FEC performance on multi-part links

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IEEE P802.3bs Task Force, Ottawa, Canada, September 2014

Introduction

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When analysing the performance of 400GbE FEC encoded links, two of the aspects that need consideration are:

- The interaction of burst errors due to decision feedback equalization (DFE) and dis-interleaving of bit-streams from higher rate lanes.
- Covering several sub-links with a single end-to-end FEC.

This contribution provides an analysis of the performance of RS(528,514) and RS(544,514) FEC for various interleaving schemes and also provides an analysis of the BER limit for a bursty sub-link to give a small penalty in a second random or bursty sub-link for various interleaving schemes.

Note: slides with a "*" in the top right corner are new or have changed compared to anslow_3bs_02_0714. For changed slides, the changes are shown in red.

Reed-Solomon FEC with random errors

For a Reed-Solomon code RS(N,k), with symbol size of **m** and symbol correction capability **t**, if the input BER is:

$$\mathrm{B}ER_{in} = \frac{1}{2} erfc\left(\sqrt{\frac{SNR}{2}}\right) \qquad (1)$$

The input symbol error ratio (or the probability that an input symbol contains errors) $\mathrm{SER}_{\mathrm{in}}$ is given by:

$$\mathrm{SER}_{\mathrm{in}} = 1-(1-\mathrm{BER}_{\mathrm{in}})^{m} \approx m \times \mathrm{BER}_{\mathrm{in}} \text{ (for small } \mathrm{BER}_{\mathrm{in}}\text{)} \qquad (2)$$

The codeword error ratio (the probability of an uncorrectable FEC codeword) is:

$$\mathrm{CER} = \sum_{i=t+1}^{N} \binom{N}{i} \mathrm{SER}^{i}{}_{\mathrm{in}} \left(1 - \mathrm{SER}_{\mathrm{in}}\right)^{N-i} \qquad (3)$$

The frame loss ratio (FLR) is then:

$$\mathrm{FLR} = \mathrm{CER} * (\mathrm{CER} * 1 + (1 - \mathrm{CER}) * (1 + \mathrm{MFC})/\mathrm{MFC}) \qquad (4)$$

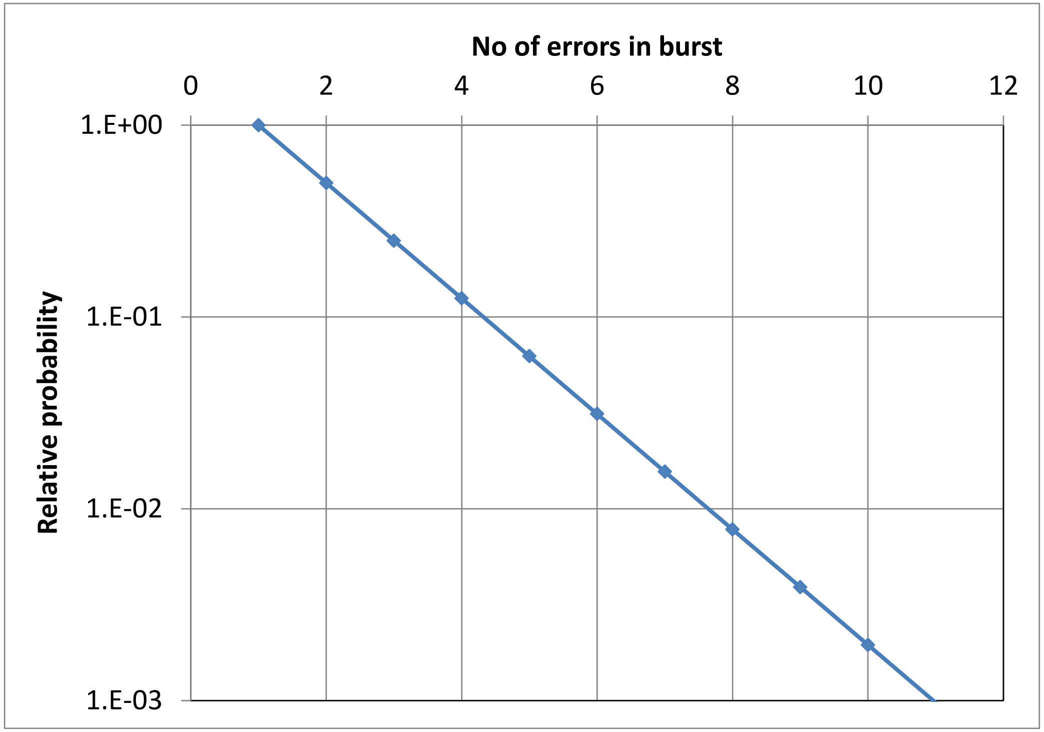
Where MFC is the number of MAC frames per codeword.

(This is an extension of slide 6 of anslow_01a_1112_mmf that still works for high CER)

Burst error model

The burst error probability is modelled in this contribution on the assumption that the probability of getting an error in the bit following an initial error (burst size of 2) is “a”, the probability of a burst of 3 errors is a^2, the probability of a burst of 4 is a^3, and so on. The worst case of a=0.5 is assumed throughout this contribution.

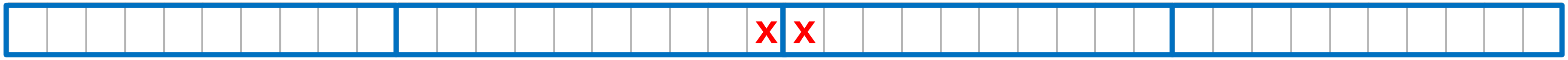
While a=0.5 may seem to be a pessimistic assumption, this simple model only applies to a 1-bit DFE. For a multi-bit DFE (particularly where the largest tap is not at 1 bit of delay) error burst patterns with several correct bits between errors are expected which makes the model used here not unreasonable.



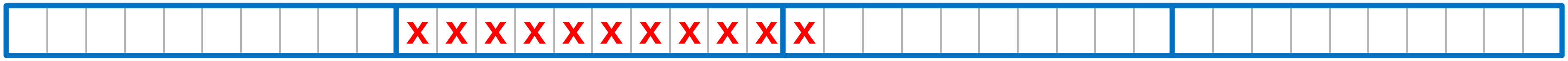
RS(528,514) performance with bursts I

For the RS(528,514) code, N=528, k=514, m=10, t=7

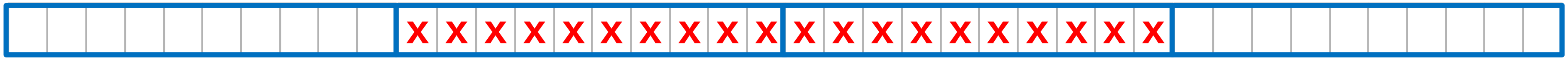
Using similar analysis as slide 5 of wang_t_3bs_01_0514, the minimum burst size that can cause exactly 2 symbol errors is 2 bits, but this only occurs if the first error is the last bit of a symbol (i.e. a probability of 0.1).



If the burst size is 11 bits, then the probability that it will cause exactly 2 symbol errors is 1.



The maximum burst size that can cause exactly 2 symbol errors is 20 bits, but this only occurs if the first error is the first bit of a symbol (i.e. a probability of 0.1).



The probability of exactly two symbol errors (given an initial error) is:

$$\mathrm{P}(2)=\sum_{i=1}^{m}\frac{i}{m}a^{i}(1-a)+\sum_{i=m+1}^{2m-1}\frac{2m-i}{m}a^{i}(1-a)=0.0998\ \text{ for m}=10,\ \text{a}=0.5 \qquad (5)$$

RS(528,514) performance with bursts II

Similar analysis to that on the previous slide gives:
P(1) = 0.9, P(2) = 0.0998, P(3) = 9.75E-5, P(4) = 9.52E-8, etc.

These probabilities can then be used to calculate the overall probability of any combination of events that results in t+1 or more symbol errors:

$$
\begin{aligned}
\mathrm{CER} = {} & \sum_{i=t+1}^{N} \binom{N}{i} \mathrm{SER}^{i}_{\mathrm{in}} \left(1-\mathrm{SER}_{\mathrm{in}}\right)^{N-i} + \binom{N}{1} \mathrm{SER}^{1}_{\mathrm{in}} \left(1-\mathrm{SER}_{\mathrm{in}}\right)^{N-1} P(8) \\
& + \binom{N}{2} \mathrm{SER}^{2}_{\mathrm{in}} \left(1-\mathrm{SER}_{\mathrm{in}}\right)^{N-2} \left(P(4)P(4) + \binom{2}{1} P(3)P(5) + \binom{2}{1} P(2)P(6) + \ldots \right) \\
& + \binom{N}{3} \mathrm{SER}^{3}_{\mathrm{in}} \left(1-\mathrm{SER}_{\mathrm{in}}\right)^{N-3} \left(\binom{3}{1} P(2)P(3)P(3) + \binom{3}{1} P(2)P(2)P(4) + \ldots \right) \\
& + \ \ldots \\
& + \binom{N}{7} \mathrm{SER}^{7}_{\mathrm{in}} \left(1-\mathrm{SER}_{\mathrm{in}}\right)^{N-7} \left(\binom{7}{1} P(1)^{6} P(2) + \binom{7}{2} P(1)^{5} P(2)P(2) + \ldots \right) \quad (7)
\end{aligned}
$$

Monte Carlo analysis

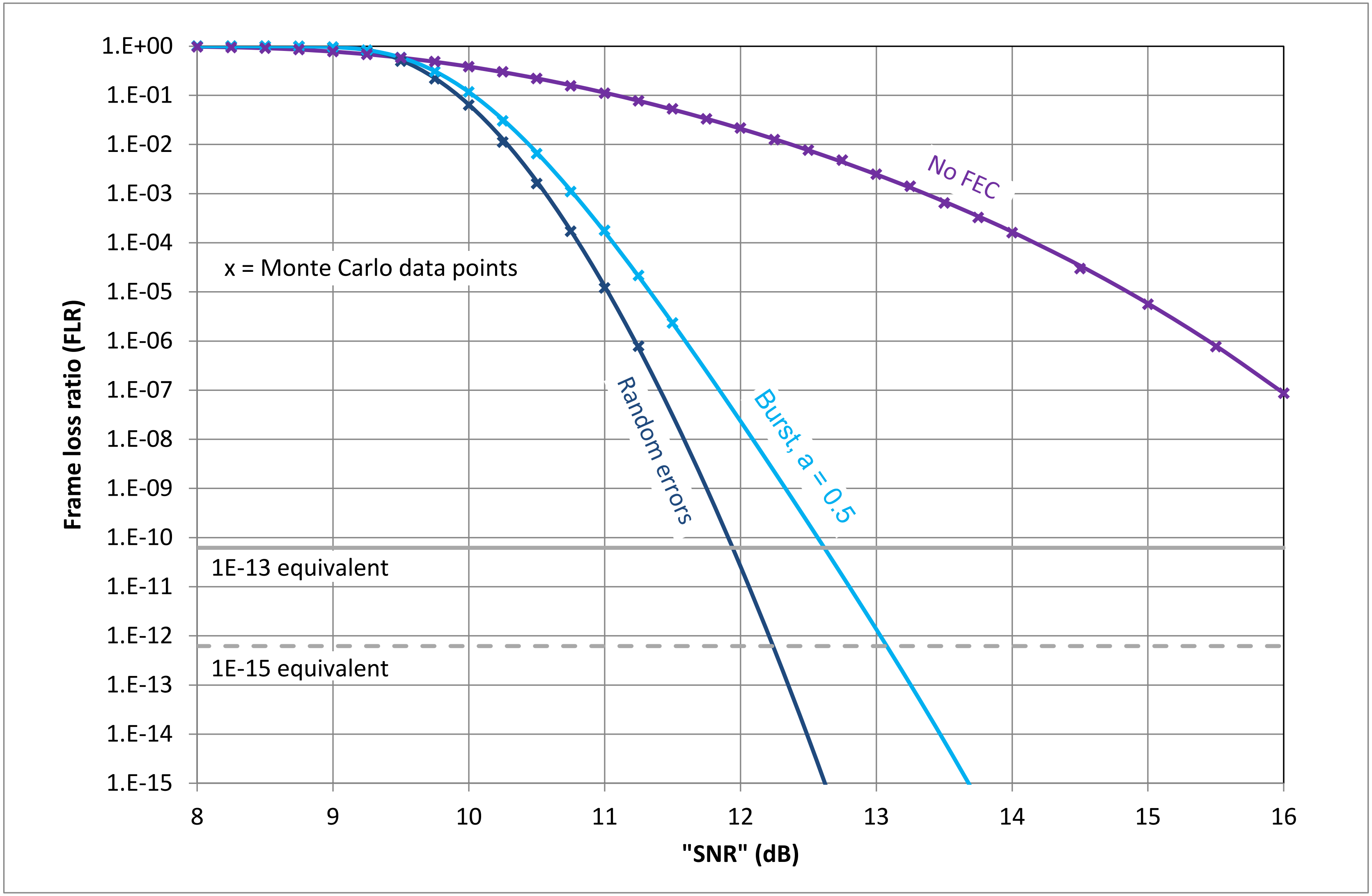
In an attempt to verify that the formulae on the previous slides give plausible results, a Monte Carlo based simulation was also performed using a random number generator to generate errors in a large number of codewords both with and without burst errors.

The results of plotting equations 1, 3, and 4 for random errors and equations 1, 5, and 4 for burst errors is shown on the next slide.

Also shown via “x” markers are the results of the Monte Carlo simulations which show good agreement with the formulae down to the probability where the simulations become too time consuming.

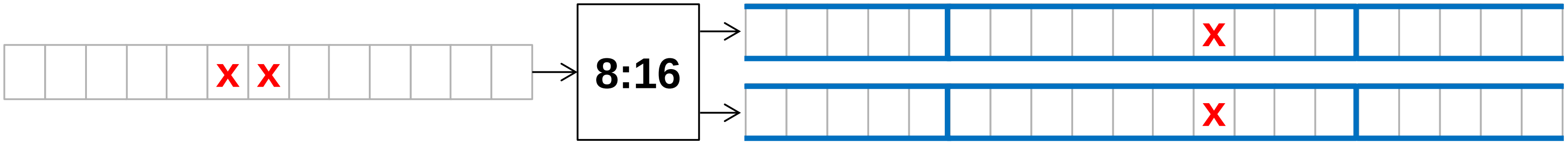
Note – the vertical axis for the plots is Frame Loss Ratio (FLR) for 64-octet MAC frames since uncorrectable codewords have to be discarded to provide an adequate Mean Time To False Packet Acceptance (MTTFPA).

RS(528,514) performance with bursts

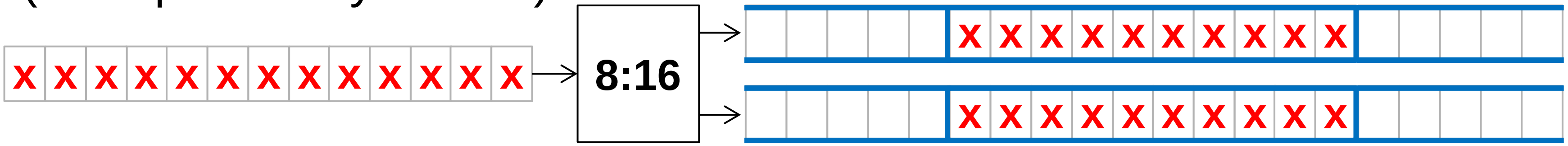


Bit interleaving – same FEC instance

For the case of bit interleaving two lanes from the same FEC instance, the minimum burst size that can cause exactly 2 symbol errors is again 2 bits, but here the probability of causing exactly 2 symbol errors is 1.



The maximum burst size that can cause exactly 2 symbol errors is 20 bits, but this only occurs if the first two errors are the first bits of a symbol (i.e. a probability of 0.05).

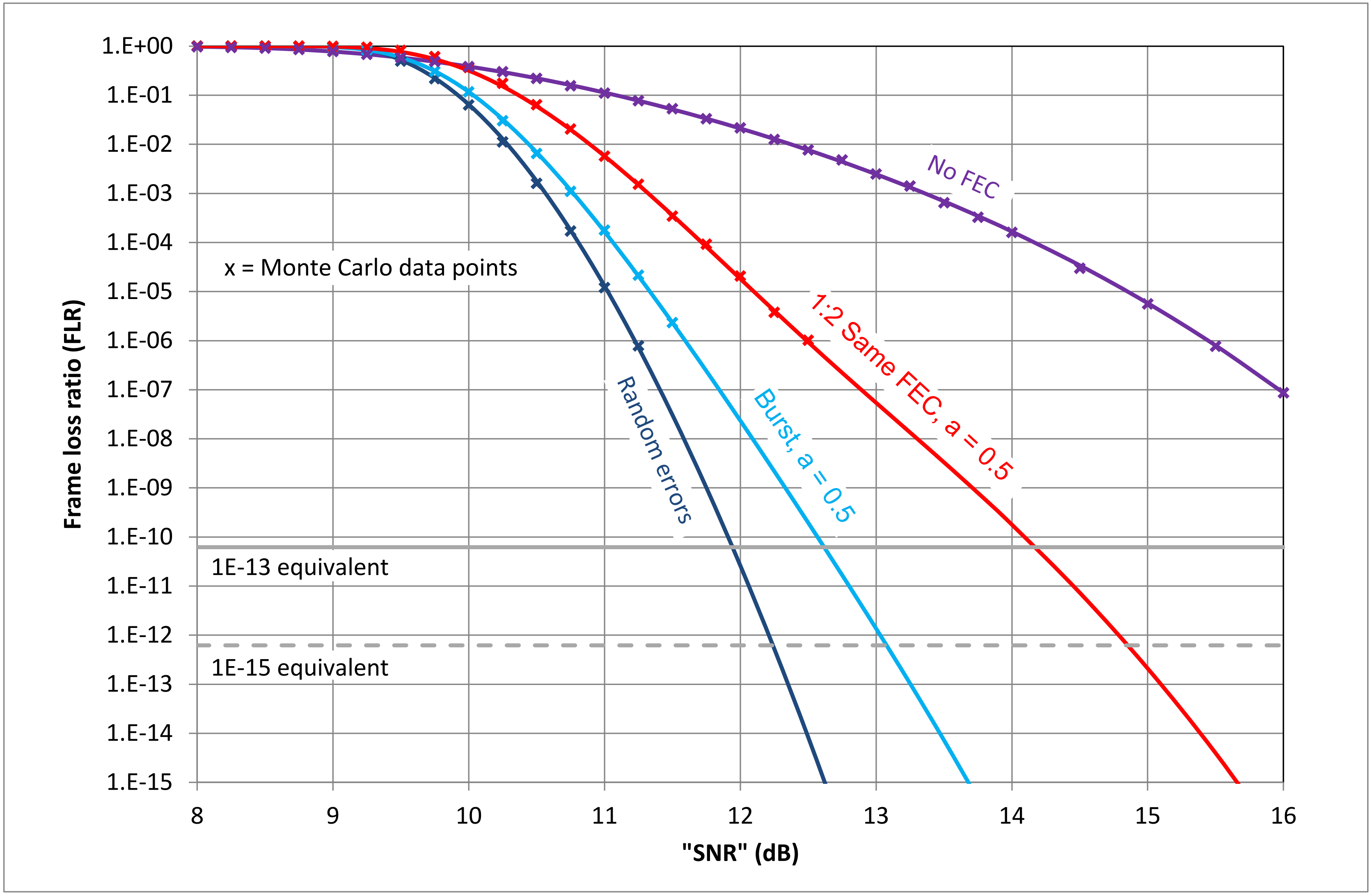


Note – this analysis assumes no skew between the two lanes as this is worse than assuming random alignment.
The probability of exactly two symbol errors (given an initial error) is:

$$\mathrm{P}(2) = a(1-a) + \sum_{i=2}^{2m-1} \frac{2m-i}{2m} a^i (1-a) = 0.4625 \ \text{ for m} = 10,\ \text{a} = 0.5 \qquad (8)$$

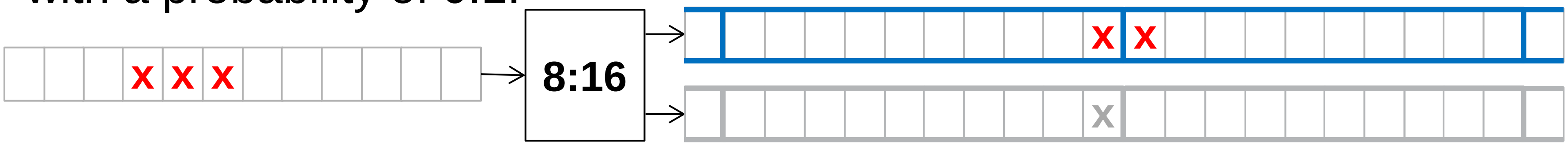
P(1) = 0.5, P(2) = 0.4625, P(3) = 0.025, P(4) = 0.0125, P(5) = 2.4E-8 etc.

RS(528,514) with 1:2 interleaving and bursts



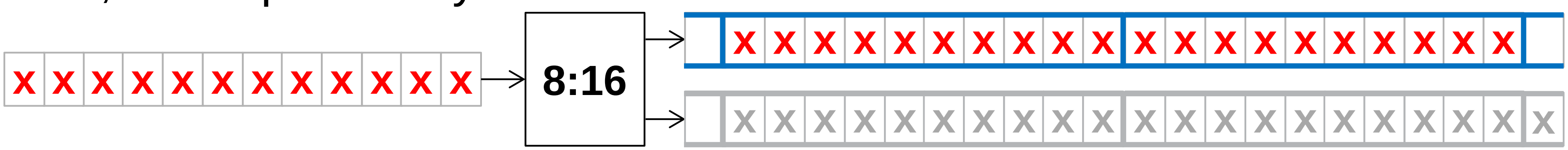
Bit interleaving – different FEC instance

The case of bit interleaving two lanes from different FEC instances can be analysed by considering just the bits from one FEC instance with an input BER increased by a factor of 1 + a.
The minimum burst size that can cause exactly 2 symbol errors is 3 bits, with a probability of 0.1.



If the burst size is 22 bits, then the probability that it will cause exactly 2 symbol errors is 1.

The maximum burst size that can cause exactly 2 symbol errors is 41 bits, with a probability of 0.1.

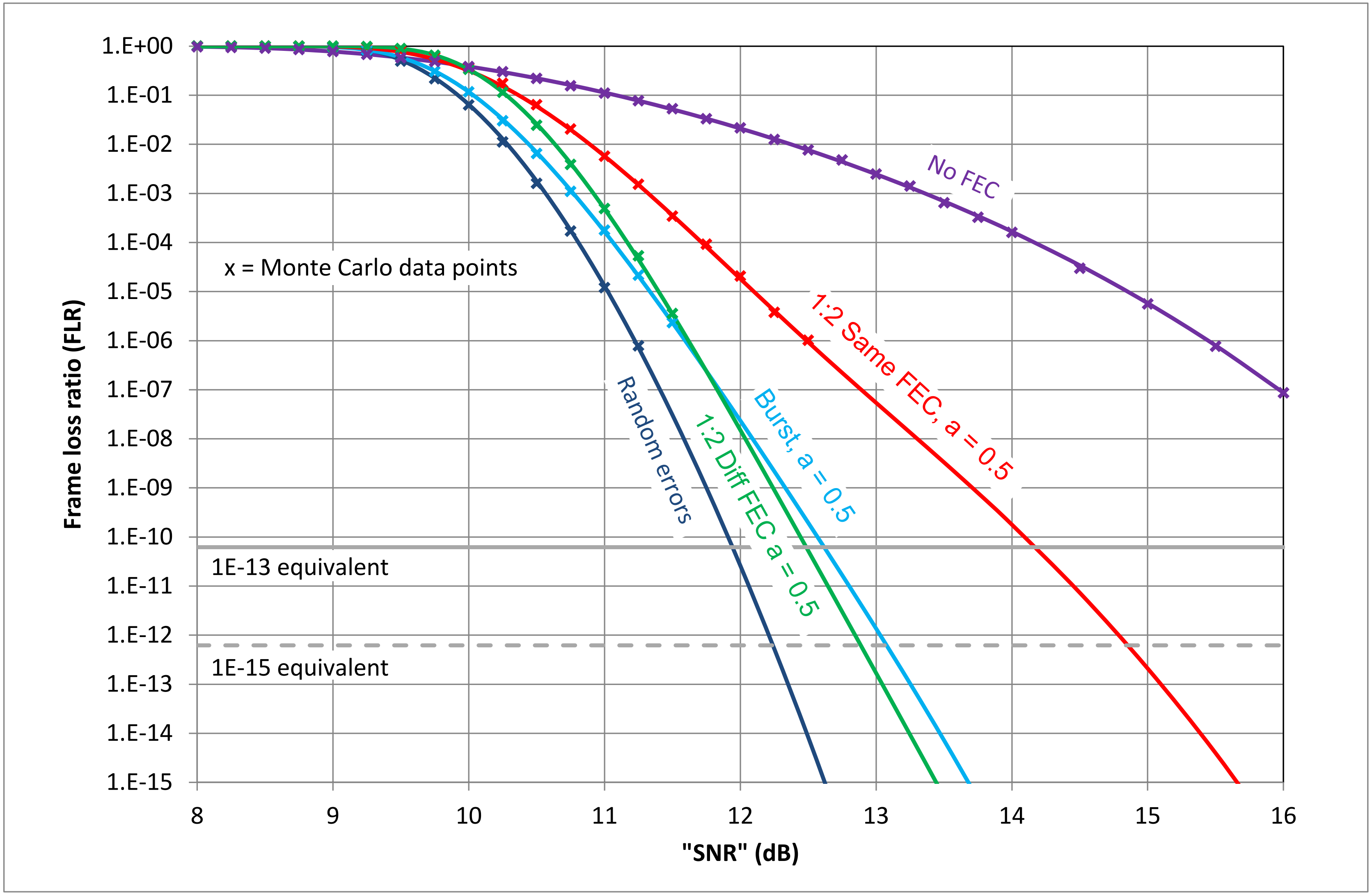


The probability of exactly two symbol errors (given an initial error) is:

$$\mathrm{P}(2) = \sum_{i=2}^{2m+1} \frac{\mathrm{floor}(i/2)}{m} a^i (1-a) + \sum_{i=2m+2}^{4m-1} \frac{2m - \mathrm{floor}(i/2)}{m} a^i (1-a) = 0.0333 \qquad (9)$$

P(1) = 0.9667, P(2) = 0.0333, P(3) = 3.2E-8 etc.

RS(528,514) all curves



RS(544,514) all curves

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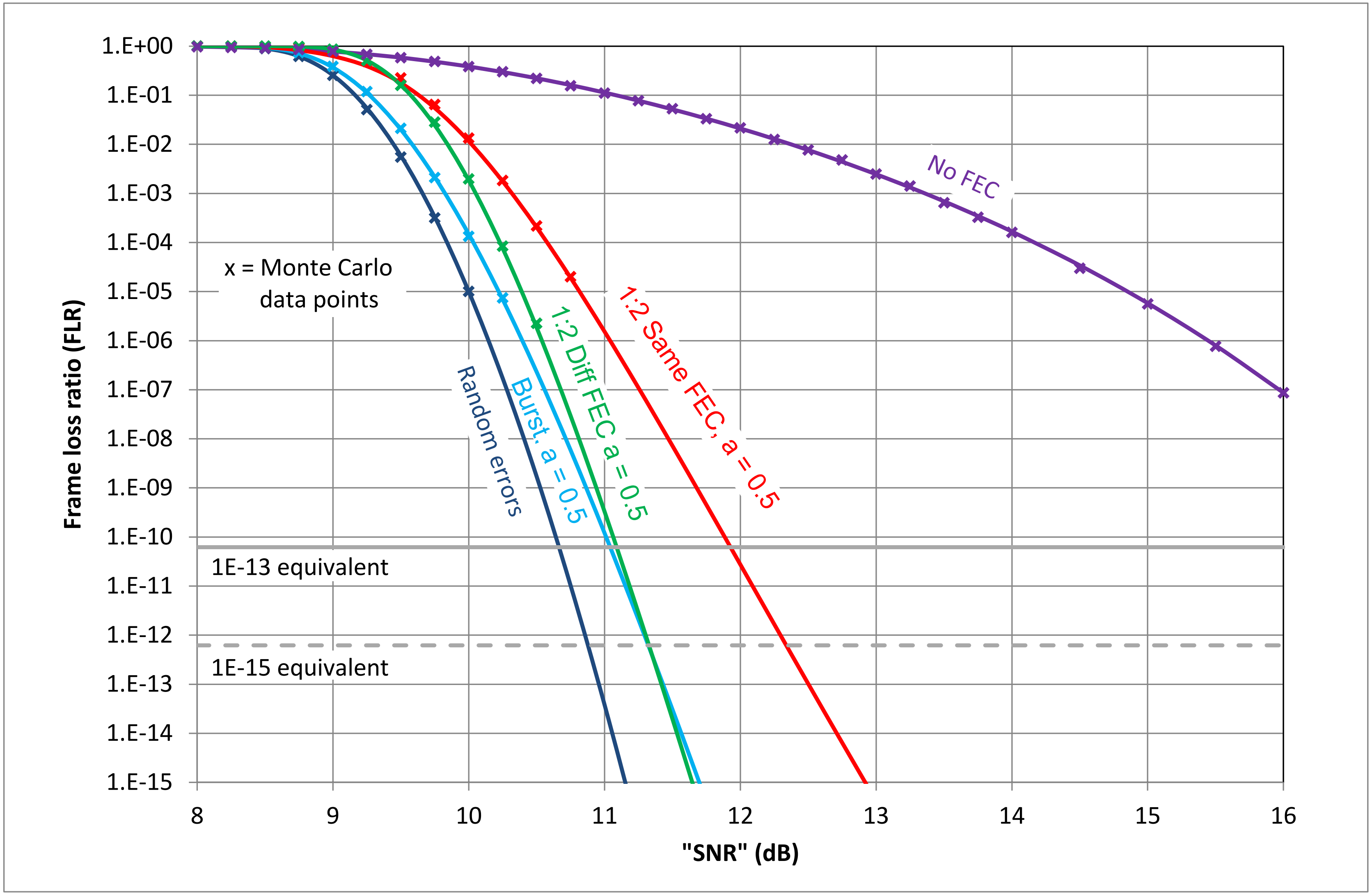
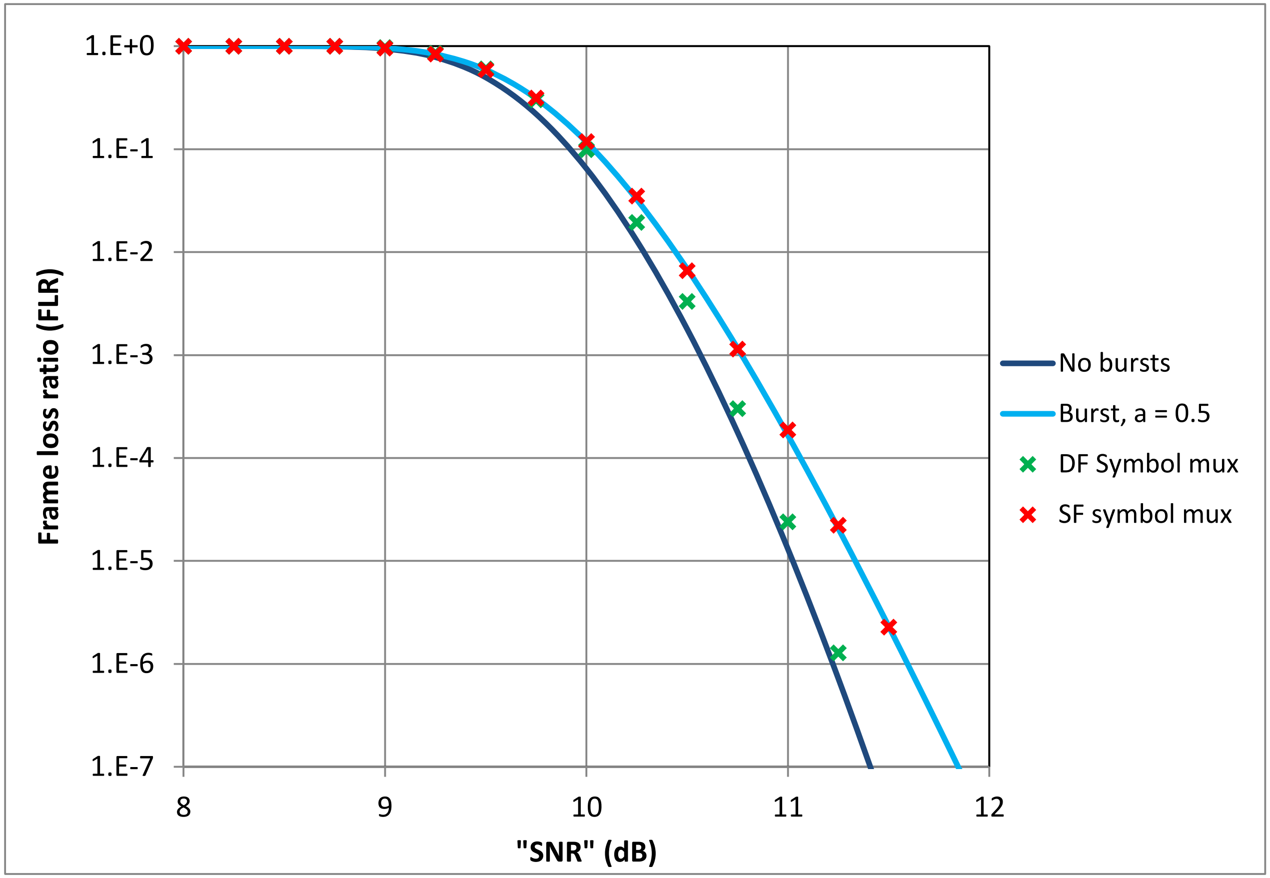


Symbol interleaving

Monte Carlo analysis of symbol interleaving shows that, as expected, 1:2 same FEC (SF) with bursts shows the same performance as for non-interleaved FEC with bursts and 1:2 different FEC (DF) with bursts has only slightly worse performance than for random errors.



Results for RS(528,514) and RS(544,514) *

From the curves shown on the previous slide, the BERs (due to the "SNR") at the input required to give FLRs equivalent to that of a BER of 1E-13 and 1E-15 are:

	RS(528,514)		RS(544,514)	
	FLR = 6.2E-11	FLR = 6.2E-13	FLR = 6.2E-11	FLR = 6.2E-13
No FEC	1E-13	1E-15	1E-13	1E-15
1:2 Same FEC, a = 0.5	1.6E-7*	1.6E-8*	3.9E-5*	1.7E-5*
Single FEC burst, a = 0.5	9.7E-6*	3.3E-6*	1.8E-4*	1.2E-4*
1:2 Different FEC, a = 0.5	1.3E-5*	5.2E-6*	1.7E-4*	1.1E-4*
Random errors	3.8E-5	2.1E-5	3.2E-4	2.3E-4

Note – these values are the BER derived from equation 1 and do not include the additional errors due to the bursts. To account for burst errors, the values marked with "*" must be multiplied by 2 when a = 0.5.

What about multi-part links with FEC?

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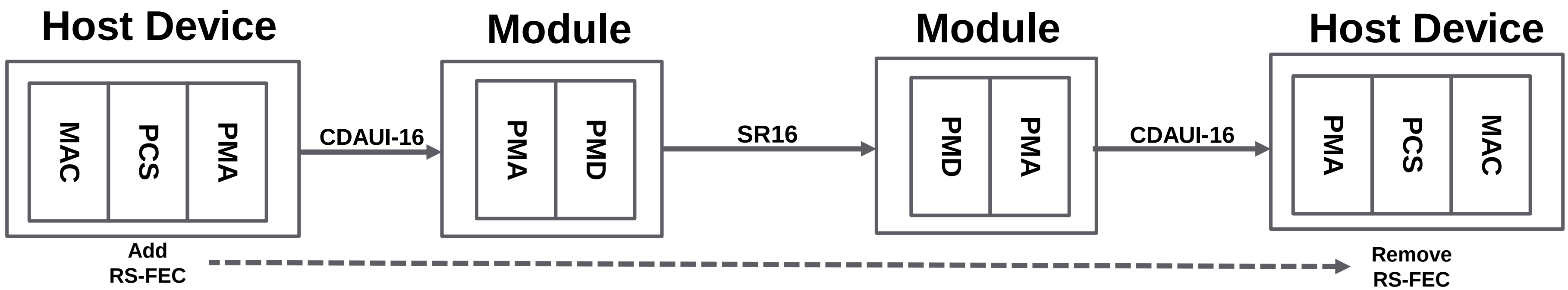
If the FEC bytes are added at the source PCS layer and then the correction is applied only at the destination PCS layer as in:



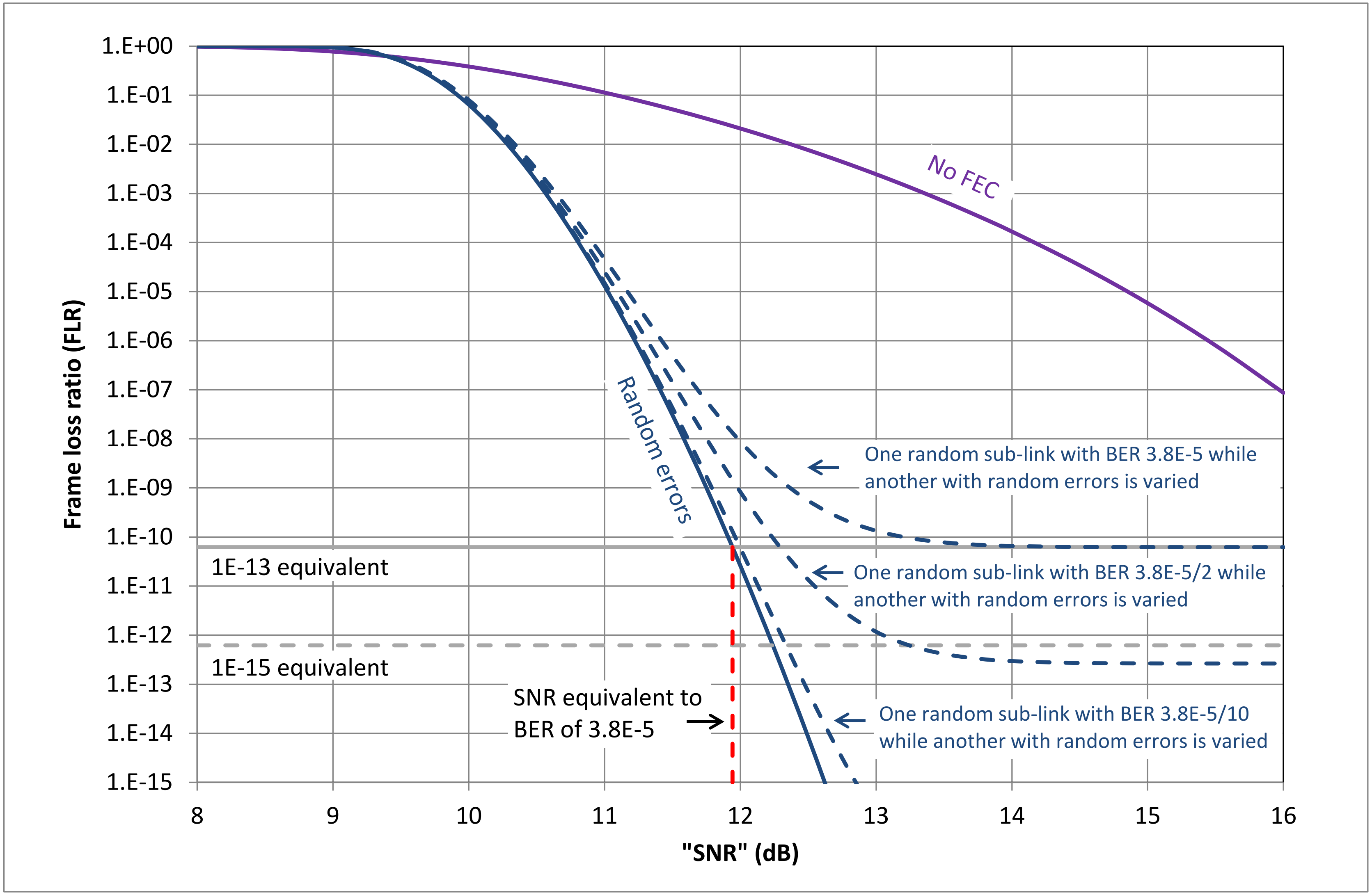
Source: [gustlin_3bs_02_0514]

Then the worst case input BER for the FEC decoder must be met by the concatenation of all of the sub-links.

In the case of CAUI-10 -> SR10 -> CAUI-10, the worst case BER for each of the sub-links is 1E-12 which is the same as the end-to-end requirement. This situation is tolerated on the basis that it is unlikely that all three sub-links will be at the worst case BER at the same time and if two of them were, then the end-to-end BER would still be 2E-12.

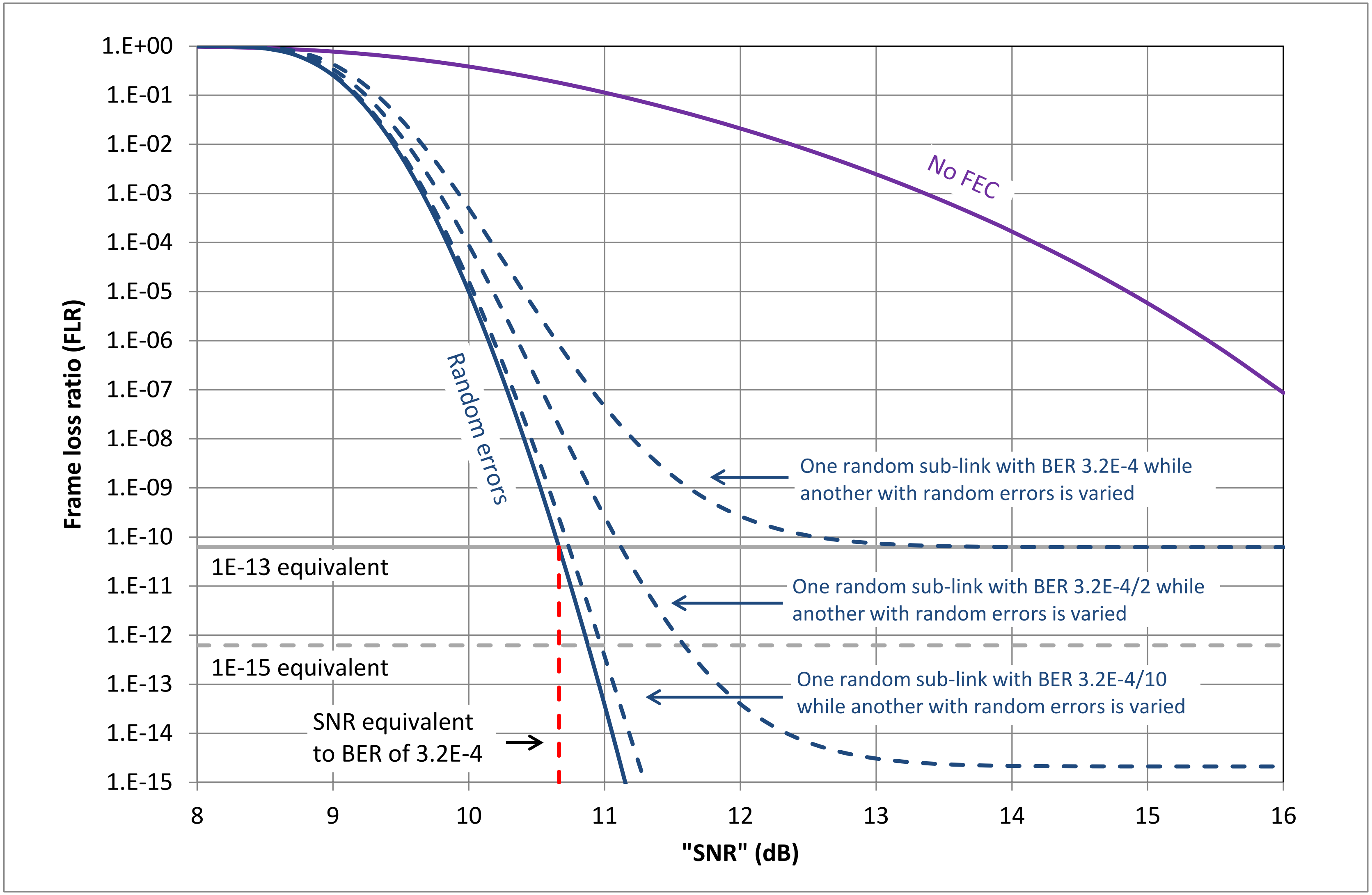
The situation for a pair of sub-links sharing the same RS(528,514) or RS(544,514) protection is shown on the next two slides.

RS(528,514) multi-part (random)



RS(544,514) multi-part (random)

*



Sub-links with bursts

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The curves on the previous slides are all for random errors.

What if there are burst errors in the electrical sub-links but random errors in the optical part of the link, or bursts in both?

Similar principles as previously can be used for this, except that instead of calculating the probability of 8 or more symbol errors, the calculation finds the probability of 7, 6, 5, 4, 3, 2, and 1 symbol errors due to the bursty sub-link. This is then combined with the probability of 1, 2, 3, 4, 5, 6, or 7 symbol errors due to the random sub-link or due to a bursty sub-link.

Plots covering mixtures of random and bursty sub-links as well as two bursty sub-links are included in Annex 1 and Annex 2 to this presentation.

Annex 1 contains plots for a BER floor that is reachable by a Monte Carlo simulation in an attempt to validate the accuracy of each model. These show a good match between the analytical model and the Monte Carlo results.

Annex 2 then uses these models to generate plots similar to slides 17 and 18 for each case.

RS(528,514) results

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The BERs (due to the “SNR”) of the electrical sub-link for a penalty of 0.1 dB optical in the optical sub-link are:

			RS(528,514)		RS(544,514)	
	Electrical	Optical	FLR = 6.2E-11	FLR = 6.2E-13	FLR = 6.2E-11	FLR = 6.2E-13
1:2 Same FEC, a = 0.5	Burst	Random	4.6E-8*	5.4E-9*	7.5E-6*	3.4E-6*
	Burst	Burst	7.5E-8*	8.4E-9*	1.4E-5*	7.1E-6*
Single FEC burst, a = 0.5	Burst	Random	2.9E-6*	1.0E-6*	4.4E-5*	2.9E-5*
	Burst	Burst	3.6E-6*	1.3E-6*	5.1E-5*	3.4E-5*
1:2 Different FEC, a = 0.5	Burst	Random	3.9E-6*	1.6E-6*	4.2E-5*	3.0E-5*
	Burst	Burst	4.6E-6*	2.0E-6*	4.7E-5*	3.4E-5*
Random errors	Random	Random	1.2E-5	7.2E-6	8.2E-5	6.2E-5

Note – these values do not include the additional errors due to the bursts. To account for burst errors, the values marked with “*” must be multiplied by 2 when a = 0.5.

Conclusions

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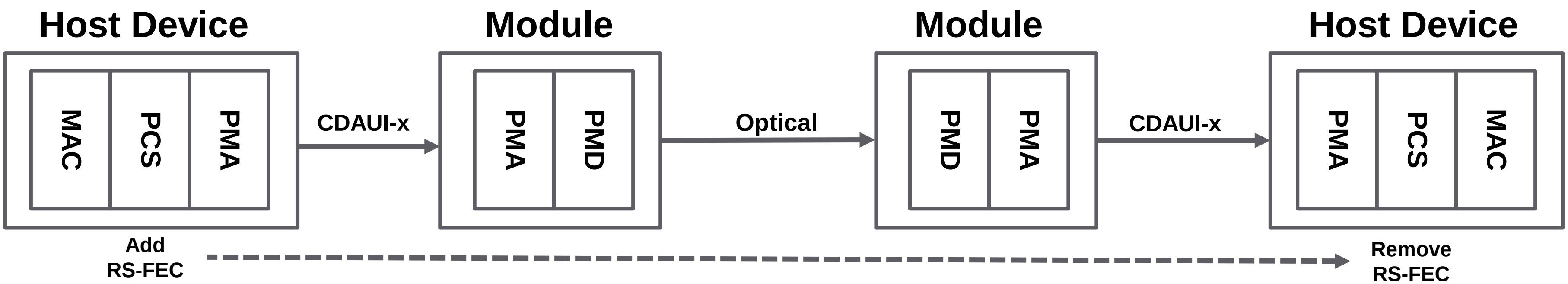
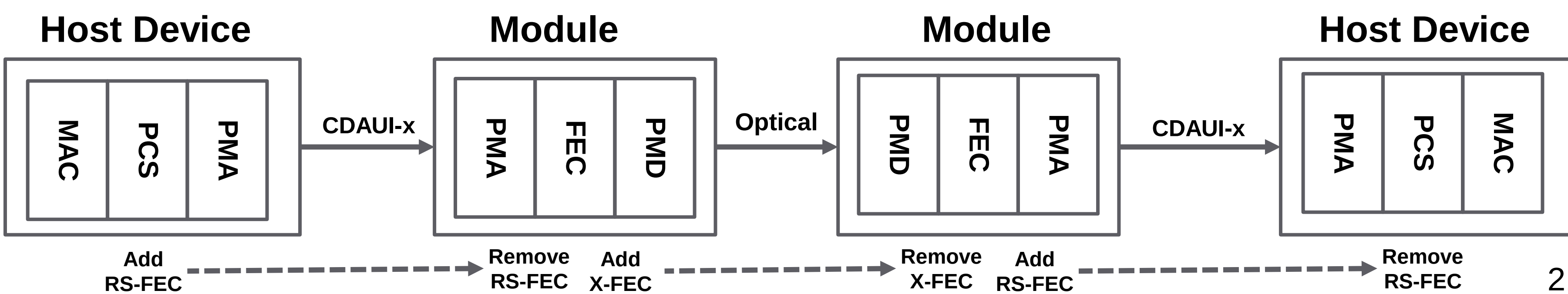
For the multi sub-link systems analysed in this contribution, the limiting BER for the electrical sub-links that cause 0.1 dB penalty for the optical sub-link are shown on the previous slide.



For all cases except one (1:2 bit dis-interleaving to the same FEC instance using RS(528,514)), a BER of 1E-6 as contained in some of the OIF baselines for 56G is low enough to give less than 0.1 dB penalty in the optical sub-link. If a stronger FEC is needed for a future PMD then this can be accommodated as below.



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Annex 1

Plots with high FLR floors and Monte Carlo simulation results to validate models

RS(528,514) multi-part (random + random)

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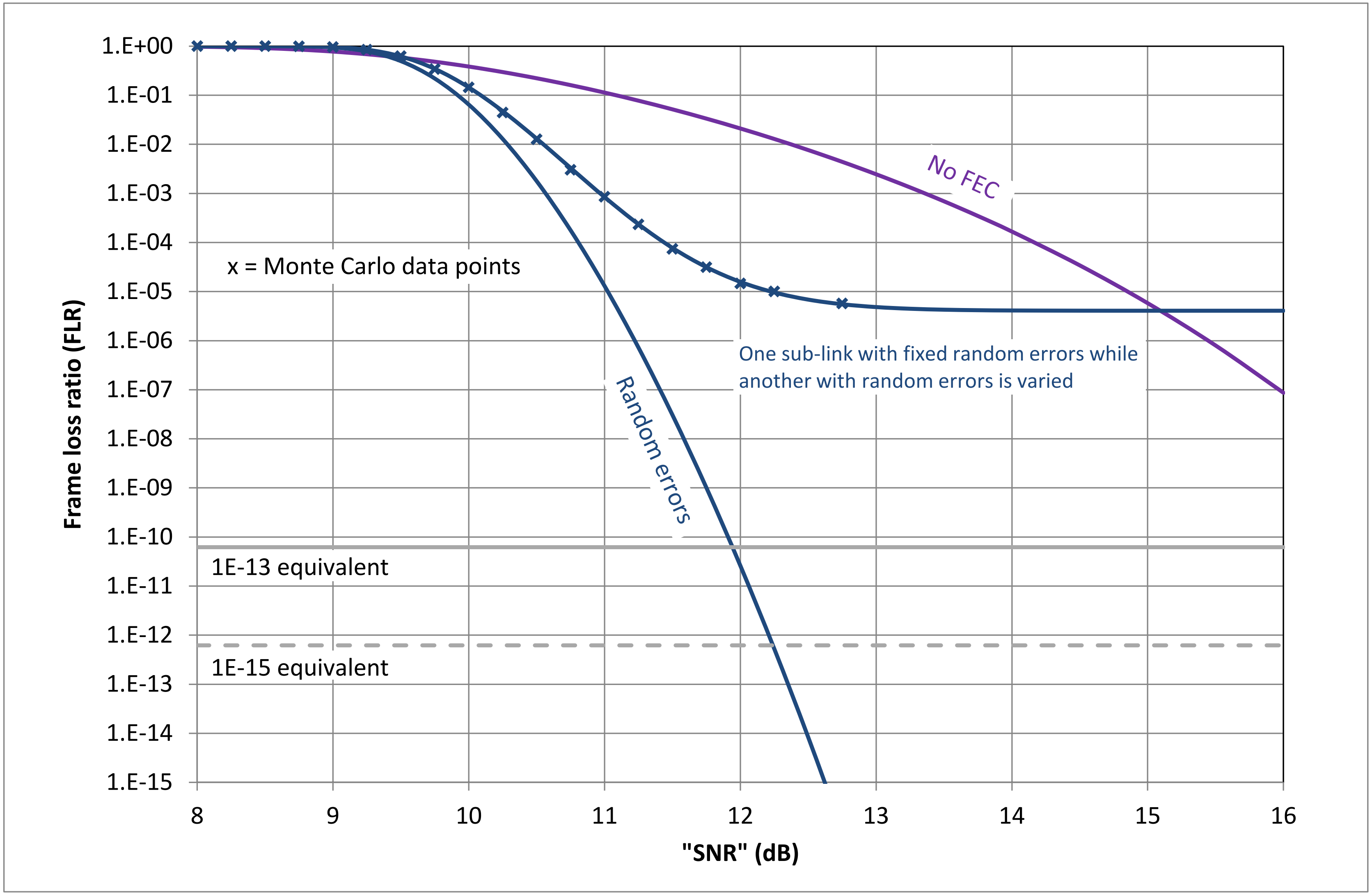
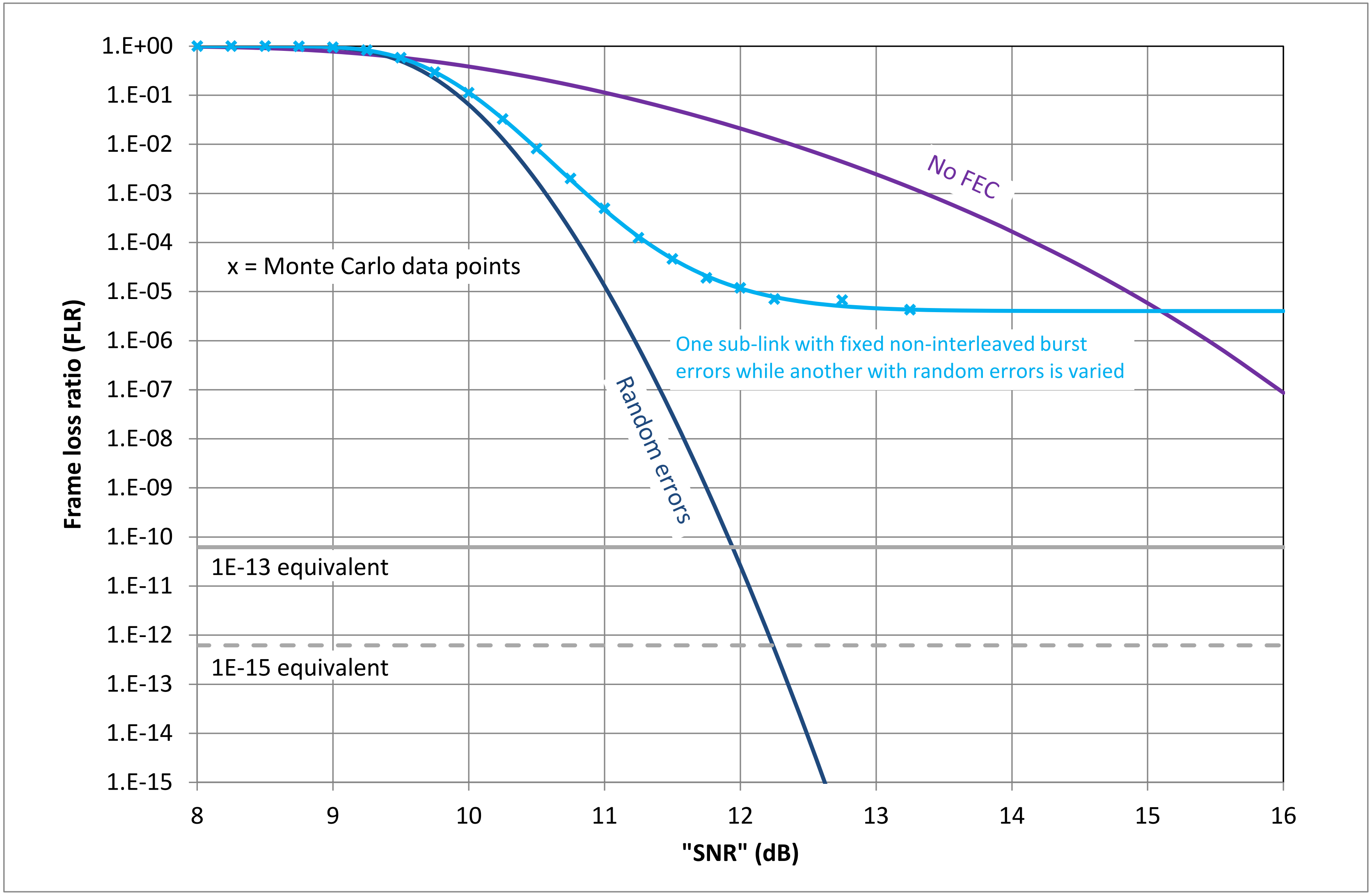
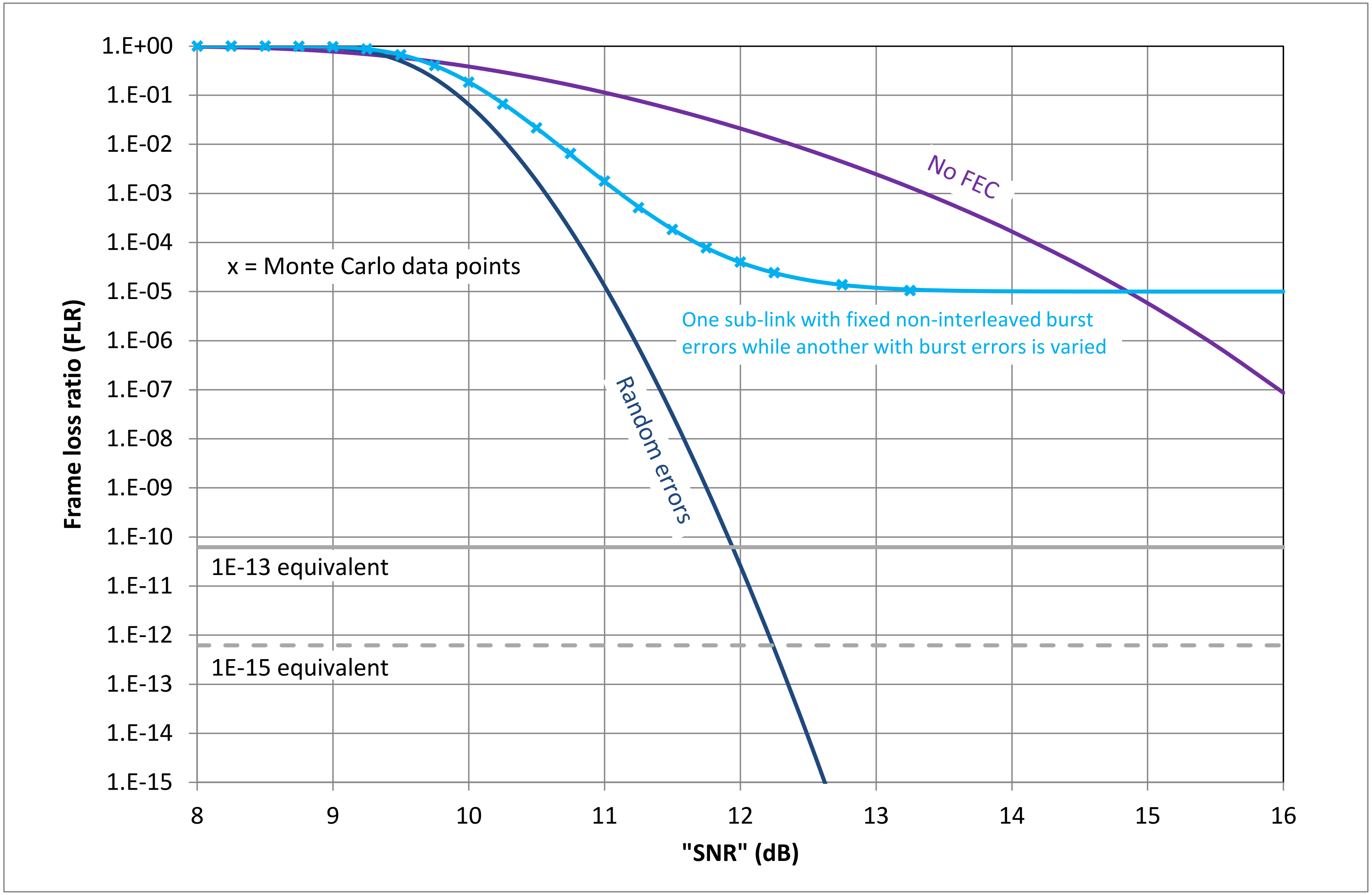


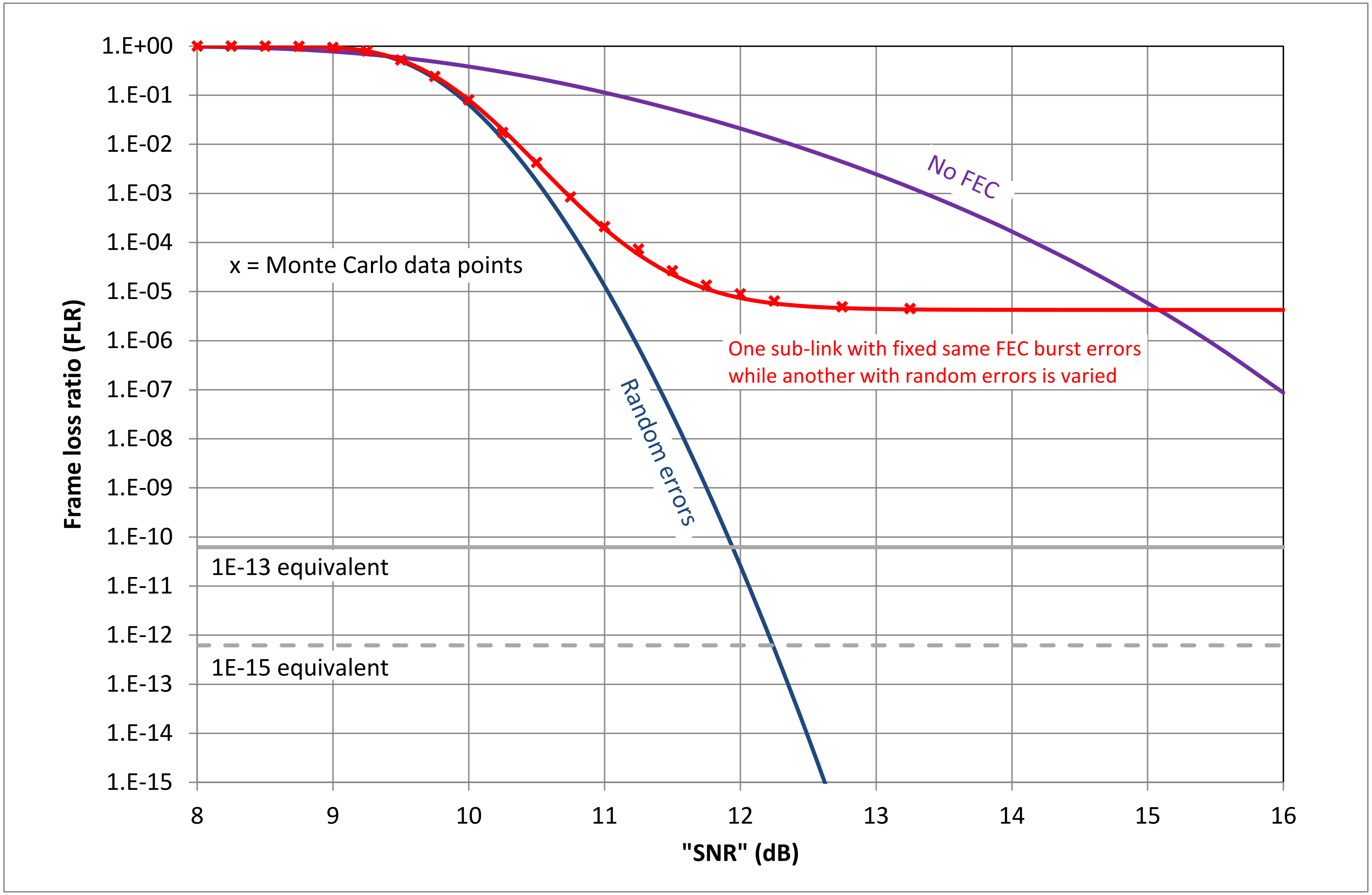
RS(528,514) multi-part non-int (burst + random) *



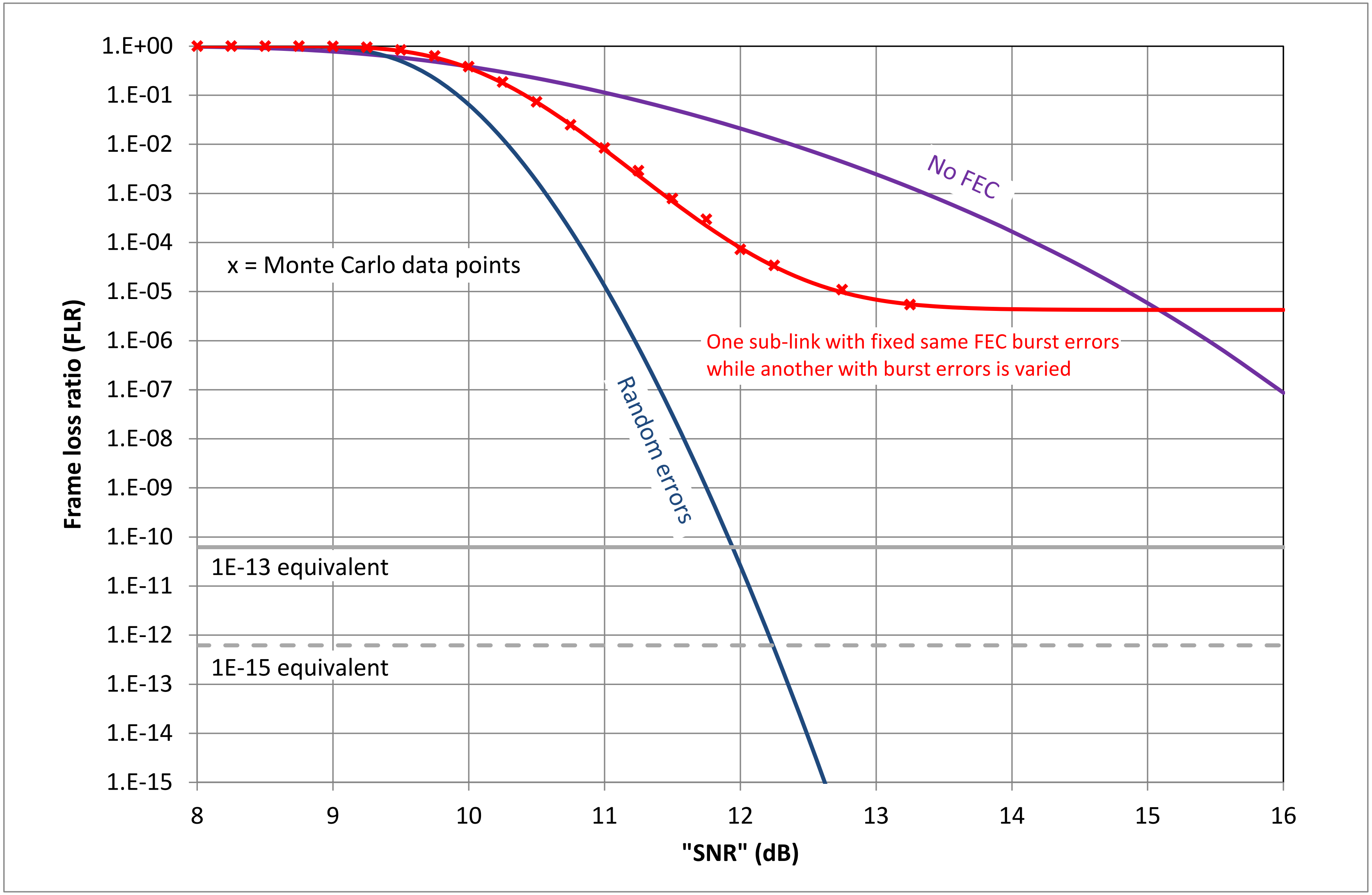
RS(528,514) multi-part non-int (burst + burst) *



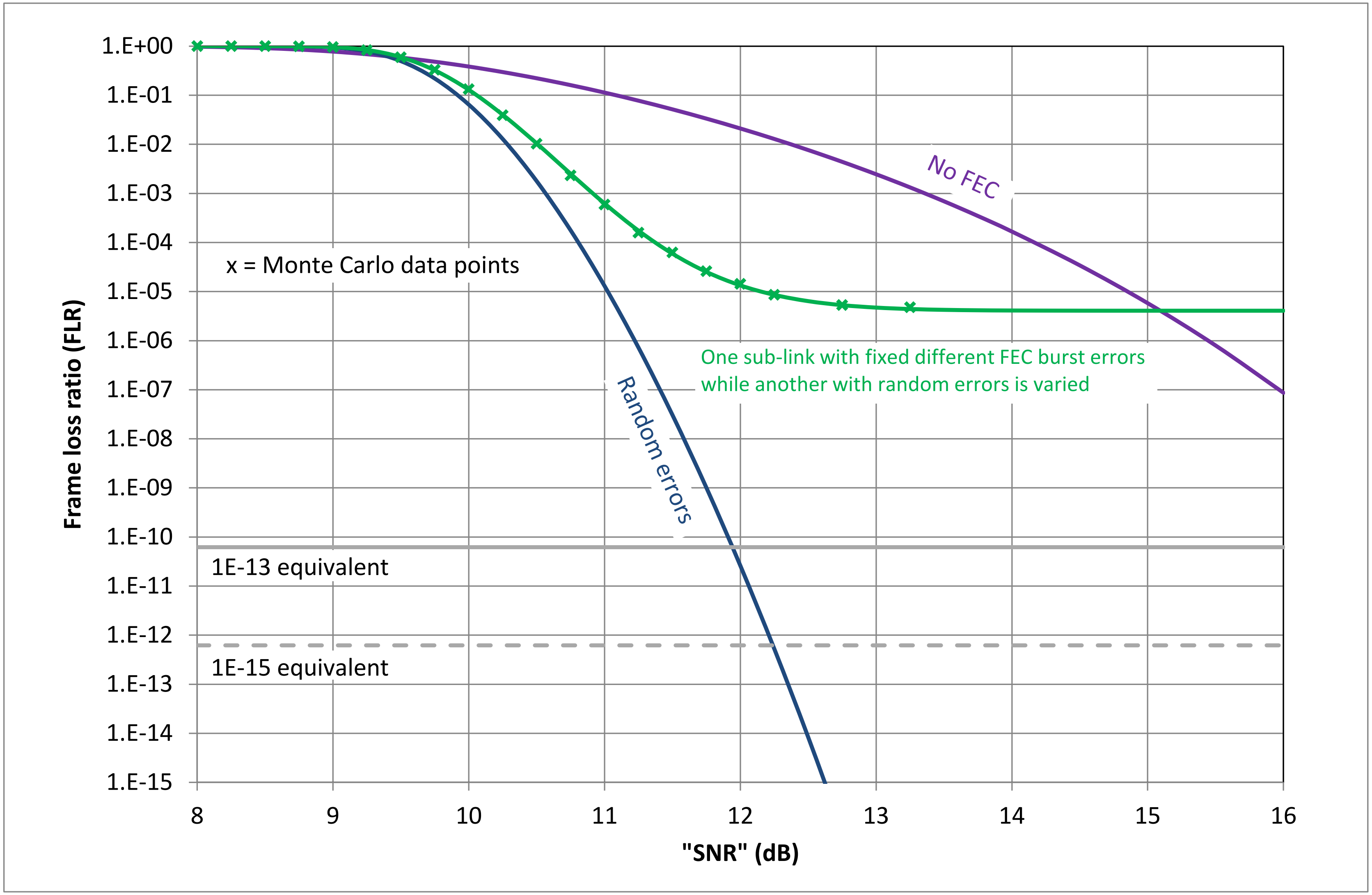
RS(528,514) multi-part (1:2 SF burst + random) *



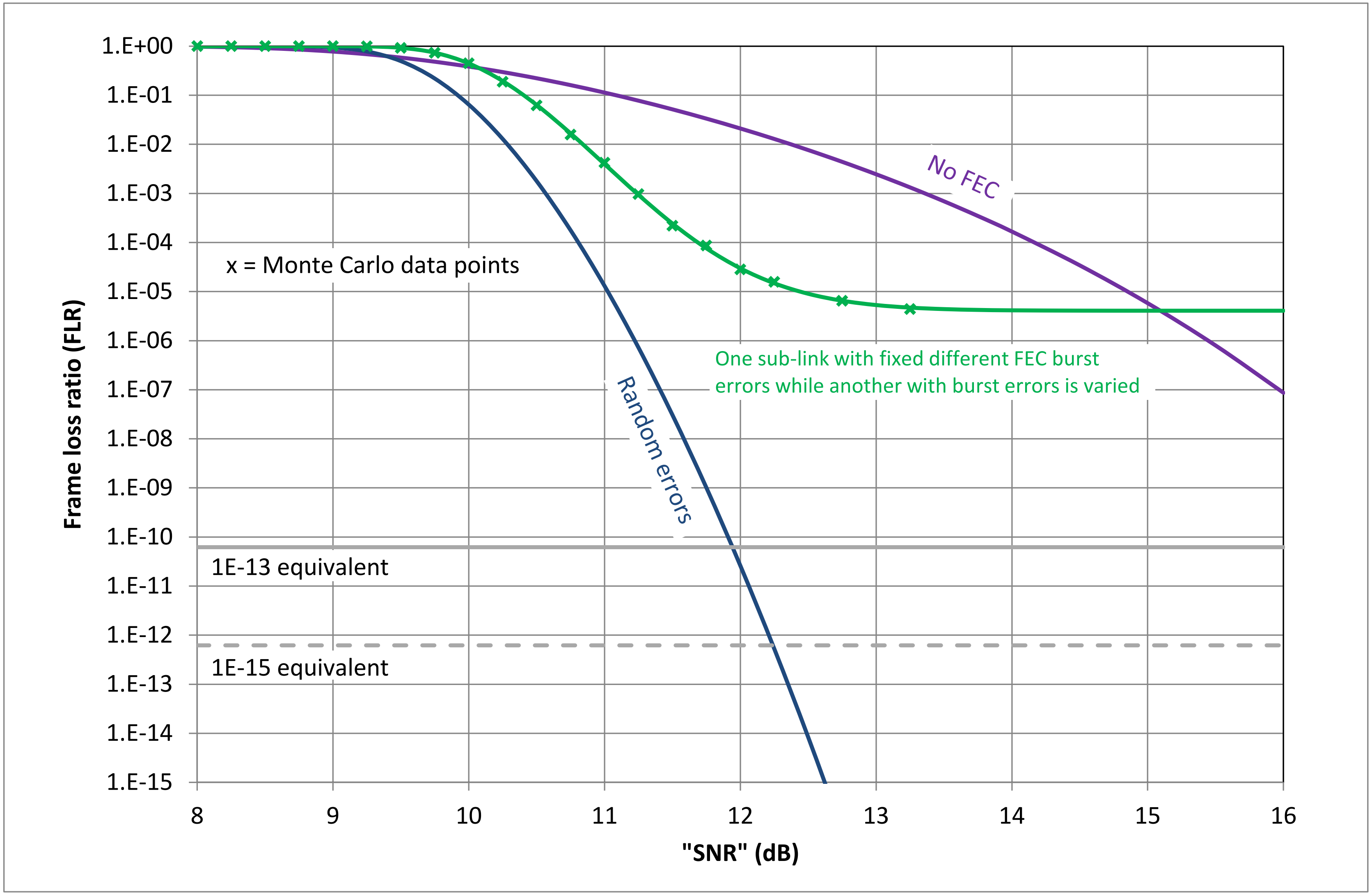
RS(528,514) multi-part (1:2 SF burst + burst) *



RS(528,514) multi-part (1:2 DF burst + random) *



RS(528,514) multi-part (1:2 DF burst + burst) *



RS(544,514) multi-part (random + random)

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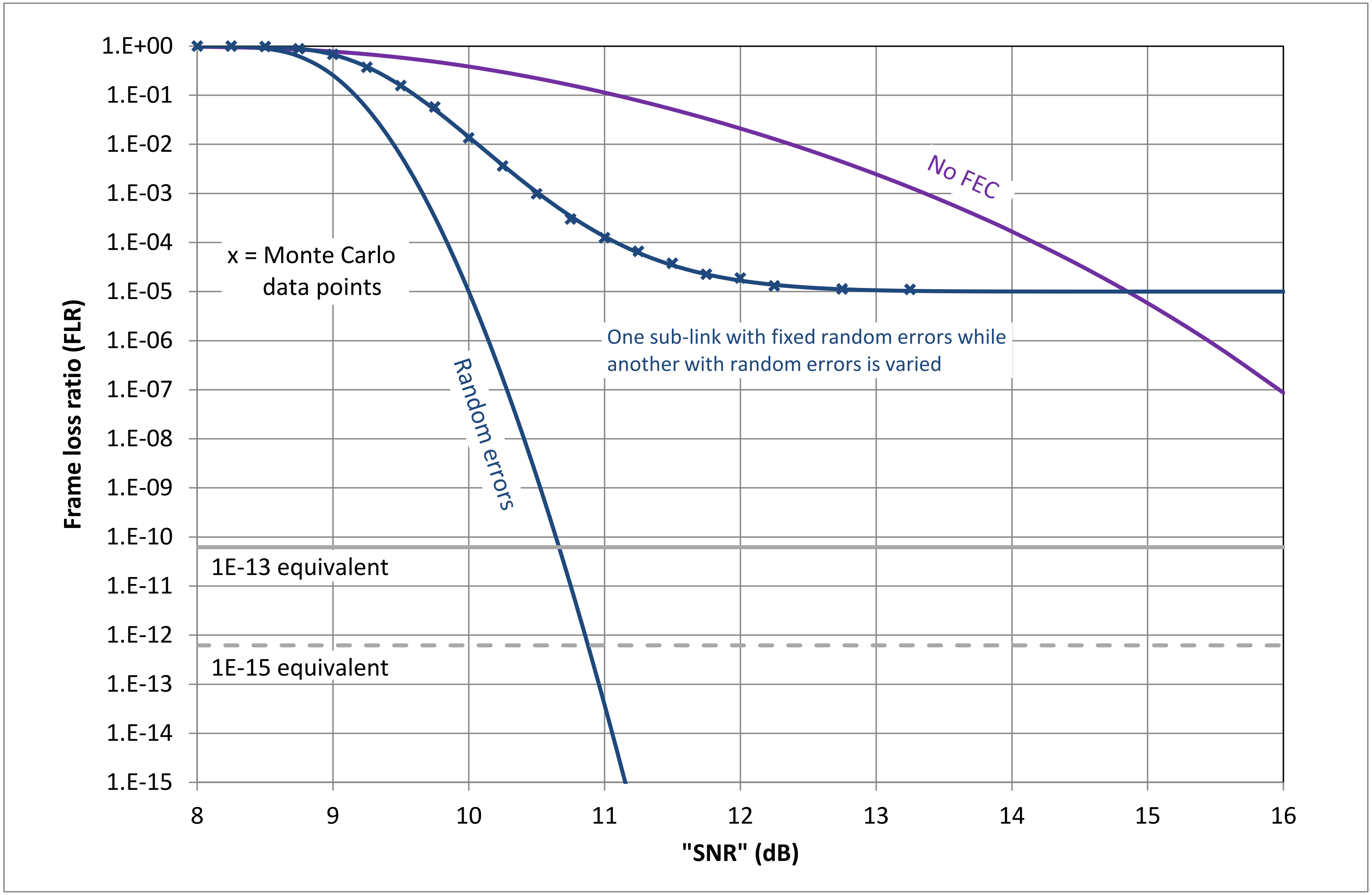
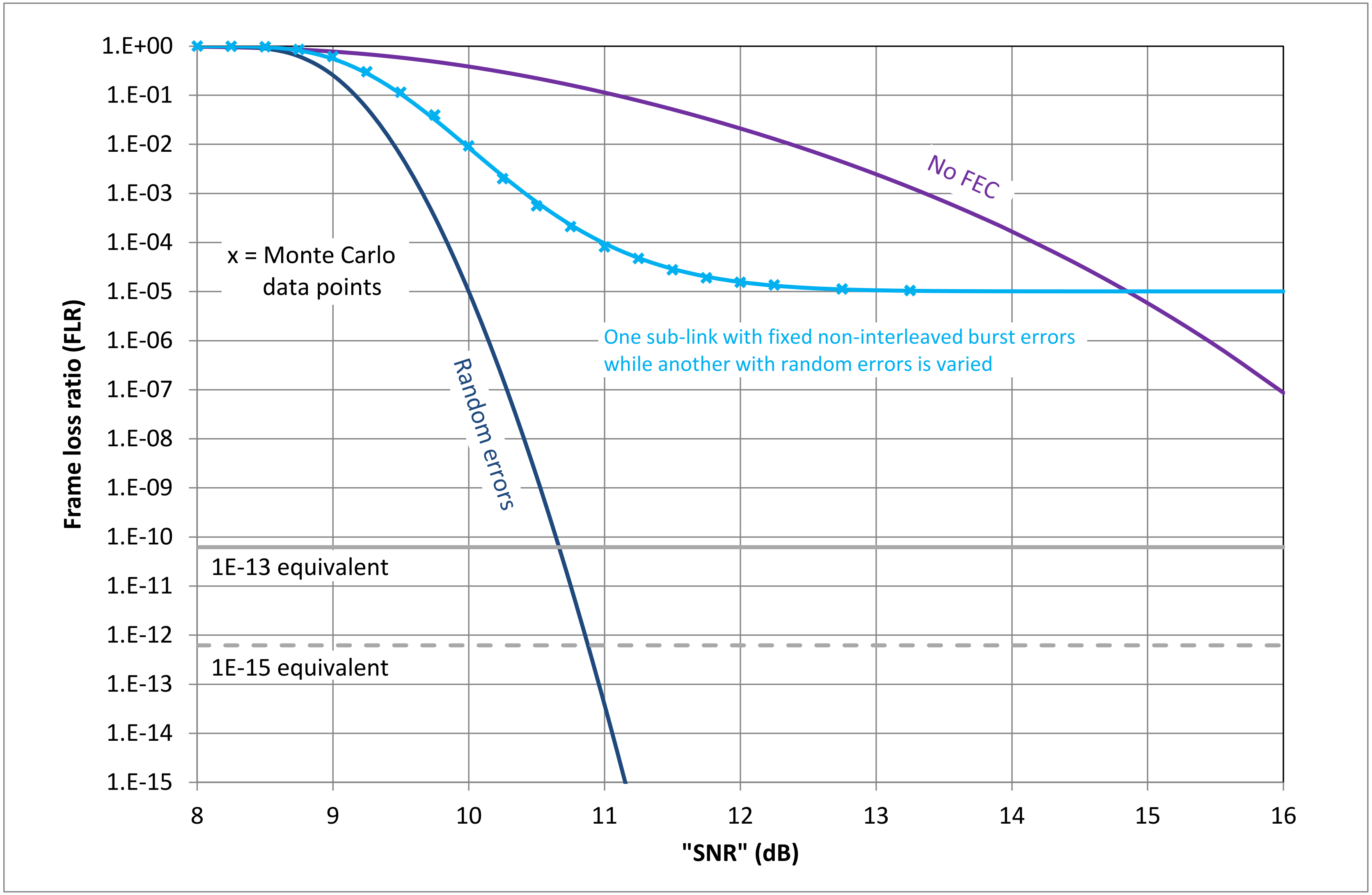
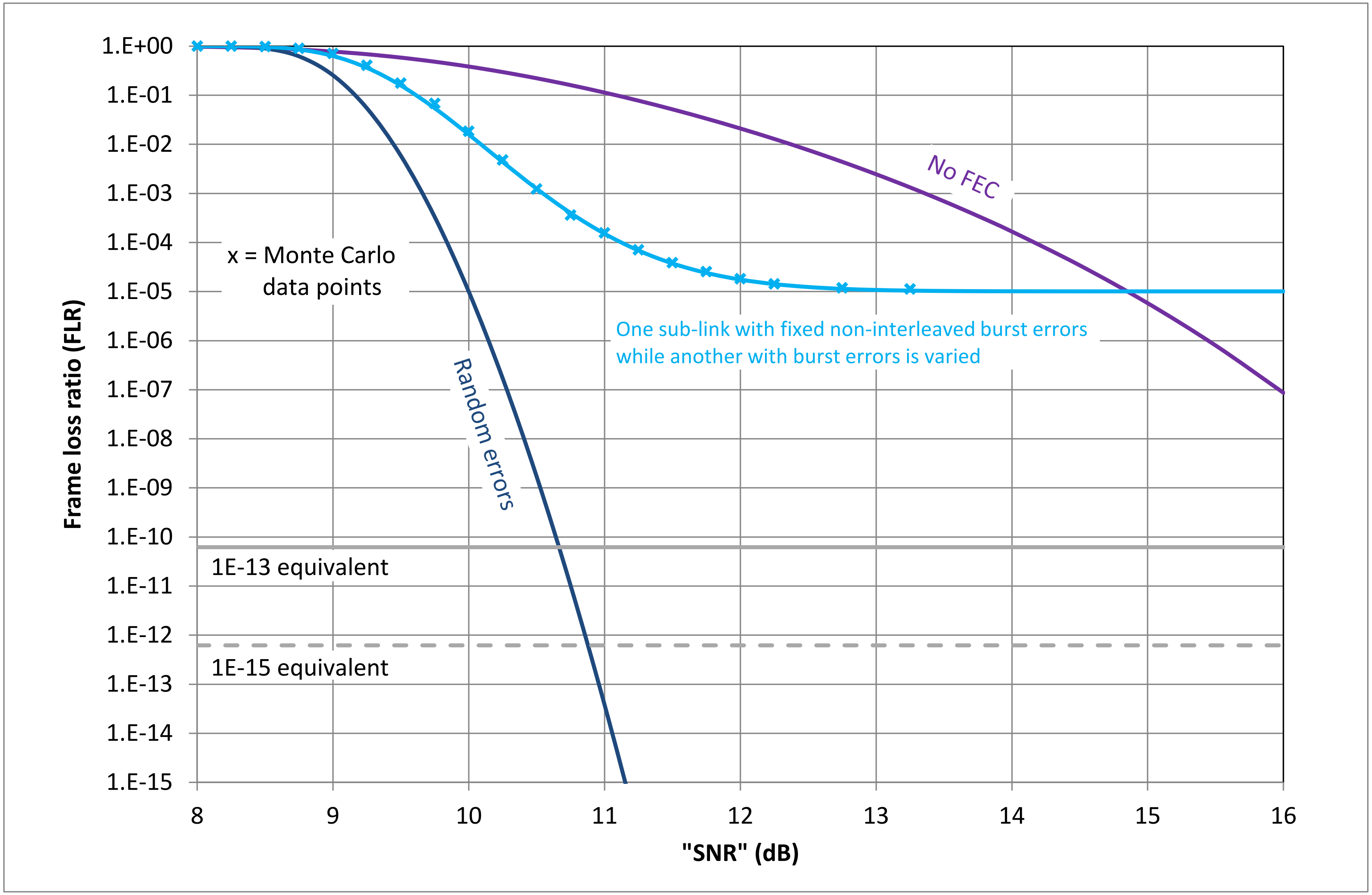


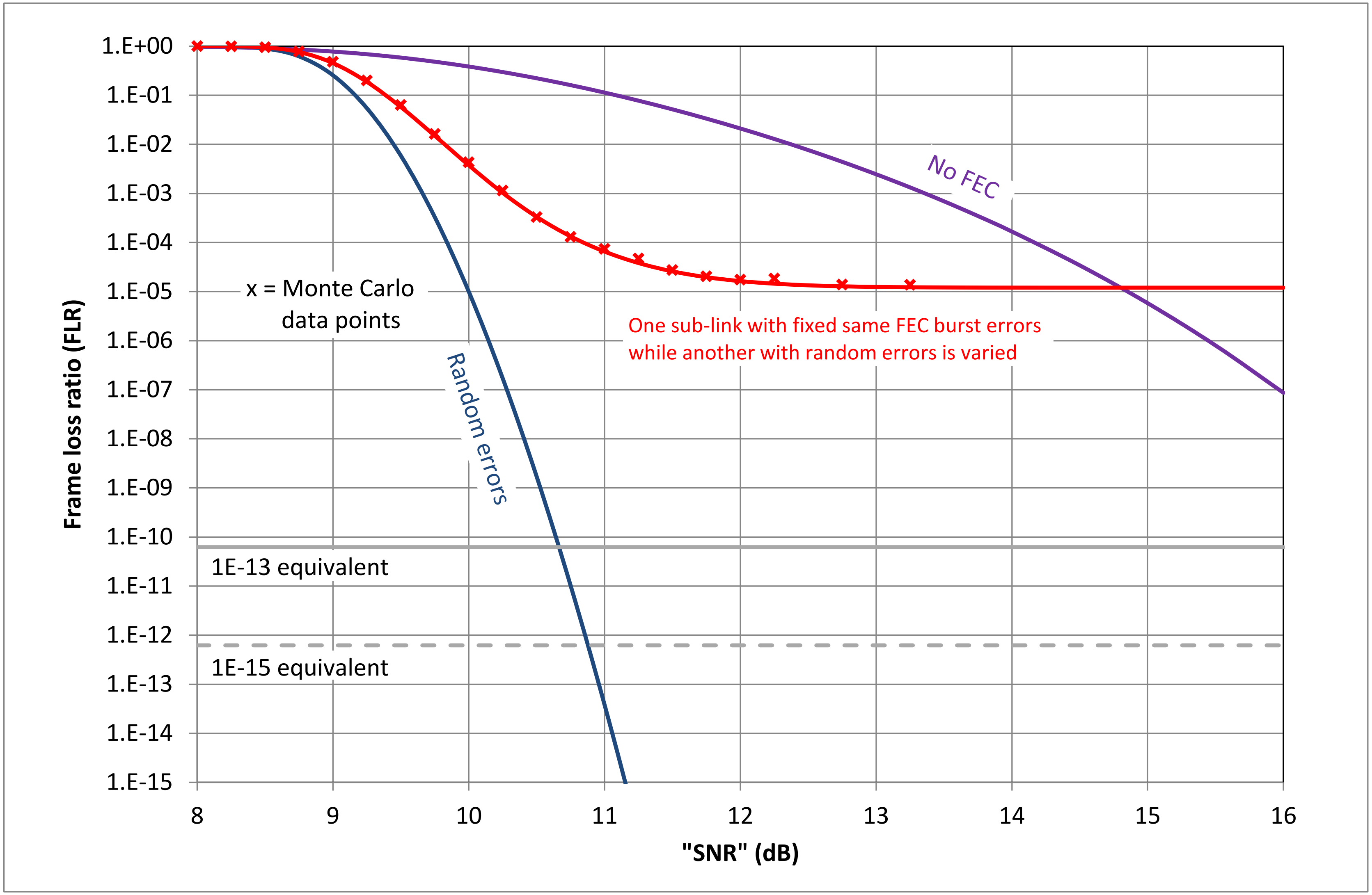
RS(544,514) multi-part non-int (burst + random) *



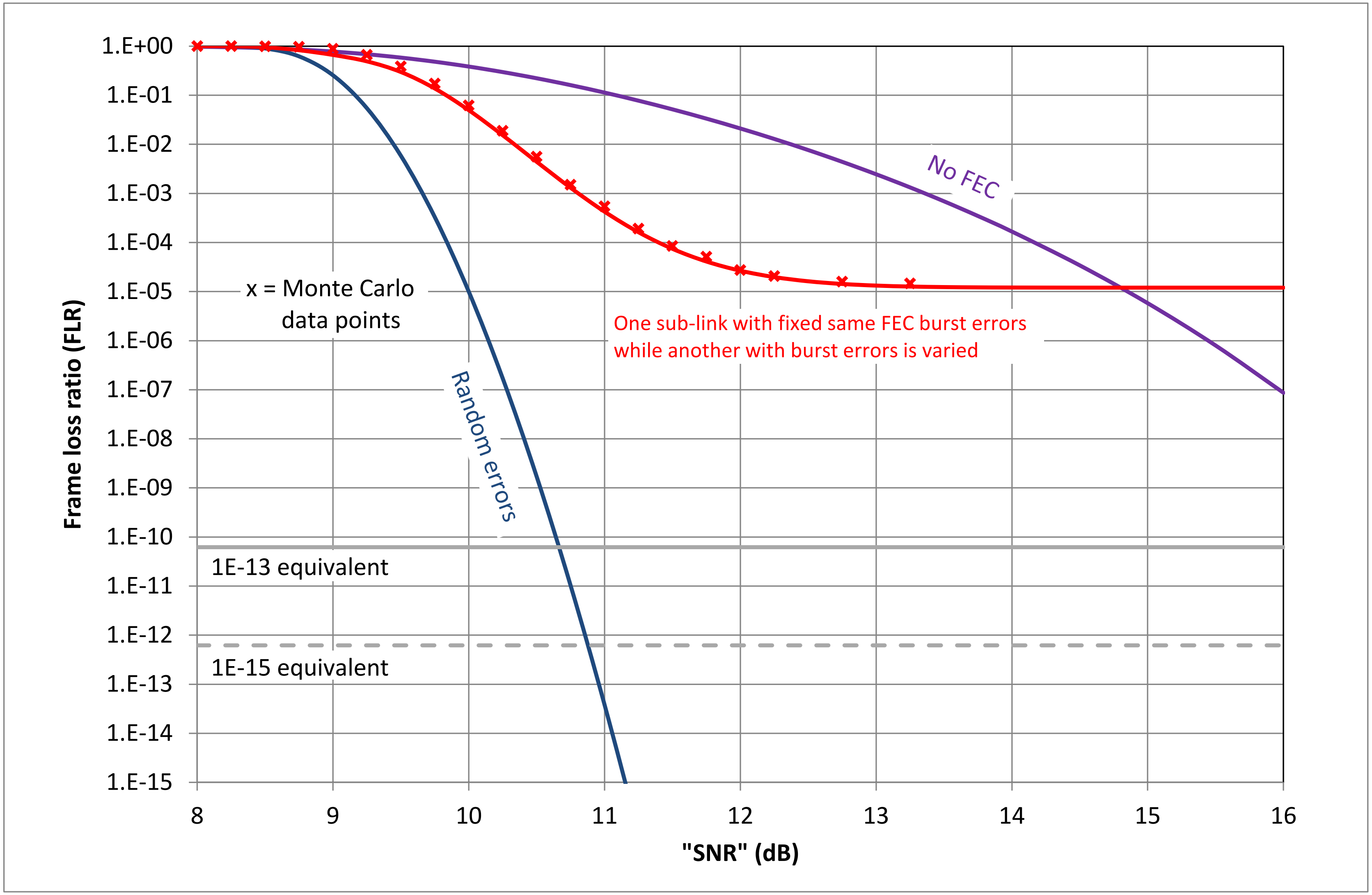
RS(544,514) multi-part non-int (burst + burst) *



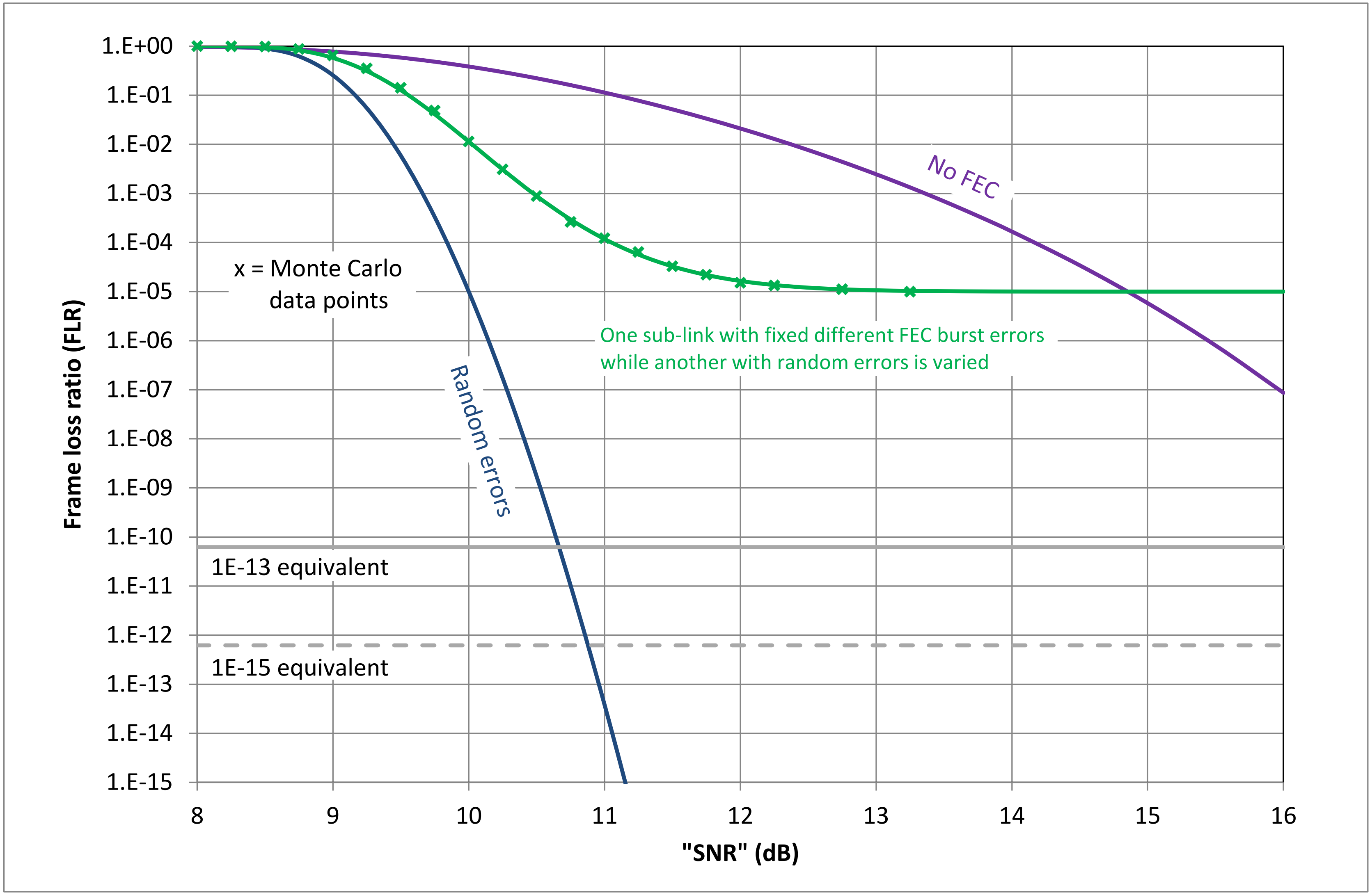
RS(544,514) multi-part (1:2 SF burst + random) *



RS(544,514) multi-part (1:2 SF burst + burst) *



RS(544,514) multi-part (1:2 DF burst + random) *



RS(544,514) multi-part (1:2 DF burst + burst)

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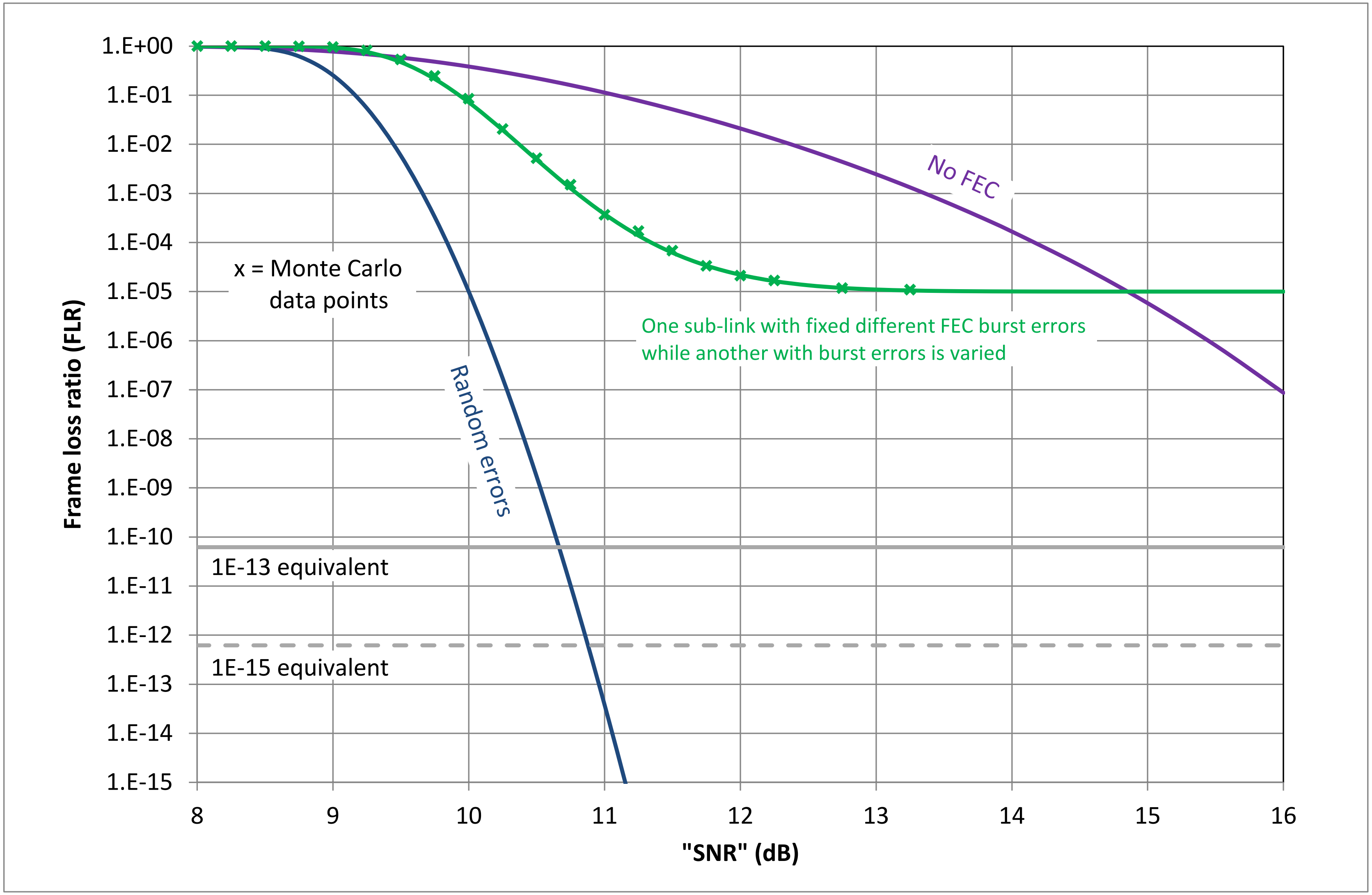

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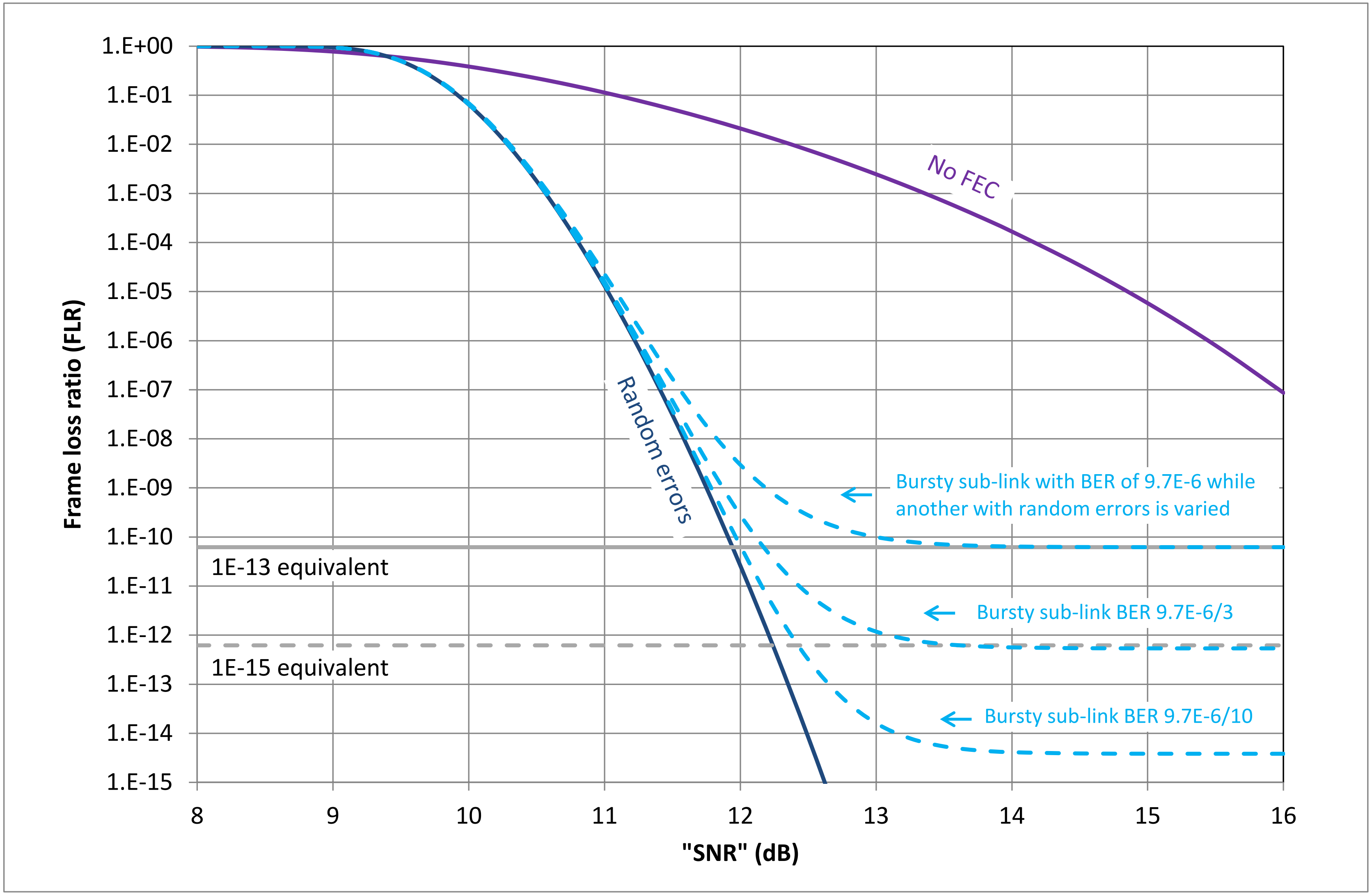
Annex 2

Plots with BER beyond the reach of Monte Carlo analysis

RS(528,514) multi-part non-int (burst + random) *



RS(528,514) multi-part non-int (burst + burst)

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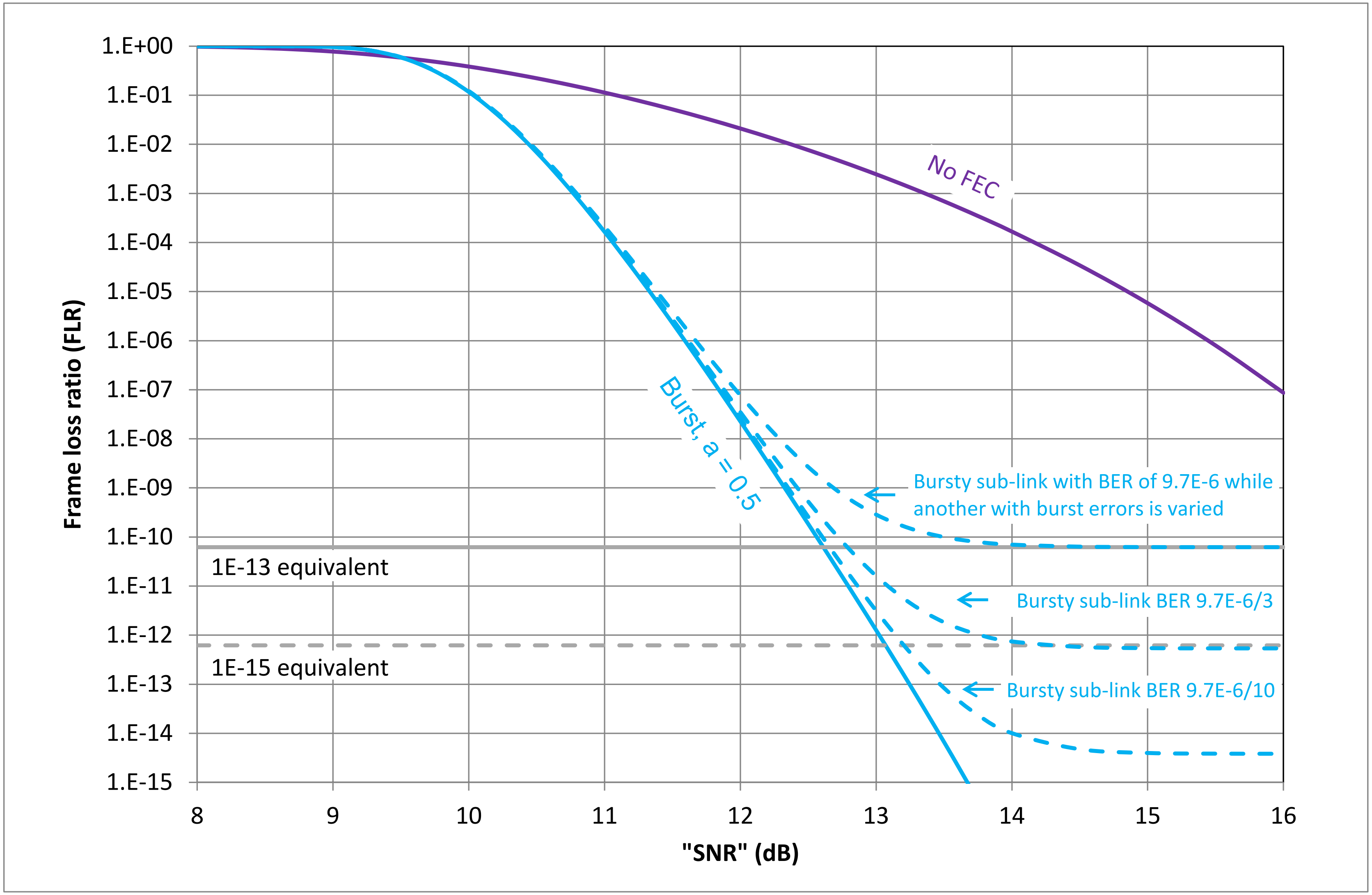
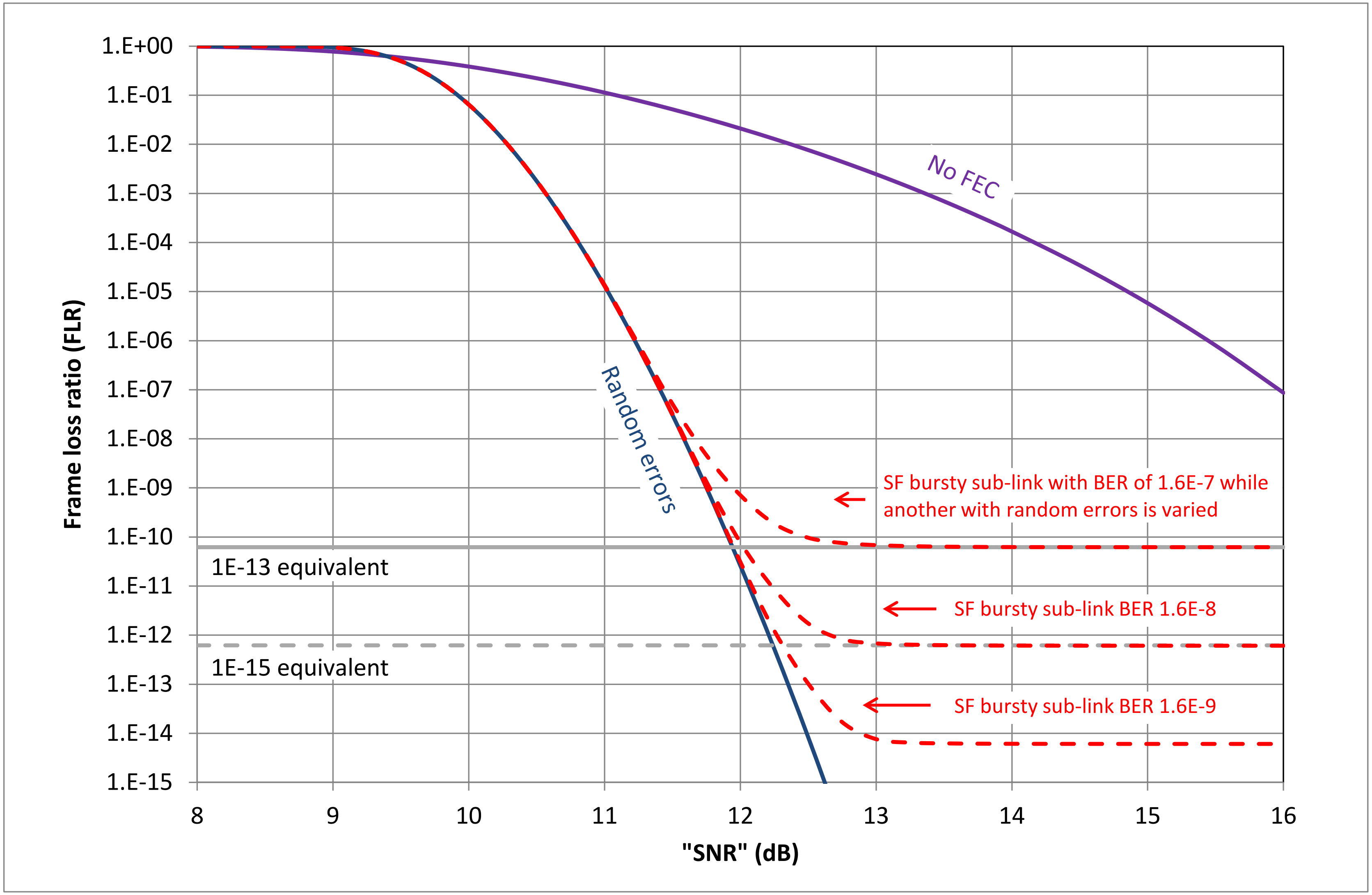


RS(528,514) multi-part (1:2 SF burst + random) *



RS(528,514) multi-part (1:2 SF burst + burst)

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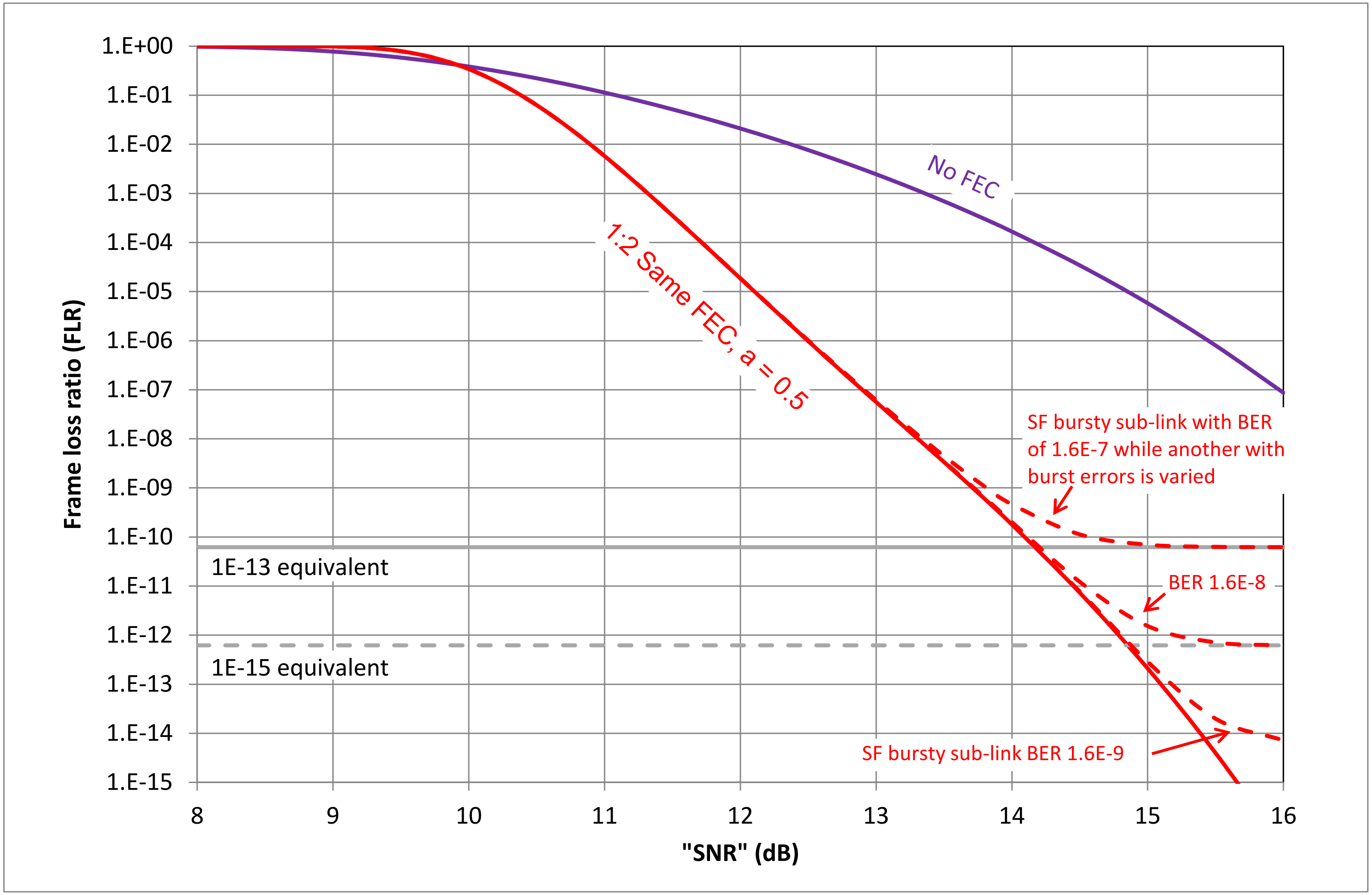
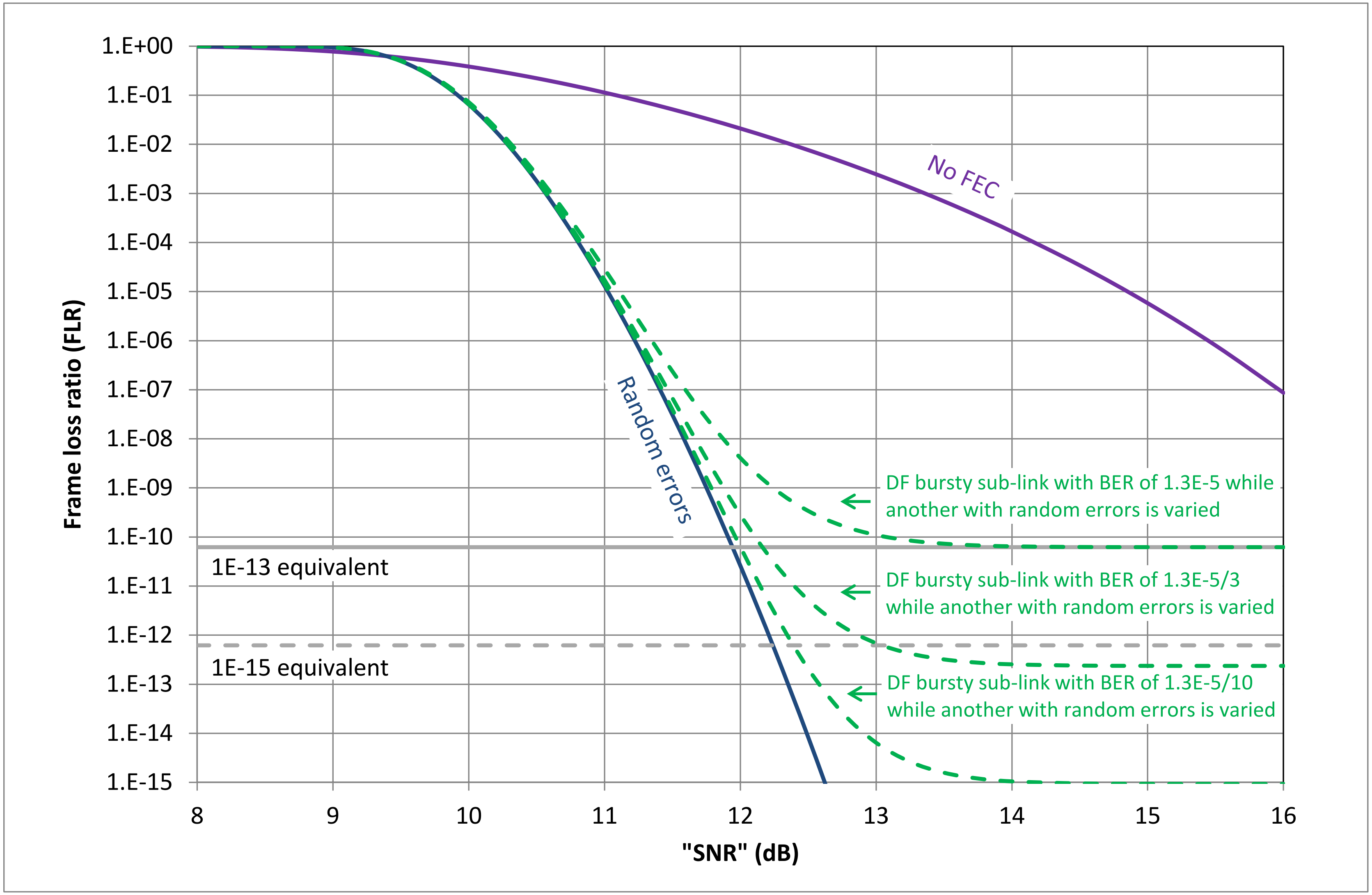
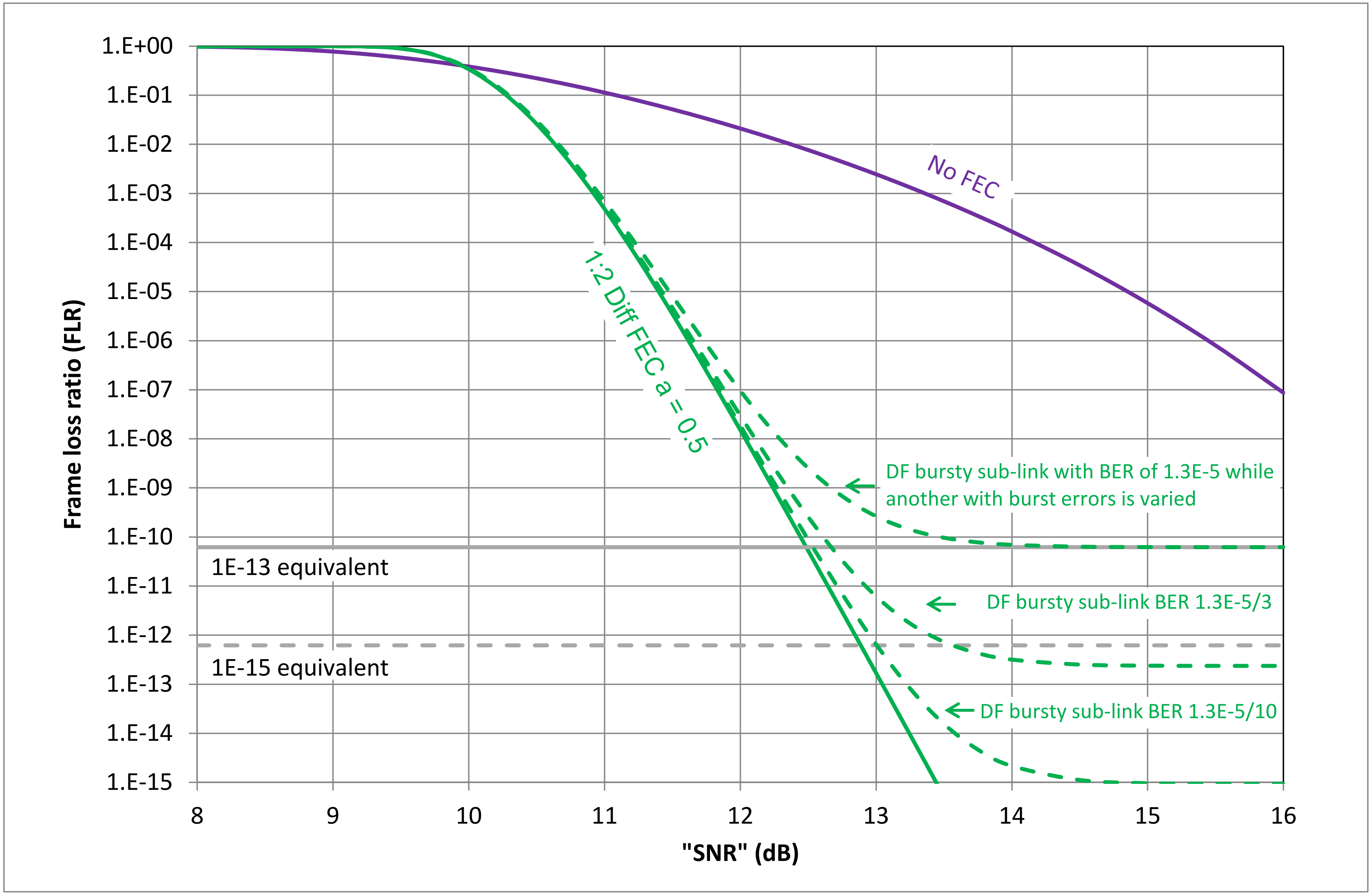


RS(528,514) multi-part (1:2 DF burst + random) *

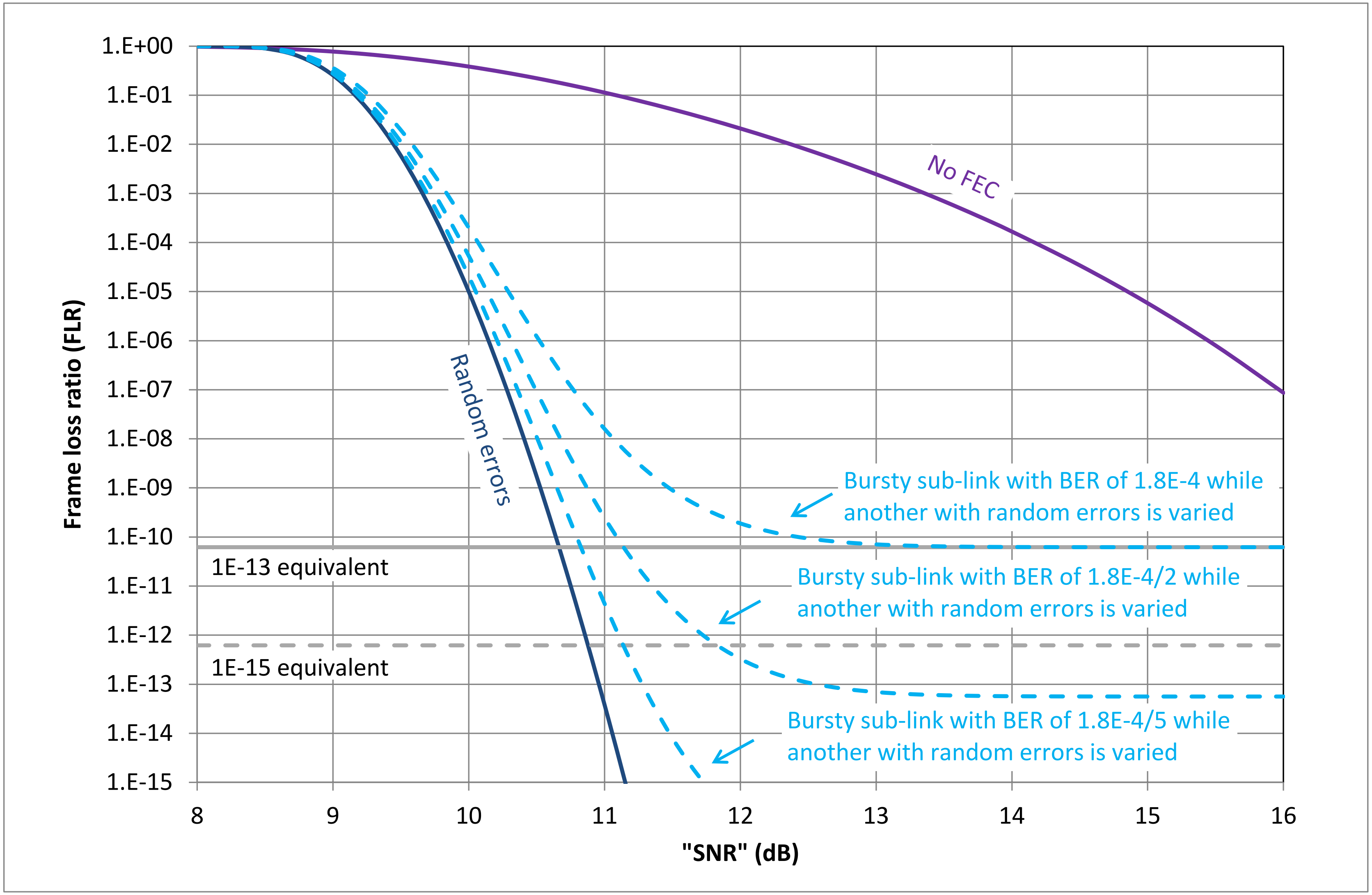


RS(528,514) multi-part (1:2 DF burst + burst)

*

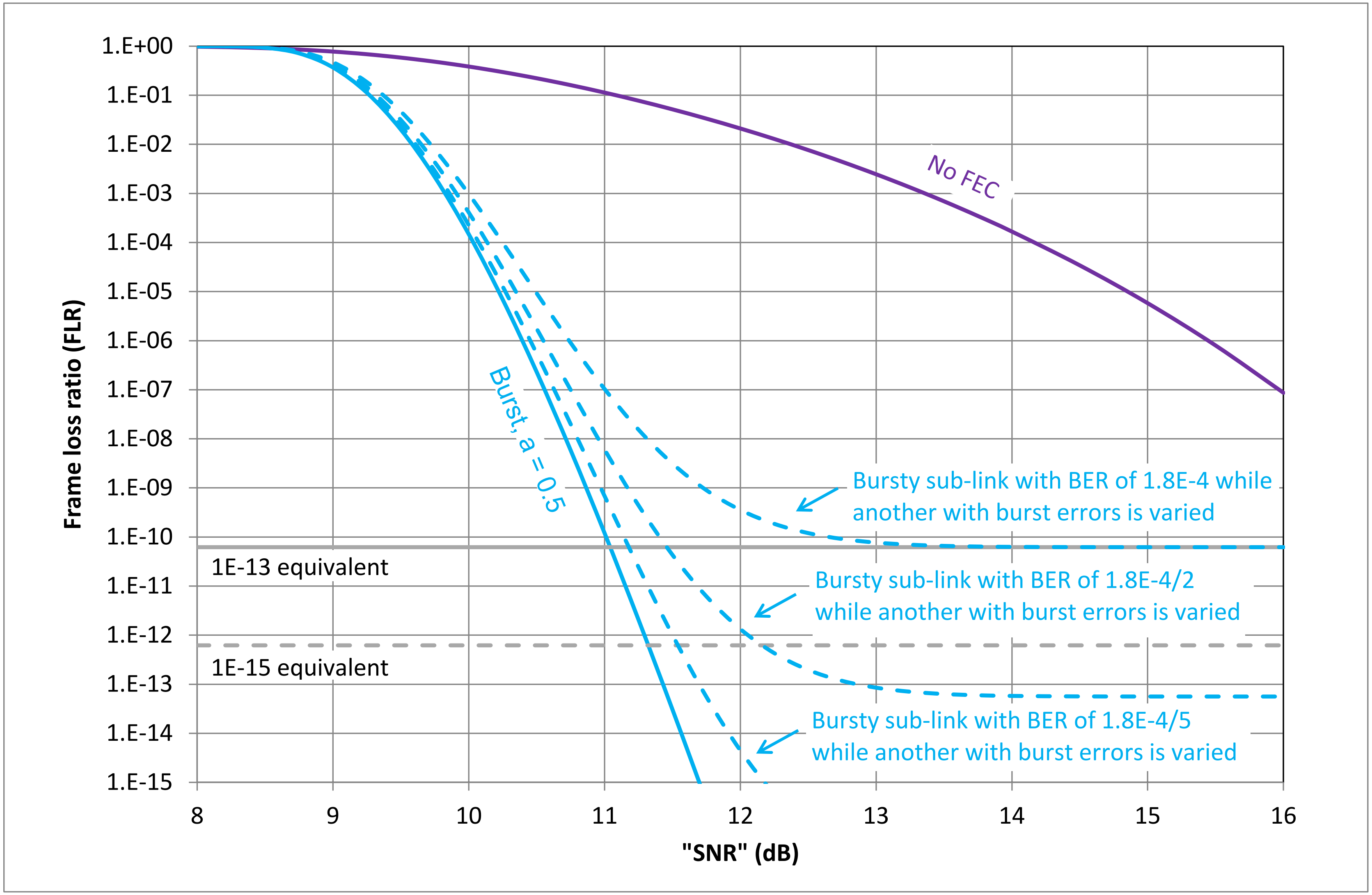


RS(544,514) multi-part non-int (burst + random) *

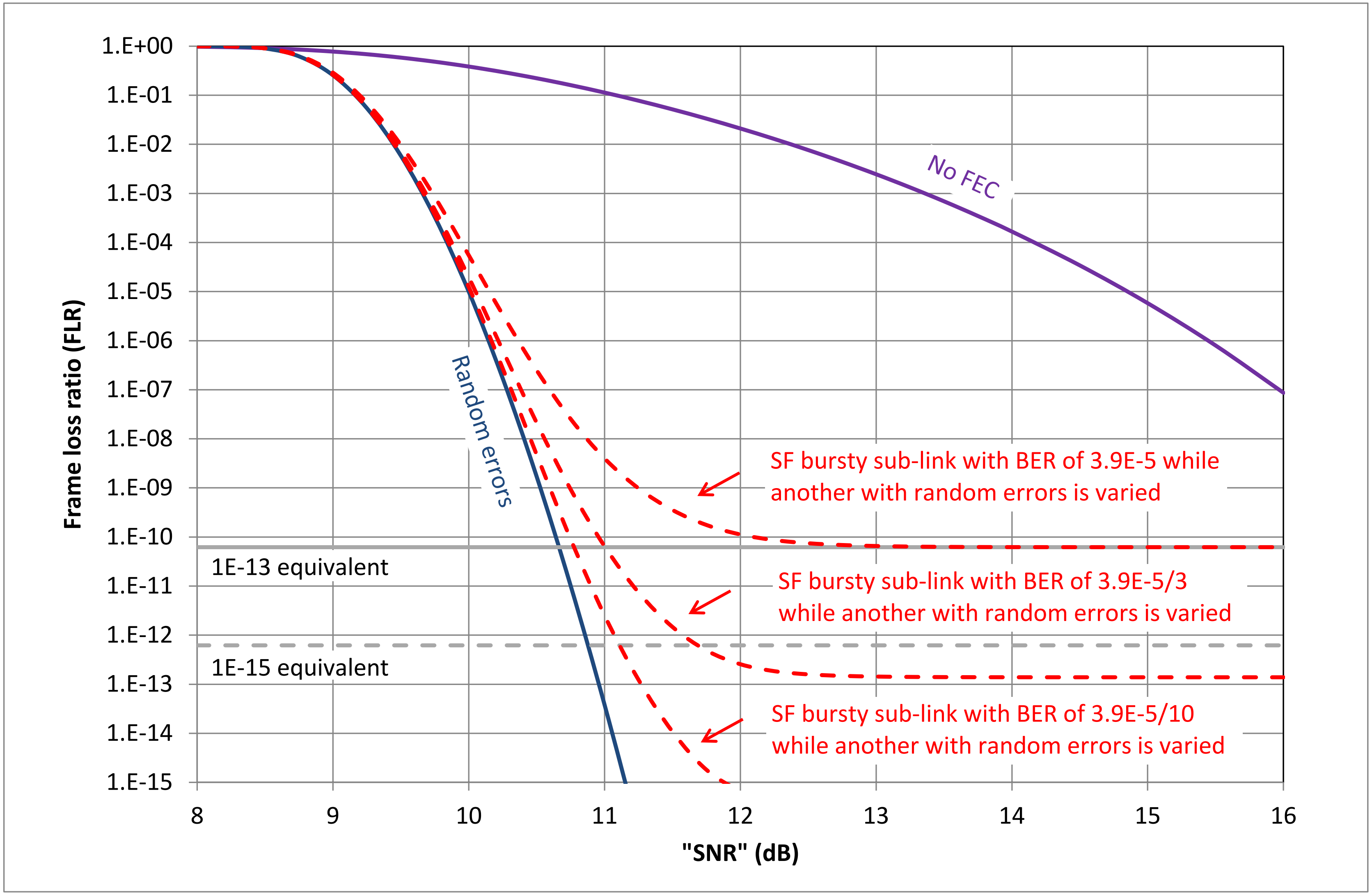


RS(544,514) multi-part non-int (burst + burst)

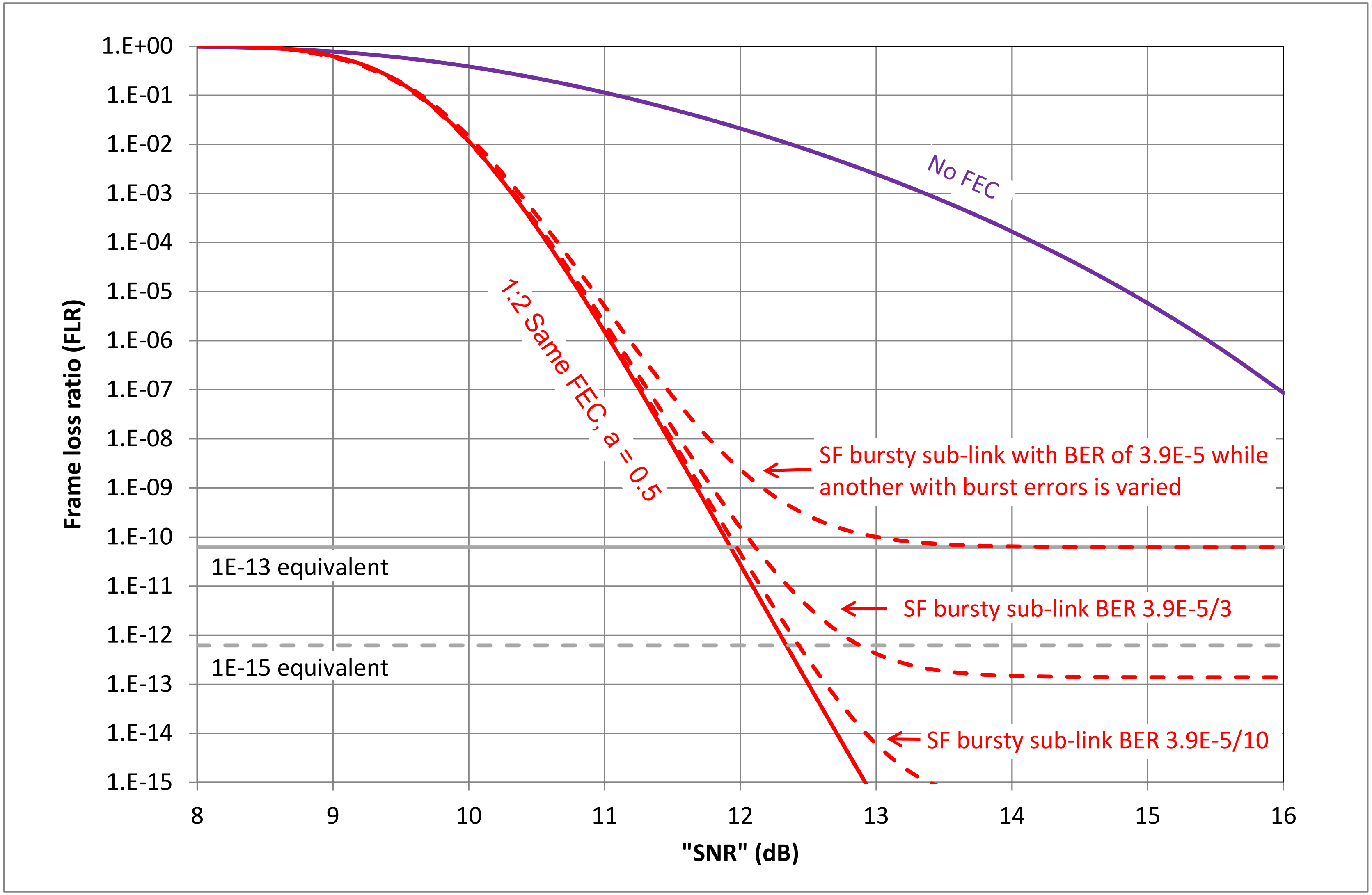
*



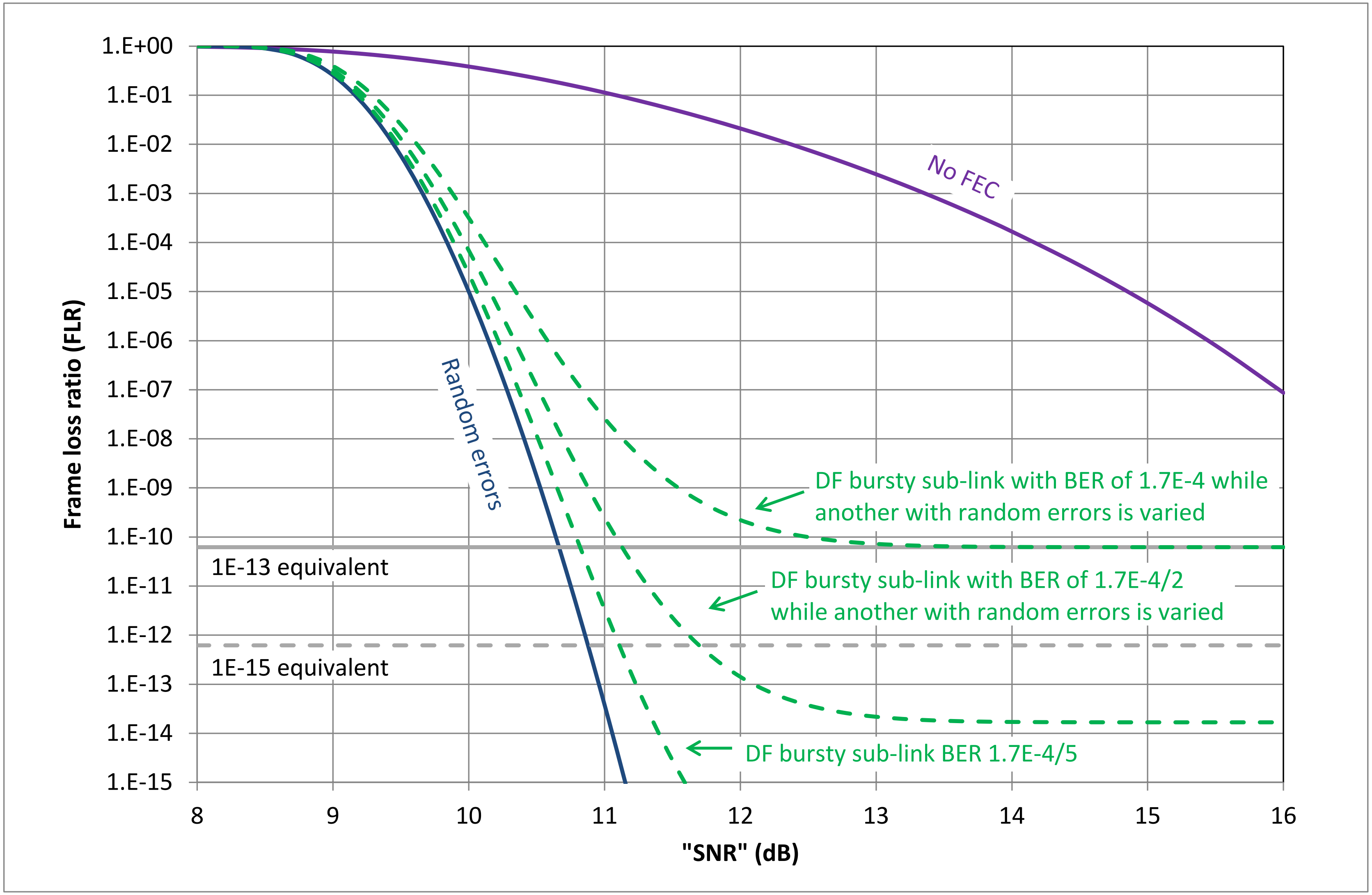
RS(544,514) multi-part (1:2 SF burst + random) *



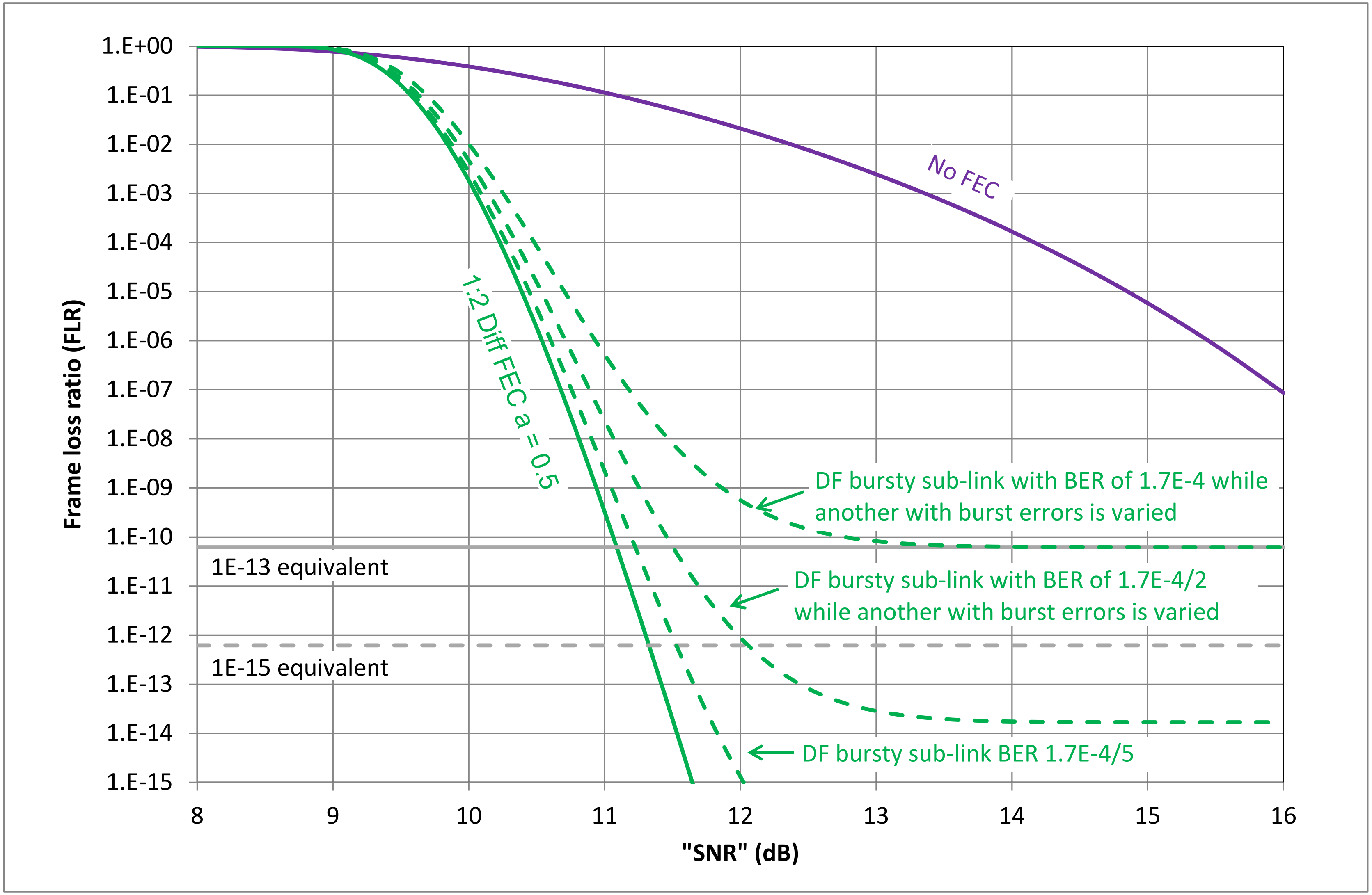
RS(544,514) multi-part (1:2 SF burst + burst) *



RS(544,514) multi-part (1:2 DF burst + random) *



RS(544,514) multi-part (1:2 DF burst + burst) *



Thanks!