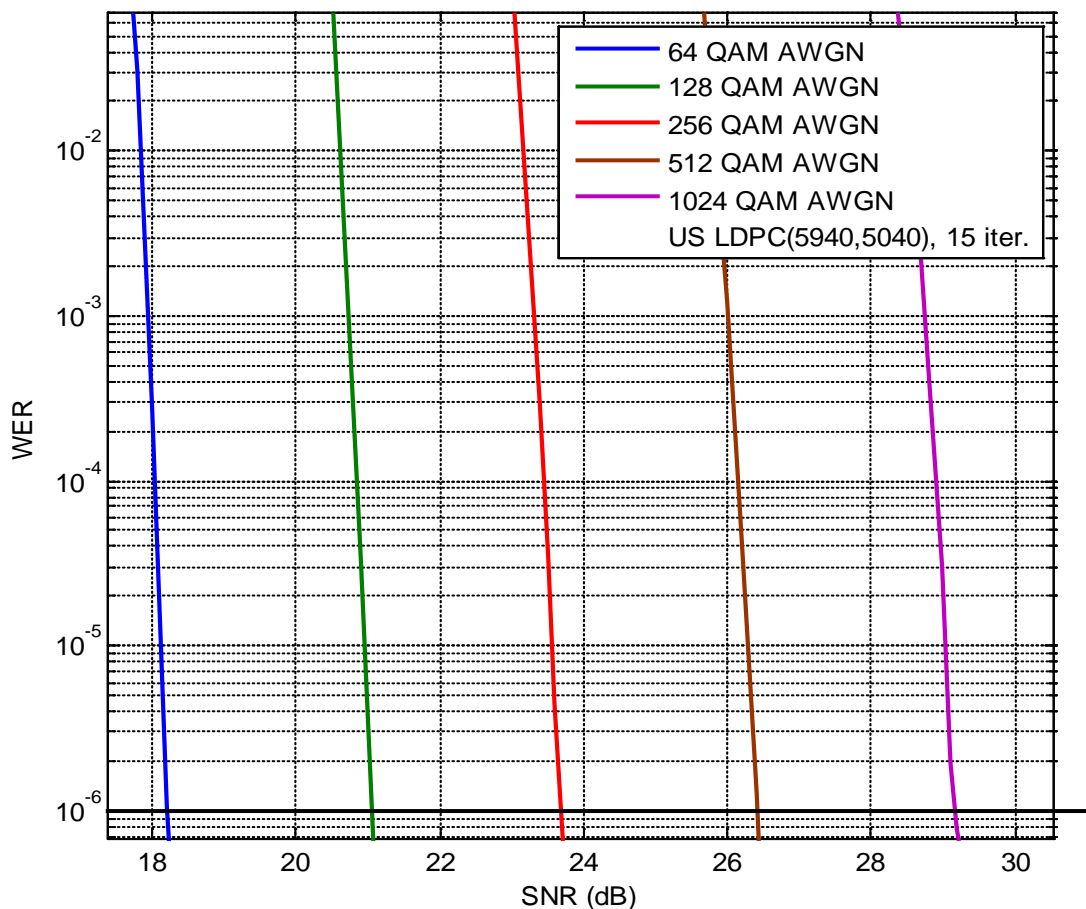
		256QAM	512QAM	1024QAM	2048QAM	4096QAM
<b>SNR@WER=1e-6</b>	<b>Max.15 iterations</b>	24.26dB	26.97dB	29.8dB	32.46dB	35.31dB
	<b>Max.30 iterations</b>	24.11dB	26.83dB	29.64dB	32.29dB	35.16dB
	<b>Difference</b>	0.15	0.14	0.16	0.17	0.15
<b>SNR@BER=1e-8</b>	<b>Max. 15 iterations</b>	24.19dB	26.89dB	29.73dB	32.38dB	35.26dB
	<b>Max. 30 iterations</b>	24.1dB	26.82dB	29.62dB	32.28dB	35.15dB
	<b>Difference</b>	0.09	0.07	0.11	0.1	0.11

# ON AWGN CHANNEL (UPSTREAM MEDIUM SIZE CODE) (MAX. 30 ITERATIONS)



	64QAM	128QAM	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	18.15dB	20.93dB	23.59dB	26.26dB	29dB
<b>SNR@BER=1e-8</b>	18.17dB	20.93dB	23.6dB	26.25dB	28.99dB

# ON AWGN CHANNEL (UPSTREAM MEDIUM SIZE CODE) (MAX. 15 ITERATIONS)



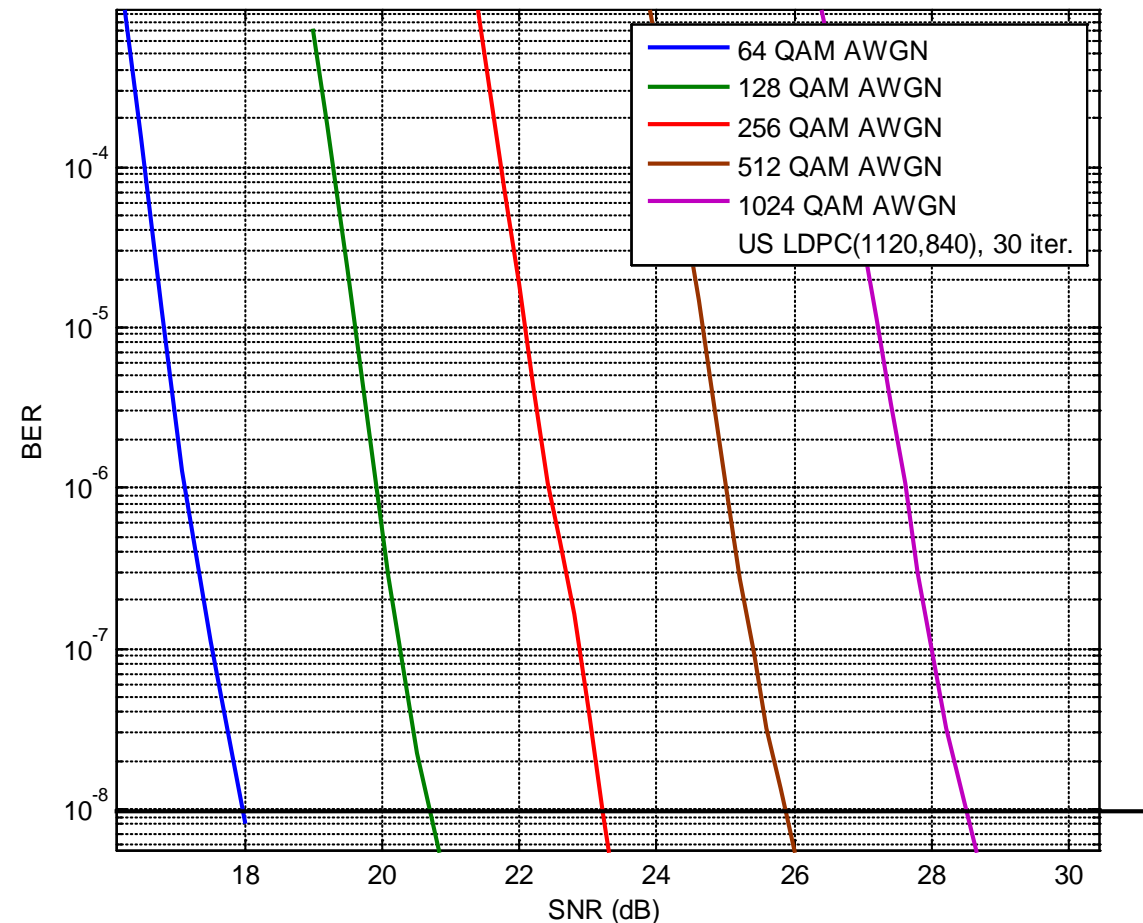
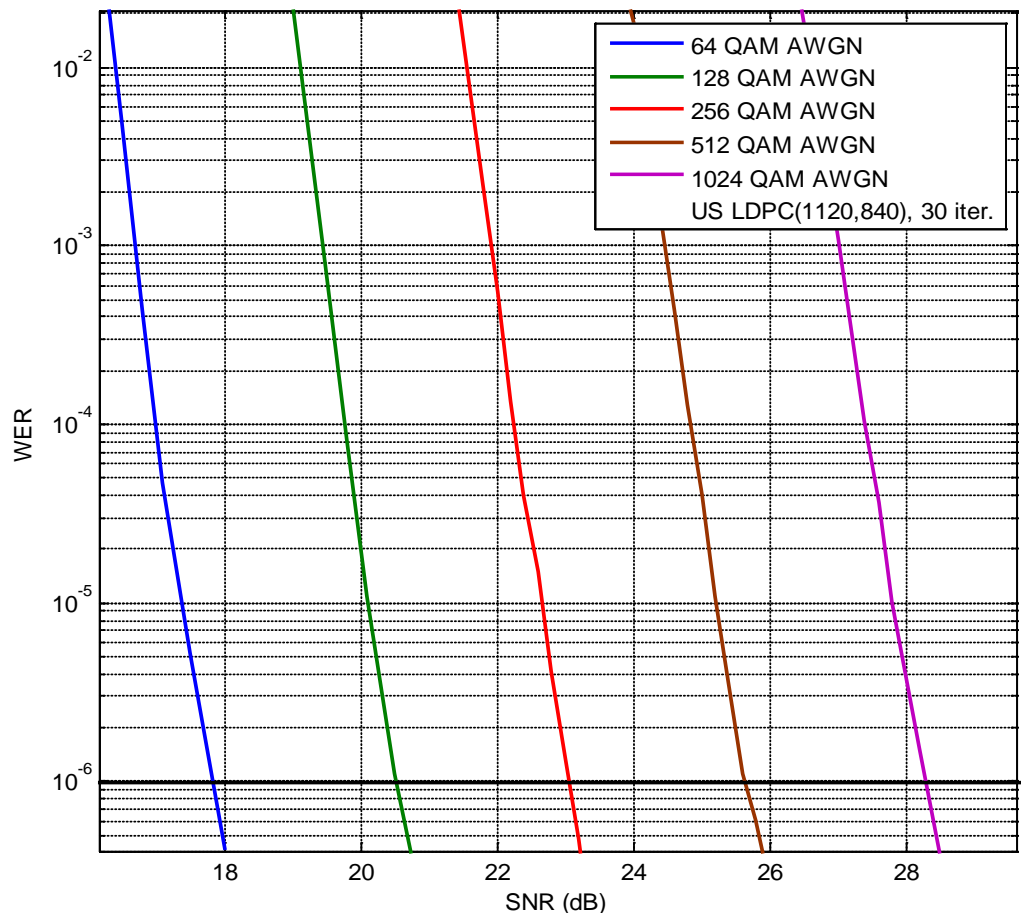
	64QAM	128QAM	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	18.2dB	21.05dB	23.69dB	26.4dB	29.18dB
<b>SNR@BER=1e-8</b>	18.2dB	21.03dB	23.69dB	26.38dB	29.16dB

# DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (UPSTREAM MEDIUM SIZE CODE)

		64QAM	128QAM	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	<b>Max.15 iterations</b>	18.2dB	21.05dB	23.69dB	26.4dB	29.18dB
	<b>Max. 30 iterations</b>	18.15dB	20.93dB	23.59dB	26.26dB	29dB
	<b>Difference</b>	0.05	0.12	0.1	0.14	0.18
<b>SNR@BER=1e-8</b>	<b>Max.15 iterations</b>	18.2dB	21.03dB	23.69dB	26.38dB	29.16dB
	<b>Max. 30 iterations</b>	18.17dB	20.93dB	23.6dB	26.25dB	28.99dB
	<b>Difference</b>	0.03	0.1	0.09	0.13	0.17

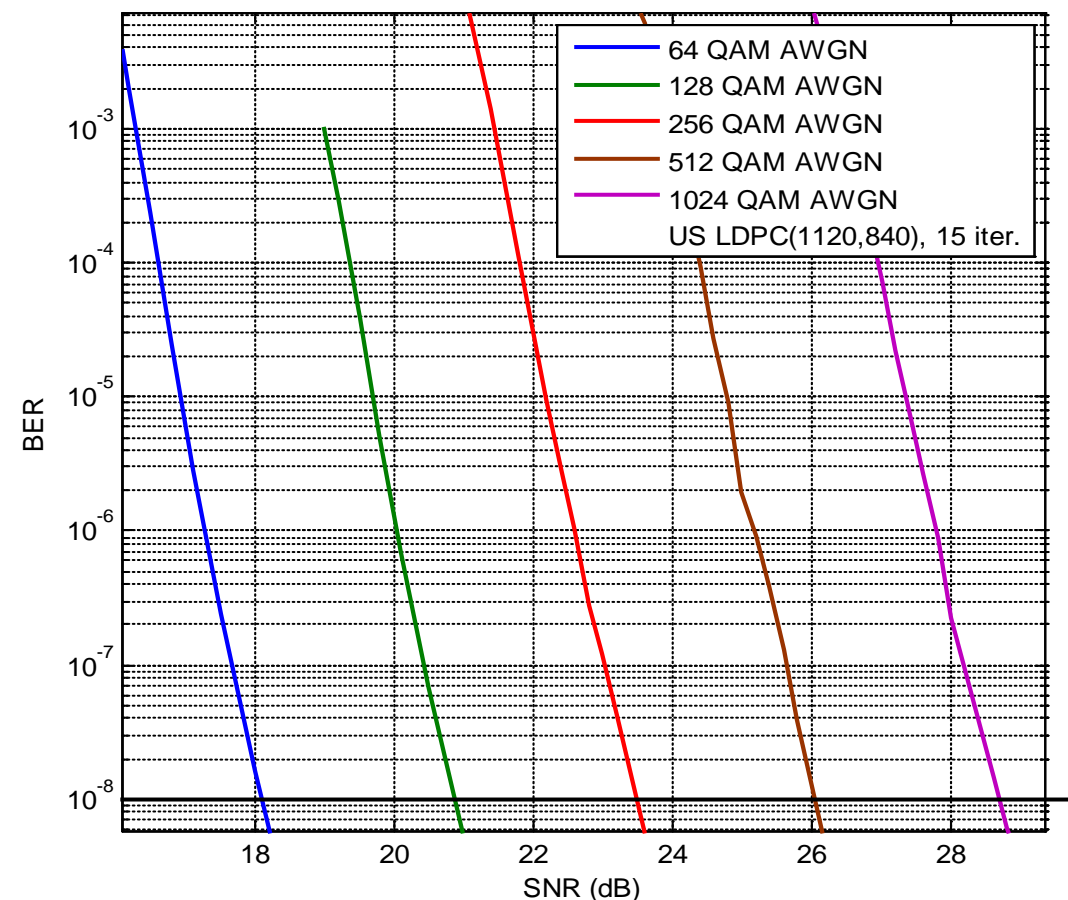
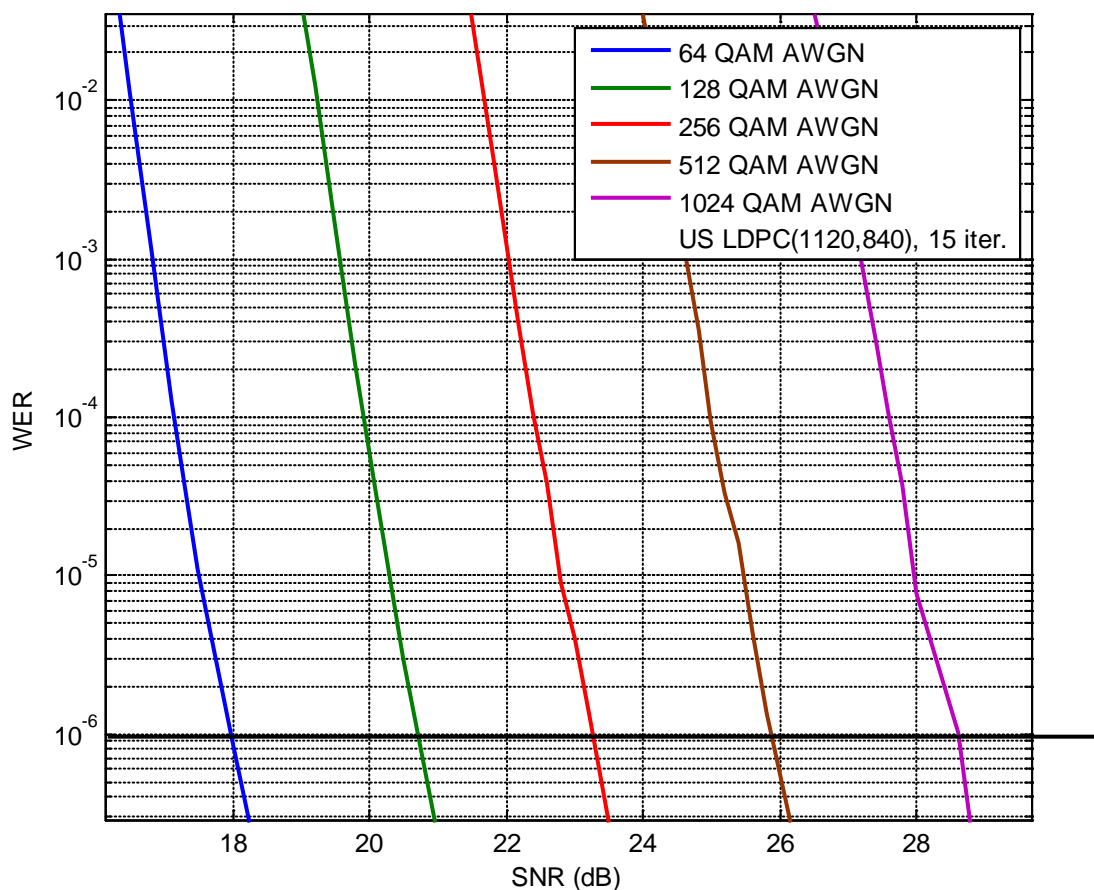


# ON AWGN CHANNEL (UPSTREAM SHORT SIZE CODE) (MAX. 30 ITERATIONS)



	64QAM	128QAM	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	17.83dB	20.52dB	23.08dB	25.69dB	28.3dB
<b>SNR@BER=1e-8</b>	17.96dB	20.69dB	23.21dB	26.01dB	28.59dB

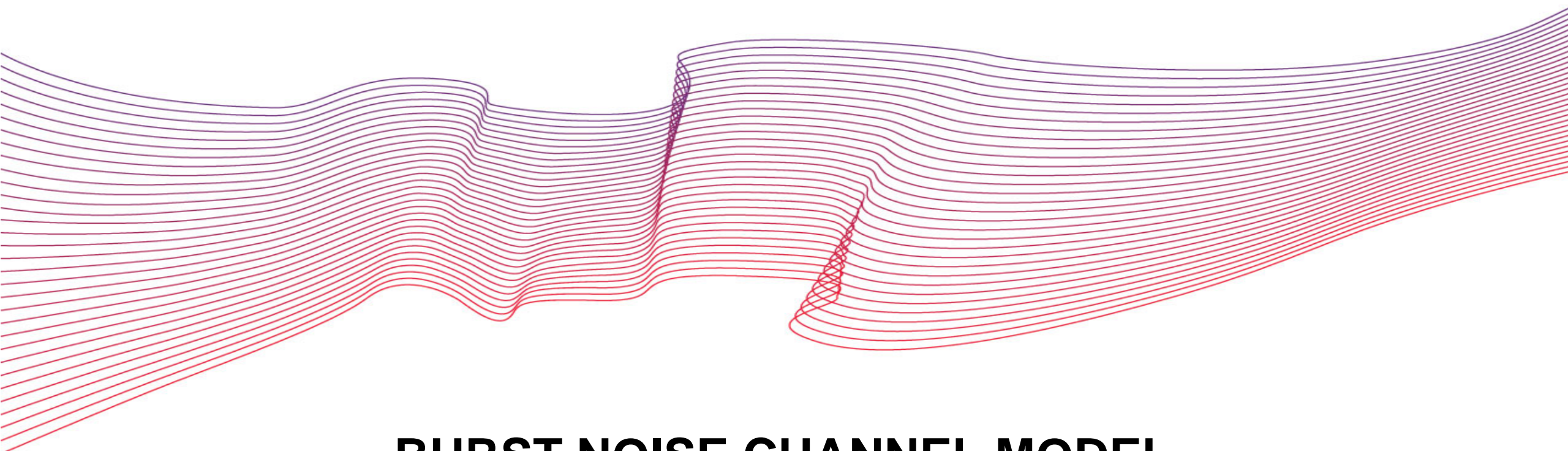
# ON AWGN CHANNEL (UPSTREAM SHORT SIZE CODE) (MAX. 15 ITERATIONS)



	64QAM	128QAM	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	17.96dB	20.71dB	23.23dB	25.91dB	28.49dB
<b>SNR@BER=1e-8</b>	18.1dB	20.87dB	23.37dB	26.11dB	28.67dB

# DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (UPSTREAM SHORT SIZE CODE)

		64QAM	128QAM	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	<b>Max.15 iterations</b>	17.96dB	20.71dB	23.23dB	25.91dB	28.49dB
	<b>Max. 30 iterations</b>	17.83dB	20.52dB	23.08dB	25.69dB	28.3dB
	<b>Difference</b>	0.13	0.19	0.15	0.22	0.19
<b>SNR@BER=1e-8</b>	<b>Max.15 iterations</b>	18.1dB	20.87dB	23.37dB	26.11dB	28.67dB
	<b>Max. 30 iterations</b>	17.96dB	20.69dB	23.21dB	26.01dB	28.59dB
	<b>Difference</b>	0.14	0.18	0.16	0.1	0.08



# BURST NOISE CHANNEL MODEL

- **Two OFDM symbol durations**

- 20  $\mu\text{s}$
- 40  $\mu\text{s}$

- **Cyclic prefix**

- 2.5  $\mu\text{s}$

- **Channel assumption**

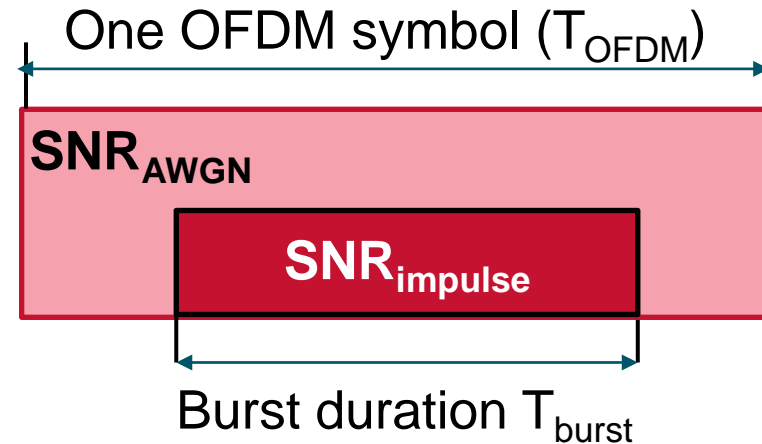
Burst noise	Duration	SNR
Downstream	16 $\mu\text{s}$	20dB
	16 $\mu\text{s}$	5dB
Upstream	10 $\mu\text{s}$	10dB
	1 $\mu\text{s}$	0dB

- **Interleaver latency threshold**

- 1 ms total PHY delay (upstream and downstream), suppose 400  $\mu\text{s}$  each
  - 20 $\mu\text{s}$  symbol:  $400/22.5=17.7$  interleave depth
  - 40 $\mu\text{s}$  symbol:  $400/42.5=9.4$  interleave depth

- **Targeted probability of error**

- BER=1e-8
- WER=1e-6



- **Case I: the burst hits one OFDM symbol**

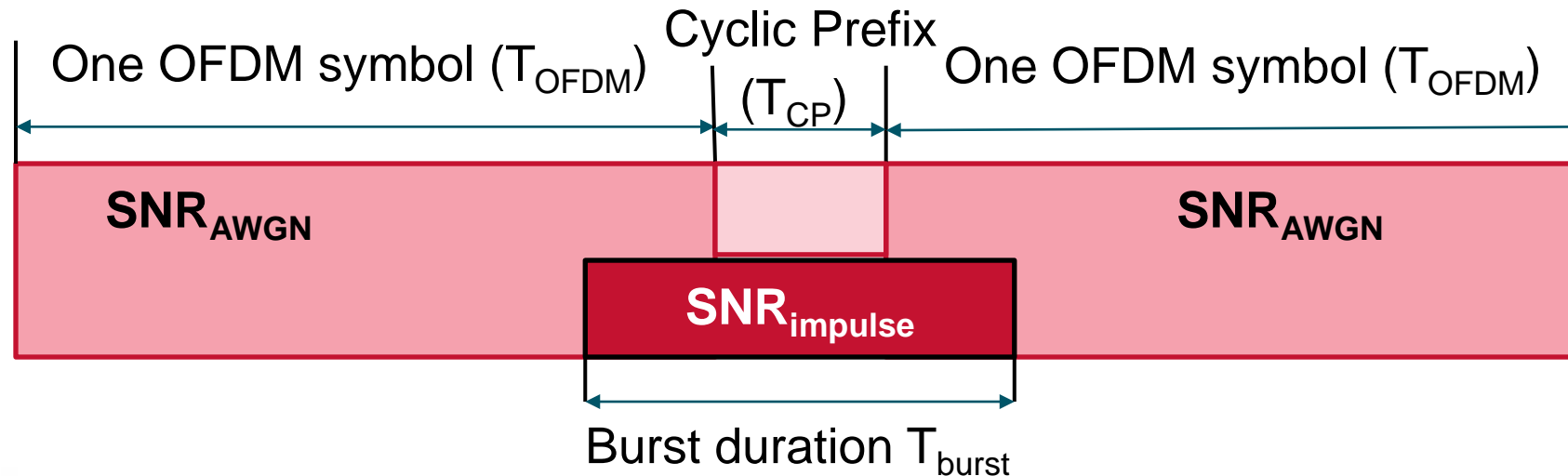
- SNR experienced by all sub-carriers in the OFDM symbol due to burst noise only is:

$$\text{SNR}_{\text{burst}} = \text{SNR}_{\text{impulse}} - 10 \log (T_{\text{burst}} / T_{\text{OFDM}})$$

$T_{\text{OFDM}}$ :	OFDM symbol duration without cyclic prefix
$T_{\text{CP}}$ :	duration of cyclic prefix
$T_{\text{burst}}$ :	burst duration
$\text{SNR}_{\text{impulse}}$ :	impulse SNR

- SNR experienced by all sub-carriers in the two OFDM symbols due to background noise only is:

$$\text{SNR}_{\text{background}} = \text{SNR}_{\text{AWGN}} - 10 \log (1 - [T_{\text{burst}} / T_{\text{OFDM}}])$$



- **Case II: the burst hits two consecutive OFDM symbols equally**

- SNR experienced by all sub-carriers in the two OFDM symbols due to burst noise only is:

$$\text{SNR}_{\text{burst}} = \text{SNR}_{\text{impulse}} - 10 \log (0.5 * (T_{\text{burst}} - T_{\text{CP}}) / T_{\text{OFDM}})$$

$T_{\text{OFDM}}$ :	OFDM symbol duration without cyclic prefix
$T_{\text{CP}}$ :	duration of cyclic prefix
$T_{\text{burst}}$ :	burst duration
$\text{SNR}_{\text{impulse}}$ :	impulse SNR

- SNR experienced by all sub-carriers in the two OFDM symbols due to background noise only is:

$$\text{SNR}_{\text{background}} = \text{SNR}_{\text{AWGN}} - 10 \log (1 - [0.5 * (T_{\text{burst}} - T_{\text{CP}}) / T_{\text{OFDM}}])$$

- **SNR on the burst noise impacted subcarrier in the presence of background AWGN is:**

- $SNR_{\text{sub-carrier}} = -10 \text{ Log} (10^{[-SNR_{\text{burst}} / 10]} + 10^{[-SNR_{\text{background}} / 10]})$

$SNR_{\text{sub-carrier}}$ : SNR experienced by all sub-carriers in the OFDM symbol

$SNR_{\text{background}}$ : Background (thermal) Additive White Gaussian noise contribution

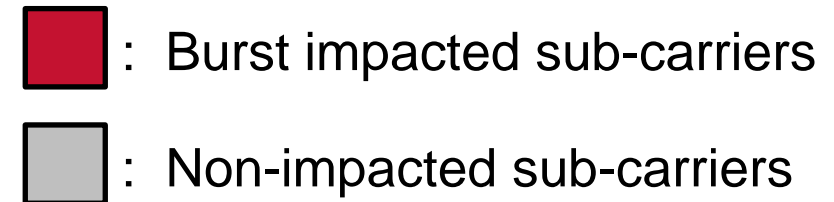
$SNR_{\text{burst}}$ : Burst SNR contribution



- Our simulations show the minimum value for interleave depth  $N$  in order for the BER to reach  $1e-8$  for a given worst case burst noise position at a specified duration and SNR and the associated minimum background AWGN SNR

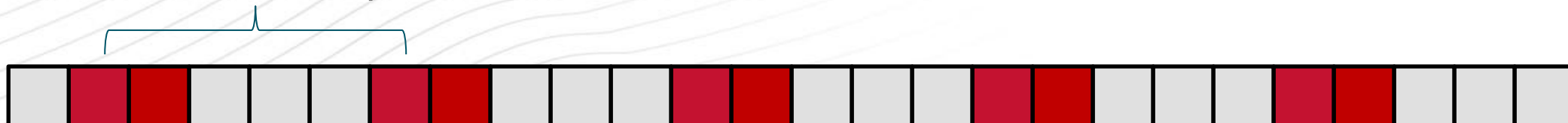
- Simulated cases**

- Case 1: one OFDM symbol is impacted:  
N sub-carriers apart



- Case 2 two consecutive OFDM symbols are impacted equally:

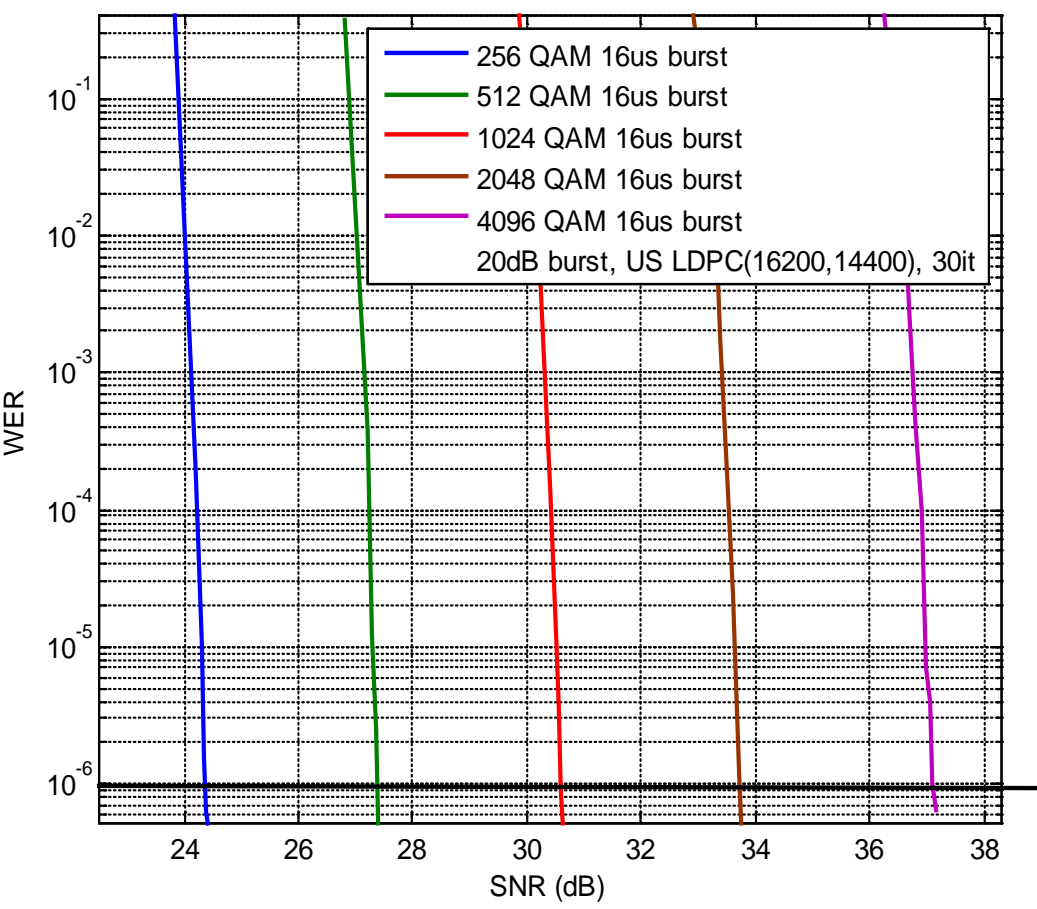
N sub-carriers apart



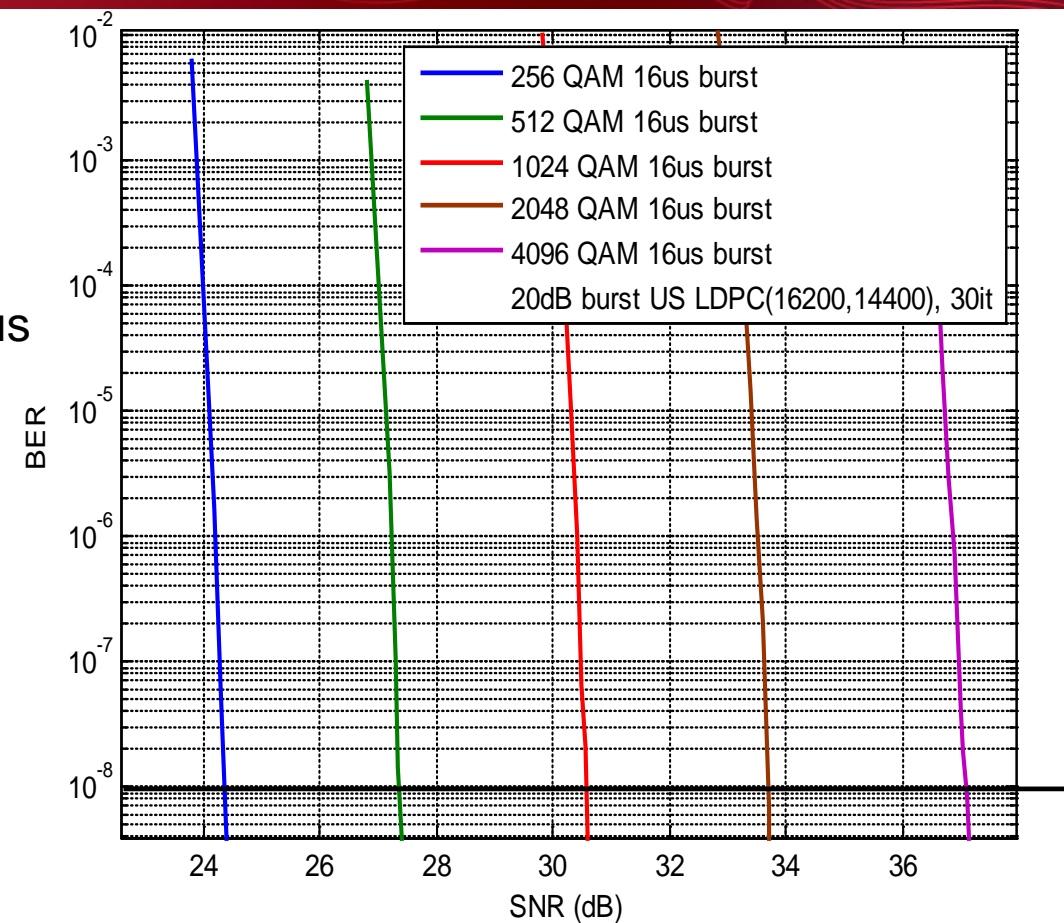
A decorative graphic consisting of numerous thin, overlapping lines in shades of purple and red, creating a wavy, ribbon-like effect that spans the width of the slide.

# BURST NOISE PERFORMANCE ON DOWNSTREAM

# 20μs SYMBOLS ON 16μs 20dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS

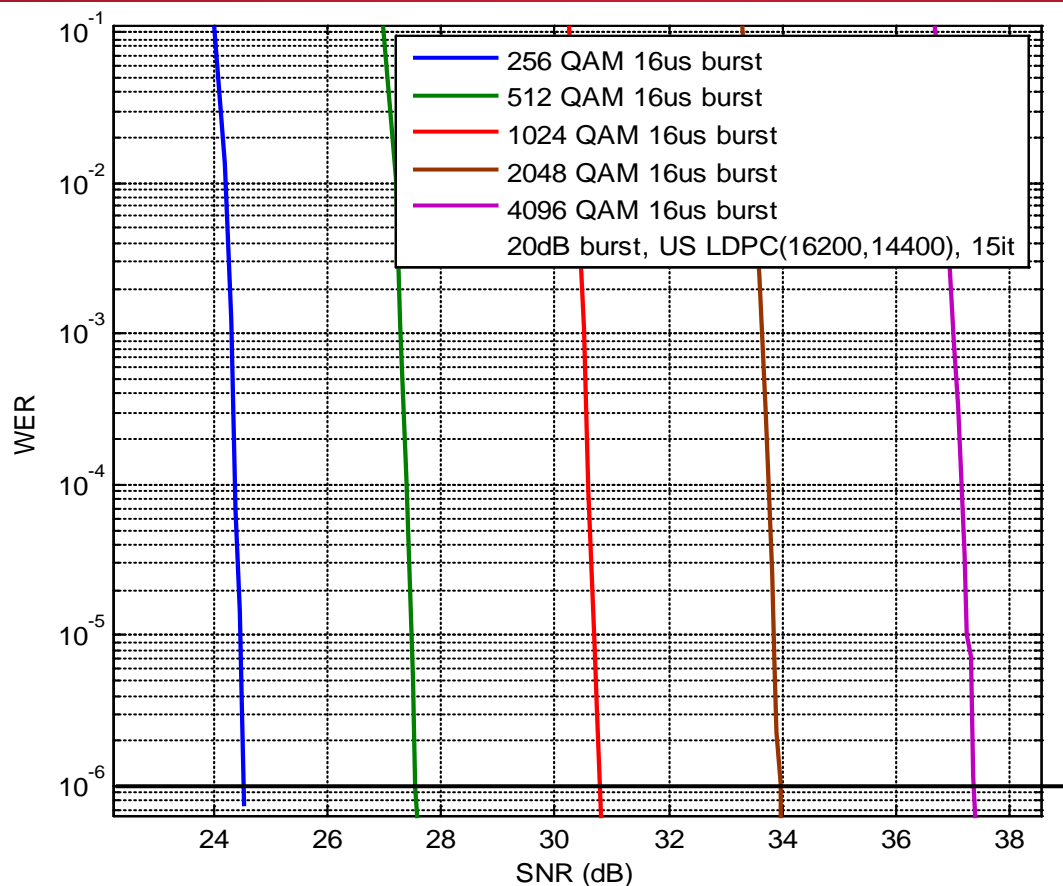


20μs symbol  
(two affected)  
Latency: 382.5μs  
(depth:17)

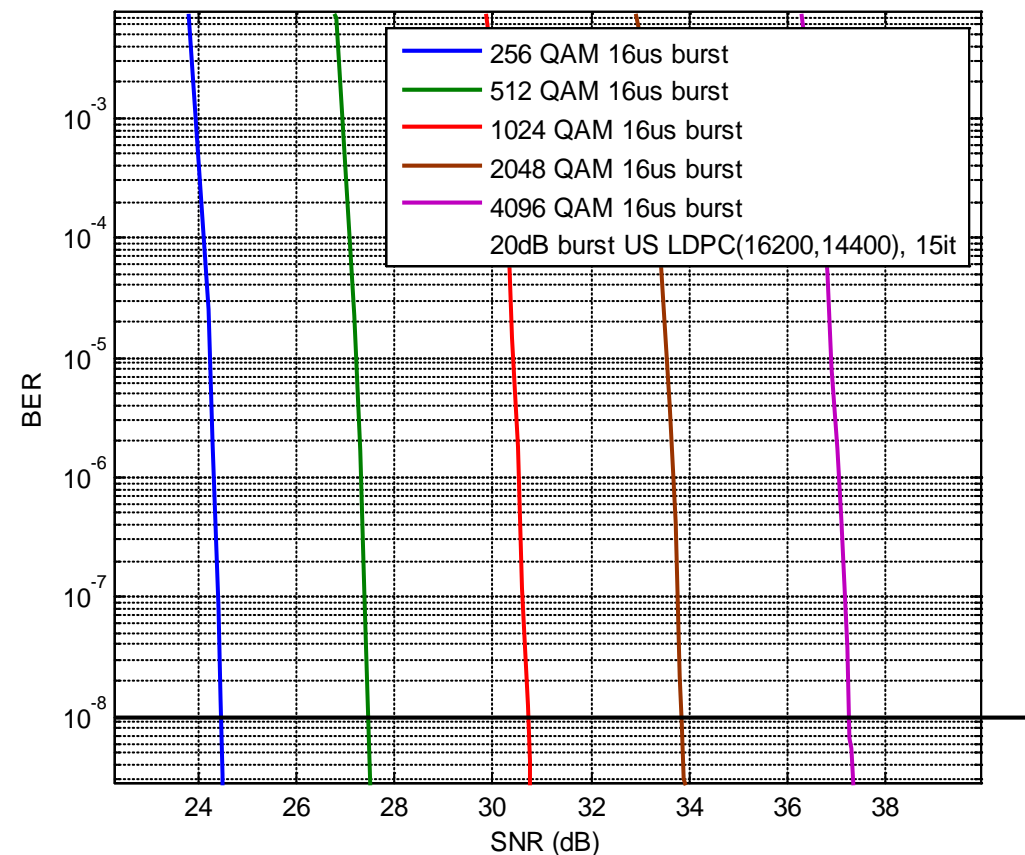


	256QAM	512QAM	1024QAM	2048QAM	4096QAM
<b>SNR@WER=1e-6</b>	24.38dB	27.38dB	30.59dB	33.71dB	37.10dB
<b>SNR@BER=1e-8</b>	24.36dB	27.36dB	30.58dB	33.69dB	37.09dB

# 20 $\mu$ s SYMBOLS ON 16 $\mu$ s 20dB BURST (LONG SIZE CODE) MAX 15 ITERATIONS

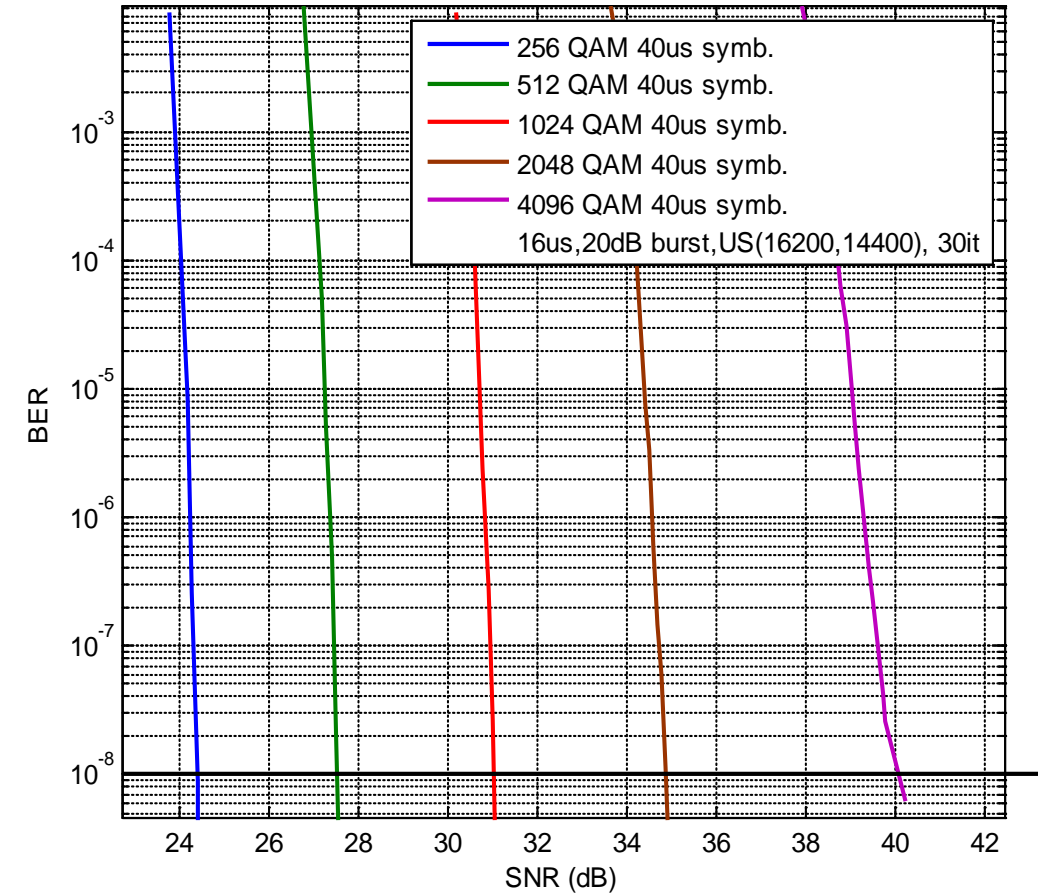
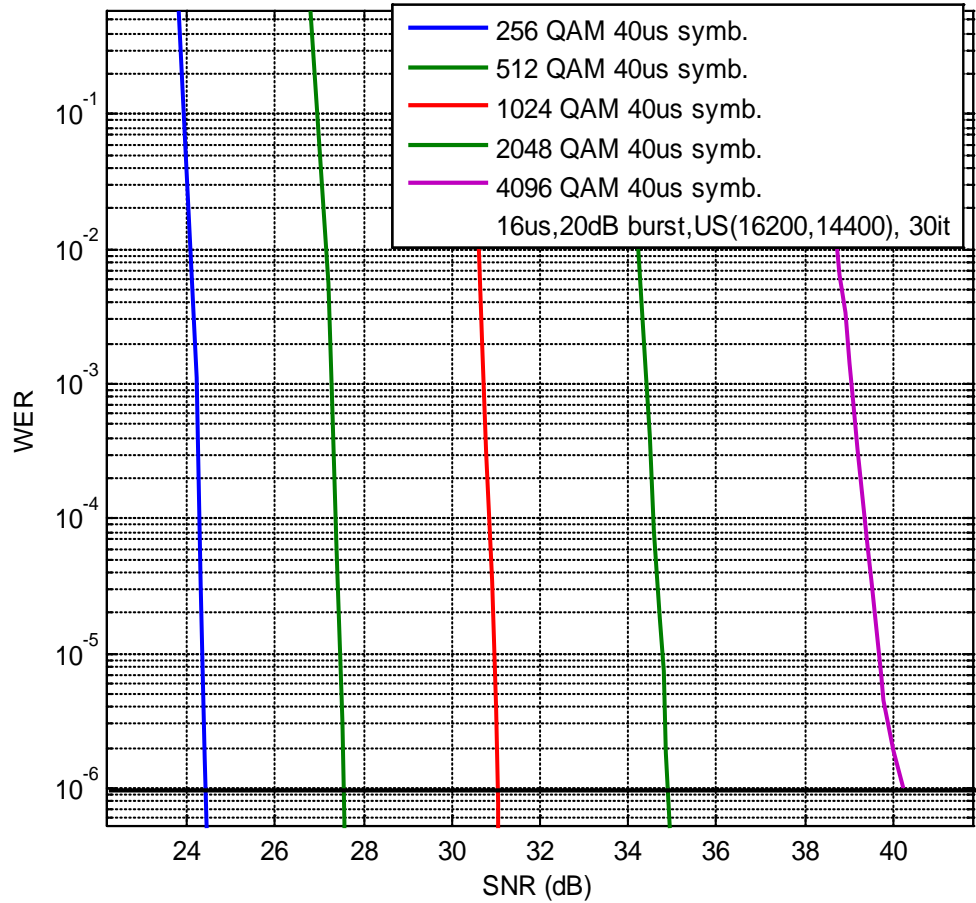


20 $\mu$ s symbol  
(two affected)  
Latency: 382.5 $\mu$ s  
(depth:17)



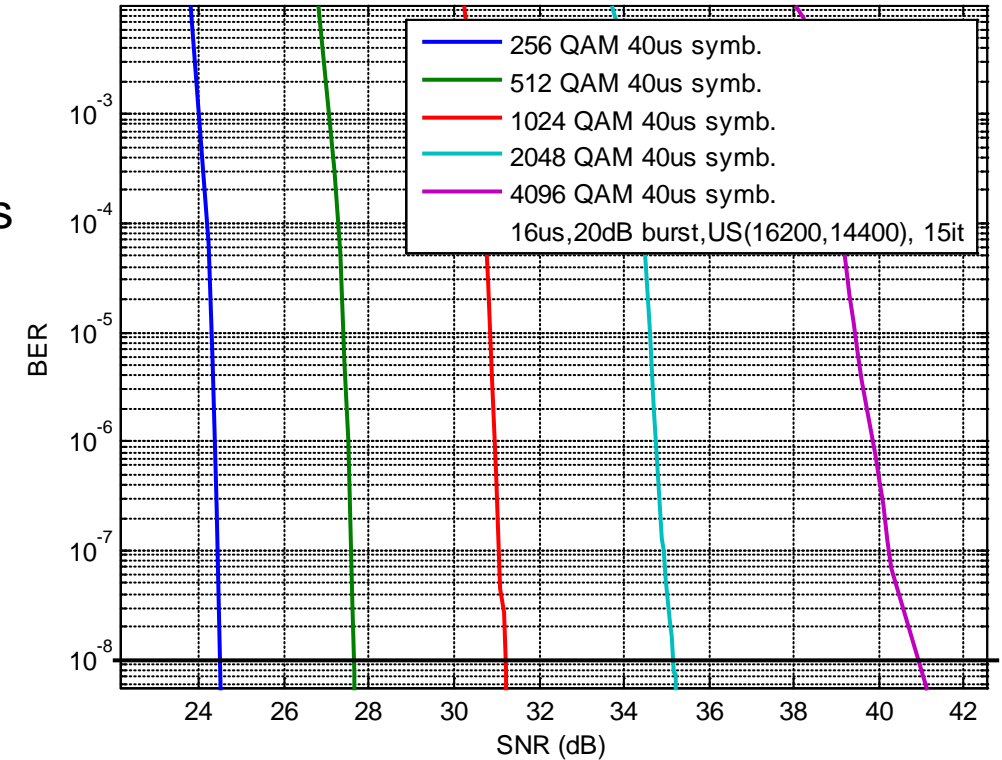
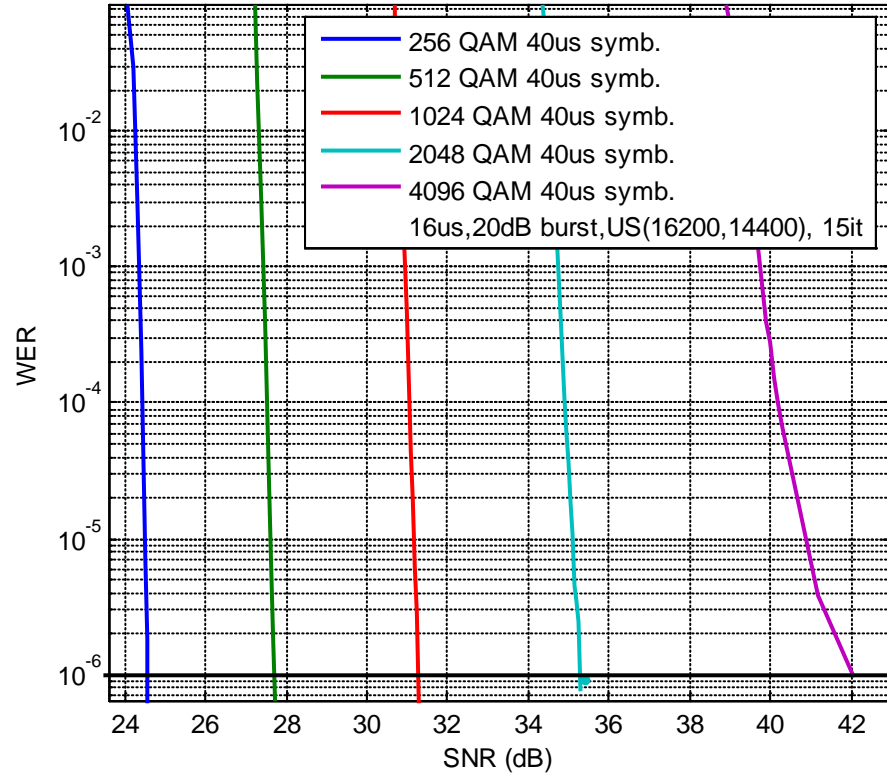
	256QAM	512QAM	1024QAM	2048QAM	4096QAM
<b>SNR@WER=1e-6</b>	24.54dB	27.55dB	30.78dB	33.95dB	37.36dB
<b>SNR@BER=1e-8</b>	24.26dB	27.46dB	30.71dB	33.84dB	37.24dB

# 40μs SYMBOLS ON 16μs 20dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS



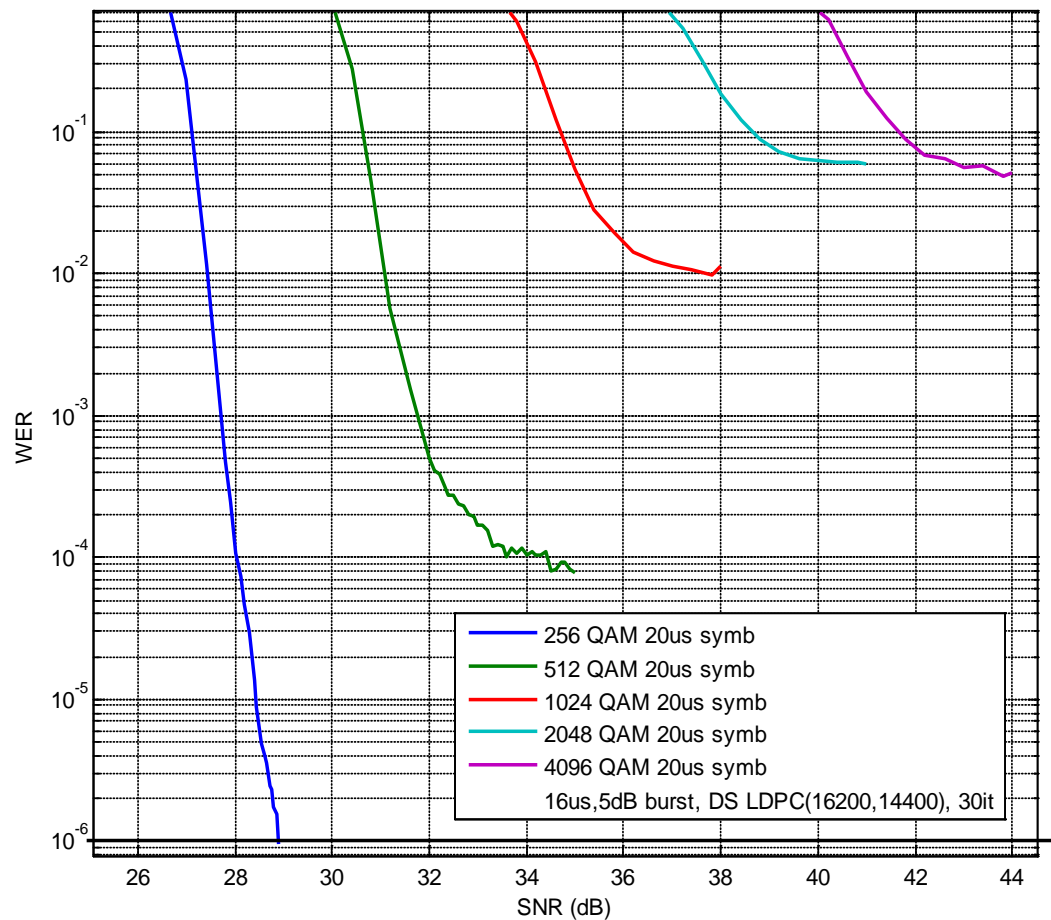
	256QAM	512QAM	1024QAM	2048QAM	4096QAM
<b>SNR@WER=1e-6</b>	24.41dB	27.53dB	31.04dB	34.9dB	40.25dB
<b>SNR@BER=1e-8</b>	24.4dB	27.52dB	31.03dB	34.86dB	40.1dB

# 40μs SYMBOLS ON 16μs 20dB BURST (UPSTREAM LONG CODE) MAX 15 ITERATIONS

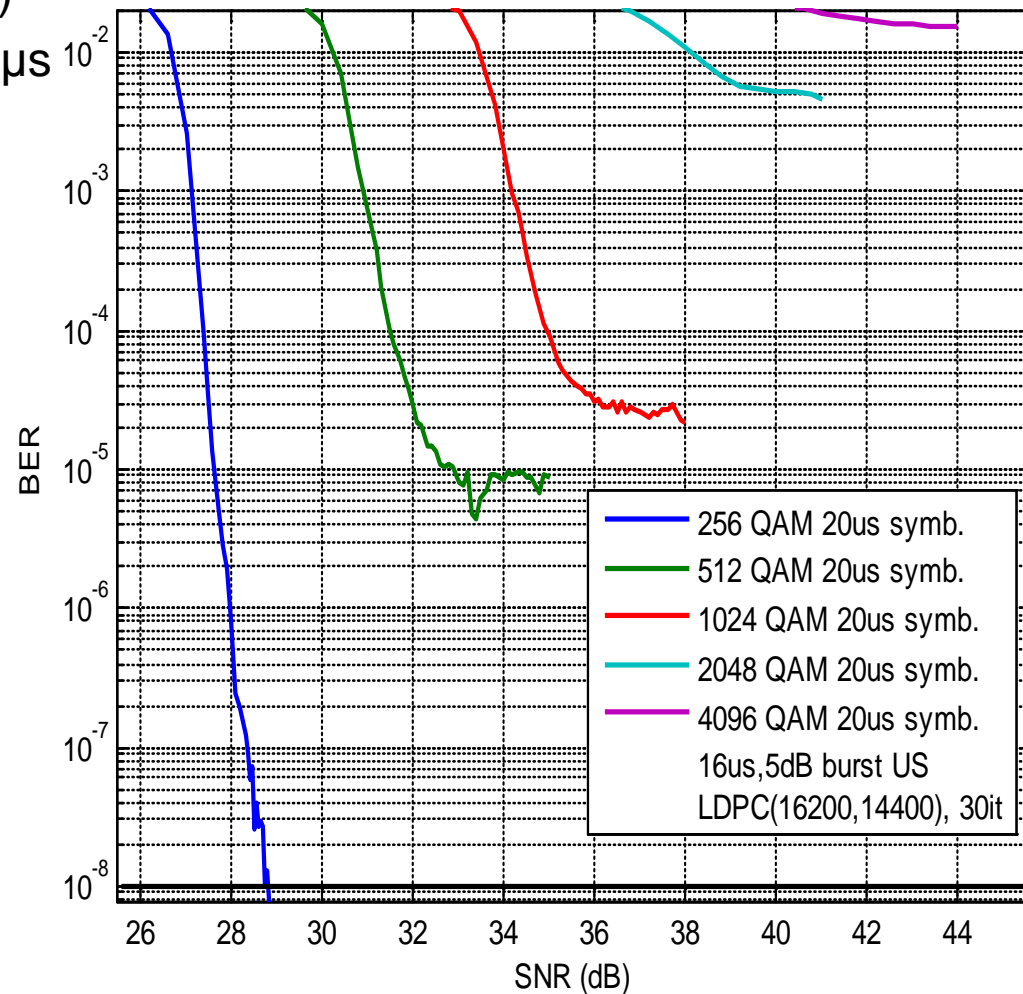


	256QAM	512QAM	1024QAM	2048QAM	4096QAM
<b>SNR@WER=1e-6</b>	24.57dB	27.73dB	31.28dB	35.29dB	42dB
<b>SNR@BER=1e-8</b>	24.48dB	27.64dB	31.19dB	35.13dB	40.9dB

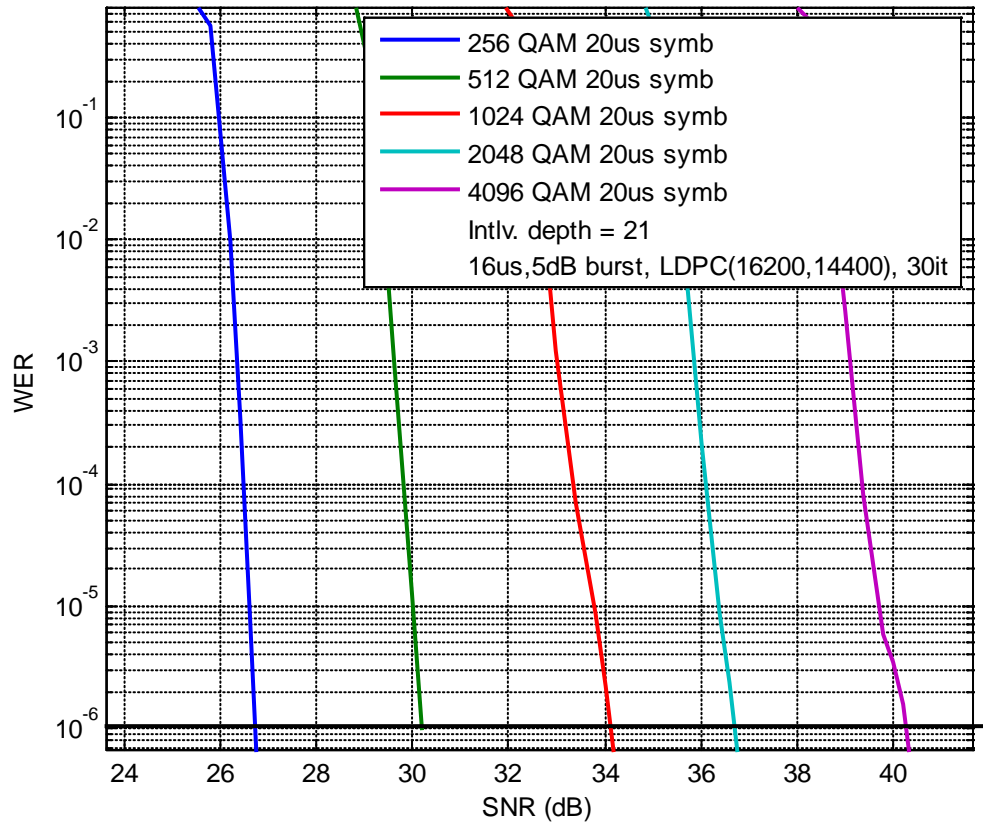
# 20μs SYMBOLS ON 16μs 5dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS



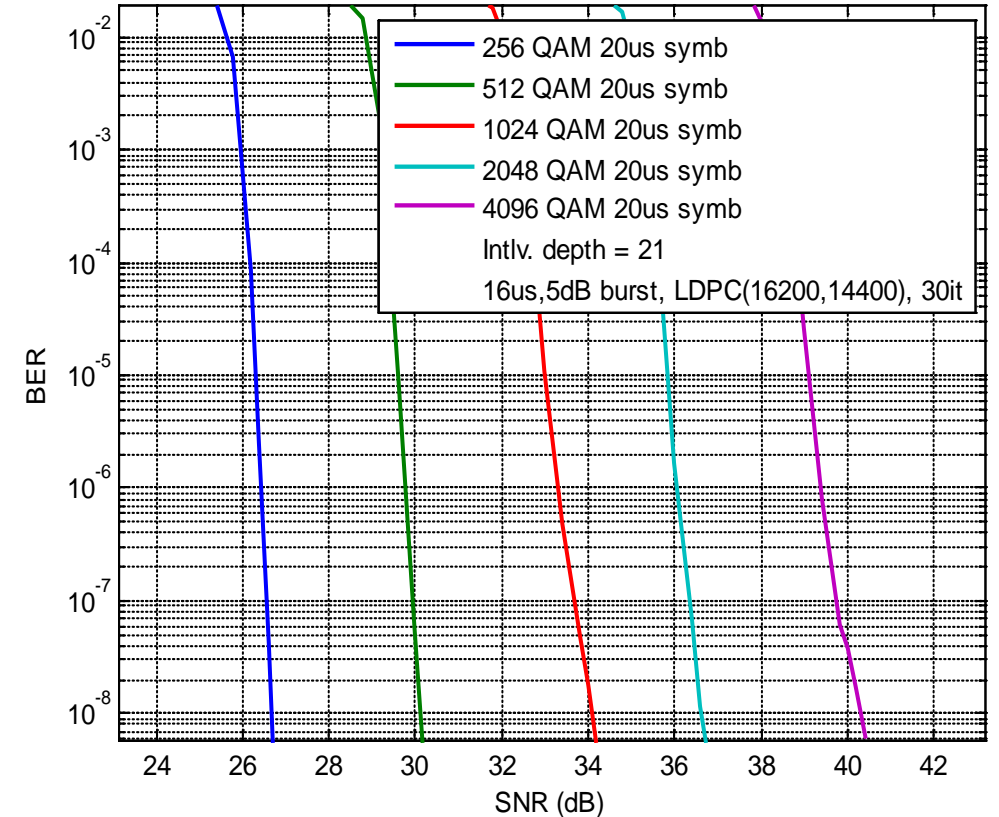
20μs symbol  
(two affected)  
Latency: 382.5μs  
(depth:17)



# 20 $\mu$ s SYMBOLS ON 16 $\mu$ s 5dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS



20 $\mu$ s symbol  
(two affected)  
Latency: 472.5 $\mu$ s  
(depth: 21)

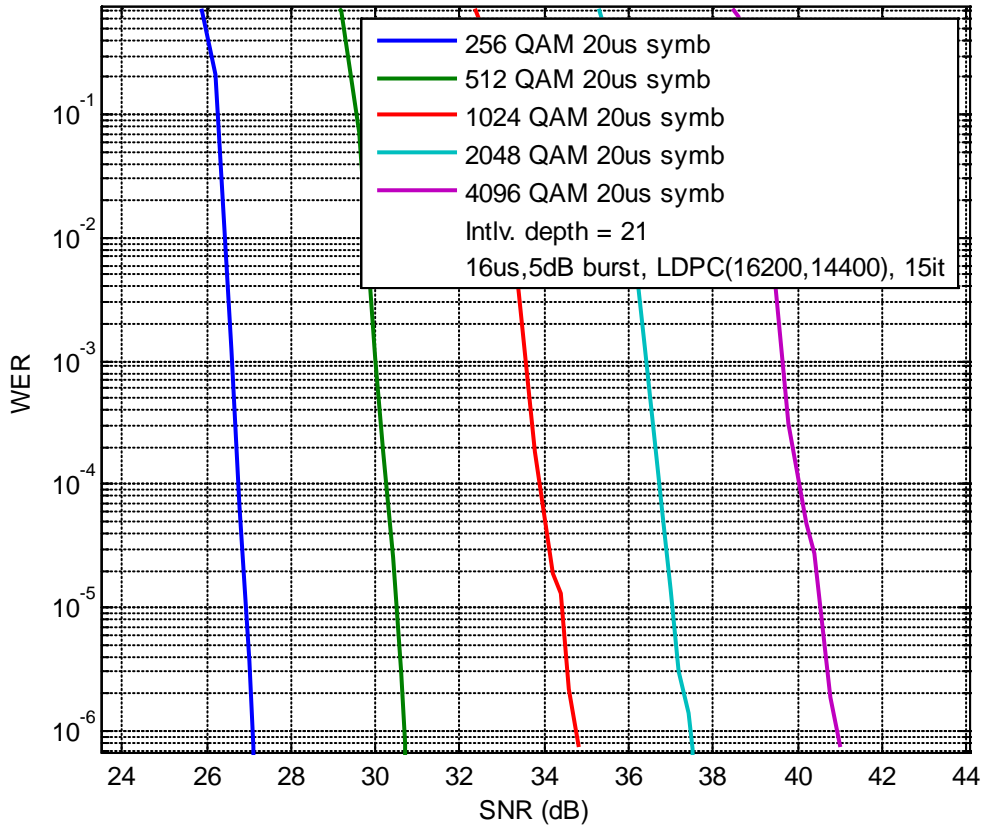


Remark: latency 450 $\mu$ s (depth 20) works for 30 iterations, but not for 15 iterations

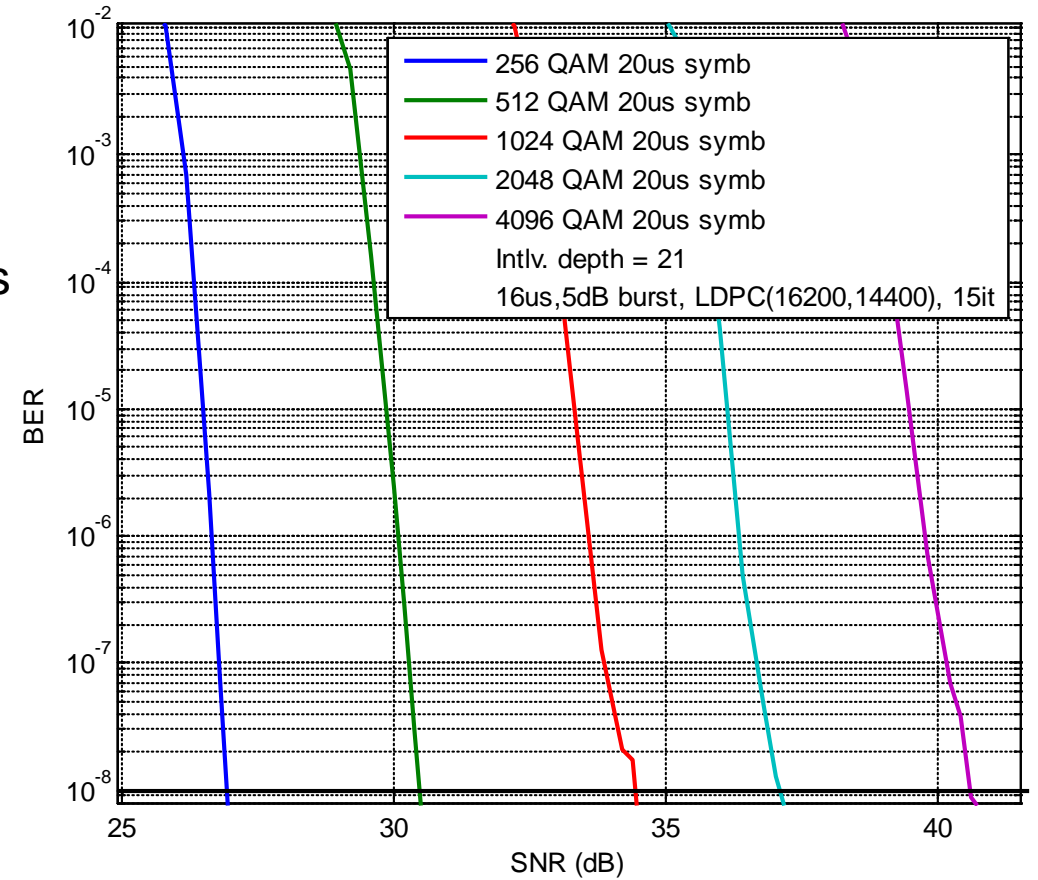
	256QAM	512QAM	1024QAM	2048QAM	4096QAM
<b>SNR@WER=1e-6</b>	26.72dB	30.20dB	34.12dB	36.72dB	40.28dB
<b>SNR@BER=1e-8</b>	26.68dB	30.13dB	34.09dB	36.62dB	40.3dB



# 20 $\mu$ s SYMBOLS ON 16 $\mu$ s 5dB BURST (LONG SIZE CODE) MAX 15 ITERATIONS

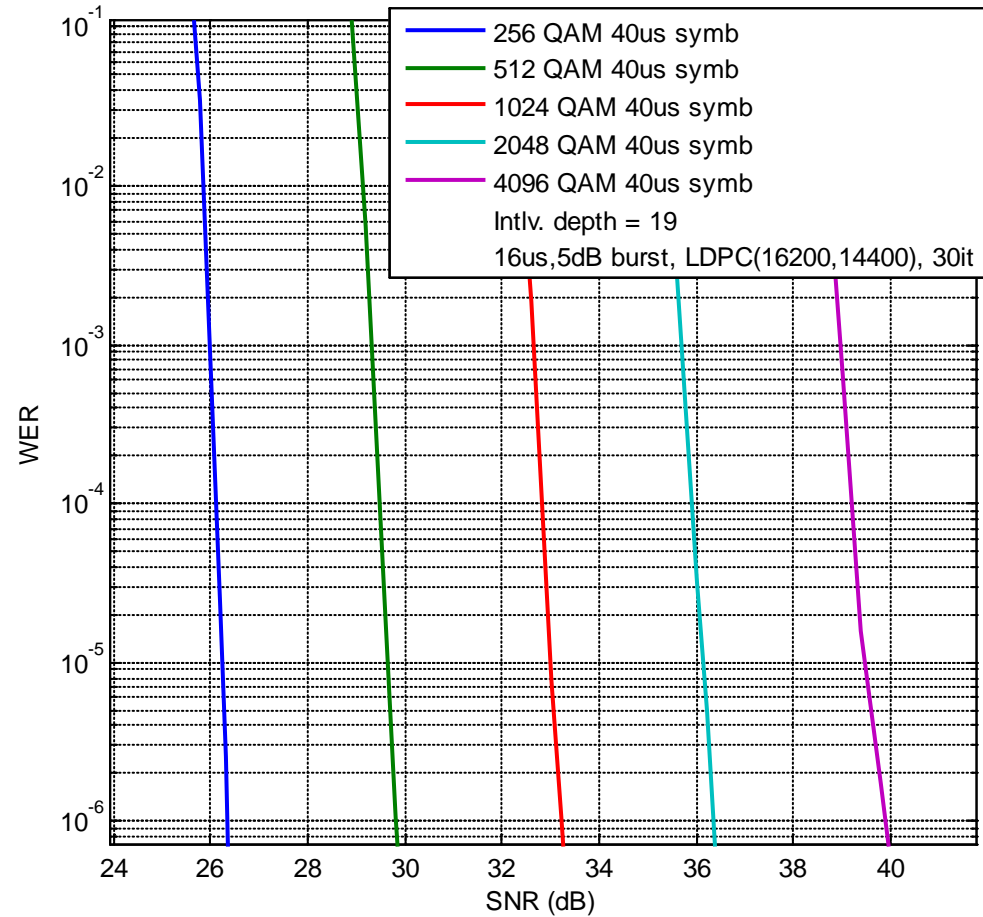


20 $\mu$ s symbol  
(two affected)  
Latency: 472.5 $\mu$ s  
(depth: 21)

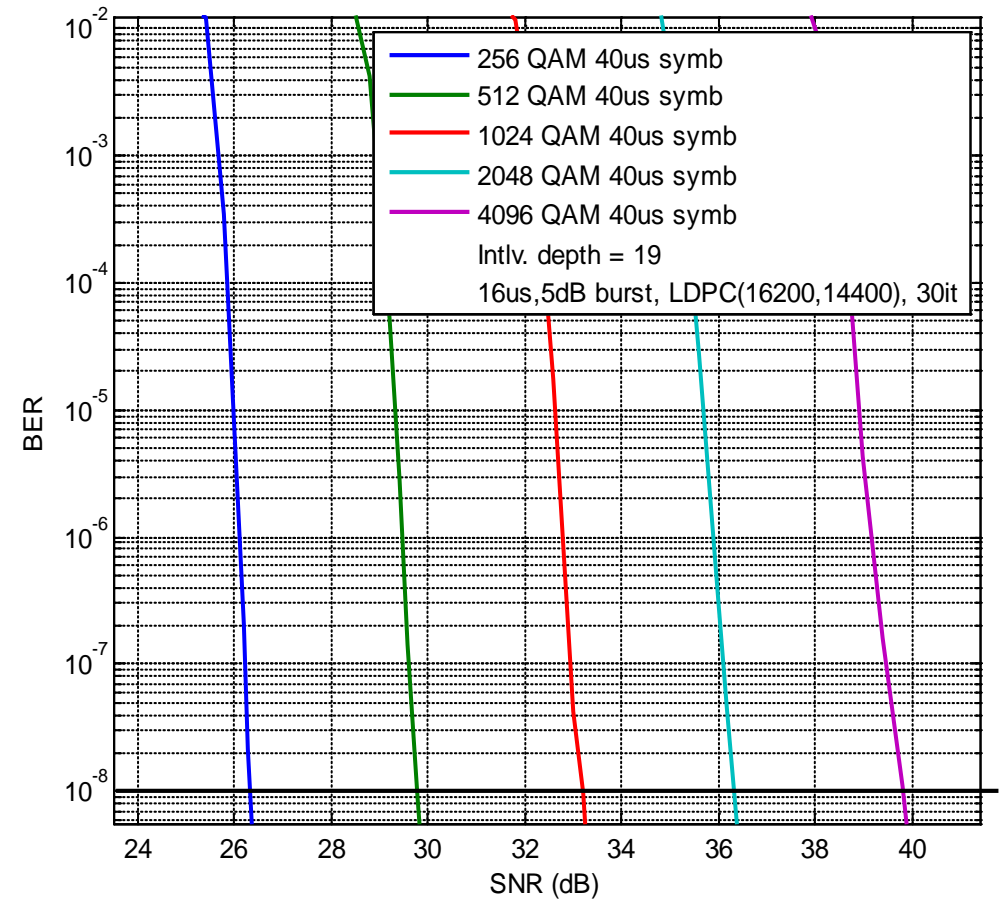


	256QAM	512QAM	1024QAM	2048QAM	4096QAM
<b>SNR@WER=1e-6</b>	27.08dB	30.68dB	34.74dB	37.45dB	40.94dB
<b>SNR@BER=1e-8</b>	26.94dB	30.49dB	34.45dB	37.07dB	40.58dB

# 40μs SYMBOLS ON 16μs 5dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS



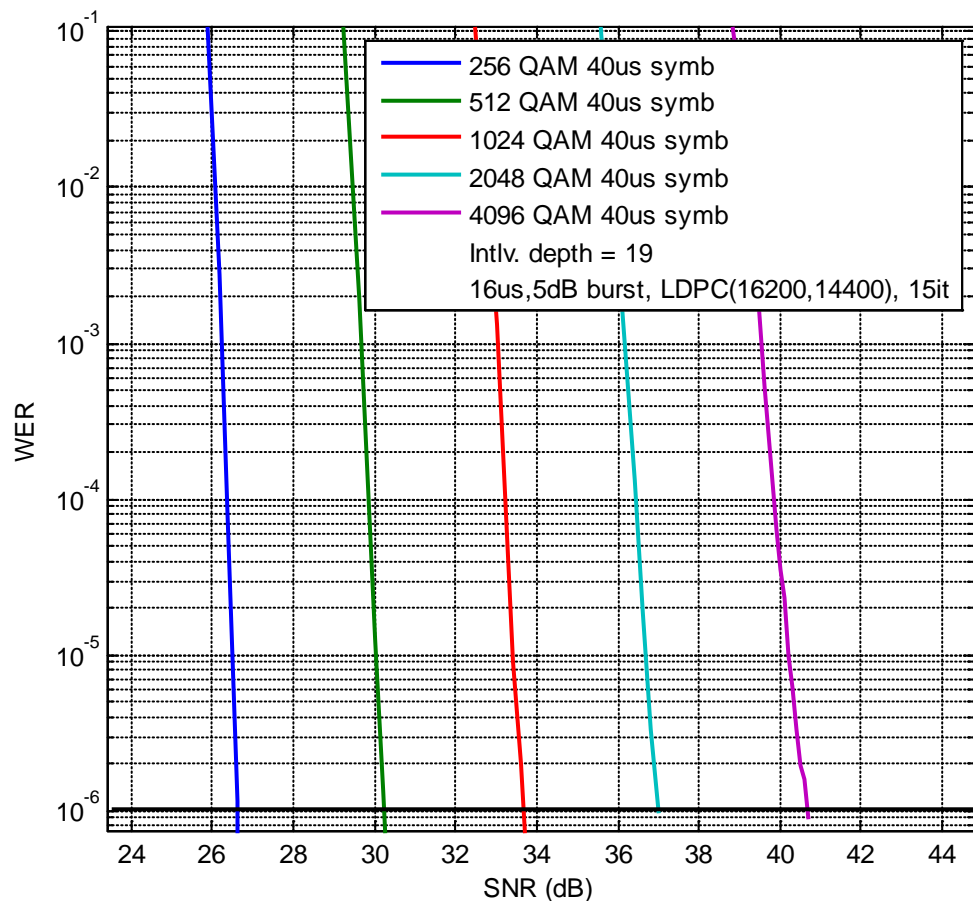
40μs symbol  
(two affected)  
Latency: 807.5.5μs  
(depth: 19)



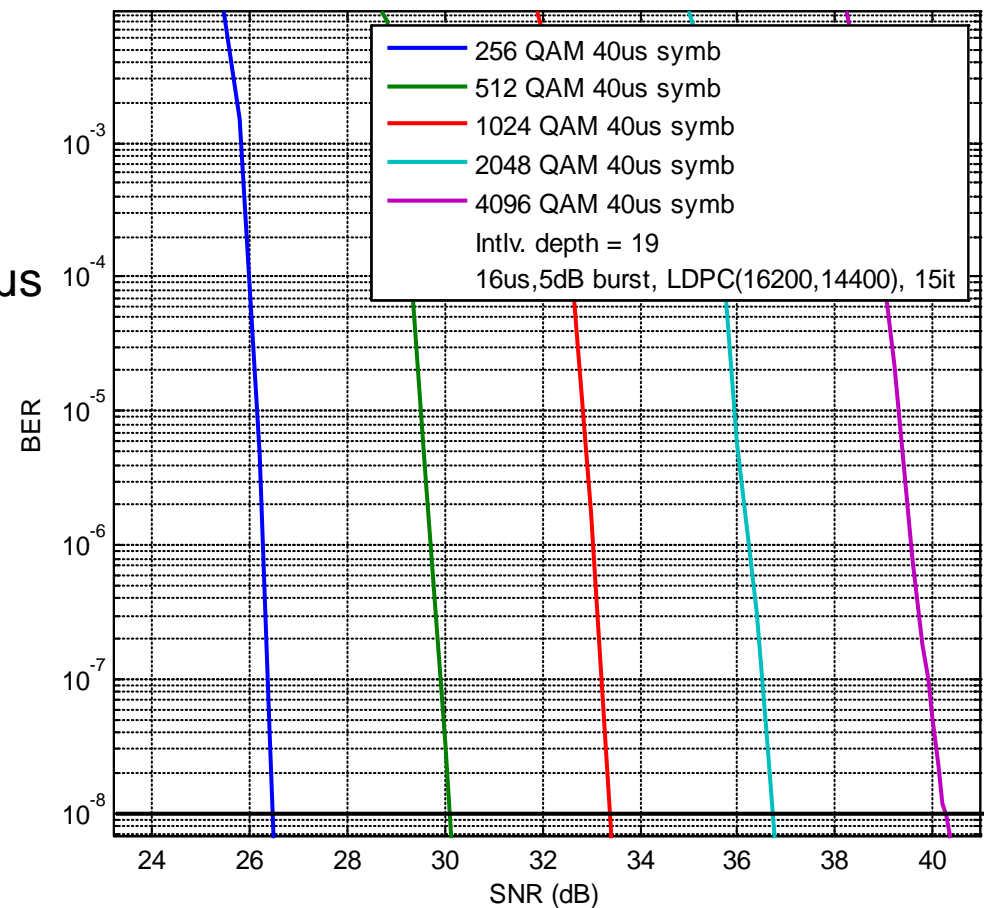
Remark: latency 765μs (depth 18) works for 30 iterations, but not for 15 iterations

	256QAM	512QAM	1024QAM	2048QAM	4096QAM
<b>SNR@WER=1e-6</b>	26.35dB	29.8dB	33.21dB	36.35dB	39.89dB
<b>SNR@BER=1e-8</b>	26,34dB	29.8dB	33.2dB	36.33dB	39.81dB

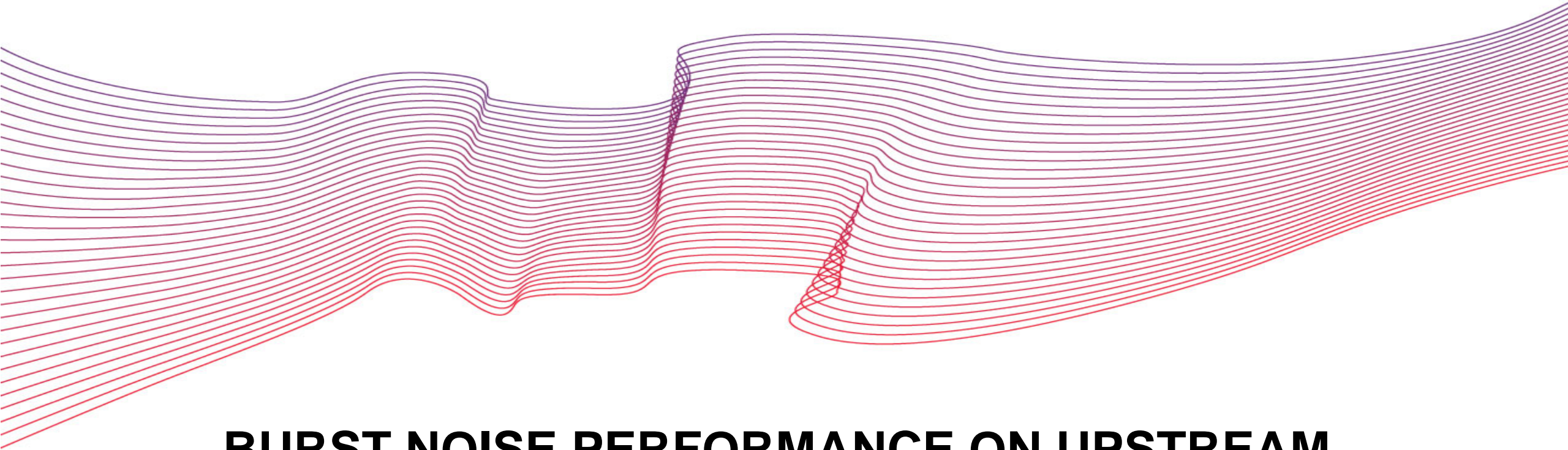
# 40μs SYMBOLS ON 16μs 5dB BURST (LONG SIZE CODE) MAX 15 ITERATIONS



40μs symbol  
(two affected)  
Latency: 807.5.5μs  
(depth: 19)

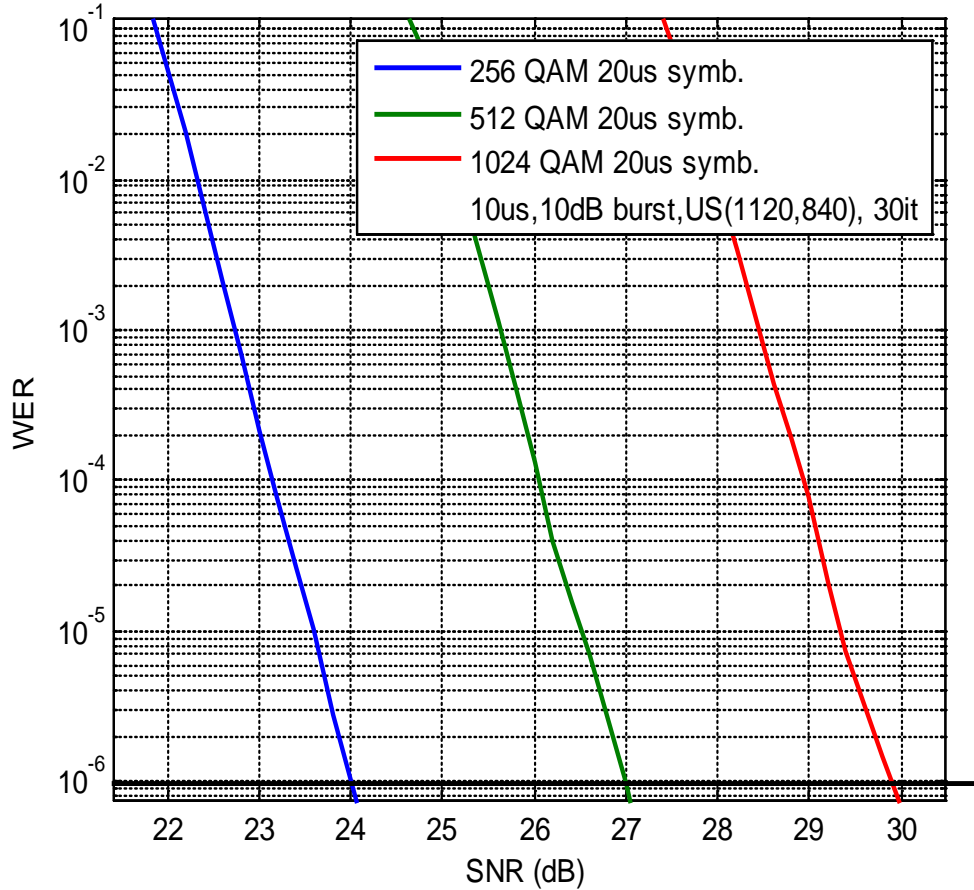


	256QAM	512QAM	1024QAM	2048QAM	4096QAM
<b>SNR@WER=1e-6</b>	26.61dB	30.22dB	33.68dB	36.99dB	40.68dB
<b>SNR@BER=1e-8</b>	26.47dB	30.09dB	33.38dB	36.73dB	40.28dB

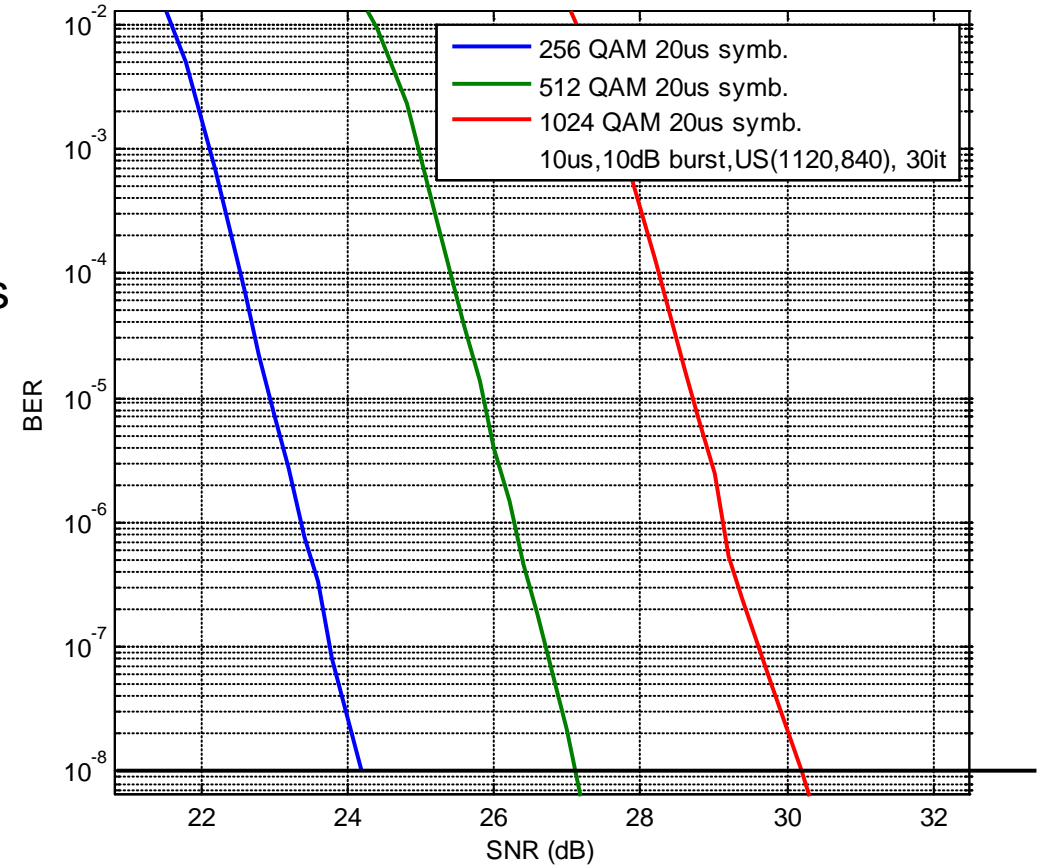


# **BURST NOISE PERFORMANCE ON UPSTREAM**

# 20μs SYMBOLS ON 10μs 10dB BURST (SHORT SIZE CODE) MAX 30 ITERATIONS

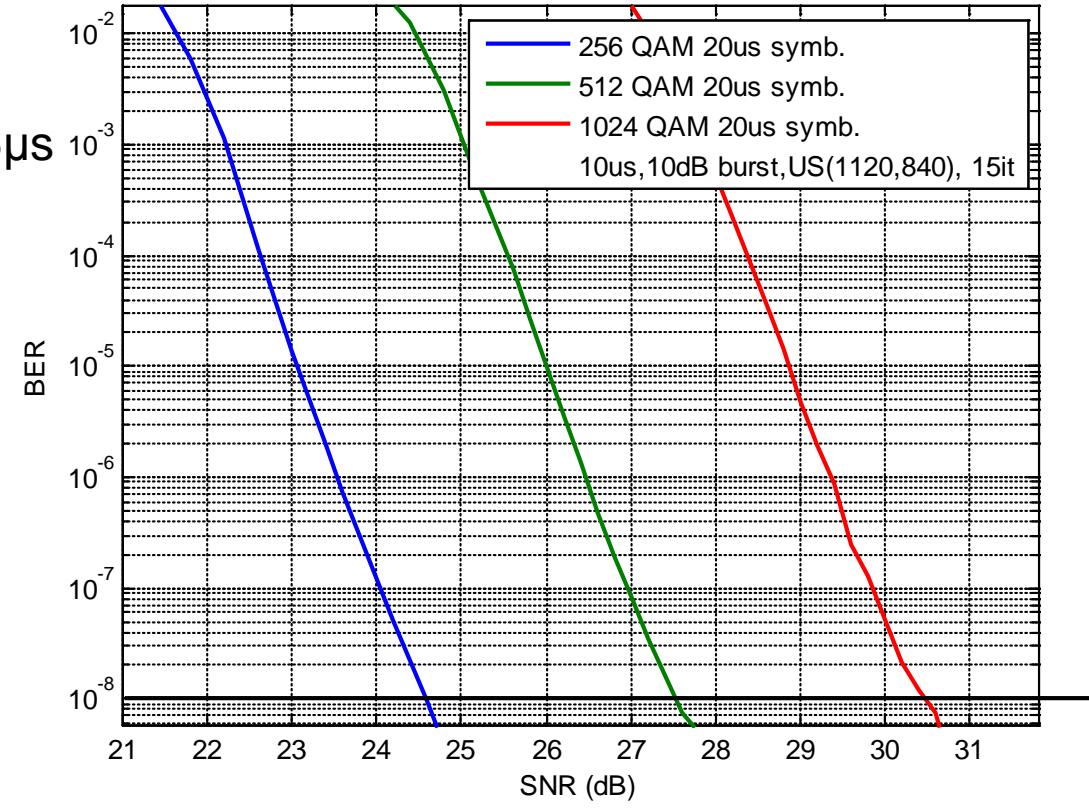
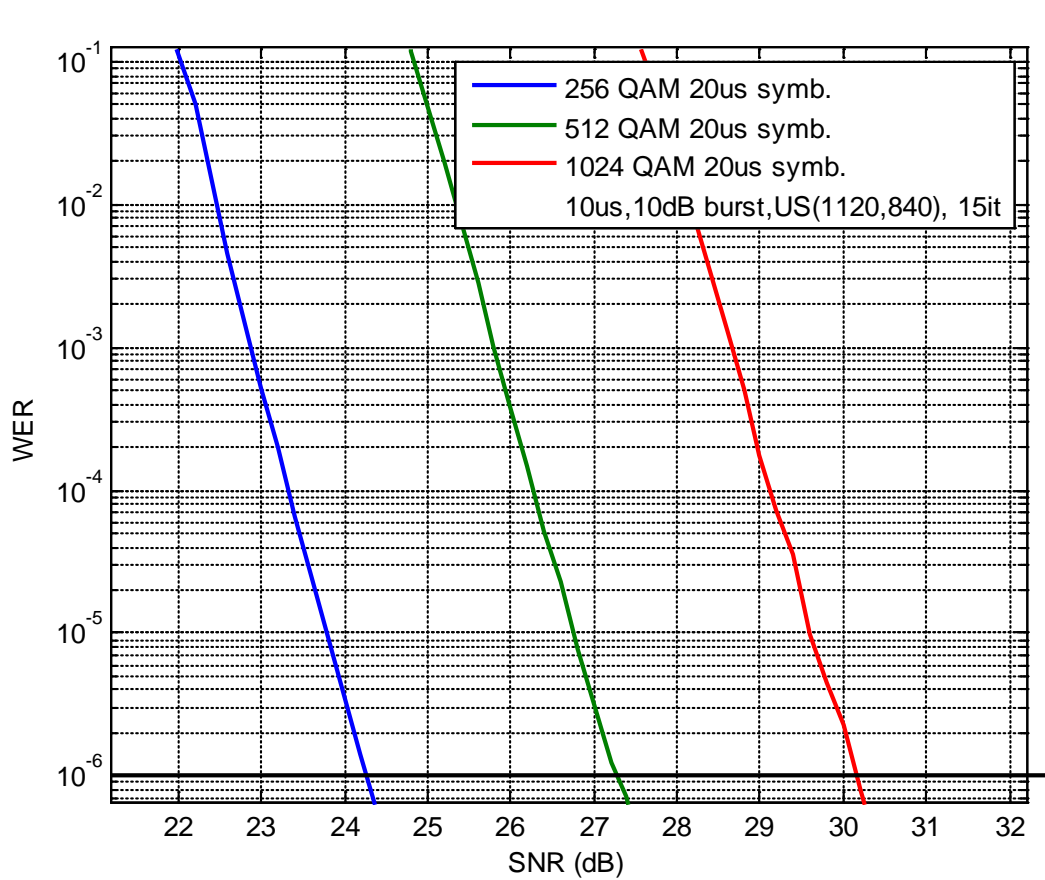


20μs symbol  
(two affected)  
Latency: 382.5μs  
(depth:17)



	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	24.08dB	27.03dB	29.91dB
<b>SNR@BER=1e-8</b>	24.27dB	27.19dB	30.30dB

# 20 $\mu$ s SYMBOLS ON 10 $\mu$ s 10dB BURST (SHORT SIZE CODE) MAX 15 ITERATIONS



	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	24.27dB	27.28dB	30.16dB
<b>SNR@BER=1e-8</b>	24.58dB	27.55dB	30.47dB

# DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (SHORT SIZE CODE)

## 20 $\mu$ s SYMBOLS ON 10 $\mu$ s 10dB BURST



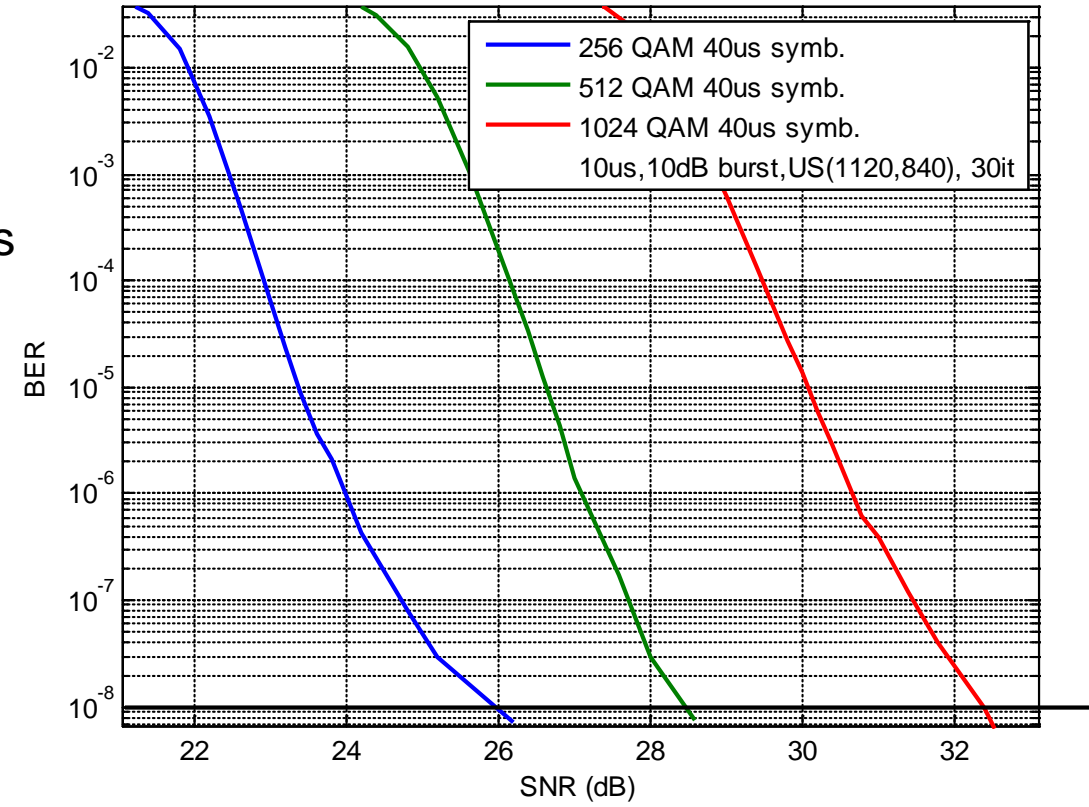
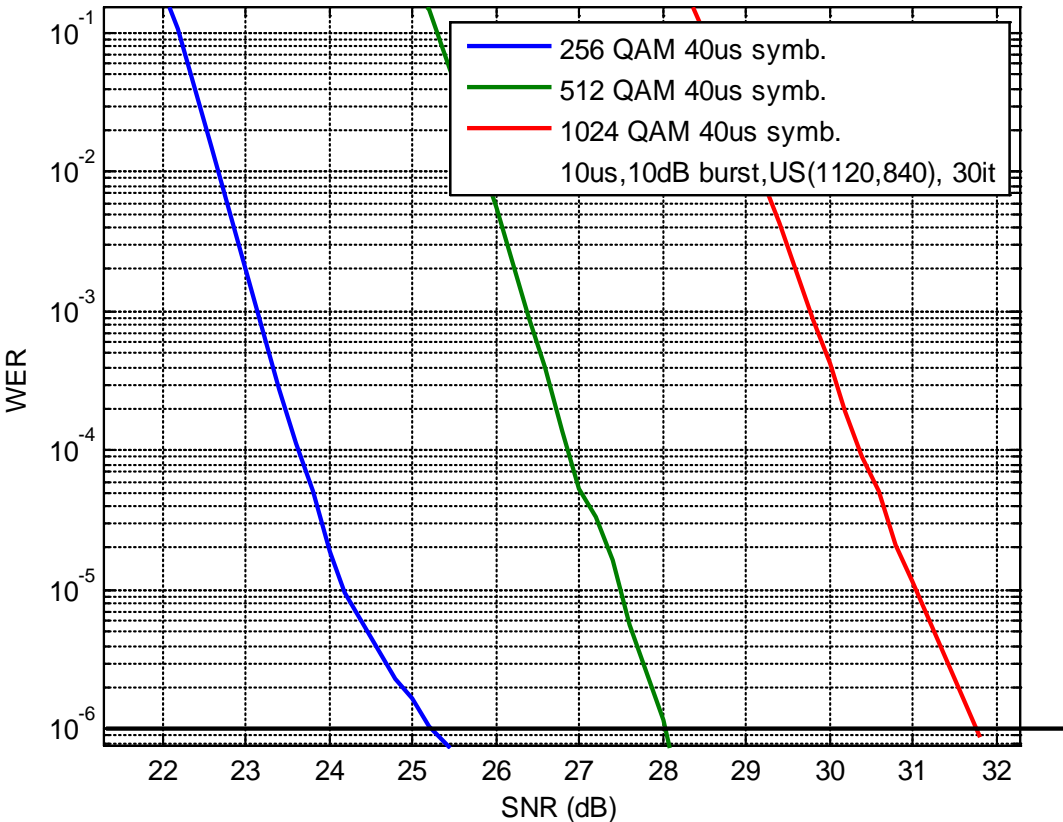
20 $\mu$ s symbol (two affected)

Latency: 382.5 $\mu$ s (depth:17)

Short size  
(1120,840)  
30 iterations  
vs.  
15 iterations

		256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	<b>Max.15 iterations</b>	24.27dB	27.28dB	30.16dB
	<b>Max.30 iterations</b>	24.08dB	27.03dB	29.91dB
	<b>Difference</b>	0.19	0.25	0.25
<b>SNR@BER=1e-8</b>	<b>Max. 15 iterations</b>	24.58dB	27.55dB	30.47dB
	<b>Max. 30 iterations</b>	24.27dB	27.19dB	30.30dB
	<b>Difference</b>	0.31	0.36	0.17

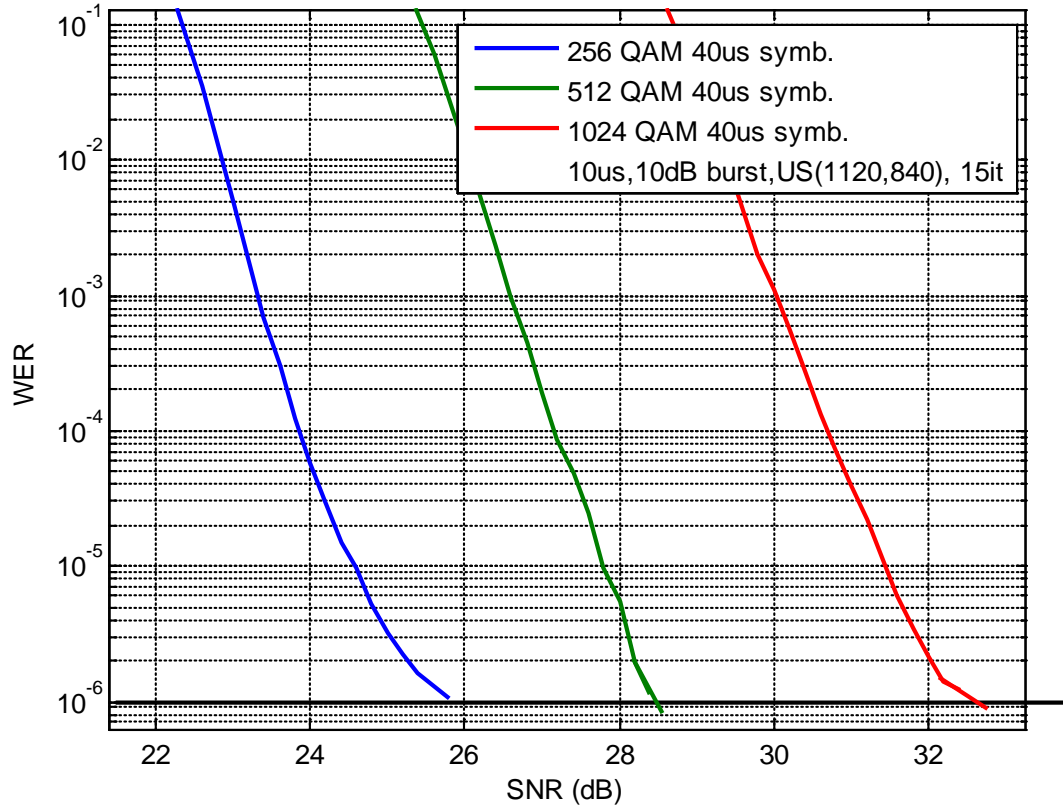
# 40μs SYMBOLS ON 10μs 10dB BURST (SHORT SIZE CODE) MAX 30 ITERATIONS



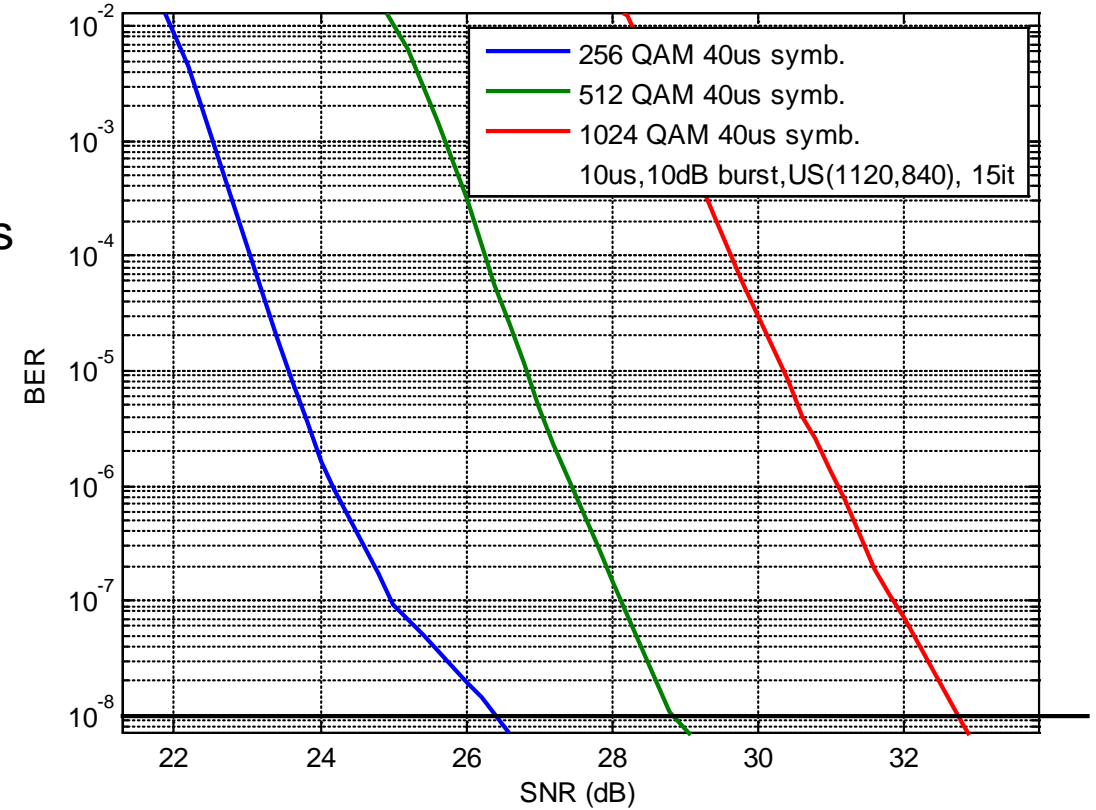
	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	25.25dB	28.03dB	31.8dB
<b>SNR@BER=1e-8</b>	25.97dB	28.52dB	32.42dB



# 40μs SYMBOLS ON 10μs 10dB BURST (SHORT SIZE CODE) MAX 15 ITERATIONS



40μs symbol  
(two affected)  
Latency: 340μs  
(depth:8)



	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	25.86dB	28.45dB	32.68dB
<b>SNR@BER=1e-8</b>	26.4dB	28.84dB	32.73dB

# DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (SHORT SIZE CODE)

## 40 $\mu$ s SYMBOLS ON 10 $\mu$ s 10dB BURST

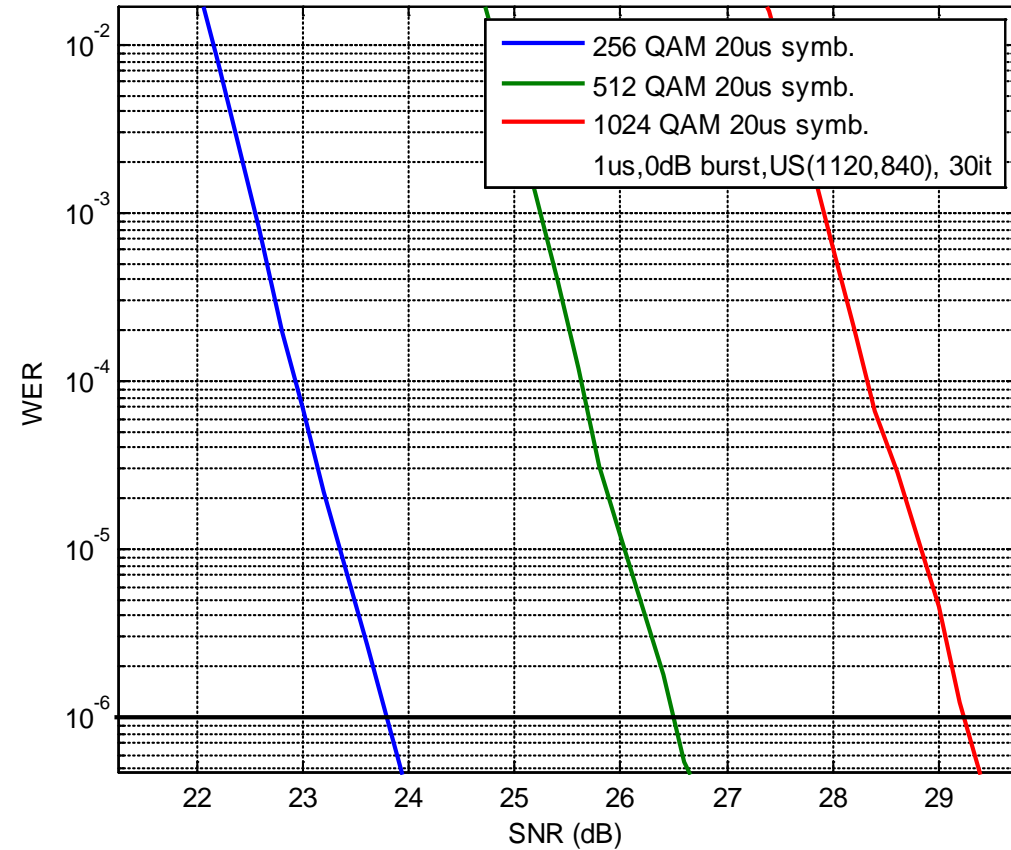
40 $\mu$ s symbol (two affected)

Latency: 340 $\mu$ s (depth:8)

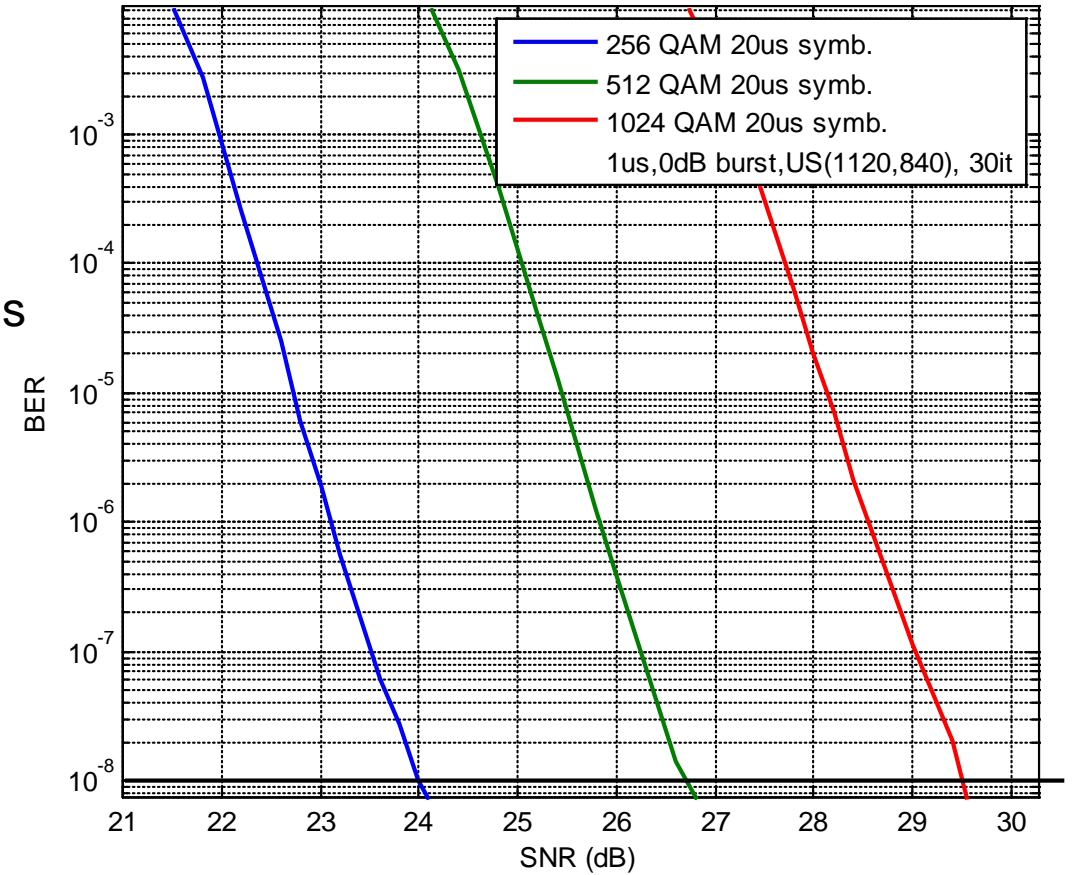
Short size  
(1120,840)  
30 iterations  
vs.  
15 iterations

		256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	<b>Max.15 iterations</b>	25.86dB	28.45dB	32.68dB
	<b>Max.30 iterations</b>	25.25dB	28.03dB	31.8dB
	<b>Difference</b>	0.61	0.42	0.88
<b>SNR@BER=1e-8</b>	<b>Max. 15 iterations</b>	26.4dB	28.84dB	32.73dB
	<b>Max. 30 iterations</b>	25.97dB	28.52dB	32.42dB
	<b>Difference</b>	0.43	0.32	0.31

# 20μs SYMBOLS ON 1μs 0dB BURST (SHORT CODE) MAX 30 ITERATIONS

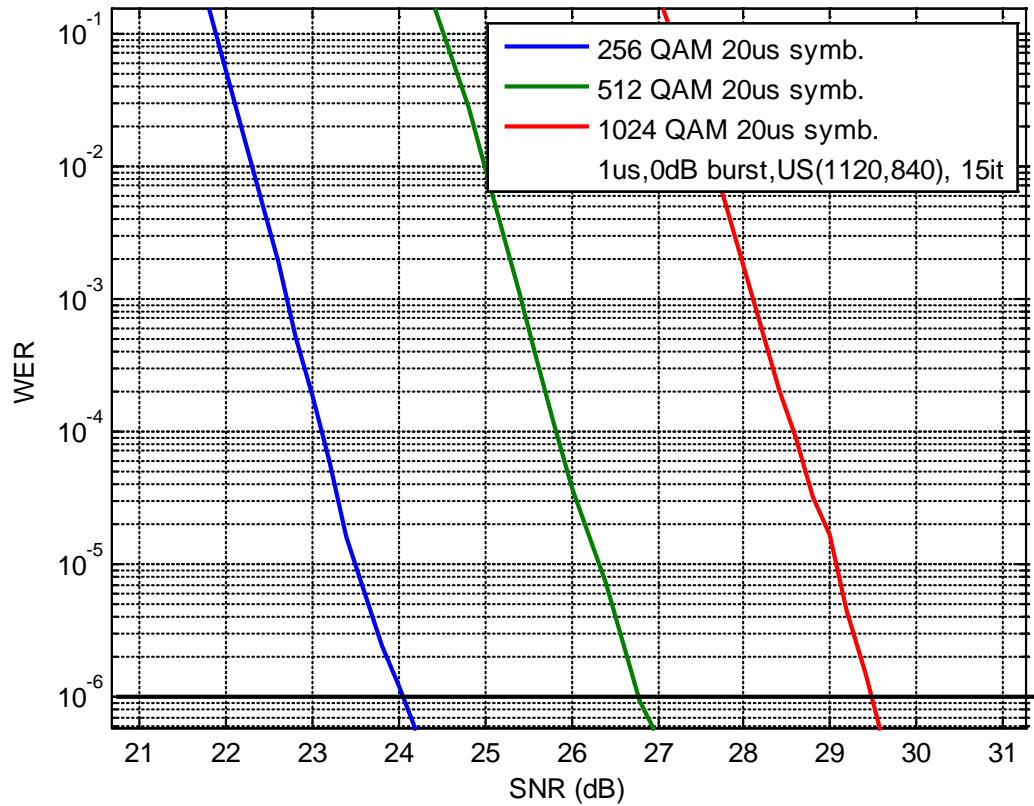


20μs symbol  
(one affected)  
Latency: 382.5μs  
(depth:17)

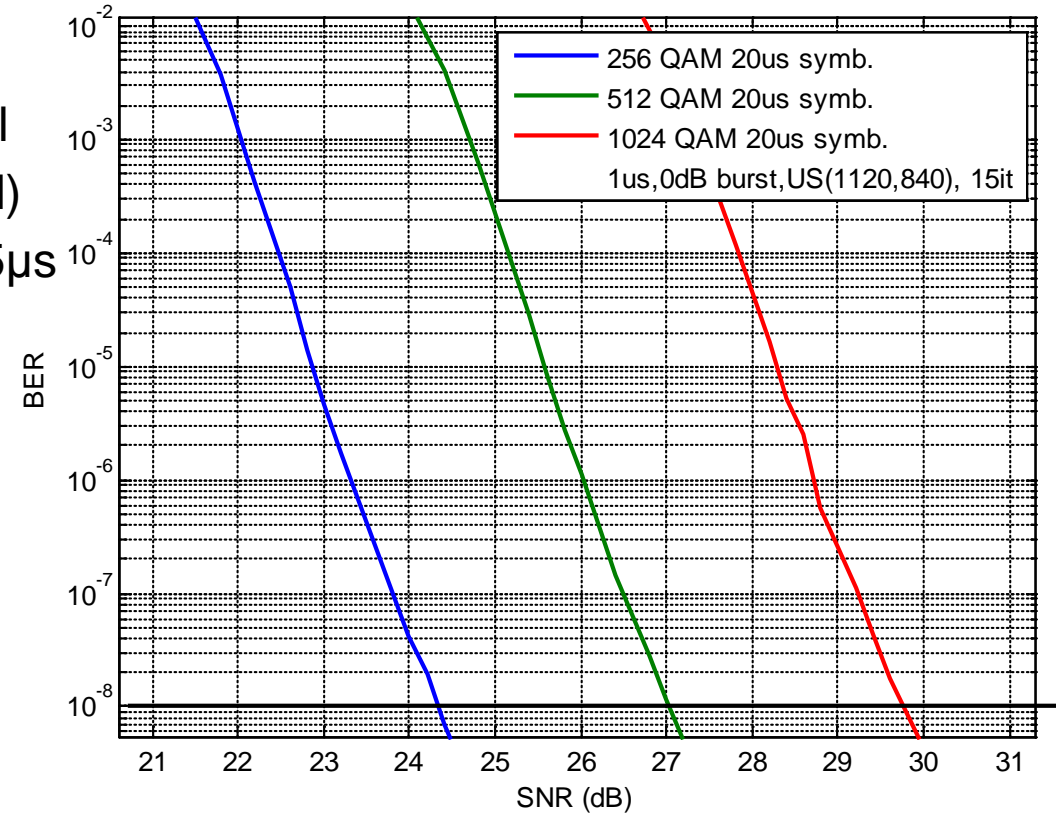


	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	23.79dB	26.5dB	29.24dB
<b>SNR@BER=1e-8</b>	24.1dB	26.71dB	29.51dB

# 20 $\mu$ s SYMBOLS ON 1 $\mu$ s 0dB BURST (SHORT CODE) MAX 15 ITERATIONS



20 $\mu$ s symbol  
(one affected)  
Latency: 382.5 $\mu$ s  
(depth:17)



	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	24.05dB	26.78dB	29.48dB
<b>SNR@BER=1e-8</b>	24.33dB	27.04dB	29.77dB

# DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (SHORT SIZE CODE)

## 20 $\mu$ s SYMBOLS ON 1 $\mu$ s 0dB BURST

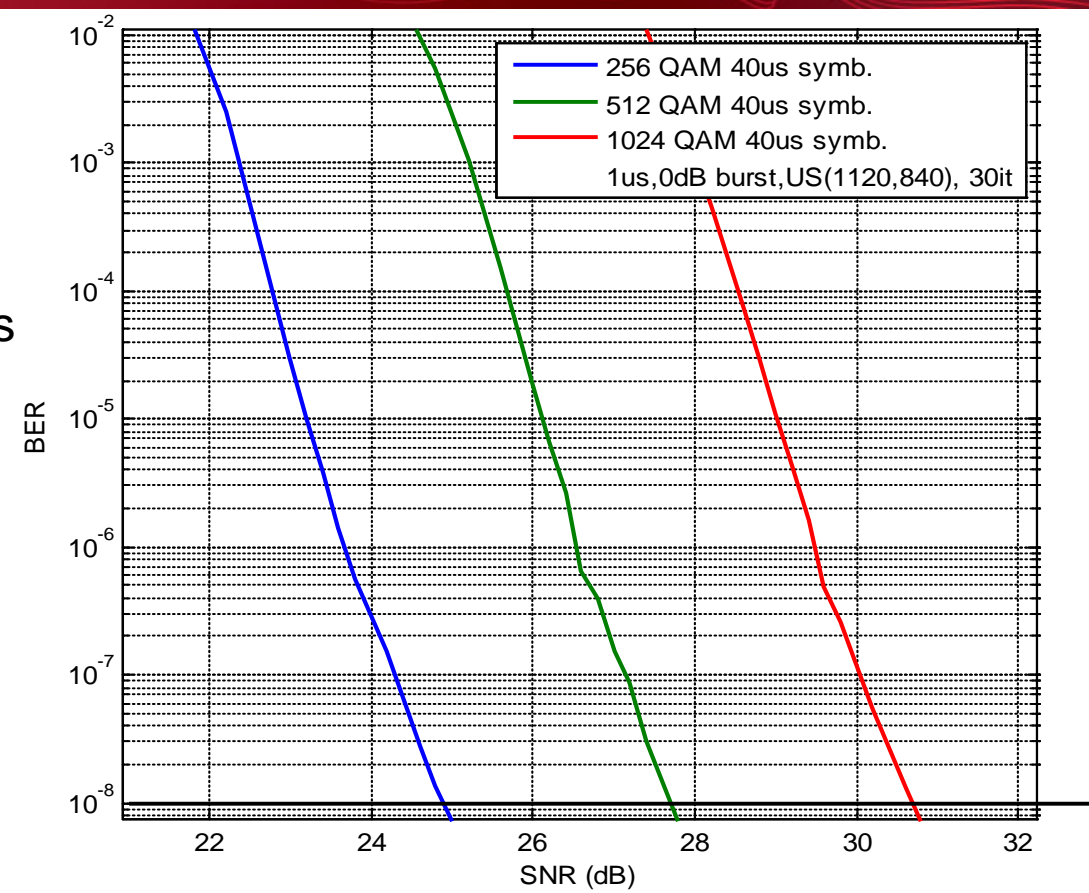
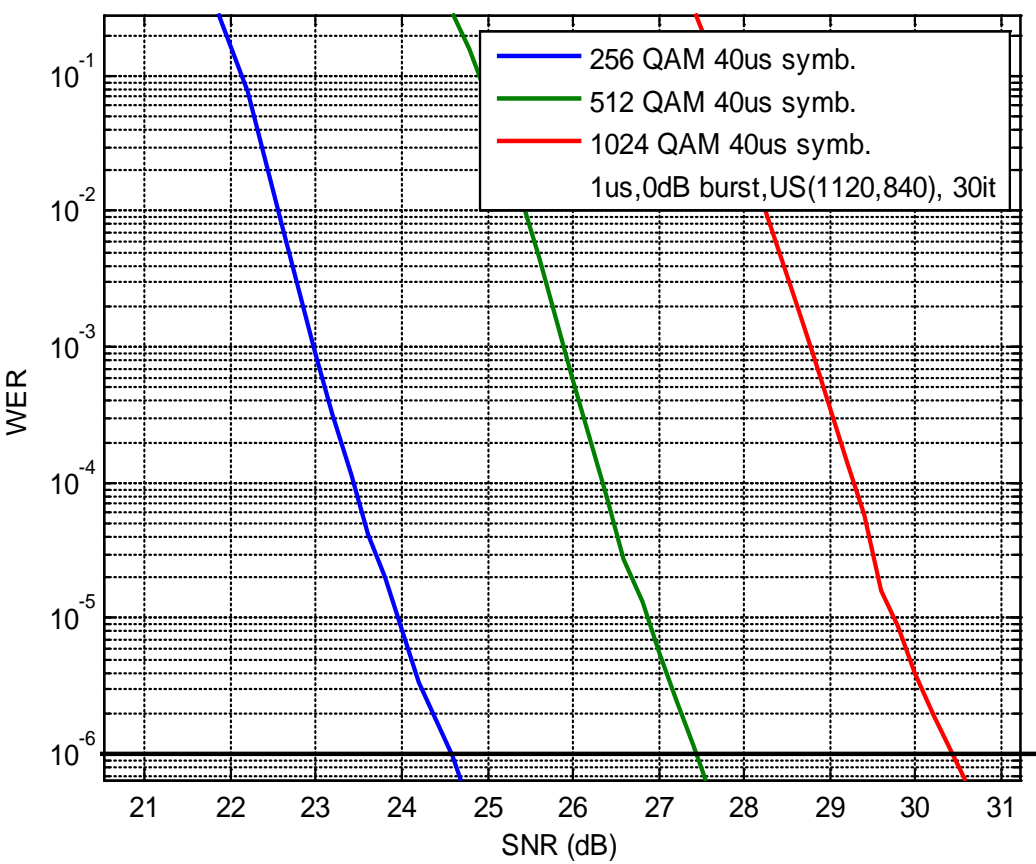
20 $\mu$ s symbol (two affected)

Latency: 382.5 $\mu$ s (depth:17)

Short size  
(1120,840)  
30 iterations  
vs.  
15 iterations

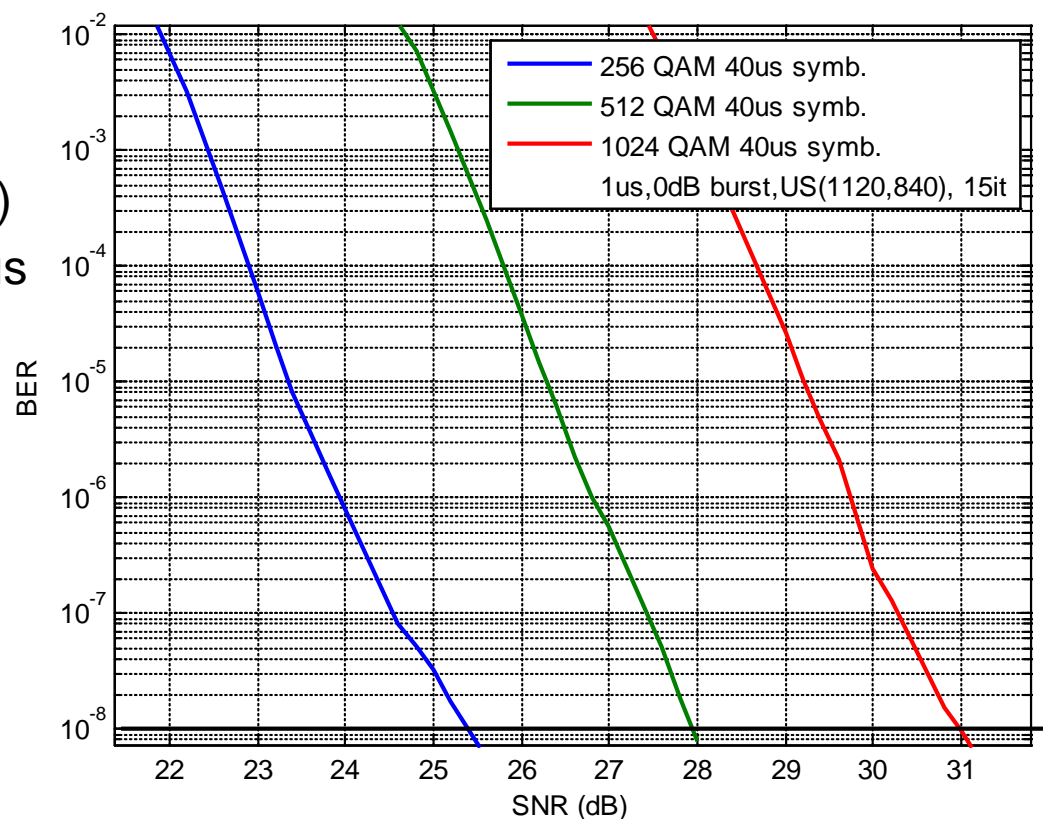
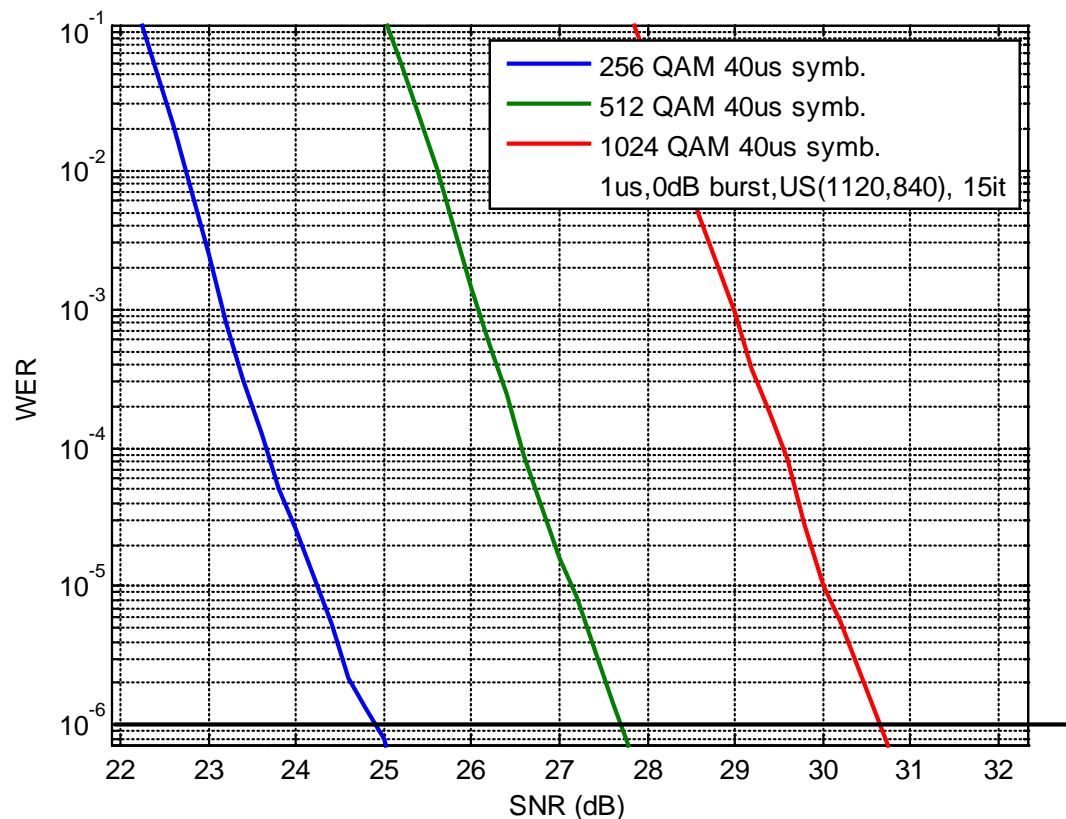
		256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	<b>Max.15 iterations</b>	24.05dB	26.78dB	29.48dB
	<b>Max.30 iterations</b>	23.79dB	26.5dB	29.24dB
	<b>Difference</b>	0.26	0.28	0.24
<b>SNR@BER=1e-8</b>	<b>Max. 15 iterations</b>	24.33dB	27.04dB	29.77dB
	<b>Max. 30 iterations</b>	24.1dB	26.71dB	29.51dB
	<b>Difference</b>	0.23	0.33	0.26

# 40μs SYMBOLS ON 1μs 0dB BURST (SHORT CODE) MAX 30 ITERATIONS



	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	24.59dB	27.45dB	30.44dB
<b>SNR@BER=1e-8</b>	24.89dB	27.71dB	30.69dB

# 40μs SYMBOLS ON 1μs 0dB BURST (SHORT CODE) MAX 15 ITERATIONS



	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	24.91dB	27.7dB	30.7dB
<b>SNR@BER=1e-8</b>	25.4dB	27.94dB	31dB

# DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (SHORT SIZE CODE)

## 40 $\mu$ s SYMBOLS ON 1 $\mu$ s 0dB BURST

40 $\mu$ s symbol (two affected)

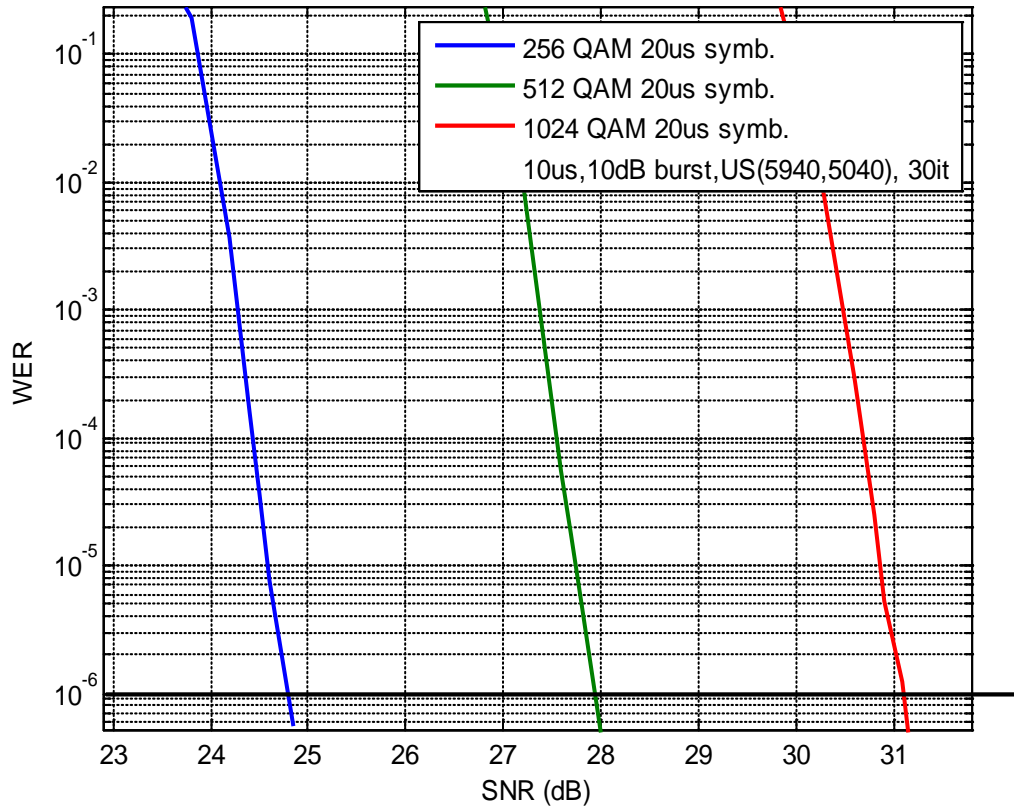
Latency: 340 $\mu$ s (depth:8)

Short size  
(1120,840)  
30 iterations  
vs.  
15 iterations

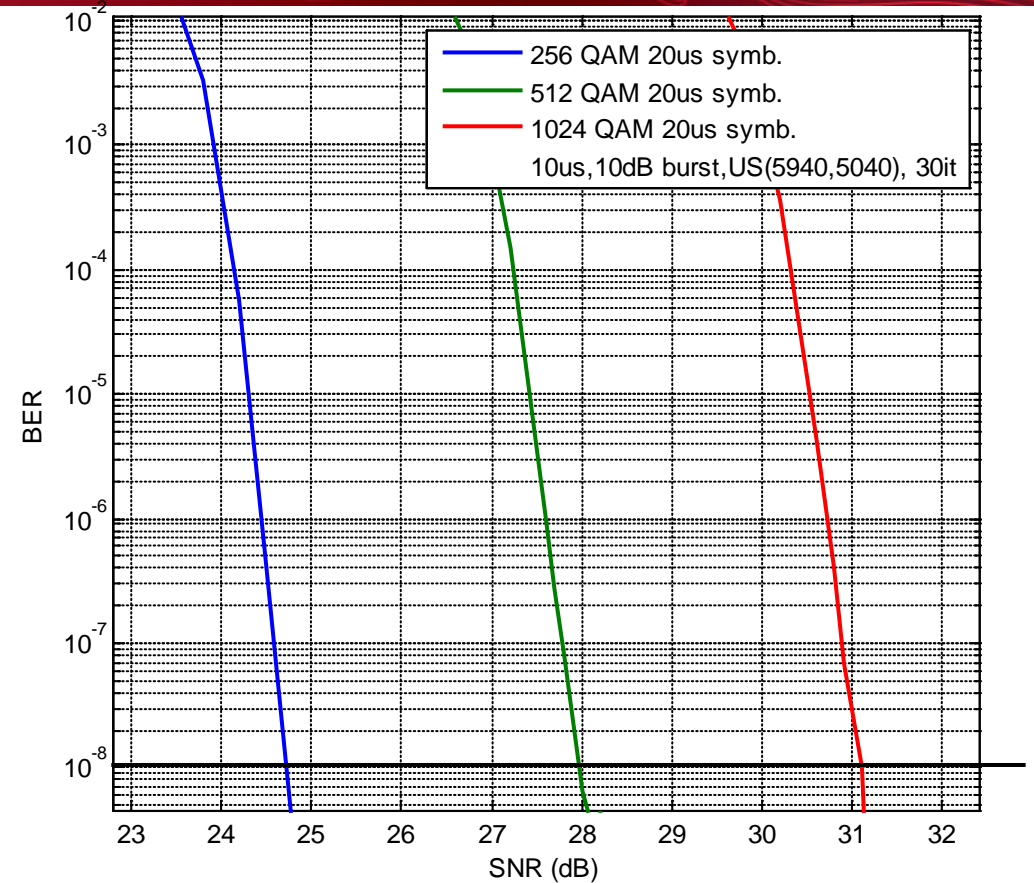
		256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	<b>Max.15 iterations</b>	24.91dB	27.7dB	30.7dB
	<b>Max.30 iterations</b>	24.59dB	27.45dB	30.44dB
	<b>Difference</b>	0.32	0.25	0.26
<b>SNR@BER=1e-8</b>	<b>Max. 15 iterations</b>	25.4dB	27.94dB	31dB
	<b>Max. 30 iterations</b>	24.89dB	27.71dB	30.69dB
	<b>Difference</b>	0.51	0.23	0.31



# 20μs SYMBOLS ON 10μs 10dB BURST (MEDIUM SIZE CODE) MAX 30 ITERATIONS

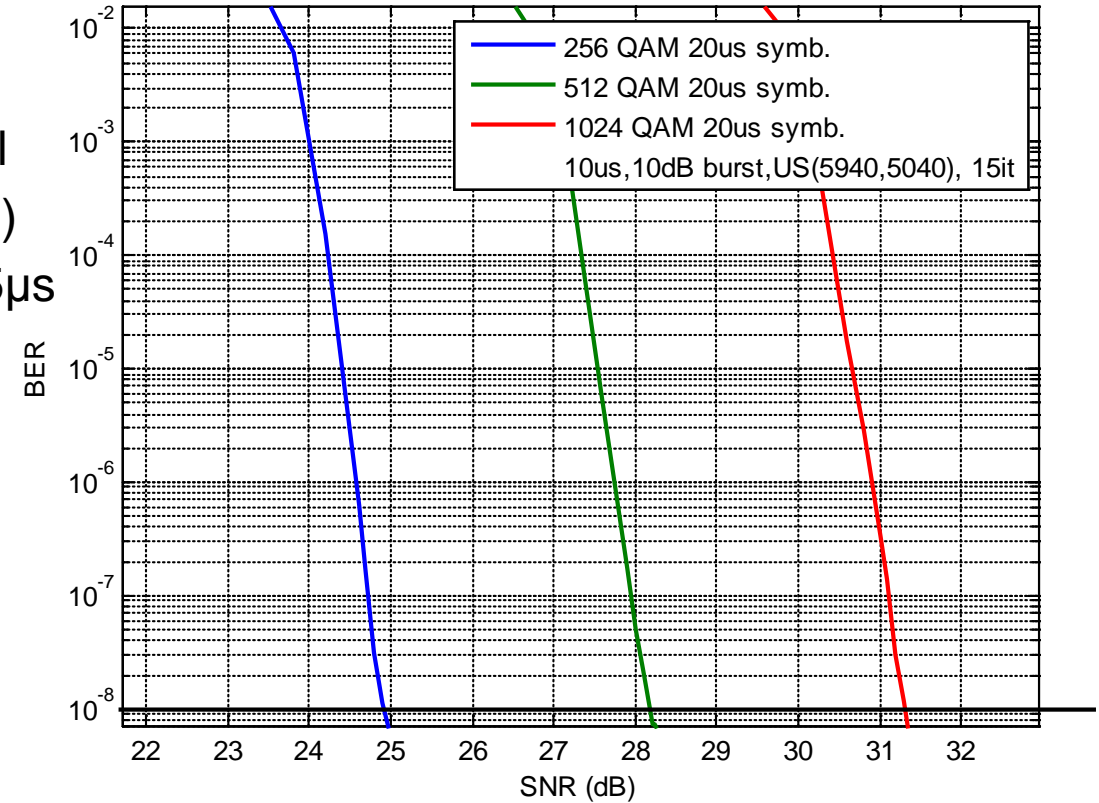
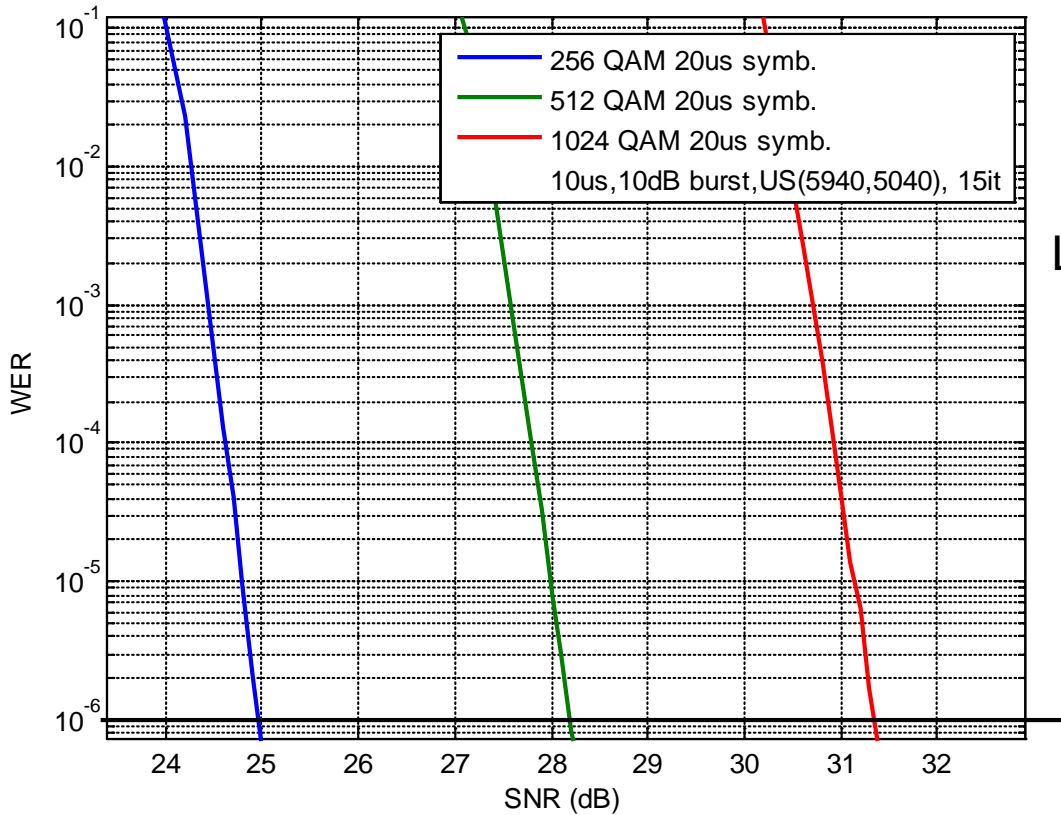


20μs symbol  
(two affected)  
Latency: 382.5μs  
(depth:17)



	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	24.77dB	27.96dB	31.08dB
<b>SNR@BER=1e-8</b>	24.75dB	27.99dB	31.08dB

# 20μs SYMBOLS ON 10μs 10dB BURST (MEDIUM SIZE CODE) MAX 15 ITERATIONS



	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	24.96dB	28.19dB	31.35dB
<b>SNR@BER=1e-8</b>	24.92dB	28.17dB	31.32dB

# DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (MEDIUM SIZE CODE)

## 20 $\mu$ s SYMBOLS ON 10 $\mu$ s 10dB BURST

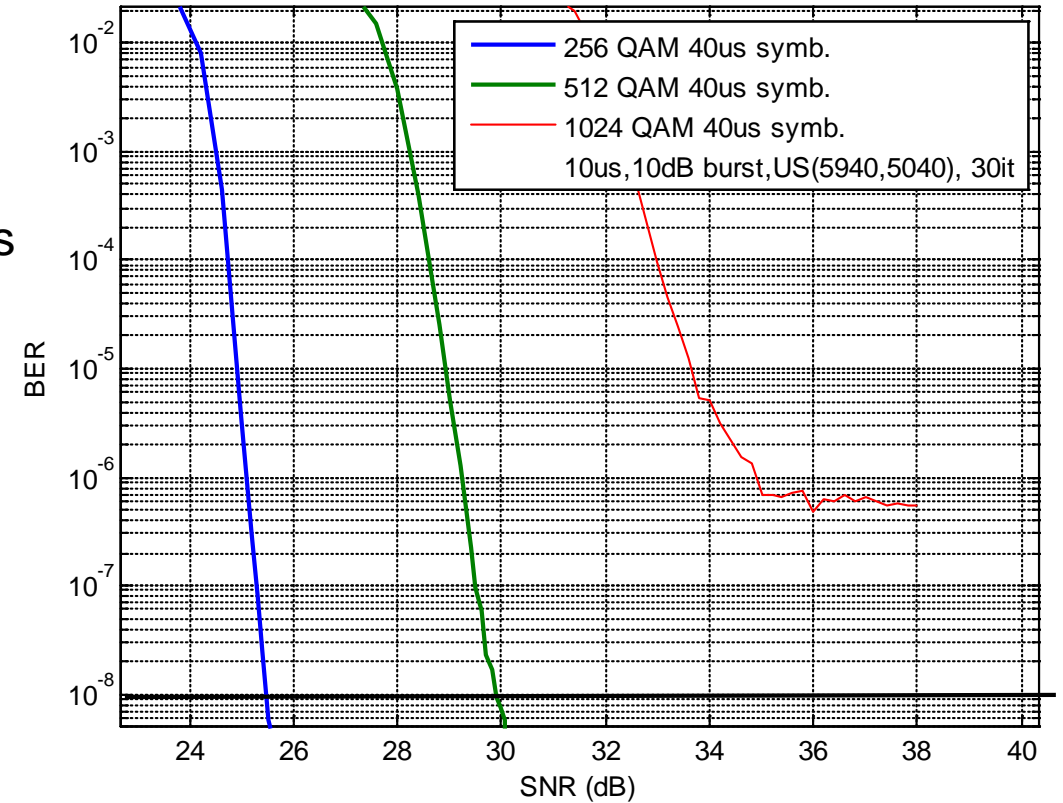
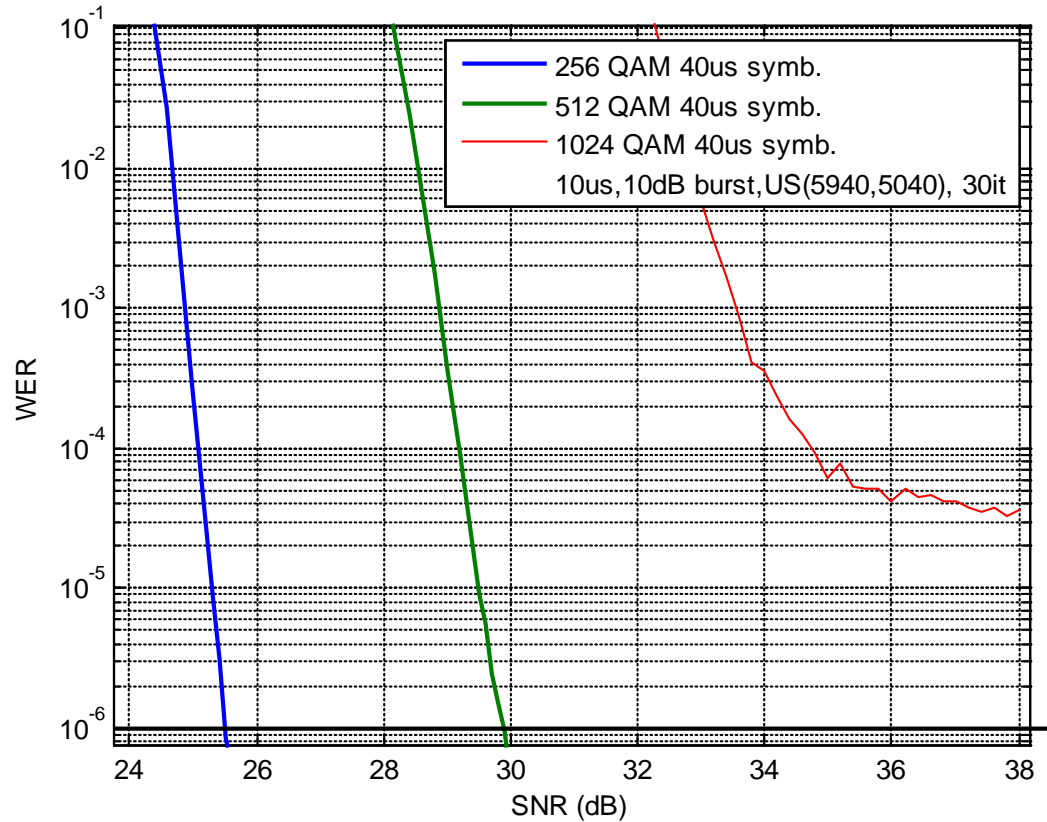


20 $\mu$ s symbol (two affected)  
 Latency: 382.5 $\mu$ s (depth:17)

Medium size  
 (5940,5040)  
 30 iterations  
 vs.  
 15 iterations

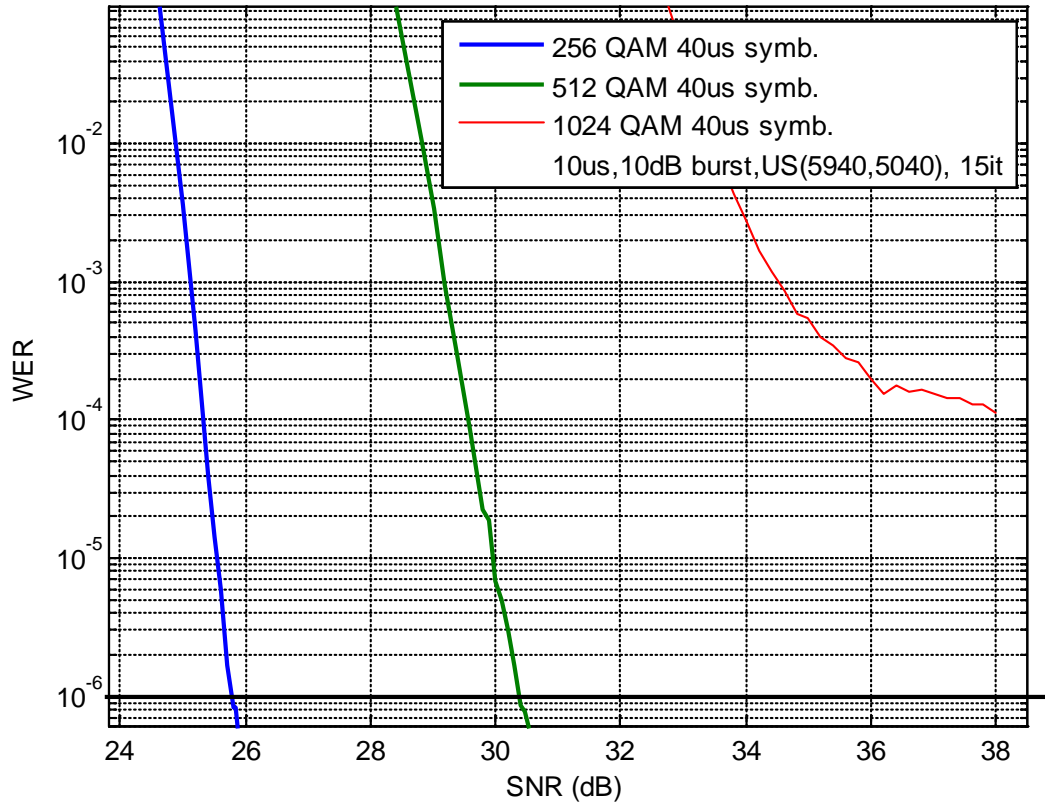
		256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	<b>Max.15 iterations</b>	24.96dB	28.19dB	31.35dB
	<b>Max.30 iterations</b>	24.77dB	27.96dB	31.08dB
	<b>Difference</b>	0.19	0.23	0.23
<b>SNR@BER=1e-8</b>	<b>Max. 15 iterations</b>	24.92dB	28.17dB	31.32dB
	<b>Max. 30 iterations</b>	24.75dB	27.99dB	31.08dB
	<b>Difference</b>	0.17	0.18	0.24

# 40μs SYMBOLS ON 10μs 10dB BURST (MEDIUM SIZE CODE) MAX 30 ITERATIONS

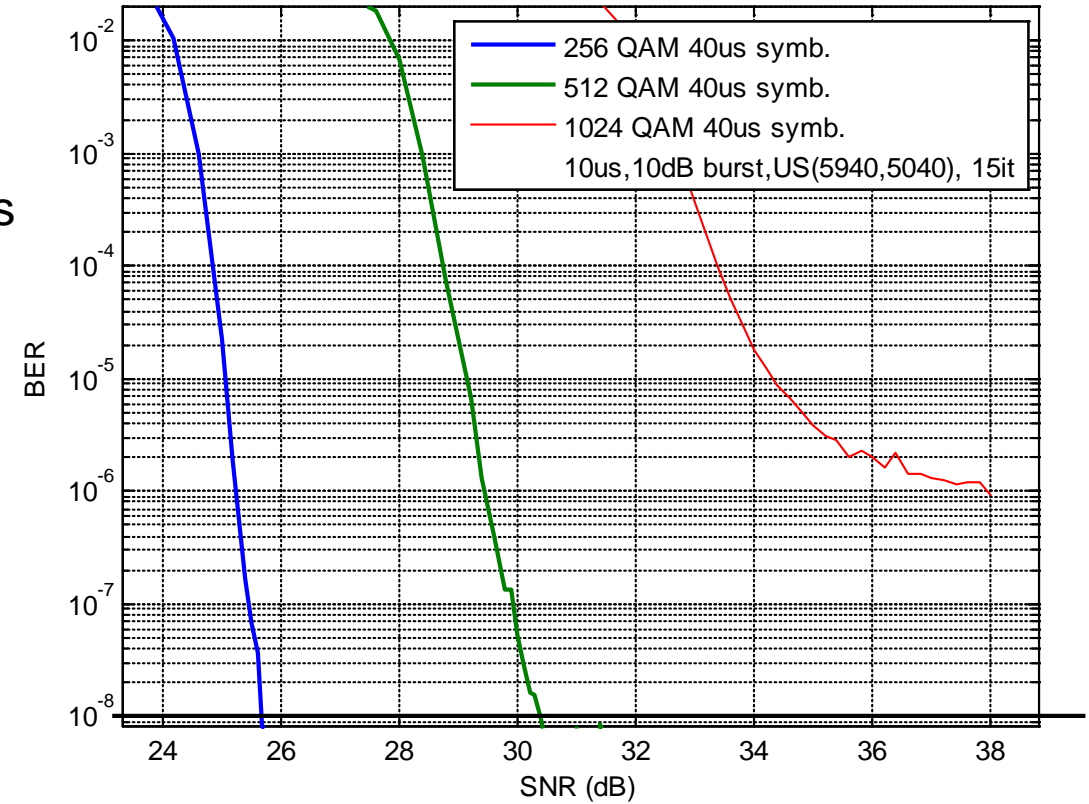


	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	25.49dB	29.8dB	
<b>SNR@BER=1e-8</b>	25.46dB	29.89dB	

# 40μs SYMBOLS ON 10μs 10dB BURST (MEDIUM SIZE CODE) MAX 15 ITERATIONS

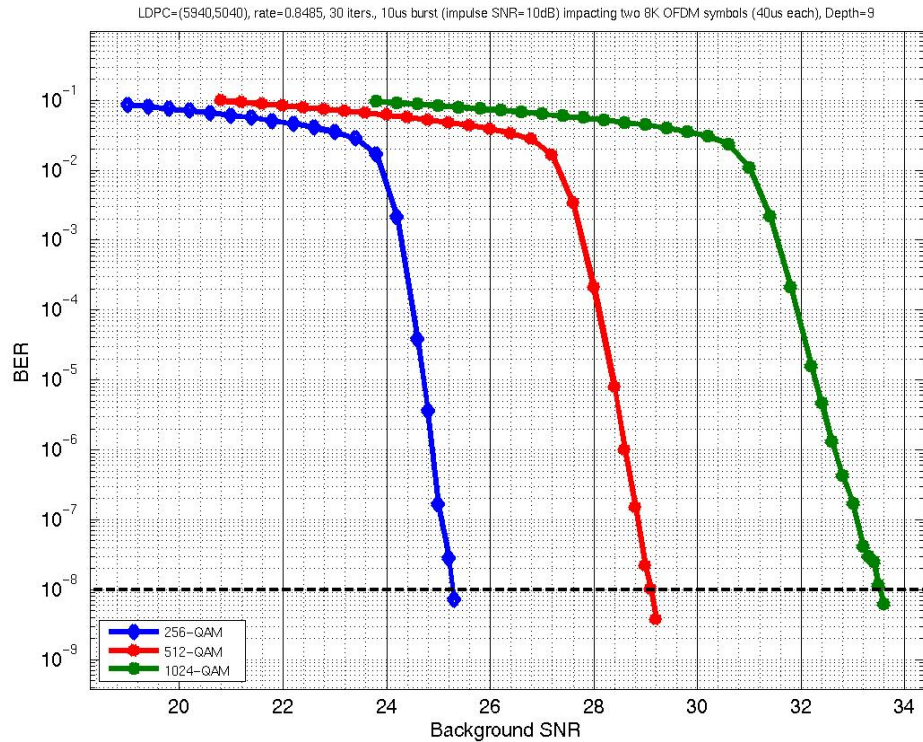


40μs symbol  
(two affected)  
Latency: 340μs  
(depth:8)



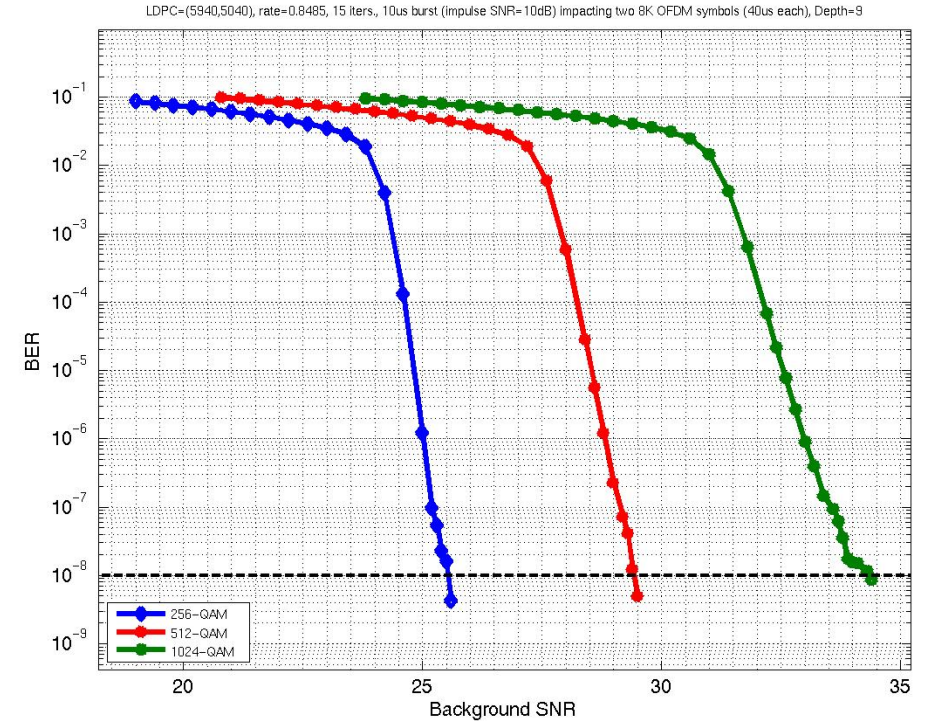
	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	25.77dB	30.38dB	
<b>SNR@BER=1e-8</b>	25.69dB	30.04dB	

# 40μs SYMBOLS ON 10μs 10dB BURST (MEDIUM SIZE CODE)



Max. 30 iterations

40μs symbol  
(two affected)  
Latency: 382.5μs  
(depth:9)



Max. 15 iterations

		256QAM	512QAM	1024QAM
SNR@BER=1e-8	Max. 15 iterations	25.5dB	29.5dB	34.3dB
	Max. 30 iterations	25.3dB	29.1dB	33.5dB

# DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (MEDIUM SIZE CODE)

## 40μs SYMBOLS ON 10μs 10dB BURST

40μs symbol (two affected) Latency: 340μs (depth:8)

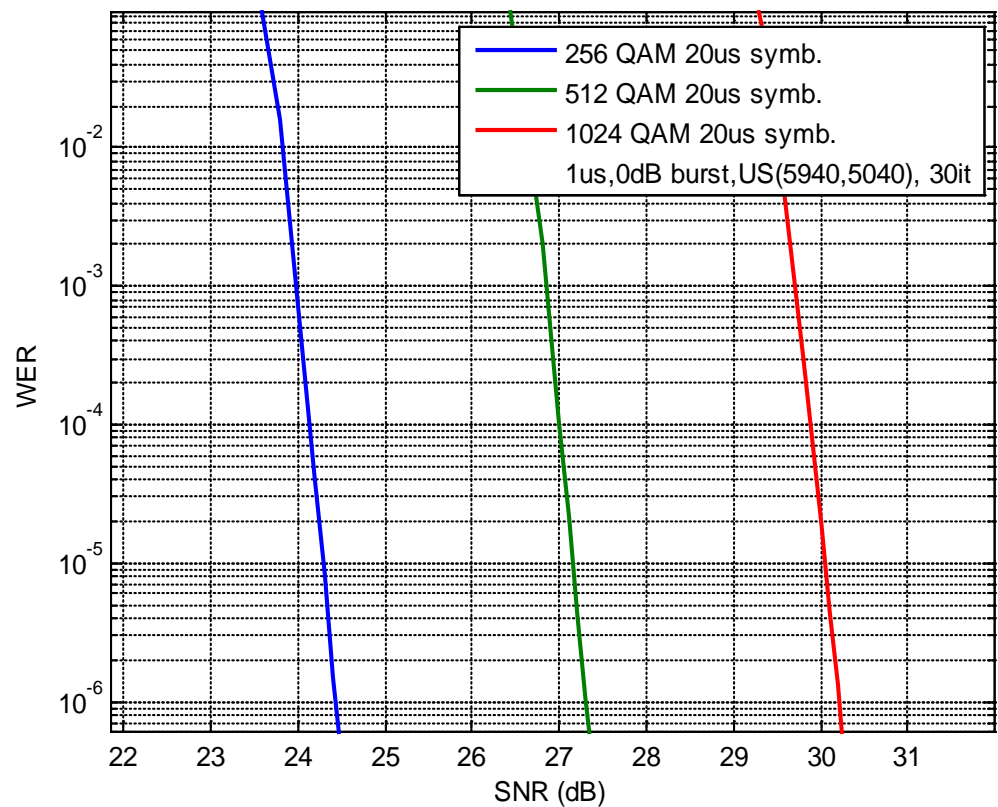
		256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	<b>Max.15 iterations</b>	25.77dB	30.38dB	
	<b>Max.30 iterations</b>	25.49dB	29.8dB	
	<b>Difference</b>	0.28	0.58	
<b>SNR@BER=1e-8</b>	<b>Max. 15 iterations</b>	25.69dB	30.04dB	
	<b>Max. 30 iterations</b>	25.46dB	29.89dB	
	<b>Difference</b>	0.23	0.15	

Medium size  
(5940,5040)  
30 iterations  
vs.  
15 iterations

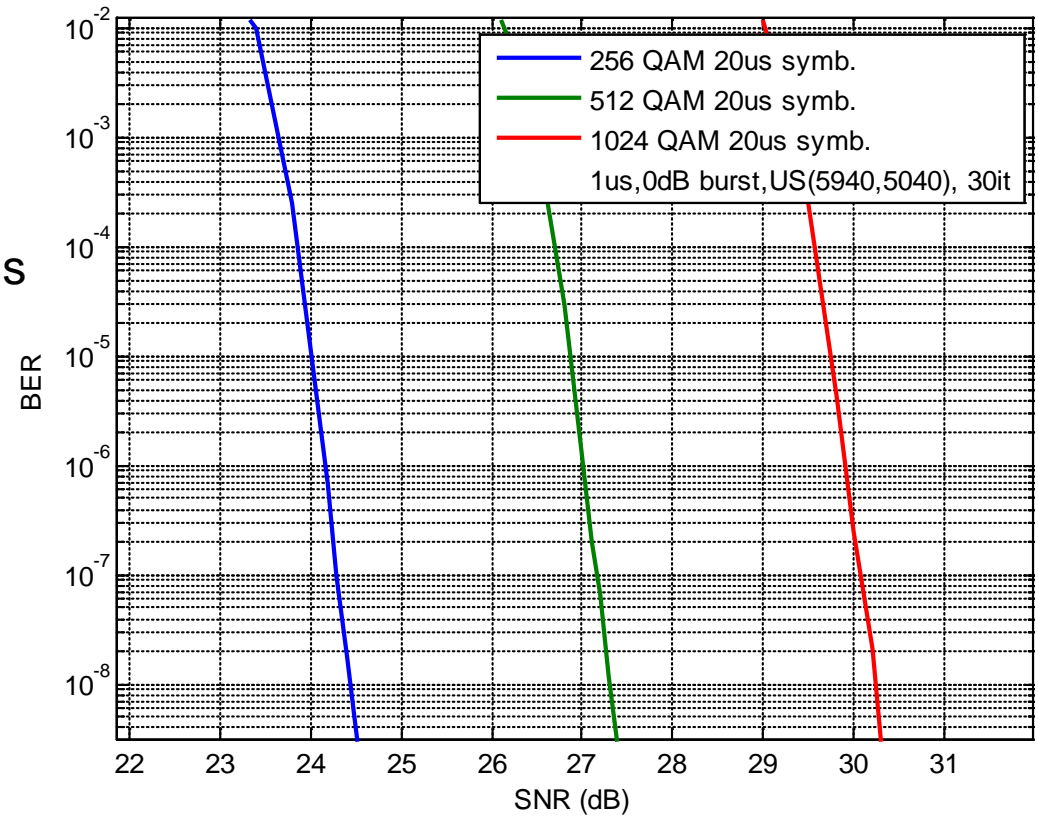
40μs symbol (two affected) Latency: 382.5μs (depth:9)

		256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	<b>Max.15 iterations</b>			
	<b>Max.30 iterations</b>			
	<b>Difference</b>			
<b>SNR@BER=1e-8</b>	<b>Max. 15 iterations</b>	25.5dB	29.5dB	34.3dB
	<b>Max. 30 iterations</b>	25.3dB	29.1dB	33.5dB
	<b>Difference</b>	0.2	0.4	0.8

# 20 $\mu$ s SYMBOLS ON 1 $\mu$ s 0dB BURST (MEDIUM CODE) MAX 30 ITERATIONS



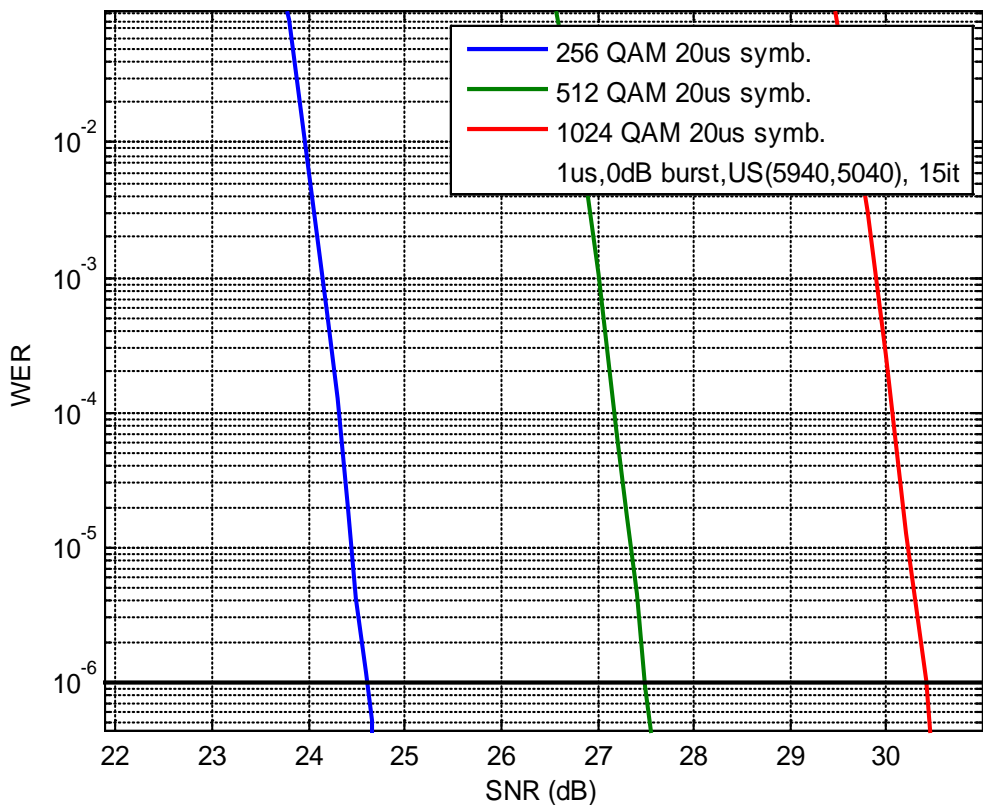
20 $\mu$ s symbol  
(one affected)  
Latency: 382.5 $\mu$ s  
(depth:17)



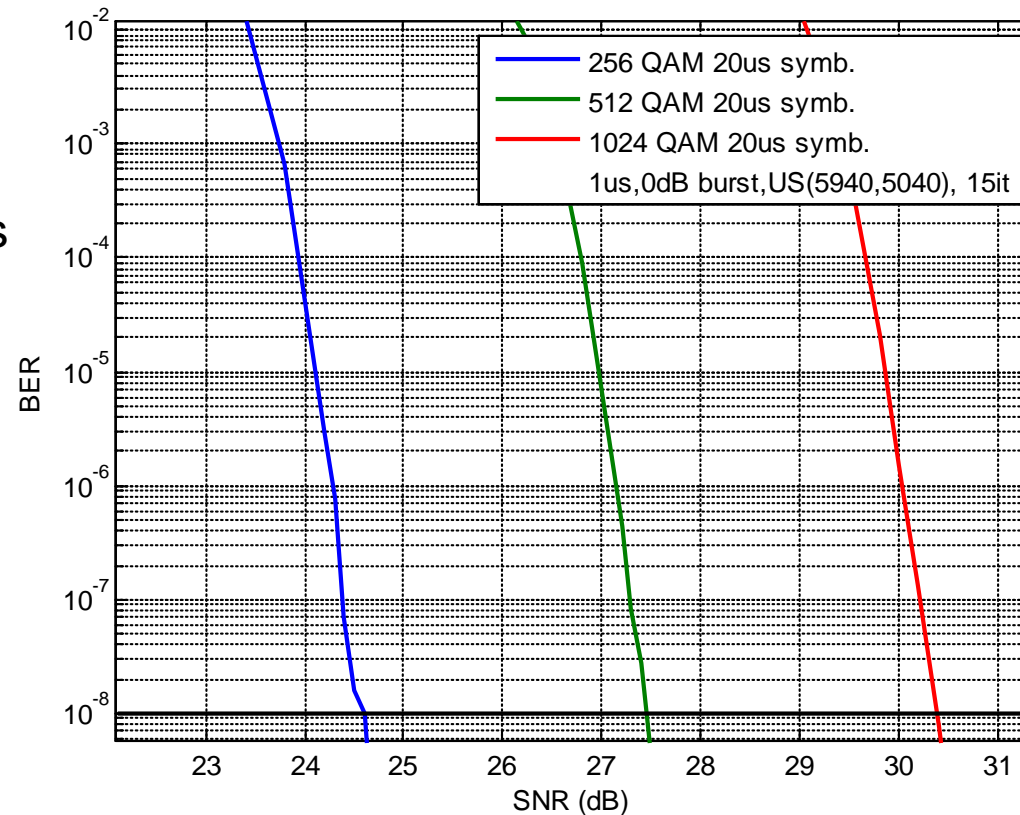
	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	24.43dB	27.30dB	30.22dB
<b>SNR@BER=1e-8</b>	24.44dB	27.31dB	30.22dB



# 20μs SYMBOLS ON 1μs 0dB BURST (MEDIUM CODE) MAX 15 ITERATIONS



20μs symbol  
(one affected)  
Latency: 382.5μs  
(depth:17)



	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	24.61dB	27.5dB	30.4dB
<b>SNR@BER=1e-8</b>	24.6dB	27.45dB	30.38dB

# DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (MEDIUM SIZE CODE)

## 20 $\mu$ s SYMBOLS ON 1 $\mu$ s 0dB BURST



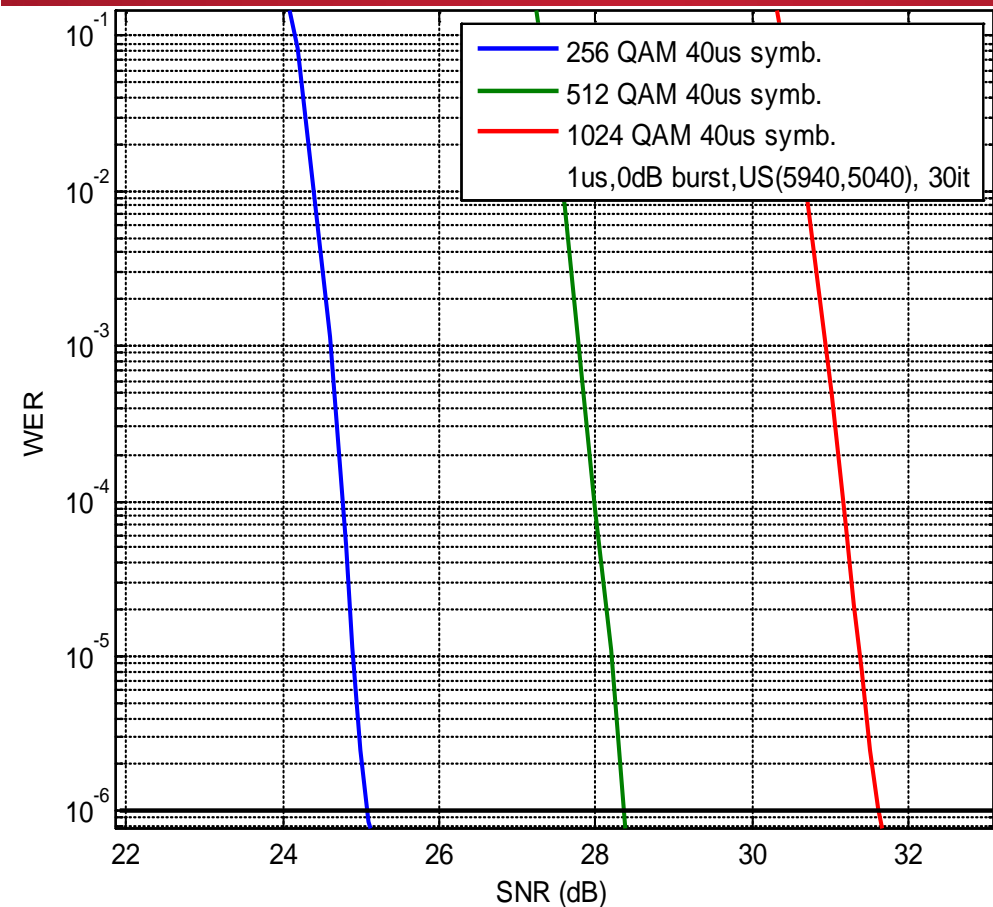
20 $\mu$ s symbol (one affected)

Latency: 382.5 $\mu$ s (depth:17)

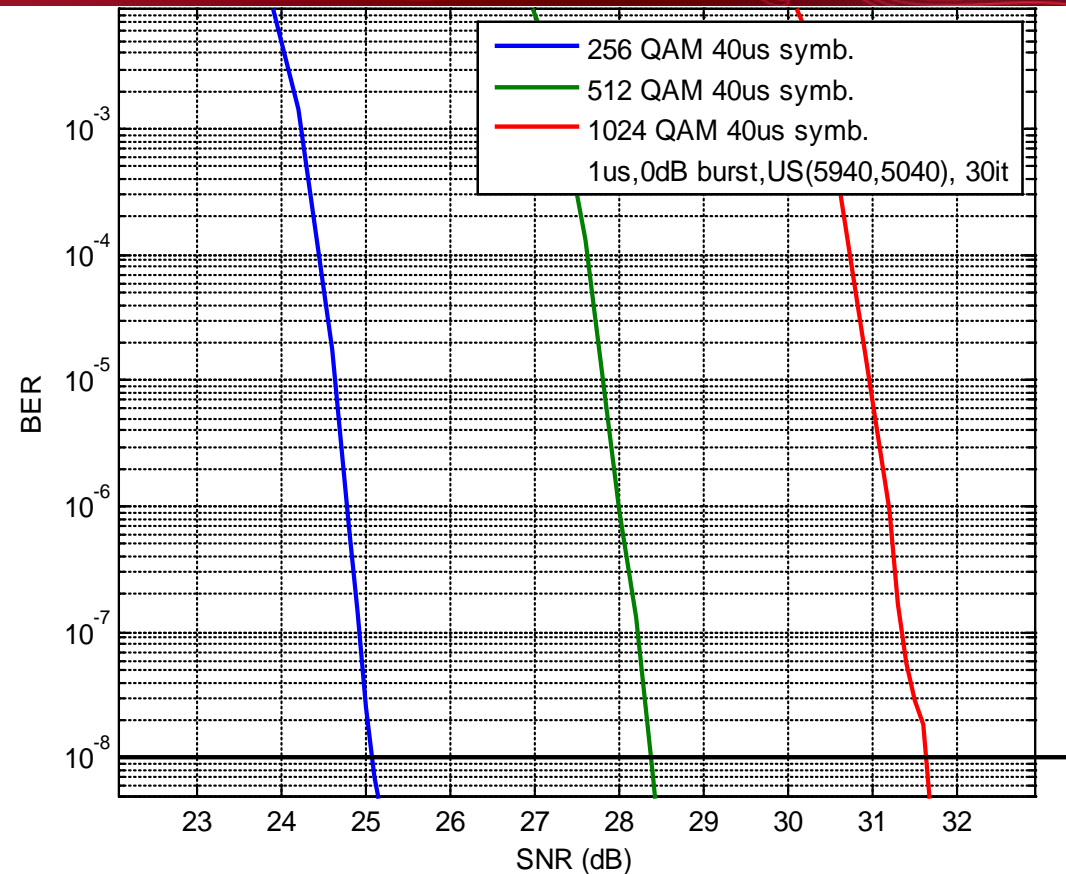
Medium size  
(5940,5040)  
30 iterations  
vs.  
15 iterations

		256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations	24.61dB	27.5dB	30.4dB
	Max.30 iterations	24.43dB	27.3dB	30.22dB
	Difference	0.18	0.2	0.18
SNR@BER=1e-8	Max. 15 iterations	24.6dB	27.45dB	30.38dB
	Max. 30 iterations	24.44dB	27.31dB	30.22dB
	Difference	0.16	0.14	0.16

# 40μs SYMBOLS ON 1μs 0dB BURST (MEDIUM SIZE CODE) MAX 30 ITERATIONS

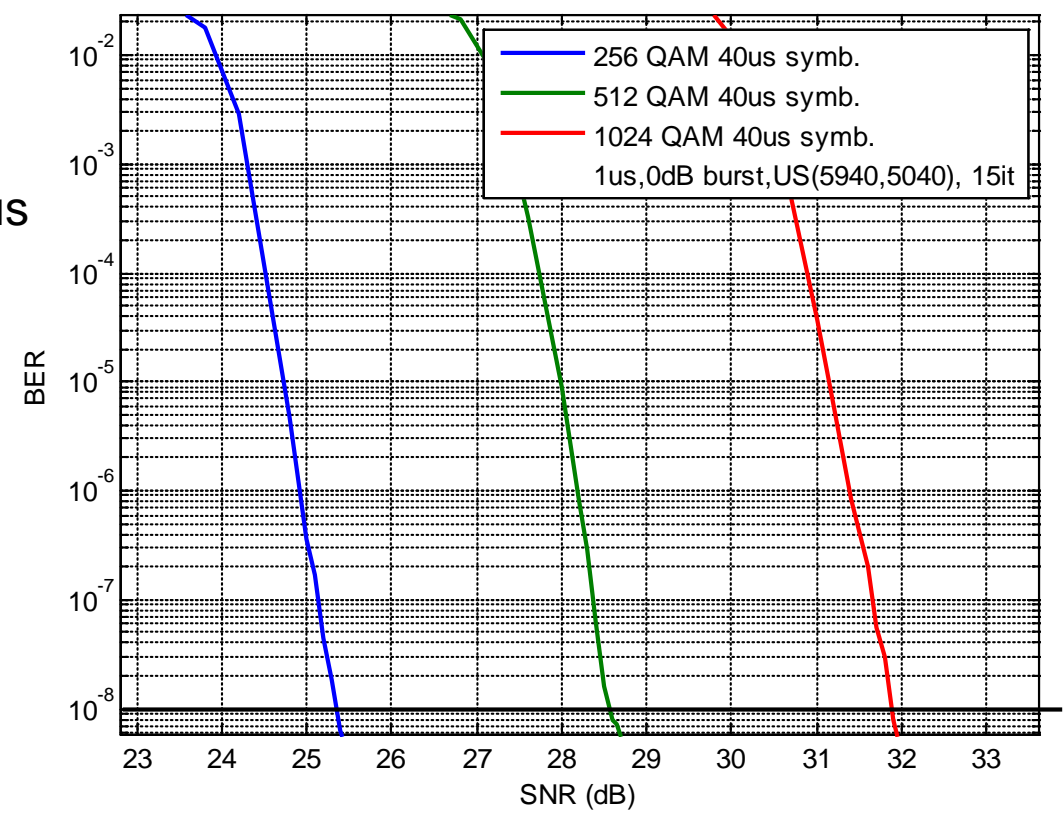
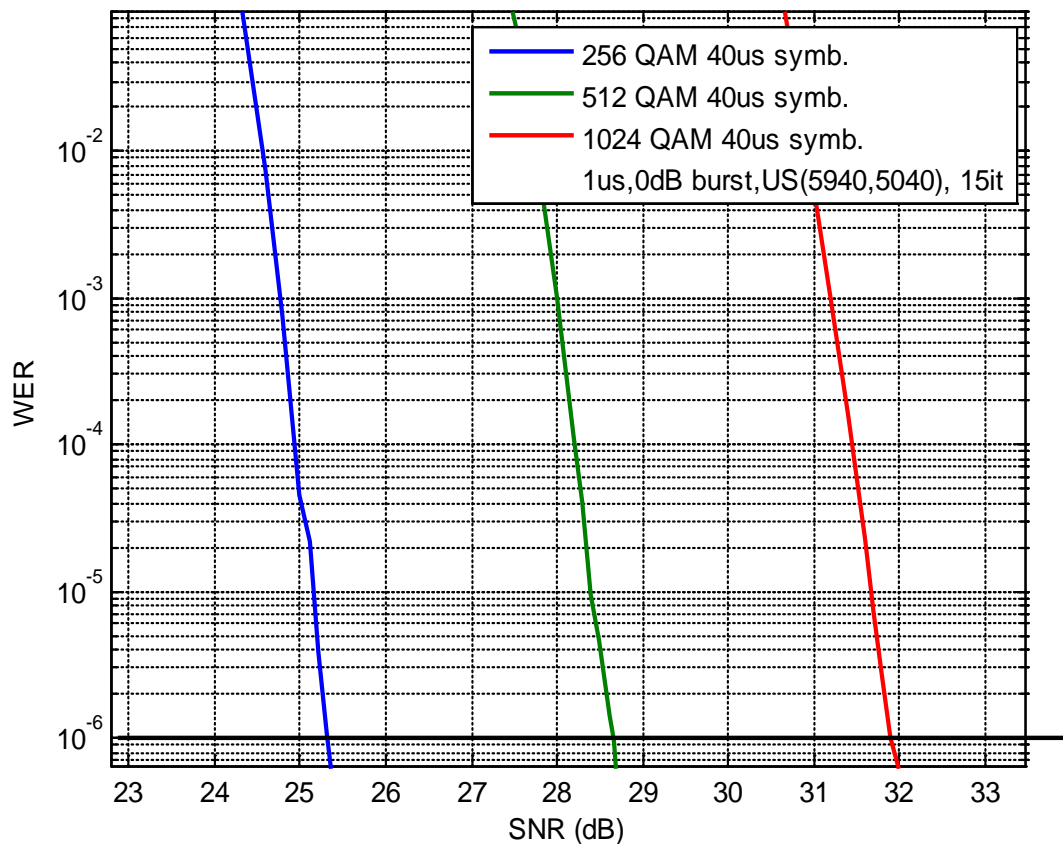


40μs symbol  
(one affected)  
Latency: 340μs  
(depth:8)



	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	25.09dB	28.35dB	31.6dB
<b>SNR@BER=1e-8</b>	25.09dB	28.37dB	31.62dB

# 40μs SYMBOLS ON 1μs 0dB BURST (MEDIUM SIZE CODE) MAX 15 ITERATIONS



	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	25.31dB	28.65dB	31.9dB
<b>SNR@BER=1e-8</b>	25.29dB	28.57dB	31.89dB

# DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (MEDIUM SIZE CODE)

## 40 $\mu$ s SYMBOLS ON 1 $\mu$ s 0dB BURST

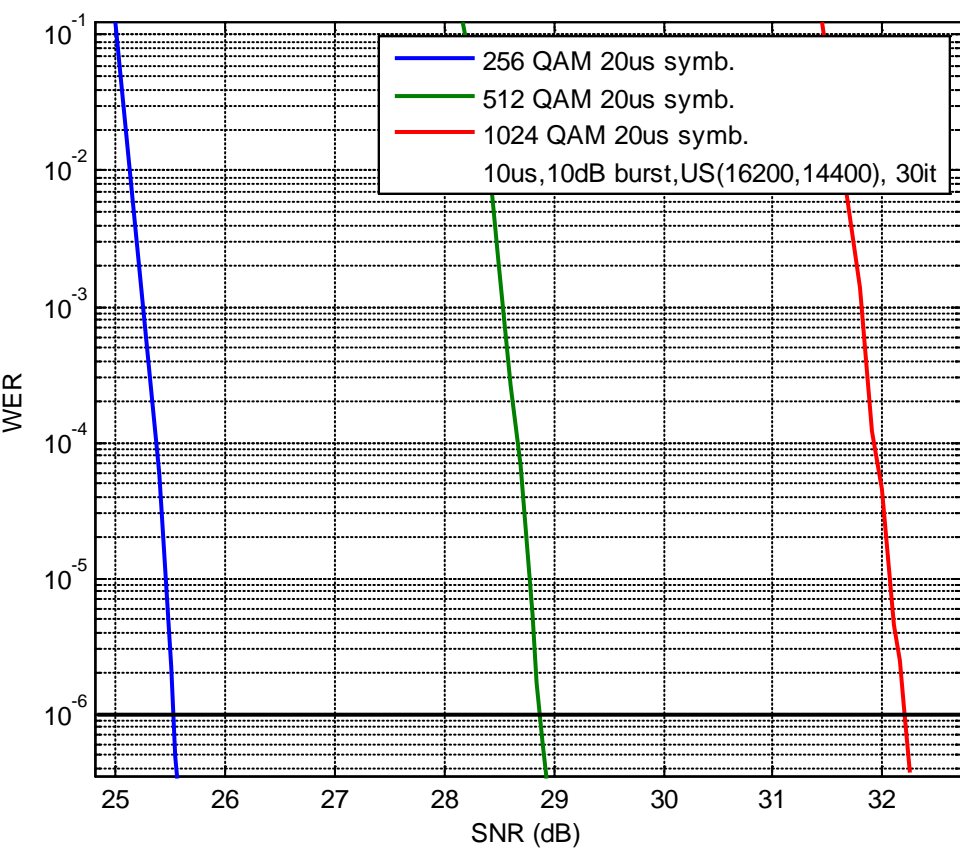


40 $\mu$ s symbol (one affected)  
 Latency: 340 $\mu$ s (depth:8)

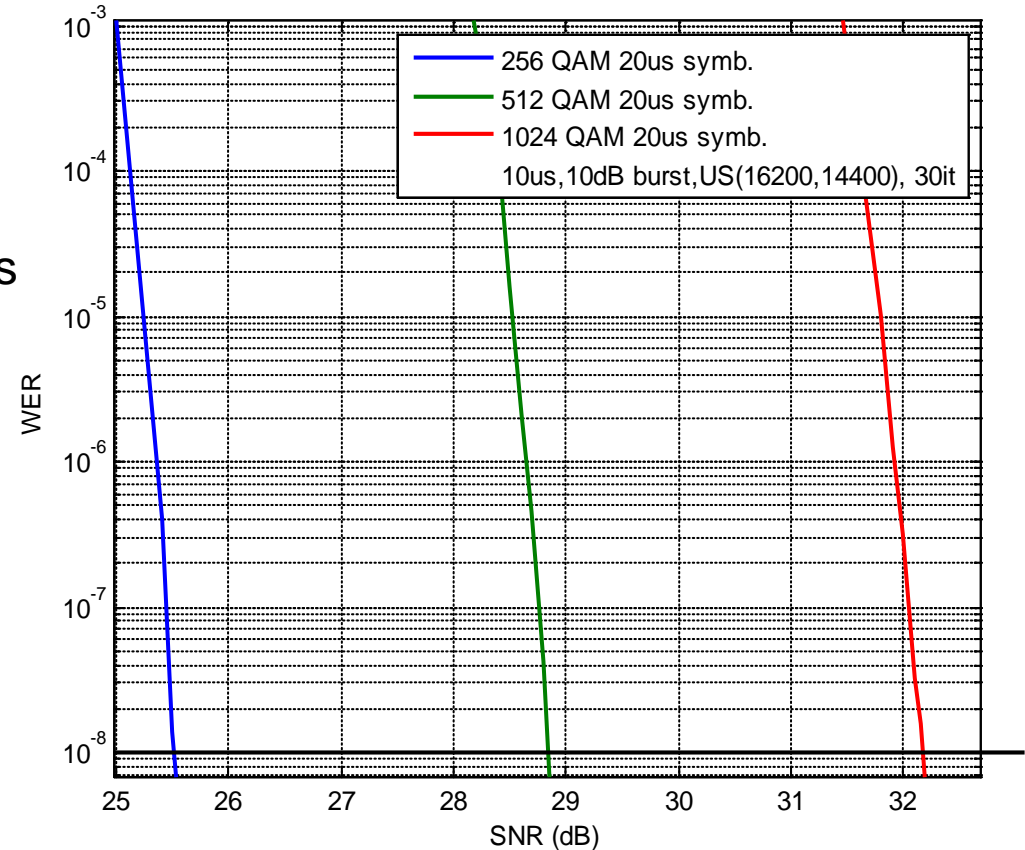
Medium size  
 (5940,5040)  
 30 iterations  
 vs.  
 15 iterations

		256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	<b>Max.15 iterations</b>	25.31dB	28.65dB	31.9dB
	<b>Max.30 iterations</b>	25.09dB	28.35dB	31.6dB
	<b>Difference</b>	0.22	0.3	0.3
<b>SNR@BER=1e-8</b>	<b>Max. 15 iterations</b>	25.29dB	28.57dB	31.89dB
	<b>Max. 30 iterations</b>	25.09dB	28.37dB	31.62dB
	<b>Difference</b>	0.2	0.2	0.27

# 20 $\mu$ s SYMBOLS ON 10 $\mu$ s 10dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS

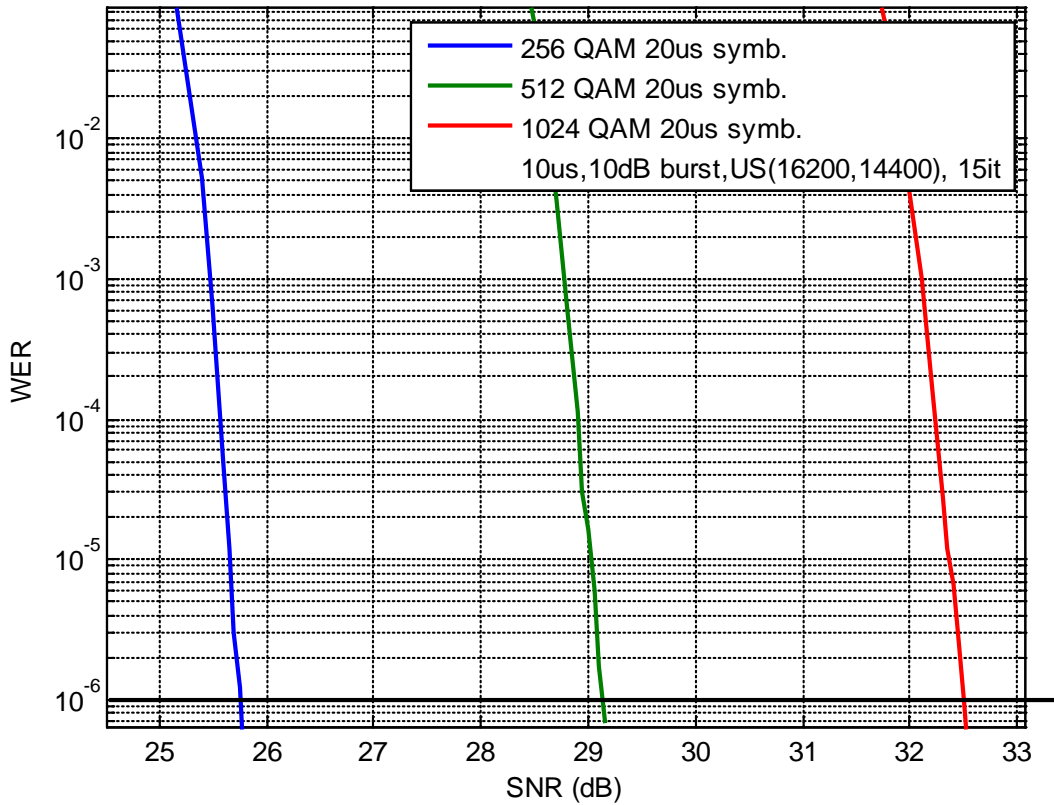


20 $\mu$ s symbol  
(two affected)  
Latency: 382.5 $\mu$ s  
(depth:17)

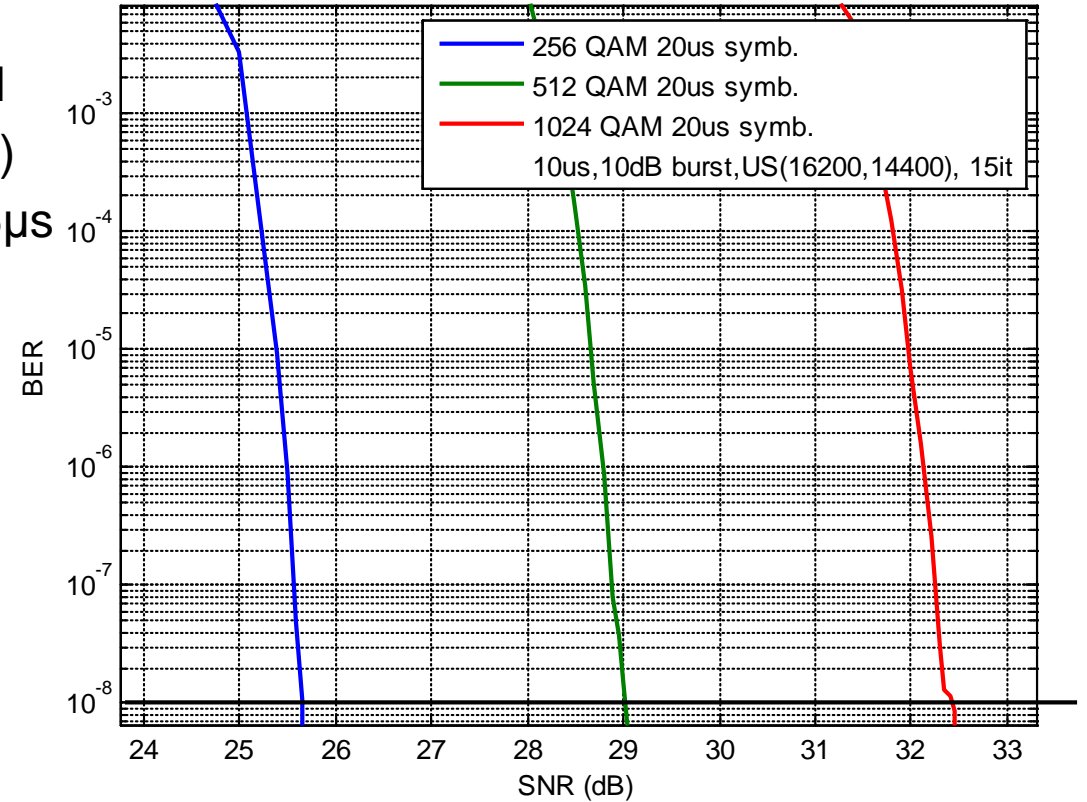


	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	25.53dB	28.88dB	32.2dB
<b>SNR@BER=1e-8</b>	25.52dB	28.86dB	32.17dB

# 20 $\mu$ s SYMBOLS ON 10 $\mu$ s 10dB BURST (LONG SIZE CODE) MAX 15 ITERATIONS



20 $\mu$ s symbol  
(two affected)  
Latency: 382.5 $\mu$ s  
(depth:17)



	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	25.76dB	29.13dB	32.5dB
<b>SNR@BER=1e-8</b>	25.65dB	29.02dB	32.42dB

# DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (LONG SIZE CODE)

## 20 $\mu$ s SYMBOLS ON 10 $\mu$ s 10dB BURST

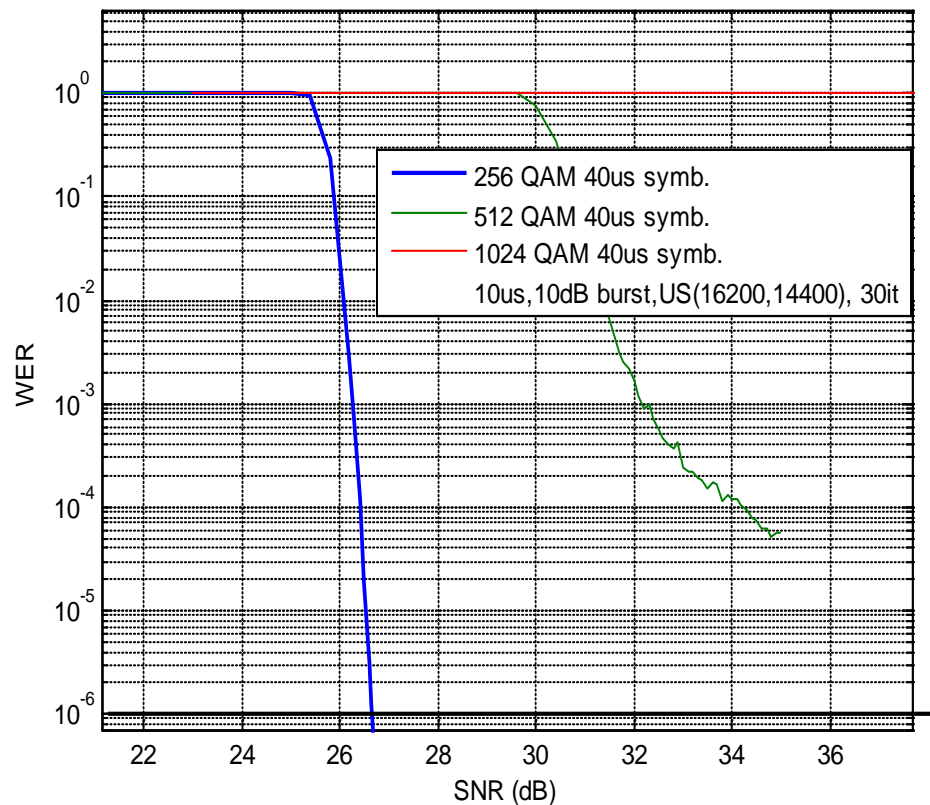
20 $\mu$ s symbol(two affected)  
 Latency: 382.5 $\mu$ s (depth:17)

Long size  
 (16200,14400)  
 30 iterations  
 vs.  
 15 iterations

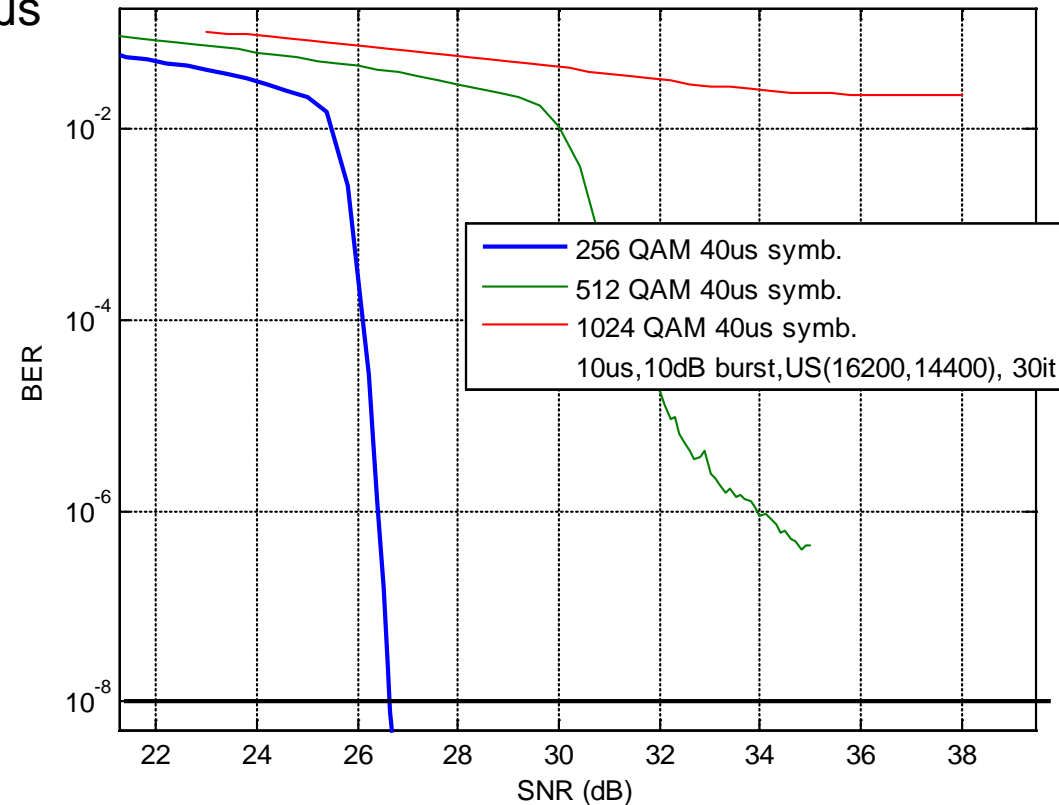
		256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	<b>Max.15 iterations</b>	25.76dB	29.13dB	32.5dB
	<b>Max.30 iterations</b>	25.53dB	28.88dB	32.2dB
	<b>Difference</b>	0.23	0.25	0.3
<b>SNR@BER=1e-8</b>	<b>Max. 15 iterations</b>	25.53dB	28.88dB	32.2dB
	<b>Max. 30 iterations</b>	25.52dB	28.86dB	32.17dB
	<b>Difference</b>	0.01	0.02	0.03



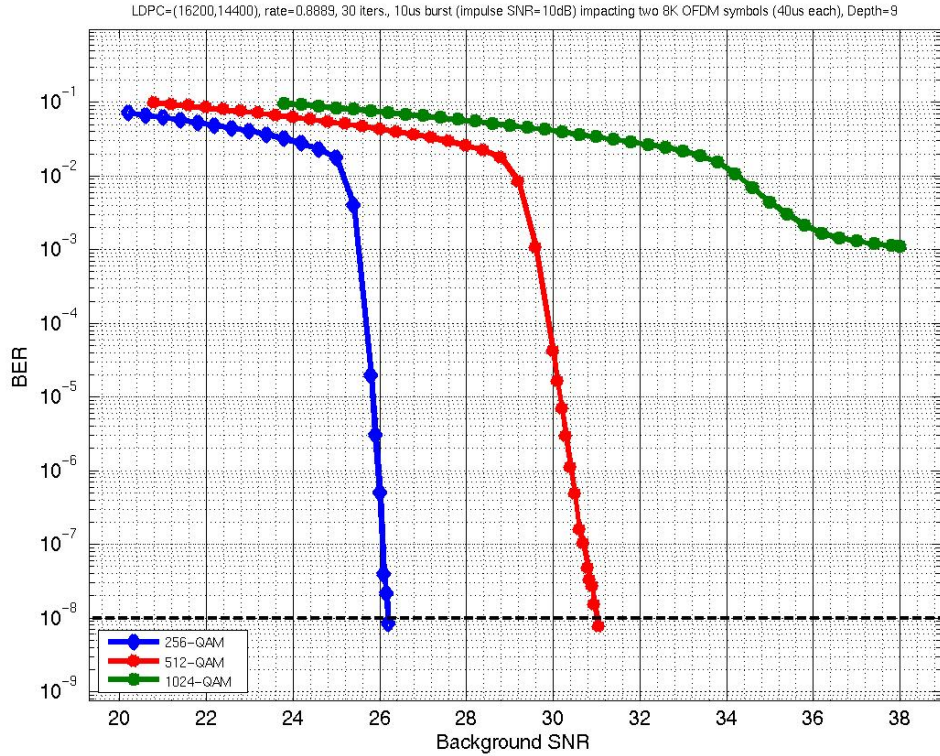
# 40 $\mu$ S SYMBOLS ON 10 $\mu$ s 10dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS



40 $\mu$ s symbol  
(two affected)  
Latency: 340 $\mu$ s  
(depth: 8)

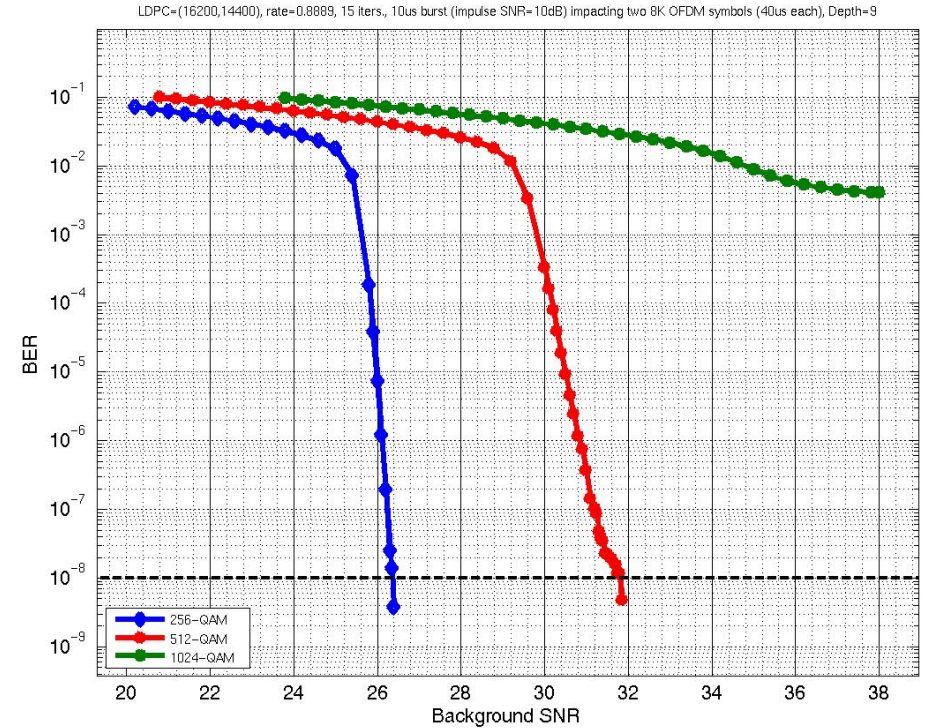


# 40μs SYMBOLS ON 10μs 10dB BURST (LONG SIZE CODE)



Max. 30 iterations

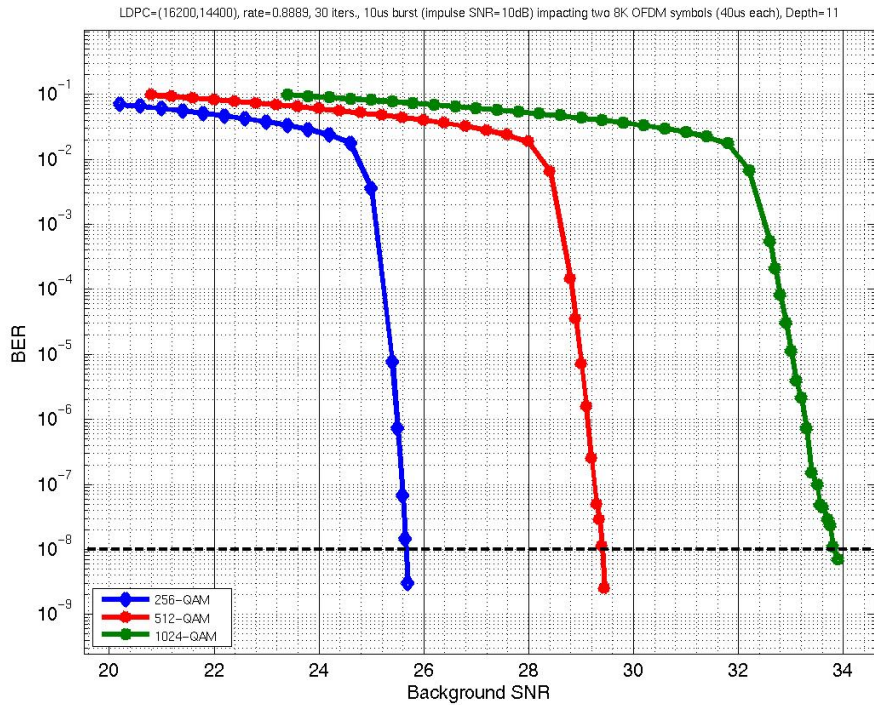
40μs symbol  
(two affected)  
Latency: 382.5μs  
(depth:9)



Max. 15 iterations

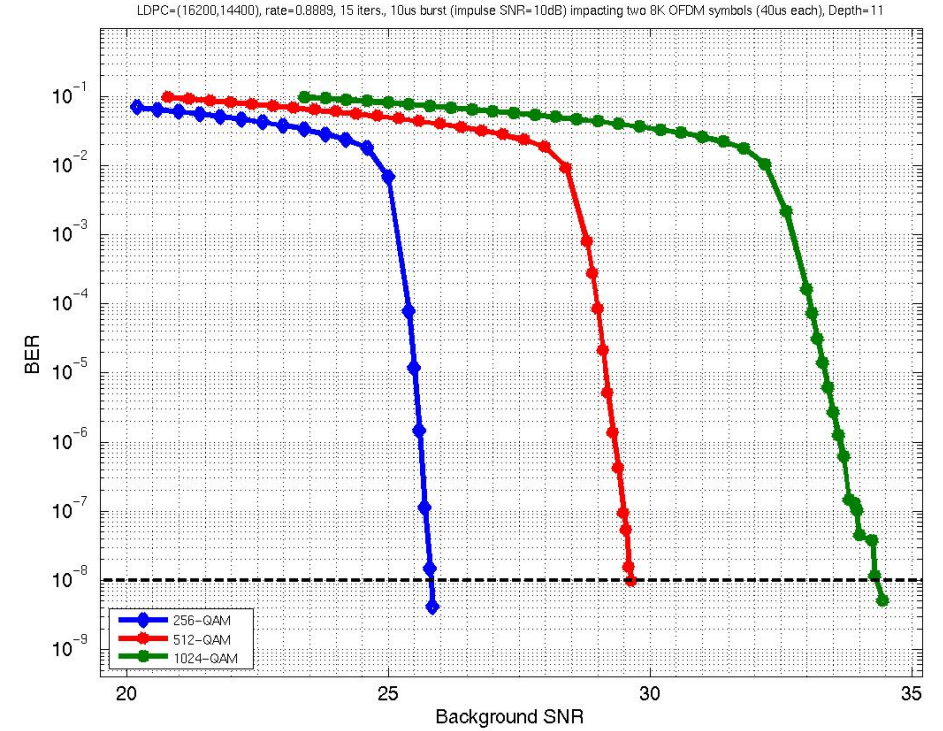
		256QAM	512QAM	1024QAM
SNR@BER=1e-8	Max. 15 iterations	26.5dB	31dB	
	Max. 30 iterations	26.4dB	31.8dB	

# 40μs SYMBOLS ON 10μs 10dB BURST (LONG SIZE CODE)



Max. 30 iterations

40μs symbol  
(two affected)  
Latency: 467.5μs  
(depth:11)



Max. 15 iterations

		256QAM	512QAM	1024QAM
SNR@BER=1e-8	Max. 15 iterations	26.5dB	29.5dB	33.8dB
	Max. 30 iterations	25.6dB	29.6dB	34.3dB

# DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (LONG SIZE CODE)

## 40μs SYMBOLS ON 10μs 10dB BURST

40μs symbol (two affected) Latency: 382.5μs (depth:9)

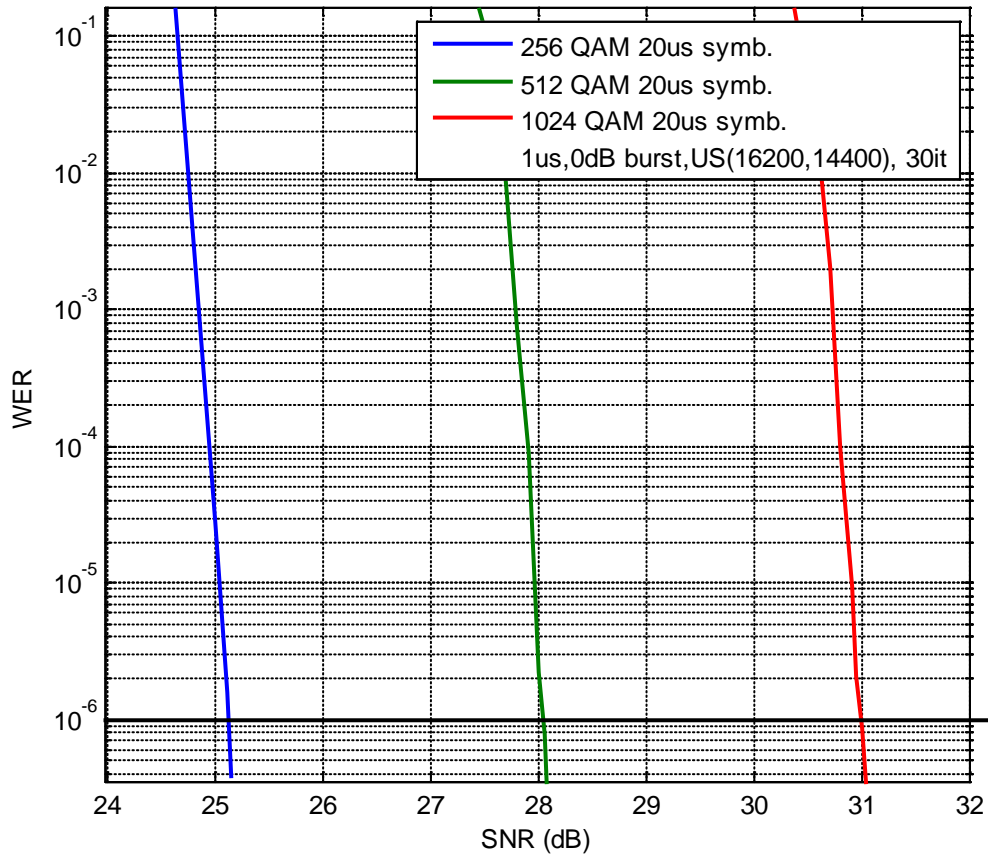
		256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations			
	Max.30 iterations			
	Difference			
SNR@BER=1e-8	Max. 15 iterations	26.5dB	31dB	
	Max. 30 iterations	26.4dB	31.8dB	
	Difference	0.1	0.8	

Long size  
(16200,14400)  
30 iterations  
vs.  
15 iterations

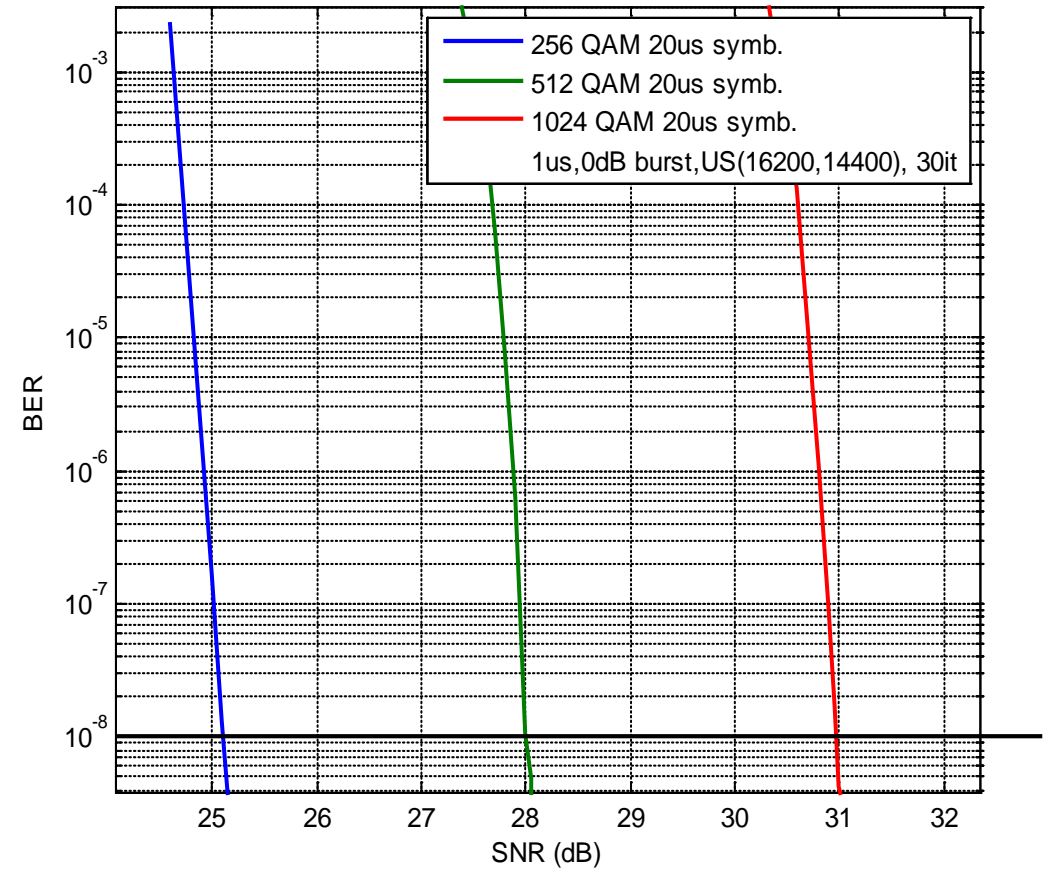
40μs symbol (two affected) Latency: 467.5μs (depth:11)

		256QAM	512QAM	1024QAM
SNR@WER=1e-6	Max.15 iterations			
	Max.30 iterations			
	Difference			
SNR@BER=1e-8	Max. 15 iterations	26.5dB	29.5dB	33.8dB
	Max. 30 iterations	25.6dB	29.6dB	34.3dB
	Difference	0.9	0.1	0.5

# 20 $\mu$ s SYMBOLS ON 1 $\mu$ s 0dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS

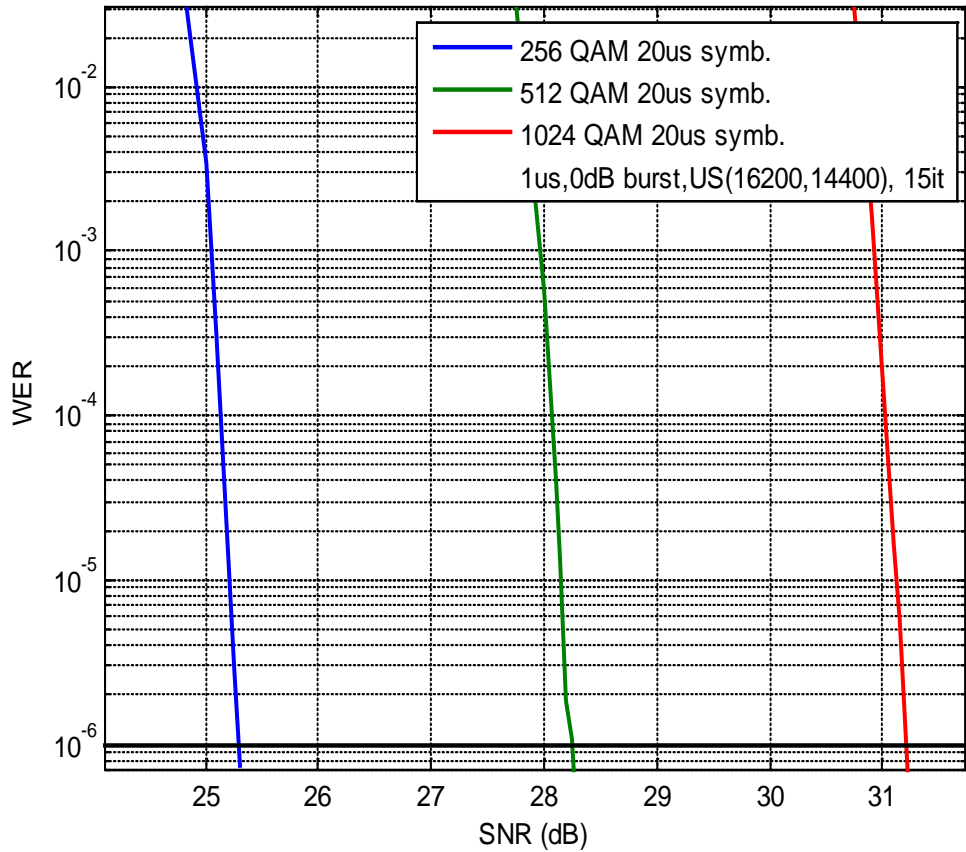


20 $\mu$ s symbol  
(one affected)  
Latency: 382.5 $\mu$ s  
(depth:17)

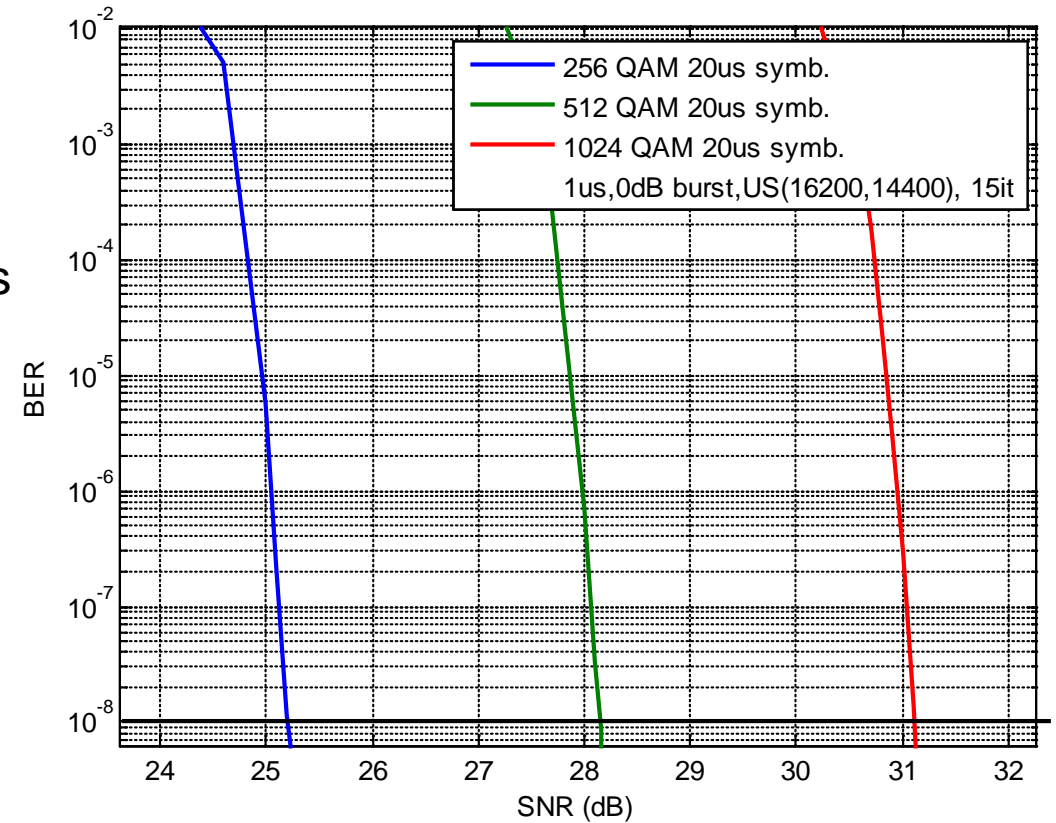


	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	25.11dB	28.04dB	30.99dB
<b>SNR@BER=1e-8</b>	25.1dB	28.01dB	30.97dB

# 20μs SYMBOLS ON 1μs 0dB BURST (LONG SIZE CODE) MAX 15 ITERATIONS



20μs symbol  
(one affected)  
Latency: 382.5μs  
(depth:17)



	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	25.29dB	28.25dB	31.21dB
<b>SNR@BER=1e-8</b>	25.21dB	28.14dB	31.11dB

# DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (LONG SIZE CODE)

20 $\mu$ s SYMBOLS ON 1 $\mu$ s 0dB BURST



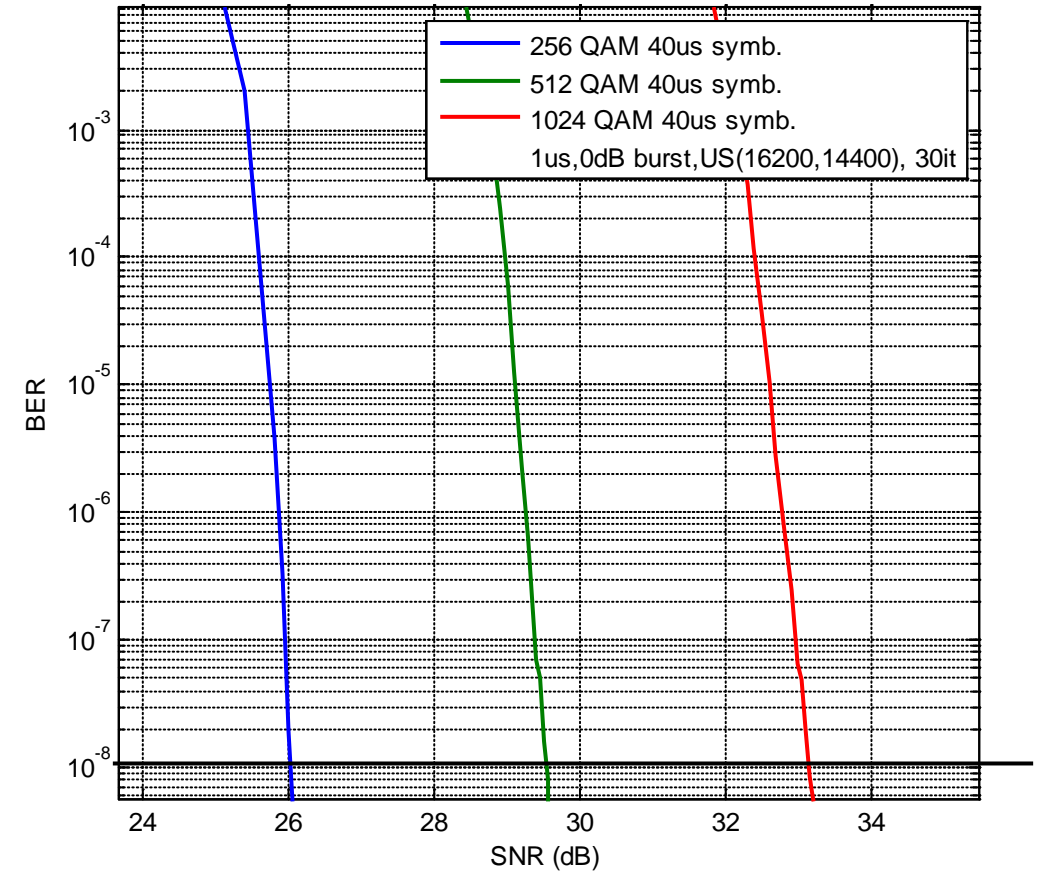
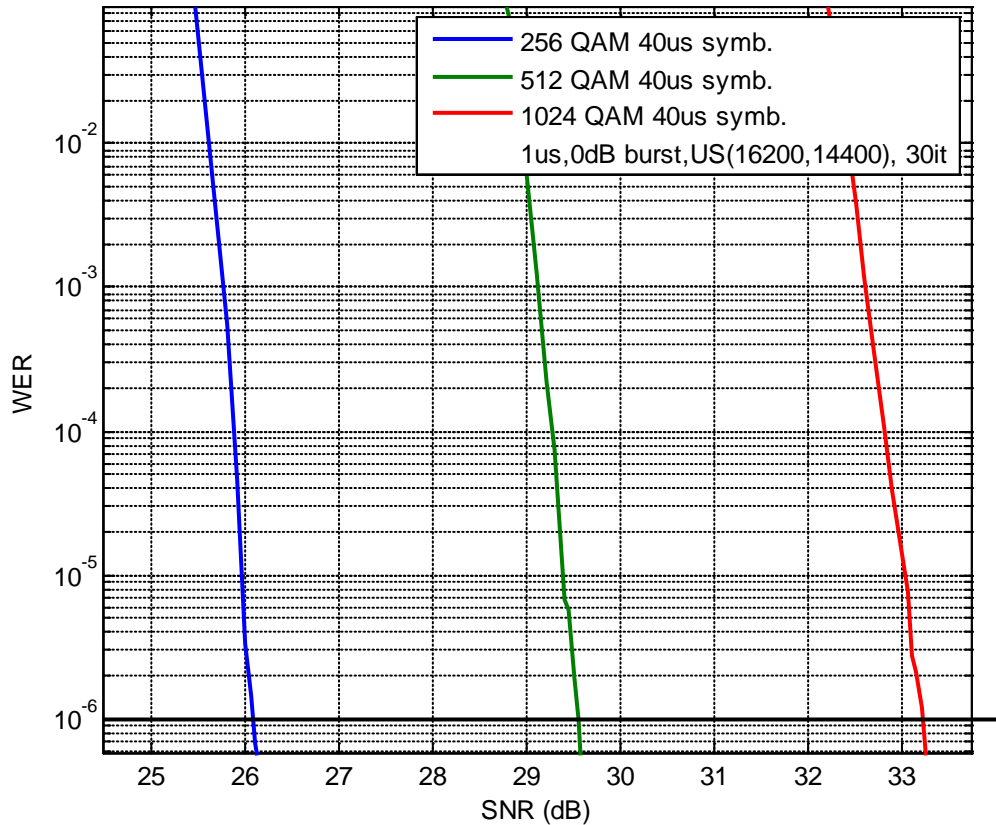
20 $\mu$ s symbol(two affected)

Latency: 382.5 $\mu$ s (depth:17)

Long size  
(16200,14400)  
30 iterations  
vs.  
15 iterations

		256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	<b>Max.15 iterations</b>	25.29dB	28.25dB	31.21dB
	<b>Max.30 iterations</b>	25.11dB	28.04dB	30.99dB
	<b>Difference</b>	0.18	0.21	0.22
<b>SNR@BER=1e-8</b>	<b>Max. 15 iterations</b>	25.21dB	28.14dB	31.11dB
	<b>Max. 30 iterations</b>	25.1dB	28.01dB	30.97dB
	<b>Difference</b>	0.2	0.13	0.14

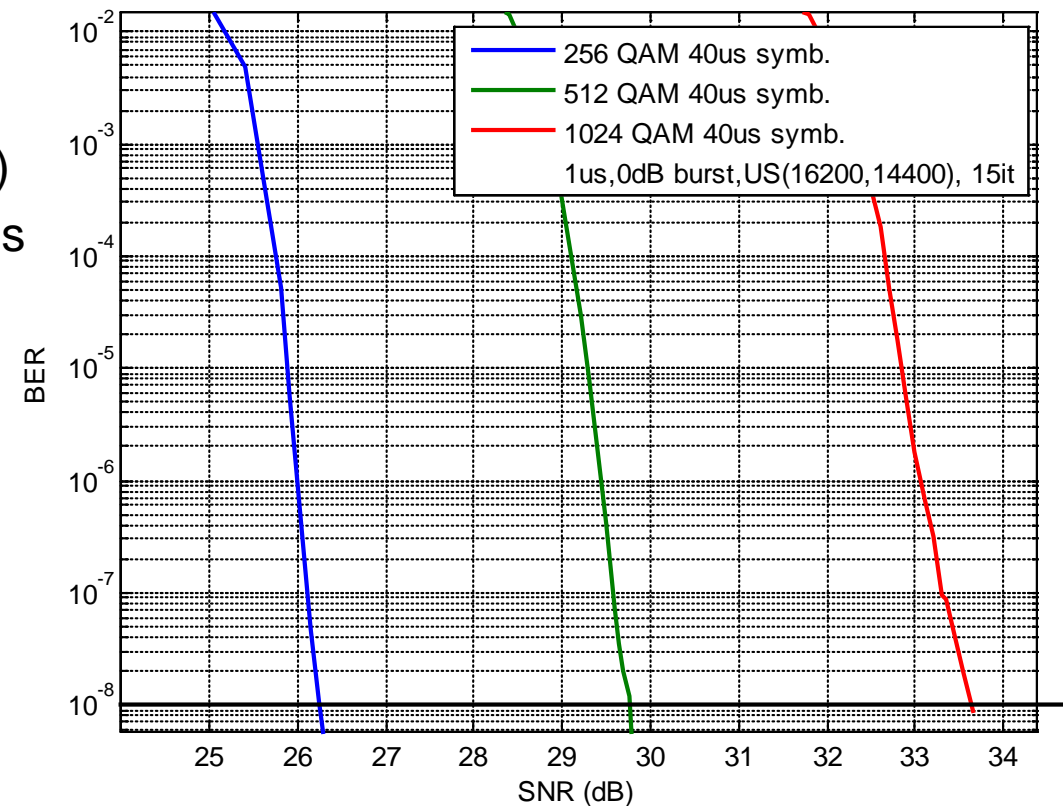
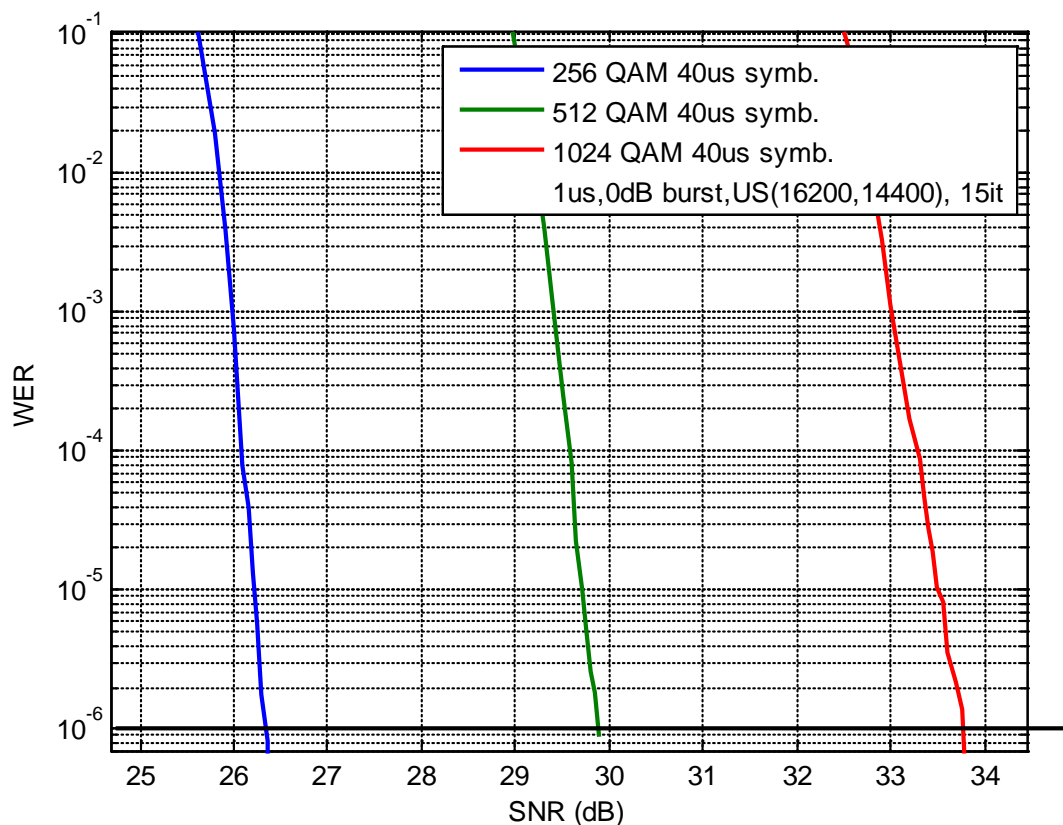
# 40μs SYMBOLS ON 1μs 0dB BURST (LONG SIZE CODE) MAX 30 ITERATIONS



	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	26.07dB	29.55dB	33.21dB
<b>SNR@BER=1e-8</b>	26.02dB	29.54dB	33.14dB



# 40 $\mu$ s SYMBOLS ON 1 $\mu$ s 0dB BURST (LONG SIZE CODE) MAX 15 ITERATIONS



	256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	26.34dB	29.89dB	33.77dB
<b>SNR@BER=1e-8</b>	26.25dB	29.76dB	33.63dB

# DIFFERENCE BETWEEN 15 AND 30 ITERATIONS (LONG SIZE CODE)

## 40μs SYMBOLS ON 1μs 0dB BURST

40μs symbol (one affected)

Latency: 340μs (depth:8)

Long size  
(16200,14400)  
30 iterations  
vs.  
15 iterations

		256QAM	512QAM	1024QAM
<b>SNR@WER=1e-6</b>	<b>Max.15 iterations</b>	26.34dB	29.89dB	33.77dB
	<b>Max.30 iterations</b>	26.07dB	29.55dB	33.21dB
	<b>Difference</b>	0.27	0.34	0.56
<b>SNR@BER=1e-8</b>	<b>Max. 15 iterations</b>	26.25dB	29.76dB	33.63dB
	<b>Max. 30 iterations</b>	26.02dB	29.54dB	33.14dB
	<b>Difference</b>	0.23	0.22	0.49

- **Three proposed LDPC codes for active plant have been presented**
- **Performance evaluation criteria including modulation orders and channel model parameters have been discussed**
- **AWGN channel performance for all proposed codes has been presented**
- **Performance under burst noise conditions in AWGN has been presented**

*Thank You*



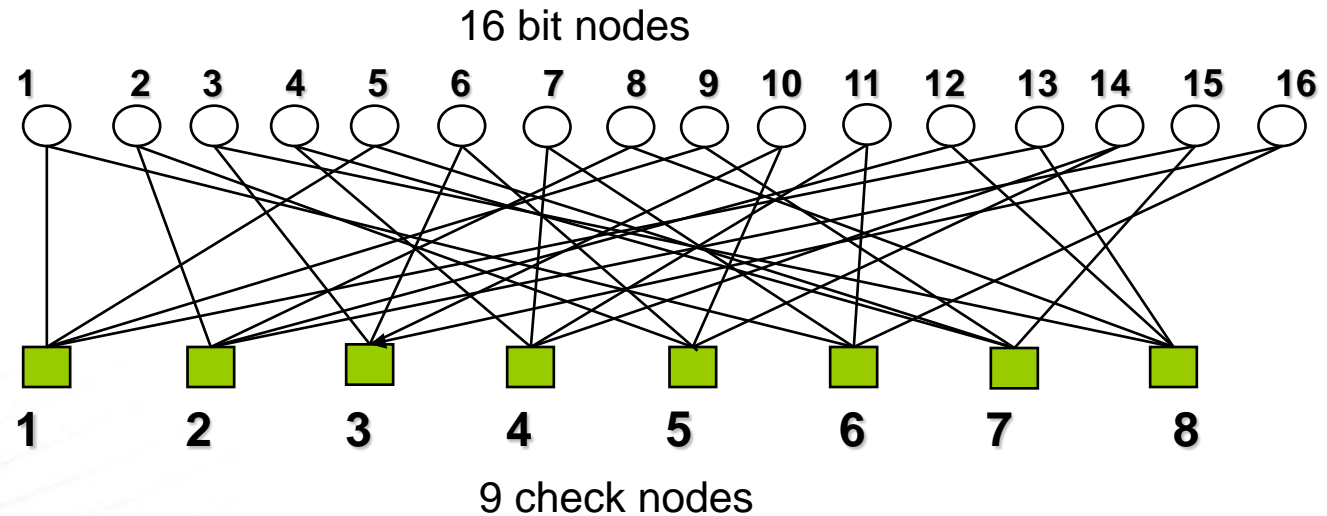
# **BACKGROUND ON LDPC CODES**

- LDPC codes are specified by a (parity-check) matrix containing mostly 0's and relatively few 1's (R. G. Gallager 1960)
- A simple example

$$H = \begin{bmatrix} 1000 & 1000 & 1000 & 1000 \\ 0100 & 0001 & 0001 & 0010 \\ 0010 & 0100 & 0100 & 0001 \\ 0001 & 0010 & 0010 & 0100 \\ 0100 & 0100 & 0100 & 0100 \\ 1000 & 0010 & 0010 & 0001 \\ 0001 & 1000 & 1000 & 0010 \\ 0010 & 0001 & 0001 & 1000 \end{bmatrix}$$

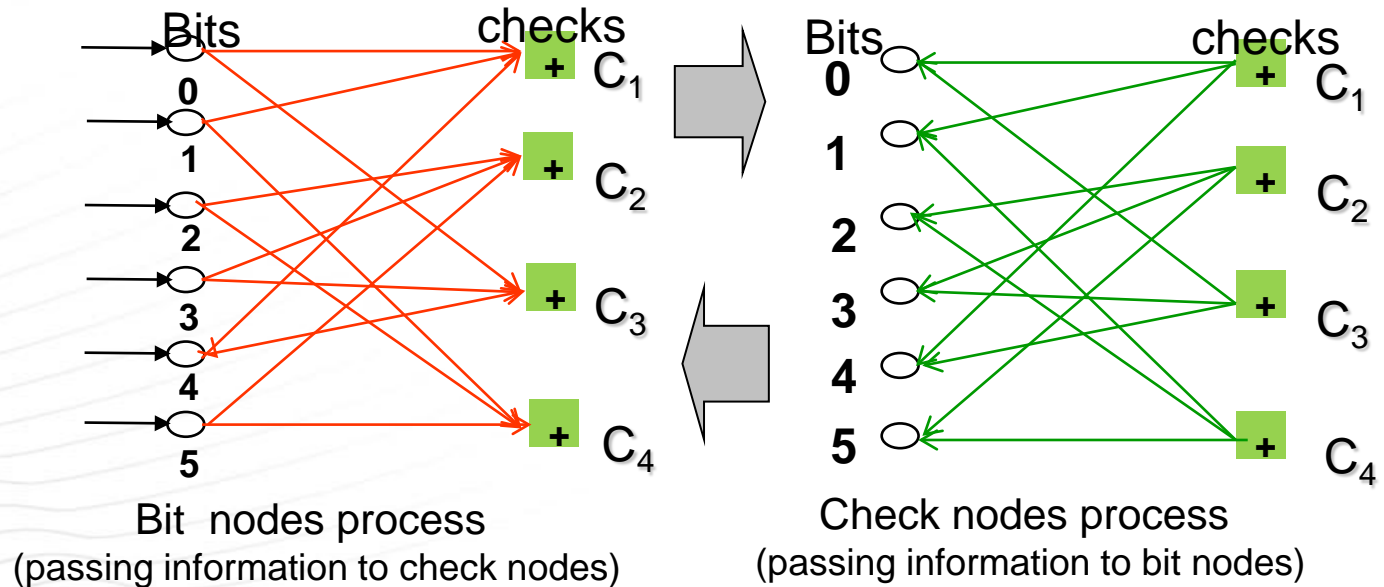
Parity check matrix of (16,9) regular LDPC code

32 1's , 128 0's  
Density of 1's = 0.25



Tanner graphs

- **Belief propagation (AKA sum-product)** is a message passing algorithm for performing inference on Tanner graph.
  - It calculates the marginal distribution for each unobserved node, conditional on any observed nodes
  - First flooding iteration:
    - Observed nodes: bit nodes with known LLR value from the demodulation
    - Unobserved nodes: check nodes
    - Extrinsic LLR; calculated marginal distribution for updated check node
  - Second flooding iteration
    - Observed nodes: check nodes with extrinsic LLR
    - Unobserved node: bit nodes
  - Etc. ....



# DECODING CAPABILITY ON (16,9) EXAMPLE CODE

tx: X=(0000000000000000)

SNR	-3dB	-1dB	0dB	0.5dB
Classical Bounded-distance decoding (hard)	Failed	Failed	Failed	<b>Failed</b>
Maximal likelihood decoding (MLD)(soft)	1010000010011 ( $H\underline{x}^T=0$ ) but wrong codeword	corrected	corrected	corrected
Bit MLD (soft)	Failed ( $H\underline{x}^T \neq 0$ )	failed ( $H\underline{x}^T \neq 0$ )	corrected	corrected
Iterative message passing decoding (soft)	not converge	not converge	not converge	corrected (5 iterations)

QPSK on AWGN channel



- back-substitution algorithm

Denote the parity check matrix  $H=[H_I, H_P]$ , where

$$H_P = \begin{array}{|c|c|c|c|c|} \hline I(u_{0,0}) & 0 & 0 & 0 & 0 \\ \hline I(u_{1,0}) & I(u_{1,1}) & 0 & 0 & 0 \\ \hline 0 & I(u_{2,1}) & I(u_{2,2}) & 0 & 0 \\ \hline 0 & 0 & I(u_{3,2}) & I(u_{3,3}) & 0 \\ \hline 0 & 0 & 0 & I(u_{4,3}) & I(u_{4,4}) \\ \hline \end{array}$$

$I$ : an  $L$  by  $L$  identity matrix ( $L=360$  for long size code,  $L=180$  for medium size code and  $L=56$  for short size code)

$I(u)$ : the matrix obtained by right cyclic shift  $I$  by  $u$  positions

$\mathbf{c}_I = (c_0, c_1, \dots, c_{k-1})$ :  $k=L(n-m)$  input (information) bits. .

$\mathbf{c}_P = (c_k, c_{k+1}, \dots, c_{Ln-1})$  :  $Lm$  parity bits need to be computed by an encoder

Encoder will output  $\mathbf{c}_P^T = \begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix}$ , where  $C_i = (c_{L(n-m+i)}, c_{L(n-m+i)+1}, \dots, c_{L(n-m+i)+L-1})^T$

## Encoding procedure:

Input:  $\mathbf{c}_I = (c_0, c_1, \dots, c_{k-1})$

Step 1: Compute  $V_i = H_{I,i} \mathbf{c}_I^T \quad i = 0, \dots, 4$

Step 2: Back-substitution

$$\begin{aligned} C_0 &= V_0(L - u_{0,0}) \\ C_1 &= V_1(L - u_{1,1}) + C_0((L - u_{1,1} + u_{1,0}) \bmod L) \\ C_2 &= V_2(L - u_{2,2}) + C_1((L - u_{2,2} + u_{2,1}) \bmod L) \\ C_3 &= V_3(L - u_{3,3}) + C_2((L - u_{3,3} + u_{3,2}) \bmod L) \\ C_4 &= V_4(L - u_{4,4}) + C_3((L - u_{4,4} + u_{4,3}) \bmod L) \end{aligned}$$

A: a size L binary column vector.

A(u): a column vector obtained by up cyclic shifting V in u positions

Output:  $\mathbf{c}_P^T = \begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix}$