CONSTELLATION MAPPING FOR EPoC LDPC CODING

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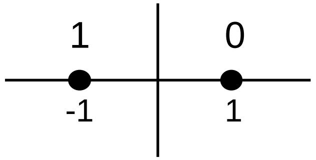
CONSTELLATION MAPPING FOR LDPC CODES

- **This presentation defines**
  - Algebraic formulaic mapping for all possible QAM constellations
    - Recursive procedure starting with BPSK
    - Can be implemented either in hardware or a look-up-table
  - Gray code mapping for even constellations
    - QPSK, 16-QAM,64-QAM,256-QAM,1024-QAM and 4096-QAM
  - The least Gray code penalty mapping for odd constellations
    - 8-QAM,32-QAM,128-QAM,512-QAM and 2048-QAM
- **The defined constellation mappings demonstrate consistently good performance on EPoC LDPC codes**
  - Long size (16200,14400) code for both downstream and upstream
  - Medium size (5940,5040) code for upstream
  - Short size (1120,840) code for upstream

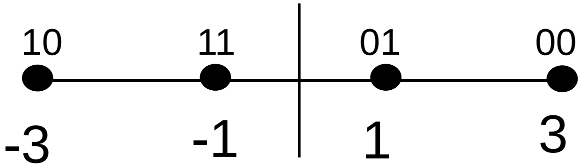
ONE DIMENSIONAL GRAY CODE MAPPING

- **Gray code for binary numbers**
  - Listing all n-bit numbers so that successive numbers differ in exactly one bit position
  - There are many Gray code mappings
- **Proposed Gray code mapping for** 2^k **integers,** $2i-(2^k-1)$ $(k=0,1,\ldots,2^k-1)$
  - k=1: $G_1(0)= 1$, $G_1(1)=-1$



  - k>1: $G_k(b_0 b_1 \cdots b_{k-1}) = (1-2b_0)\left(2^{k-1} + G_{k-1}(b_1 \cdots b_{k-1})\right)$, $\mathrm{b_i} = 0 \text{ or } 1 \text{ for } i = 0,1,\cdots,k-1$
  - **Example :** k=2:

$$\begin{cases} G_2(00) = (2 + G_1(0)) = 3 \\ G_2(01) = (2 + G_1(1)) = 1 \\ G_2(10) = -(2 + G_1(0)) = -3 \\ G_2(11) = -(2 + G_1(1)) = -1 \end{cases}$$



GRAY CODE MAPPING ON RECTANGULAR CONSTELLATION

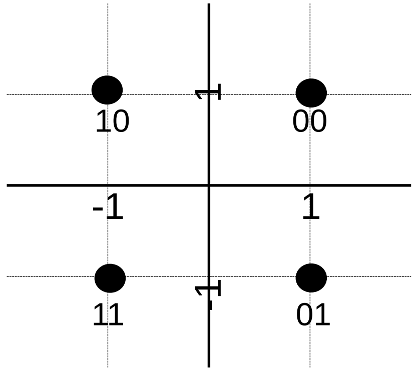
- **2^{n+m} Rectangular constellation points**
  - $\{(2i-(2^n-1), 2j-(2^m-1)) \mid i=0,1,\ldots,2^n-1, j=0,1,\ldots,2^m-1\}$
- **Mapping 2-D symbols with (n+m) binary bits using 1-D Gray code mapping**

$$\text{Let } \mathbf{d} = a_0 a_1 \cdots a_{n-1} b_0 b_1 \cdots b_{m-1}$$
$$(I_{rct}(\mathbf{d}), Q_{rct}(\mathbf{d})) = \left(G_n(a_0 a_1 \cdots a_{n-1}), G_m(b_0 b_1 \cdots, b_{m-1})\right)$$

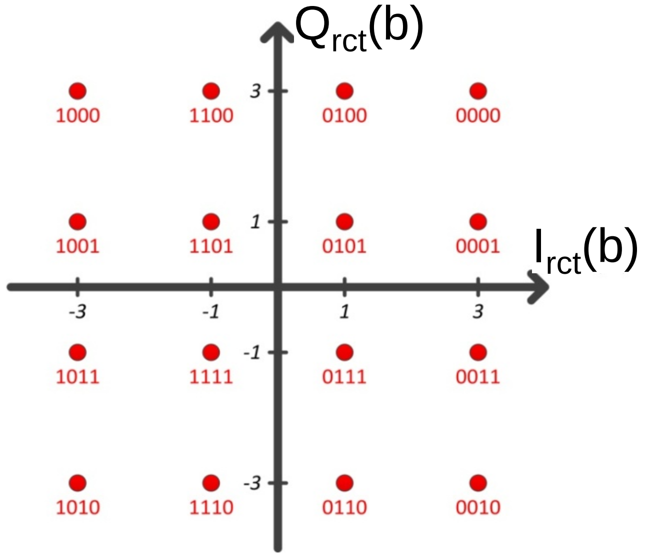
  - I and Q are mapped independently (or orthogonal)
- **Example**
  - n=m=1(QPSK)

$$\left(I_{rct}(00), Q_{rct}(00)\right) = (G_1(0), G_1(0)) = (1,1), \left(I_{rct}(01), Q_{rct}(01)\right) = (G_1(0), G_1(1)) = (1,-1)$$
$$\left(I_{rct}(10), Q_{rct}(10)\right) = (G_1(1), G_1(0)) = (-1,1), \left(I_{rct}(11), Q_{rct}(11)\right) = (G_1(1), G_1(1)) = (-1,-1)$$

- **Special case: square constellation**
  - 2^{2n} -QAM points
    - $\{(2i-(2^n-1), 2j-(2^n-1)) \mid i=0,1,\ldots,2^n-1, j=0,1,\ldots,2^n-1\}$
  - Example: 16-QAM (see figure on the right)



QPSK



16-QAM

GRAY CODE PENALTY

- **Defined by Joel G. Smith**
  - “Odd-bit quadrature amplitude shift keying,” *IEEE Trans. Commun.*, vol. COMM-23, no. 3, pp. 385–389, Mar. 1975
  - All one-dimensional Gray codes and two-dimensional Gray codes of orthogonal components are pure. All other Gray codes are “impure,” and suffer a “Gray code penalty”
- **Mathematic definition**

$$2^n-\text{QAM, there are } 2^n \text{ symbols, } S_i, i=0,1,\ldots,2^n-1$$

$$N(S_i)=\{S_j \mid S_j \text{ is the nearest (in Euclidean distance) neighbors of } S_i\}$$

$$l(S_i): \text{binary labeling given by the mapping}$$

$$wt\big(l(S_i),l(S_j)\big): \text{hamming distance between } l(S_i) \text{ and } l(S_j)$$

$$\text{Gray Code Penalty}: G_P(l)=\frac{1}{2^n}\sum_{i=0}^{2^n-1}\frac{\sum_{S_j\in N(S_i)} wt\big(l(S_j),l(S_i)\big)}{|N(S_i)|}$$

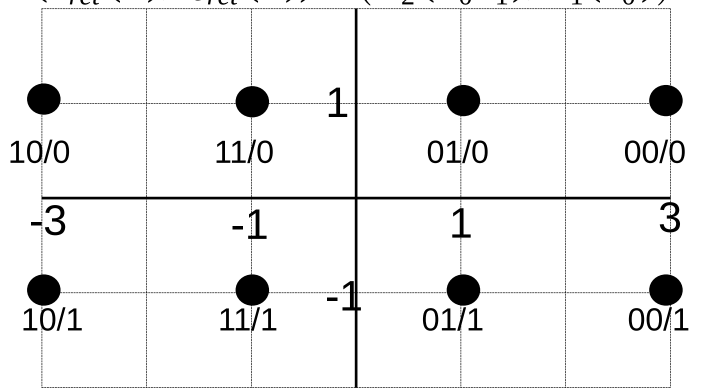
- **If l provides a Gray code mapping then $G_P(l)=1$, i.e. no penalty**
  - The proposed mapping on rectangular constellation has $G_P=1$

MAPPING ON CROSS CONSTELLATION
CASE I: 8-QAM

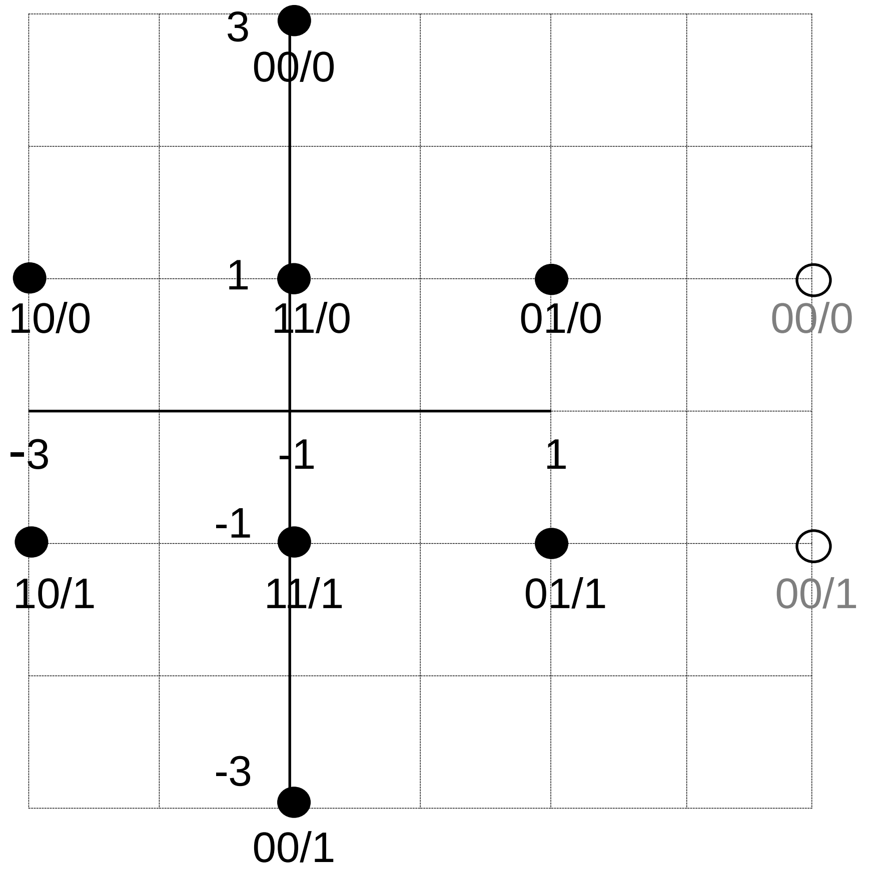
Starts with rectangular constellation mapping

$$\mathbf{d} = a_0 a_1 b_0$$

$$(I_{rct}(\mathbf{d}), Q_{rct}(\mathbf{d})) = \left(G_2(a_0 a_1), G_1(b_0)\right)$$



transform to cross constellation



Transform the right side wing to cross constellation mapping

$$\begin{cases} \begin{cases} I_{cr}(\mathbf{d}) = I_{rct}(\mathbf{d}) \\ Q_{cr}(\mathbf{d}) = Q_{rct}(\mathbf{d}) \end{cases} & I_{rect}(\mathbf{d}) < 3 \\ \begin{cases} I_{cr}(\mathbf{d}) = -sign(I_{rct}(\mathbf{d})\left(4 - \left|I_{rct}(\mathbf{d})\right|\right) \\ Q_{cr}(\mathbf{d}) = sign\left(Q_{rct}(\mathbf{d})\right)\left(\left|Q_{rct}(\mathbf{d})\right| + 2\right) \end{cases} & \text{otherwise} \end{cases}$$

$$\begin{cases} I_{cr}(000) = (4-3) = -1 \\ Q_{cr}(000) = (1+2) = 3 \end{cases} \qquad \begin{cases} I_{cr}(001) = -(4-3) = 1 \\ Q_{cr}(001) = -(1+2) = 3 \end{cases}$$

$$G_P = \frac{1}{8}(2+1+\frac{5}{4}+1+1+\frac{5}{4}+1+2) = \frac{21}{16} = 1.3125$$

MAPPING ON CROSS CONSTELLATION
CASE II: 2^{2n+1}-QAM (n>1)

- **Starts with an n x (n+1) rectangular constellation**
- **Transform two vertical wings to two horizontal wings**
- **Pseudo Gray code mapping on two horizontal wings (with the least Gray Code Penalty)**
- **Algebraic formula**

$$d = a_0 a_1 \cdots a_n b_0 b_1 \cdots d_{n-1}$$

$$(I_{rct}(\mathbf{d}), Q_{rct}(\mathbf{d})) = \left(G_{n+1}(a_0 a_1 \cdots a_{n-1}), G_n(b_0 b_1 \cdots, b_{n-1})\right)$$

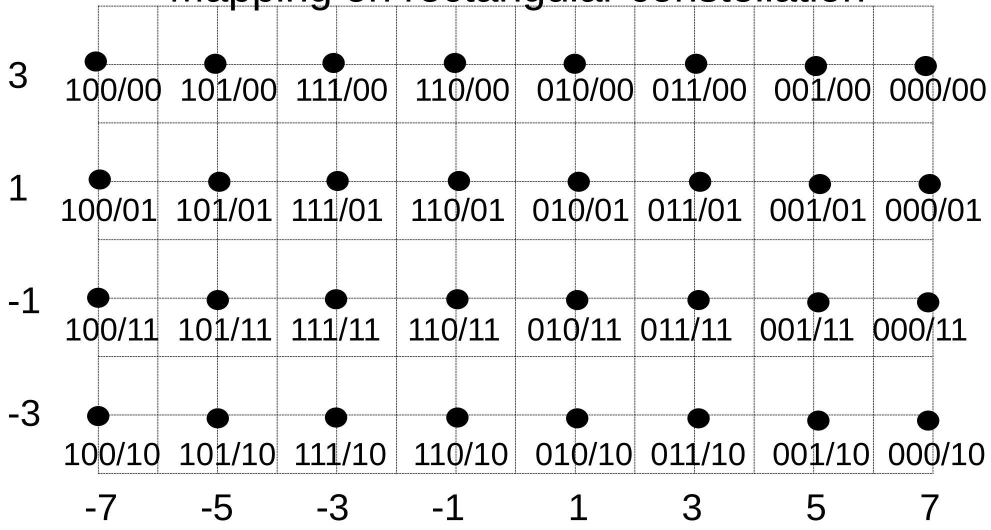
$$\text{Let } \; s = 2^{n-1}$$

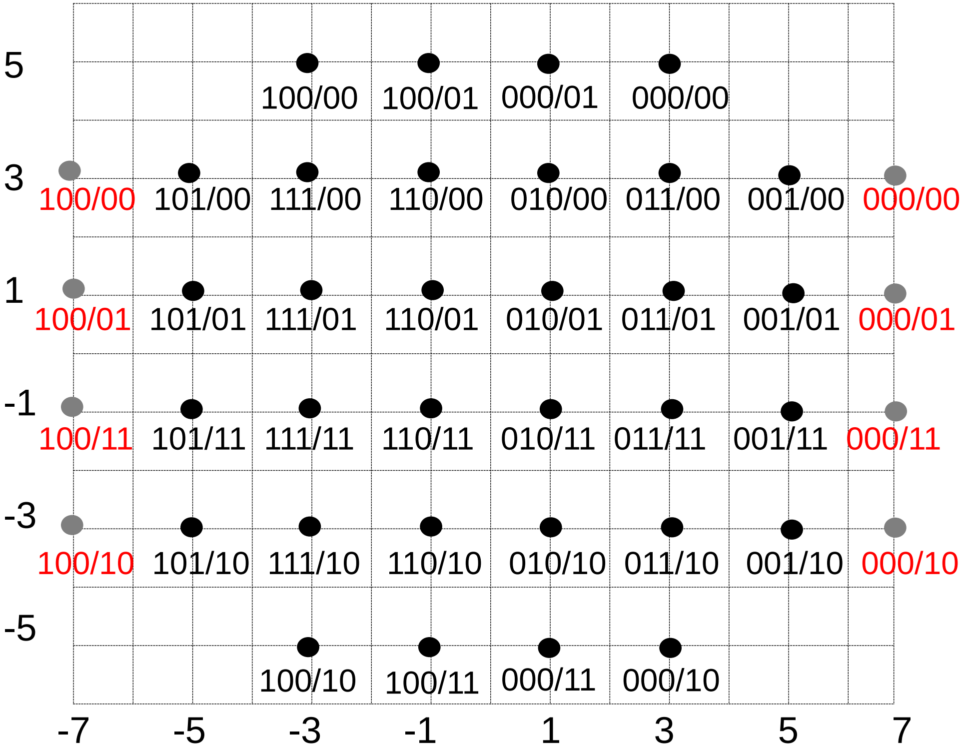
$$\begin{cases} \begin{cases} I_{cr}(\mathbf{d}) = I_{rct}(\mathbf{d}) \\ Q_{cr}(\mathbf{d}) = Q_{rct}(\mathbf{d}) \end{cases} & \text{if } \left|I_{rct}(\mathbf{d})\right| < 3s \\ \begin{cases} \begin{cases} I_{cr}(\mathbf{d}) = sign\left(I_{rct}(\mathbf{d})\right)\left(\left|I_{rct}(\mathbf{d})\right| - 2s\right) \\ Q_{cr}(\mathbf{d}) = sign\left(Q_{rct}(\mathbf{d})\right)\left(4s - \left|Q_{rct}(\mathbf{d})\right|\right) \end{cases} & \left|\mathrm{Q}_{\mathrm{rct}}(\mathbf{d})\right| > s \\ \begin{cases} I_{cr}(\mathbf{d}) = sign\left(I_{rct}(\mathbf{d})\right)\left(4s - \left|I_{rct}(\mathbf{d})\right|\right) \\ Q_{cr}(\mathbf{d}) = sign\left(Q_{rct}(\mathbf{d})\right)\left(\left|Q_{rct}(\mathbf{d})\right| + 2s\right) \end{cases} & \left|\mathrm{Q}_{\mathrm{rct}}(\mathbf{d})\right| \le s \end{cases} & \text{otherwise} \end{cases}$$

- See an example in the next page

MAPPING ON 32 CROSS CONSTELLATION QAM

Mapping on rectangular constellation





Transform two wings to cross constellation mapping

$$d = a_0 a_1 a_2 b_0 b_1$$

$$(I_{rct}(\mathbf{d}), Q_{rct}(\mathbf{d})) = \left(G_3(a_0 a_1 a_2), G_2(b_0 b_1)\right)$$

Let $s = 2$

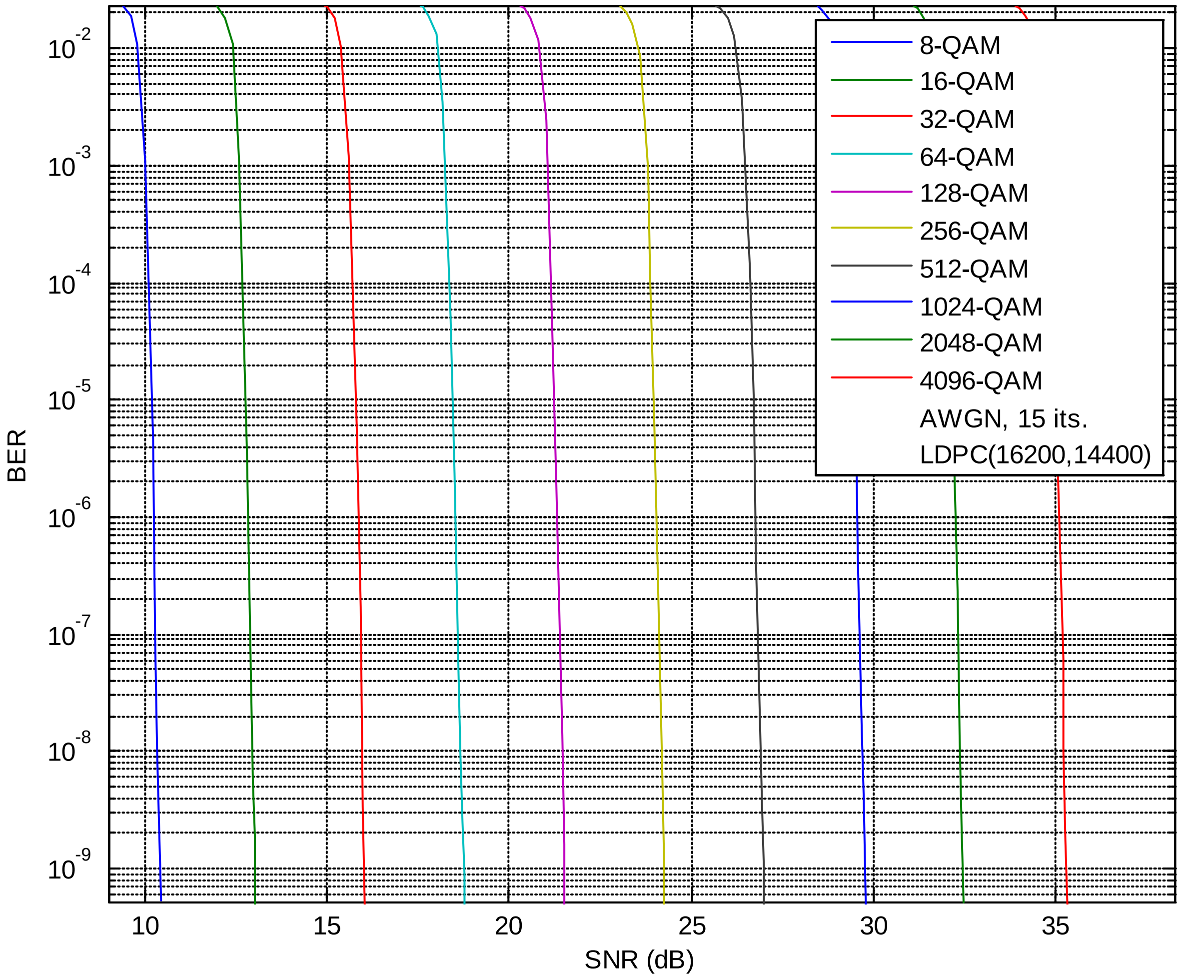
$$\begin{cases} \begin{cases} I_{cr}(\mathbf{d}) = I_{rct}(\mathbf{d}) \\ Q_{cr}(\mathbf{d}) = Q_{rct}(\mathbf{d}) \end{cases} & \text{if } |I_{rct}(\mathbf{d})| < 6 \\ \begin{cases} \begin{cases} I_{cr}(\mathbf{d}) = sign(I_{rct}(\mathbf{d}))\left(|I_{rct}(\mathbf{d})| - 4\right) \\ Q_{cr}(\mathbf{d}) = sign(Q_{rct}(\mathbf{d}))\left(8 - |Q_{rct}(\mathbf{d})|\right) \end{cases} & |Q_{rct}(\mathbf{d})| > 2 \\ \begin{cases} I_{cr}(\mathbf{d}) = sign(I_{rct}(\mathbf{d}))\left(8 - |I_{rct}(\mathbf{d})|\right) \\ Q_{cr}(\mathbf{d}) = sign(Q_{rct}(\mathbf{d}))\left(|Q_{rct}(\mathbf{d})| + 4\right) \end{cases} & |Q_{rct}(\mathbf{d})| \le 2 \end{cases} & \text{otherwise} \end{cases}$$

$$\begin{cases} I_{cr}(00000) = (7-4) = 3 \\ Q_{cr}(0000) = (8-3) = 5 \end{cases} \quad \begin{cases} I_{cr}(00001) = (8-7) = 1 \\ Q_{cr}(00001) = (1+4) = 5 \end{cases} \quad \begin{cases} I_{cr}(00011) = (8-7) = 1 \\ Q_{cr}(00011) = -(1+4) = -5 \end{cases} \quad \begin{cases} I_{cr}(00010) = (7-4) = 3 \\ Q_{cr}(00010) = -(8-3) = -5 \end{cases}$$

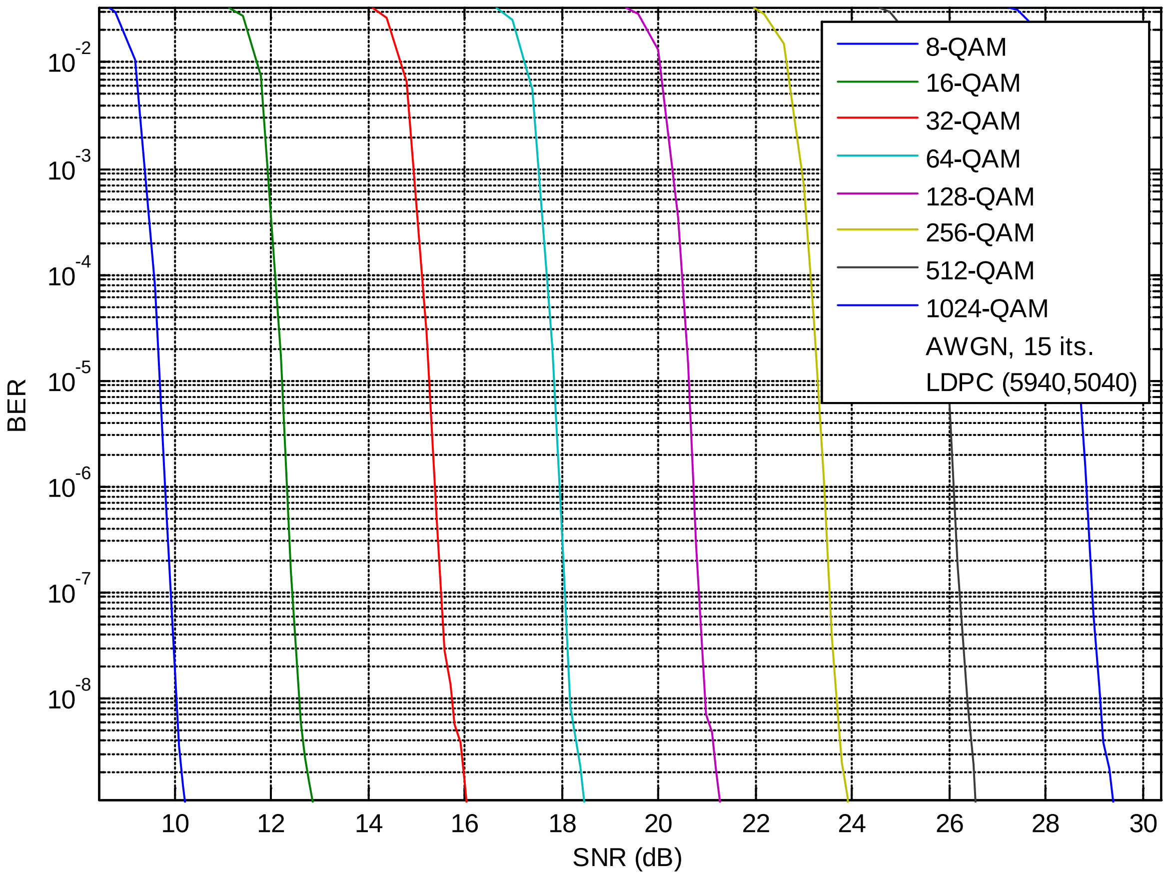
$$\begin{cases} I_{cr}(10000) = -(7-4) = -3 \\ Q_{cr}(10000) = (8-3) = 5 \end{cases} \quad \begin{cases} I_{cr}(10001) = -(8-7) = -1 \\ Q_{cr}(10001) = (1+4) = 5 \end{cases} \quad \begin{cases} I_{cr}(10011) = -(8-7) = -1 \\ Q_{cr}(10011) = -(1+4) = -5 \end{cases} \quad \begin{cases} I_{cr}(10010) = -(7-4) = -3 \\ Q_{cr}(10010) = -(8-3) = -5 \end{cases}$$

$$G_P = \frac{2}{32}\left(3/2 + 4/3 + 4/3 + 3/2 + 1 + 5/4 + 5/4 + 5/4 + 5/4 + 1 + 6\right) = 7/6 = 1.1667$$

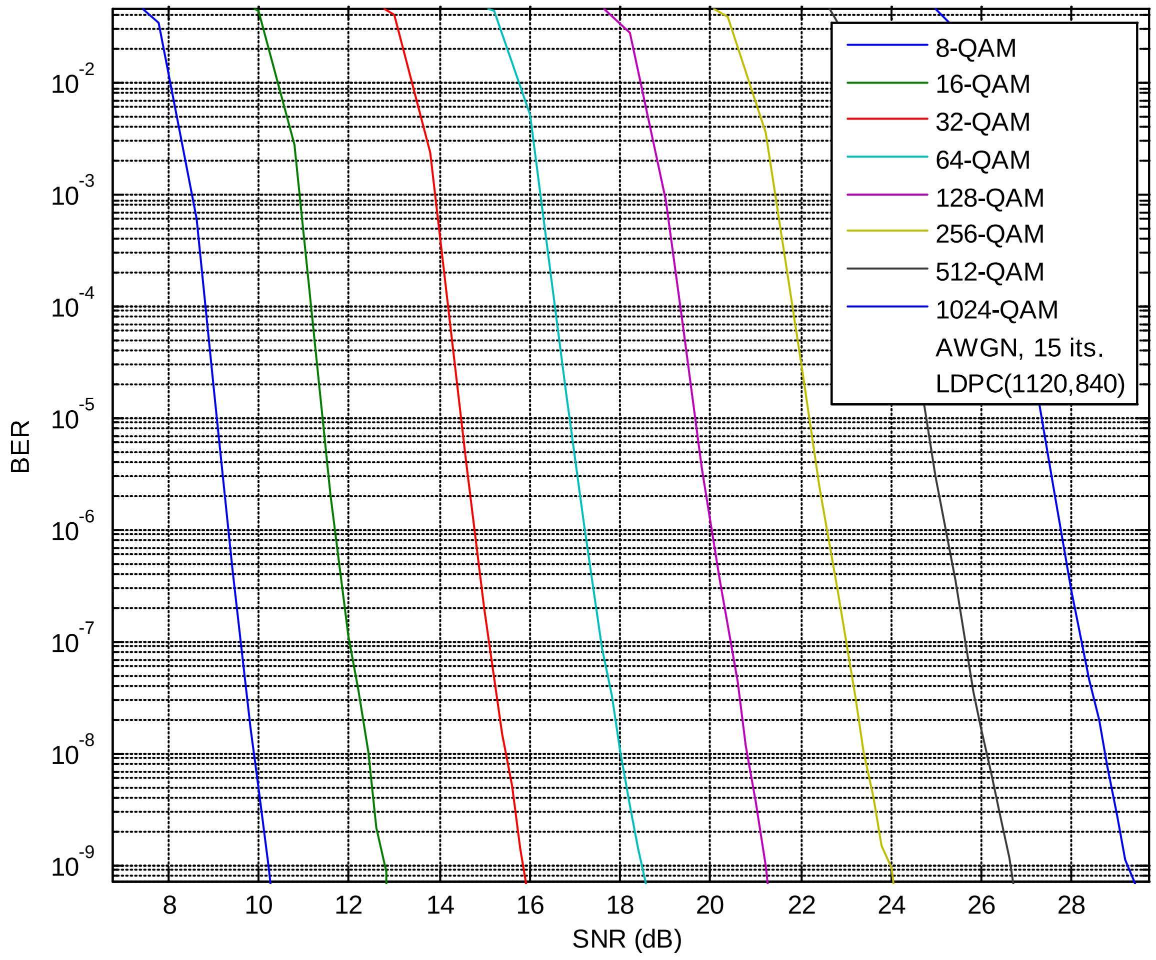
ON LONG SIZE LDPC CODE (W/ 15 ITERATIONS) (DOWNSTREAM AND UPSTREAM)



ON UPSTREAM MEDIUM SIZE LDPC CODE (W/ 15 ITERATIONS)



ON UPSTREAM SHORT SIZE LDPC CODE (W/ 15 ITERATIONS)



CONCLUSION

- **A recursive constellation mapping procedure for both downstream and upstream LDPC codes has been presented**
  - Procedure can be implemented either by hardware logic or by look-up-tables
  - On even-bit QAM constellations, i.e. 2^{2n}-QAM
    - Gray code mapping
  - On odd-bit QAM constellations, i.e. 2^{2n+1}-QAM
    - Cross constellation defined
    - Transform from the Gray code mapping of same size rectangular constellation
    - Mapping with the least Gray code penalty (pseudo-Gray code)
- **Performance**
  - Proposed constellation mappings to all LDPC codes were thoroughly simulated
  - Performance across all codes is good and comparable
    - SNR for threshold Bit Error Ratio is approximately 3 dB apart for two consecutive constellations
- **Proposal to adopt this constellation mapping procedure for EPoC LDPC coding**

PROPOSED MOTION

Move to:

Adopt the constellation mapping procedure in prodan_3bn_02_1113.pdf for EPoC.

- **Moved: BZ Shen**
- **Second:**

Thank You