

FEC performance on multi-part links

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Introduction

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When analysing the performance of 400GbE FEC encoded links, two of the aspects that need consideration are:

- The interaction of burst errors due to decision feedback equalization (DFE) and dis-interleaving of bit-streams from higher rate lanes.
- Covering several sub-links with a single end-to-end FEC.

This contribution provides an analysis of the performance of RS(528,514), RS(544,514) and BCH(2858,2570) FEC for various interleaving schemes and also provides an analysis of the BER limit for a bursty sub-link to give a small penalty in a second random or bursty sub-link for various interleaving schemes.

Note: slides with a "*" in the top right corner are new or have changed compared to [anslow_3bs_02_0914](#). For changed slides, the changes are shown in red.

Structure

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The slides giving details of how the calculations have been performed have been moved to Annex 1.

Performance plots for the various FEC schemes follow this slide for:

- Random errors
- Burst errors according to the model in Annex 1 with no interleaving
- 1:2 bit interleaving two lanes from the same FEC instance with bursts
- 1:2 bit interleaving two lanes from different FEC instances with bursts

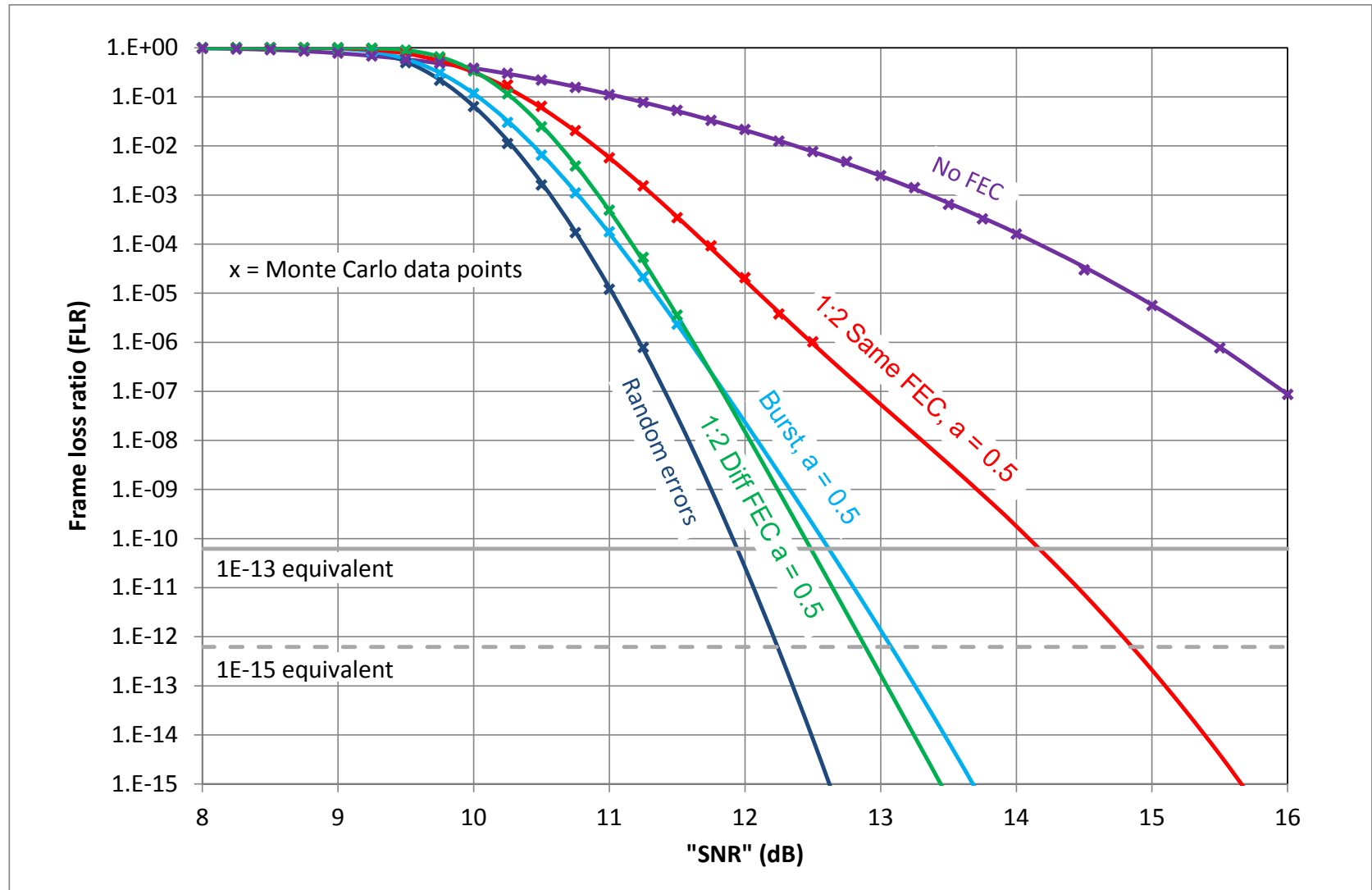
Next is a summary table that gives the pre-FEC BERs required to give FLRs equivalent to that of a BER of $1E-13$ and $1E-15$ for the scenarios given above.

This is followed by a summary table that gives the BERs for the electrical sub-links that would give a penalty of 0.1 dB optical in an optical sub-link in the same FEC domain.

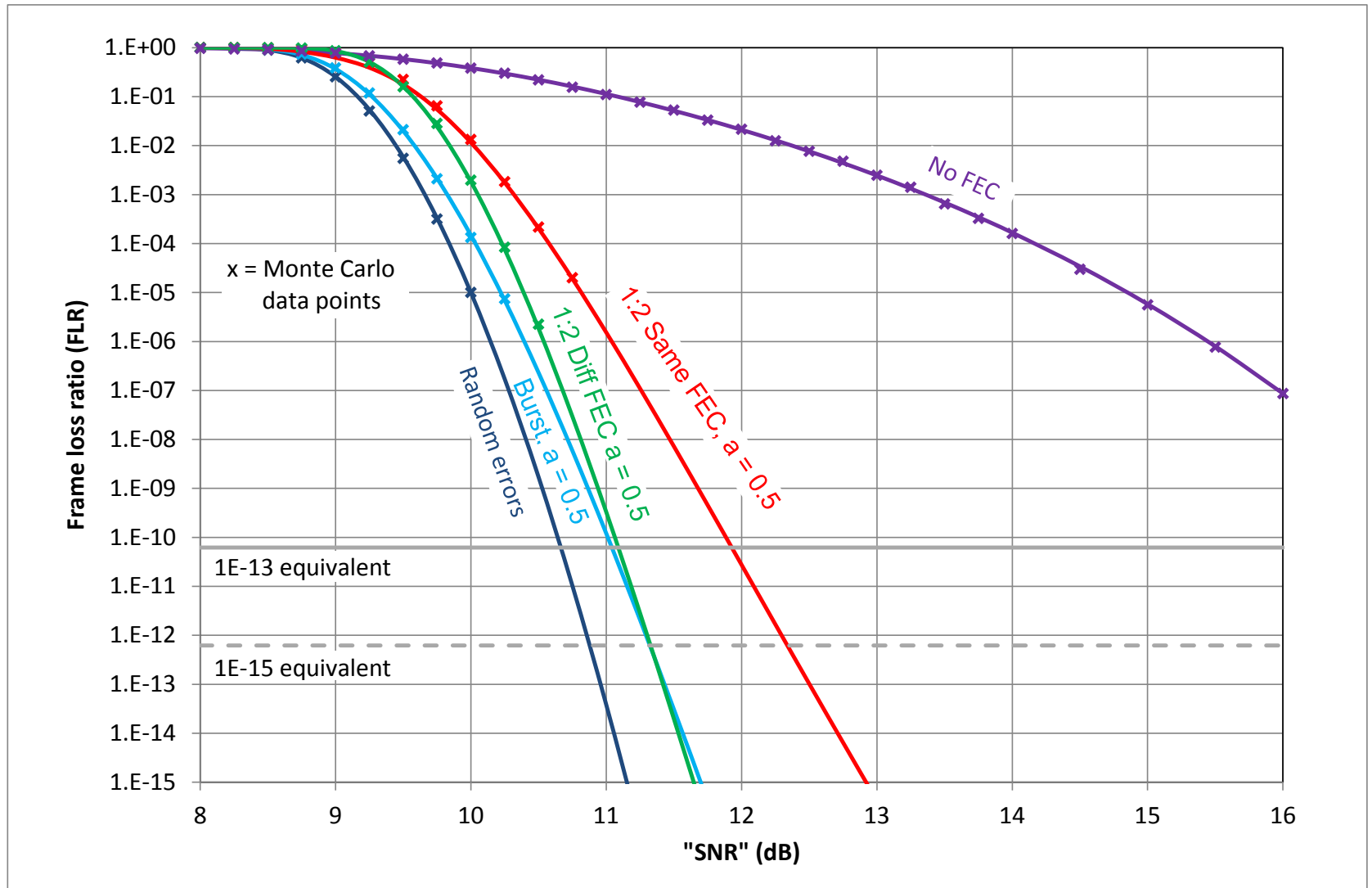
Annex 2 contains plots for a BER floor that is reachable by a Monte Carlo simulation in an attempt to validate the accuracy of each model.

Annex 3 then uses these models to generate plots down to more relevant FLRs for each case.

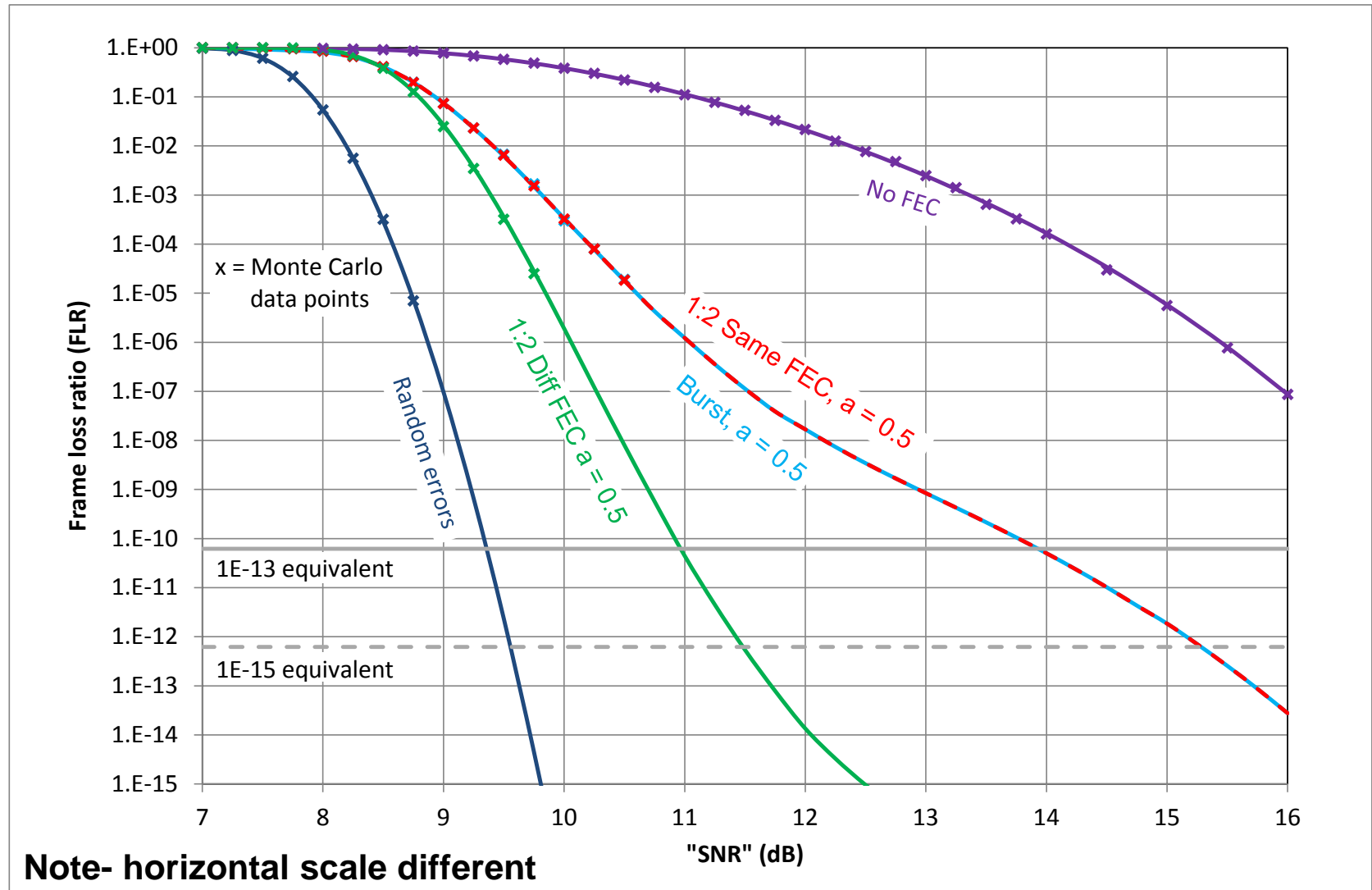
RS(528,514) all curves



RS(544,514) all curves



BCH(2858,2570) all curves



Note- horizontal scale different from previous two slides

Results for RS(528,514), RS(544,514), BCH(2858,2570) *

From the curves shown on the previous slide, the BERs (due to the “SNR”) at the FEC input required to give FLRs equivalent to that of a BER of 1E-13 and 1E-15 are:

	RS(528,514)		RS(544,514)		BCH(2858,2570) t=24	
	FLR = 6.2E-11	FLR = 6.2E-13	FLR = 6.2E-11	FLR = 6.2E-13	FLR = 6.2E-11	FLR = 6.2E-13
No FEC	1E-13	1E-15	1E-13	1E-15	1E-13	1E-15
1:2 Same FEC, a = 0.5	1.6E-7*	1.6E-8*	3.9E-5*	1.7E-5*	3.1E-7*	3.1E-9*
Single FEC burst, a = 0.5	9.7E-6*	3.3E-6*	1.8E-4*	1.2E-4*	3.1E-7*	3.1E-9*
1:2 Different FEC, a = 0.5	1.3E-5*	5.2E-6*	1.7E-4*	1.1E-4*	2.0E-4*	8.8E-5*
Random errors	3.8E-5	2.1E-5	3.2E-4	2.3E-4	1.7E-3	1.3E-3

Note – these values are the BER derived from equation 1 and do not include the additional errors due to the bursts. To account for burst errors, the values marked with “*” must be multiplied by 2 when a = 0.5.

(Yellow highlighted cells require BER below 1E-6)

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Multi-part link results

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The BERs (due to the “SNR”) of the electrical sub-link for a penalty of 0.1 dB optical in the optical sub-link are:

			RS(528,514)		RS(544,514)		BCH(2858,2570) t=24	
	Electrical	Optical	FLR = 6.2E-11	FLR = 6.2E-13	FLR = 6.2E-11	FLR = 6.2E-13	FLR = 6.2E-11	FLR = 6.2E-13
1:2 Same FEC, a = 0.5	Burst	Random	4.6E-8*	5.4E-9*	7.5E-6*	3.4E-6*	6.6E-9*	1.4E-10*
	Burst	Burst	7.5E-8*	8.4E-9*	1.4E-5*	7.1E-6*	1.4E-7*	1.7E-9*
Single FEC burst, a = 0.5	Burst	Random	2.9E-6*	1.0E-6*	4.4E-5*	2.9E-5*	6.6E-9*	1.4E-10*
	Burst	Burst	3.6E-6*	1.3E-6*	5.1E-5*	3.4E-5*	1.4E-7*	1.7E-9*
1:2 Different FEC, a = 0.5	Burst	Random	3.9E-6*	1.6E-6*	4.2E-5*	3.0E-5*	1.7E-5*	3.1E-6*
	Burst	Burst	4.6E-6*	2.0E-6*	4.7E-5*	3.4E-5*	5.5E-5*	2.6E-5*
Random errors	Random	Random	1.2E-5	7.2E-6	8.2E-5	6.2E-5	3.3E-4	2.8E-4

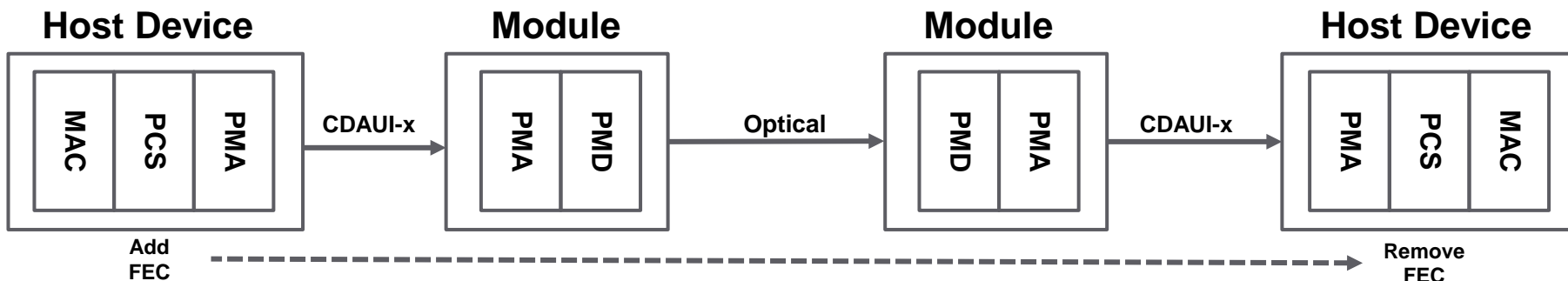
Note – these values do not include the additional errors due to the bursts. To account for burst errors, the values marked with “*” must be multiplied by 2 when a = 0.5.

(Yellow highlighted cells require electrical BER below 1E-6)

Conclusions

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For the multi sub-link systems analysed in this contribution, the limiting BER for the electrical sub-links that cause 0.1 dB penalty for the optical sub-link are shown on the previous slide.



For **many cases** a BER of $1E-6$ as contained in some of the OIF [baselines for 56G](#) is low enough to give less than 0.1 dB penalty in the optical sub-link.

However, error bursts can cause RS(528,514) or BCH(2858,2570) to need a pre-FEC BER of less than $1E-6$ which is not the case for RS(544,514).

Annex 1

Calculation principles

Reed-Solomon FEC with random errors

For a Reed-Solomon code RS(N,k), with symbol size of **m** and symbol correction capability **t**, if the input BER is:

$$BER_{in} = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{SNR}{2}} \right) \quad (1)$$

The input symbol error ratio (or the probability that an input symbol contains errors) SER_{in} is given by:

$$SER_{in} = 1 - (1 - BER_{in})^m \approx m \times BER_{in} \quad (\text{for small } BER_{in}) \quad (2)$$

The codeword error ratio (the probability of an uncorrectable FEC codeword) is:

$$CER = \sum_{i=t+1}^N \binom{N}{i} SER_{in}^i (1 - SER_{in})^{N-i} \quad (3)$$

The frame loss ratio (FLR) is then:

$$FLR = CER * (CER * 1 + (1 - CER) * (1 + MFC)/MFC) \quad (4)$$

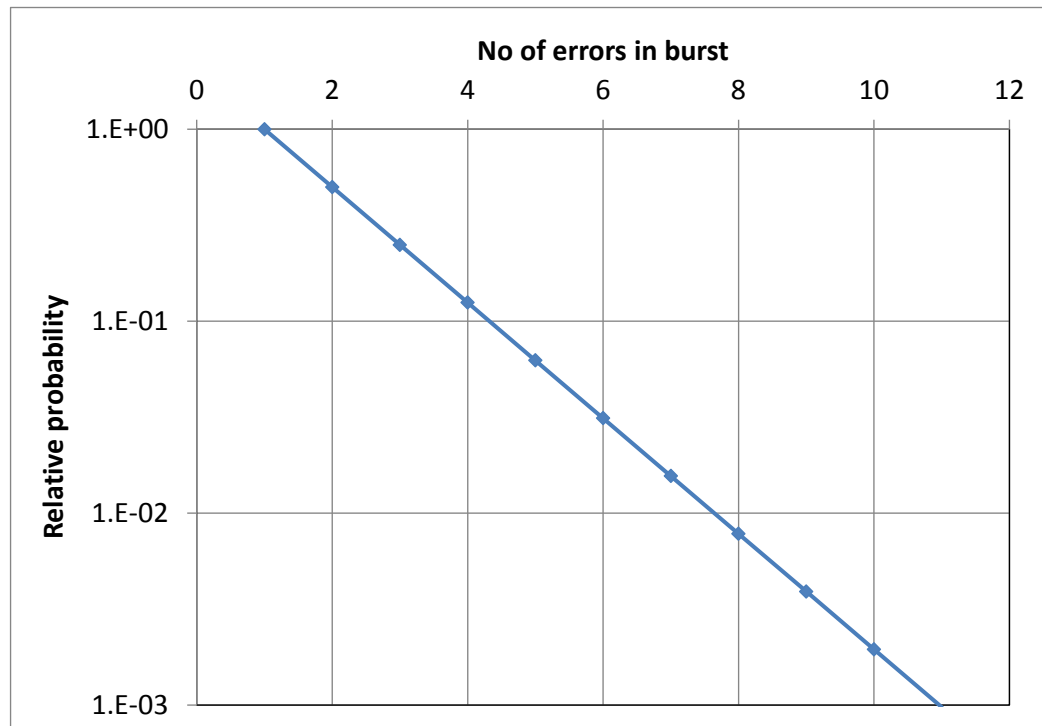
Where MFC is the number of MAC frames per codeword.

(This is an extension of slide 6 of [anslow_01a_1112_mmf](#) that still works for high CER)

Burst error model

The burst error probability is modelled in this contribution on the assumption that the probability of getting an error in the bit following an initial error (burst size of 2) is “a”, the probability of a burst of 3 errors is a^2 , the probability of a burst of 4 is a^3 , and so on. The worst case of $a=0.5$ is assumed throughout this contribution.

While $a=0.5$ may seem to be a pessimistic assumption, this simple model only applies to a 1-bit DFE. For a multi-bit DFE (particularly where the largest tap is not at 1 bit of delay) error burst patterns with several correct bits between errors are expected which makes the model used here not unreasonable.



RS(528,514) performance with bursts I

For the RS(528,514) code, $N=528$, $k=514$, $m=10$, $t=7$

Using similar analysis as slide 5 of [wang t 3bs 01 0514](#), the minimum burst size that can cause exactly 2 symbol errors is 2 bits, but this only occurs if the first error is the last bit of a symbol (i.e. a probability of 0.1).



If the burst size is 11 bits, then the probability that it will cause exactly 2 symbol errors is 1.



The maximum burst size that can cause exactly 2 symbol errors is 20 bits, but this only occurs if the first error is the first bit of a symbol (i.e. a probability of 0.1).



The probability of exactly two symbol errors (given an initial error) is:

$$P(2) = \sum_{i=1}^m \frac{i}{m} a^i (1-a) + \sum_{i=m+1}^{2m-1} \frac{2m-i}{m} a^i (1-a) = 0.0998 \text{ for } m=10, a=0.5 \quad (5)$$

RS(528,514) performance with bursts II

Similar analysis to that on the previous slide gives:

$P(1) = 0.9$, $P(2) = 0.0998$, $P(3) = 9.75E-5$, $P(4) = 9.52E-8$, etc.

These probabilities can then be used to calculate the overall probability of any combination of events that results in $t+1$ or more symbol errors:

$$\begin{aligned}
 \text{CER} = & \sum_{i=t+1}^N \binom{N}{i} \text{SER}_{\text{in}}^i (1 - \text{SER}_{\text{in}})^{N-i} + \binom{N}{1} \text{SER}_{\text{in}}^1 (1 - \text{SER}_{\text{in}})^{N-1} P(8) \\
 & + \binom{N}{2} \text{SER}_{\text{in}}^2 (1 - \text{SER}_{\text{in}})^{N-2} \left(P(4)P(4) + \binom{2}{1} P(3)P(5) + \binom{2}{1} P(2)P(6) + \dots \right) \\
 & + \binom{N}{3} \text{SER}_{\text{in}}^3 (1 - \text{SER}_{\text{in}})^{N-3} \left(\binom{3}{1} P(2)P(3)P(3) + \binom{3}{1} P(2)P(2)P(4) + \dots \right) \\
 & + \dots \\
 & + \binom{N}{7} \text{SER}_{\text{in}}^7 (1 - \text{SER}_{\text{in}})^{N-7} \left(\binom{7}{1} P(1)^6 P(2) + \binom{7}{2} P(1)^5 P(2)P(2) + \dots \right) \quad (7)
 \end{aligned}$$

Monte Carlo analysis

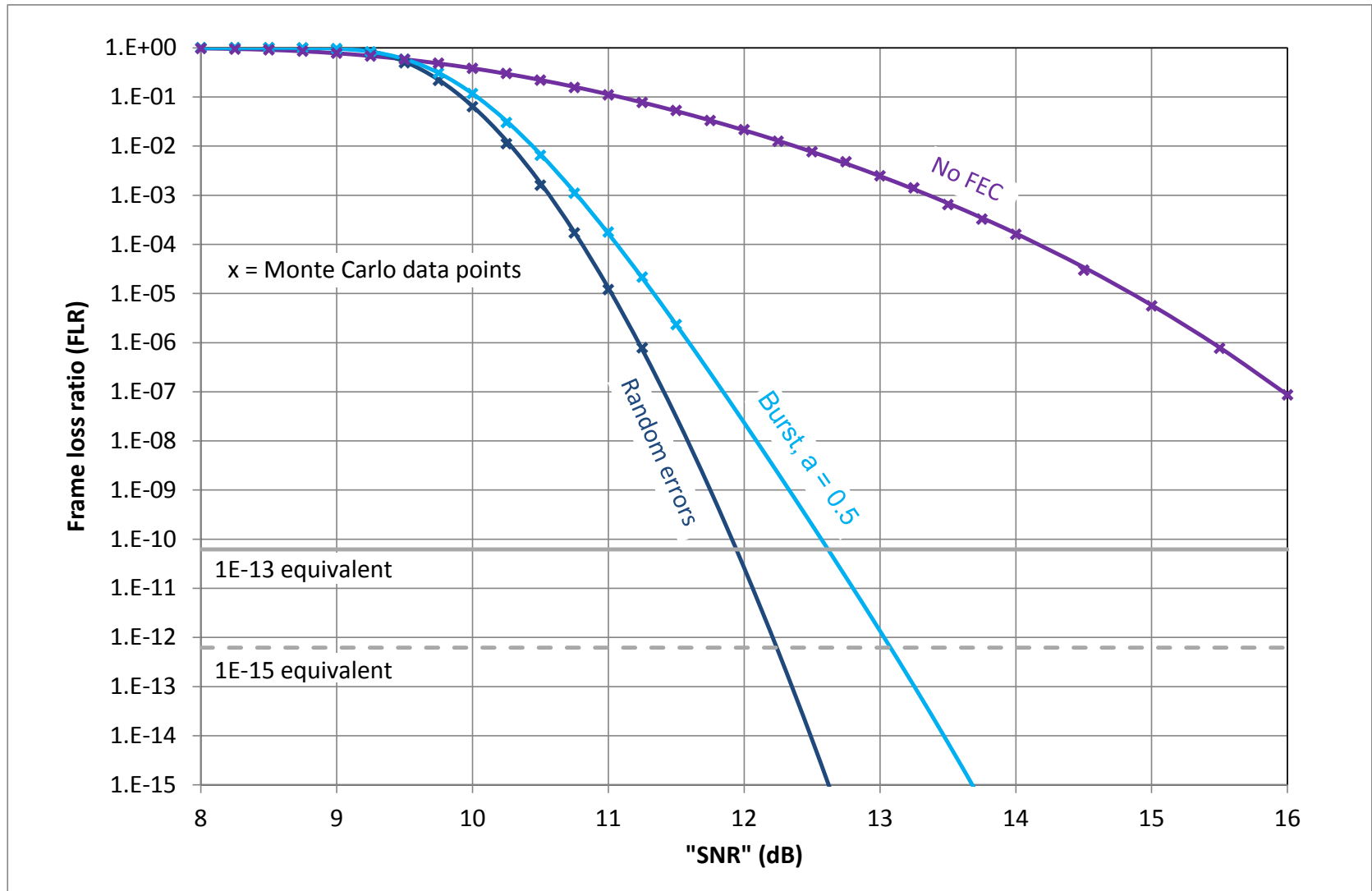
In an attempt to verify that the formulae on the previous slides give plausible results, a Monte Carlo based simulation was also performed using a random number generator to generate errors in a large number of codewords both with and without burst errors.

The results of plotting equations 1, 3, and 4 for random errors and equations 1, 5, and 4 for burst errors is shown on the next slide.

Also shown via “x” markers are the results of the Monte Carlo simulations which show good agreement with the formulae down to the probability where the simulations become too time consuming.

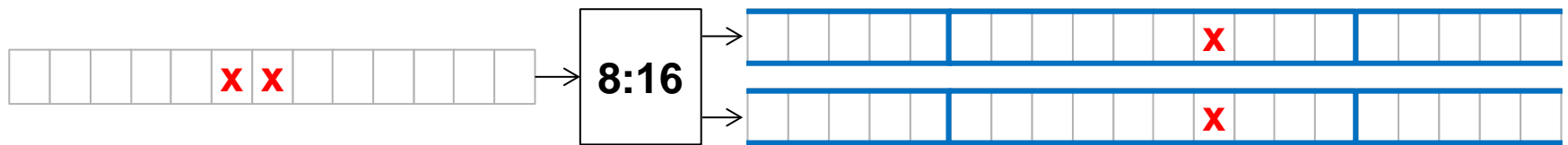
Note – the vertical axis for the plots is Frame Loss Ratio (FLR) for 64-octet MAC frames since uncorrectable codewords have to be discarded to provide an adequate Mean Time To False Packet Acceptance (MTTFPA).

RS(528,514) performance with bursts

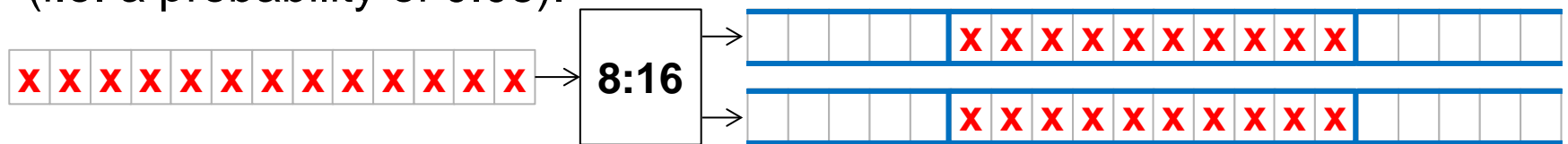


Bit interleaving – same FEC instance

For the case of bit interleaving two lanes from the same FEC instance, the minimum burst size that can cause exactly 2 symbol errors is again 2 bits, but here the probability of causing exactly 2 symbol errors is 1.



The maximum burst size that can cause exactly 2 symbol errors is 20 bits, but this only occurs if the first two errors are the first bits of a symbol (i.e. a probability of 0.05).



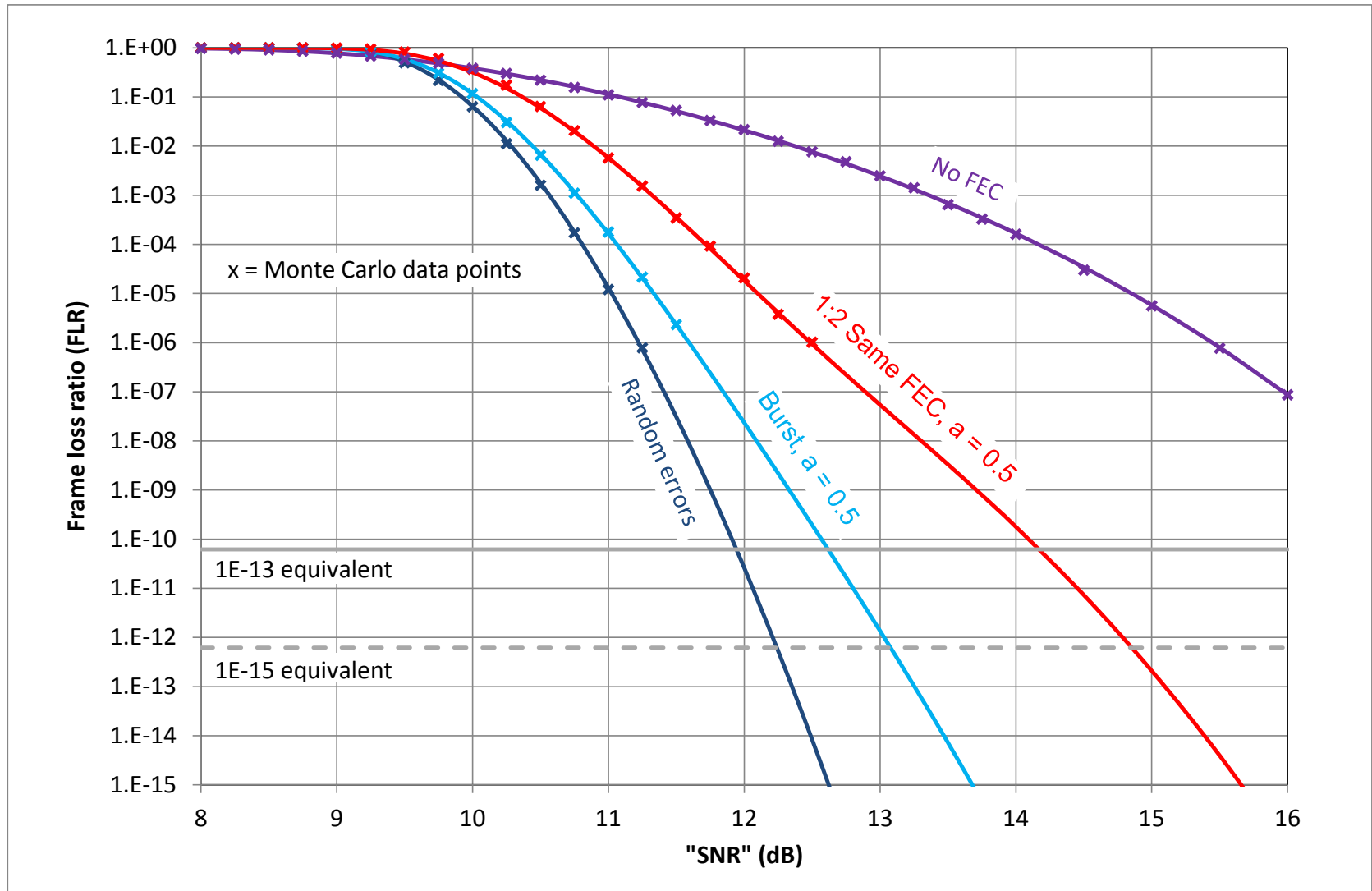
Note – this analysis assumes no skew between the two lanes as this is worse than assuming random alignment.

The probability of exactly two symbol errors (given an initial error) is:

$$P(2) = a(1-a) + \sum_{i=2}^{2m-1} \frac{2^{m-i}}{2^m} a^i (1-a) = 0.4625 \text{ for } m = 10, a = 0.5 \quad (8)$$

$P(1) = 0.5, P(2) = 0.4625, P(3) = 0.025, P(4) = 0.0125, P(5) = 2.4E-8$ etc.

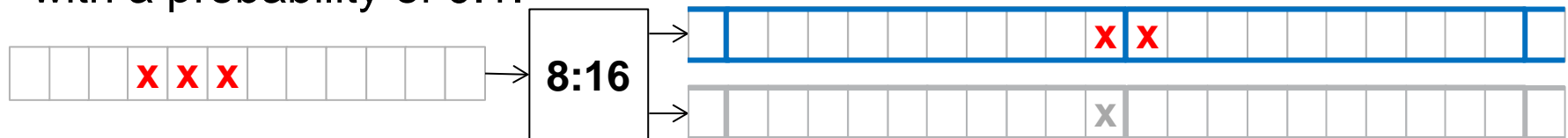
RS(528,514) with 1:2 interleaving and bursts



Bit interleaving – different FEC instance

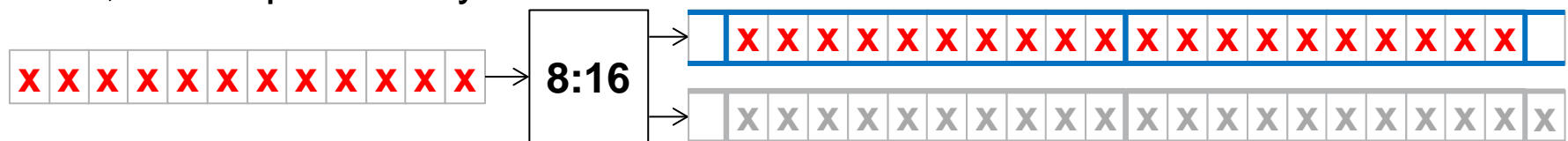
The case of bit interleaving two lanes from different FEC instances can be analysed by considering just the bits from one FEC instance with an input BER increased by a factor of $1 + a$.

The minimum burst size that can cause exactly 2 symbol errors is 3 bits, with a probability of 0.1.



If the burst size is 22 bits, then the probability that it will cause exactly 2 symbol errors is 1.

The maximum burst size that can cause exactly 2 symbol errors is 41 bits, with a probability of 0.1.



The probability of exactly two symbol errors (given an initial error) is:

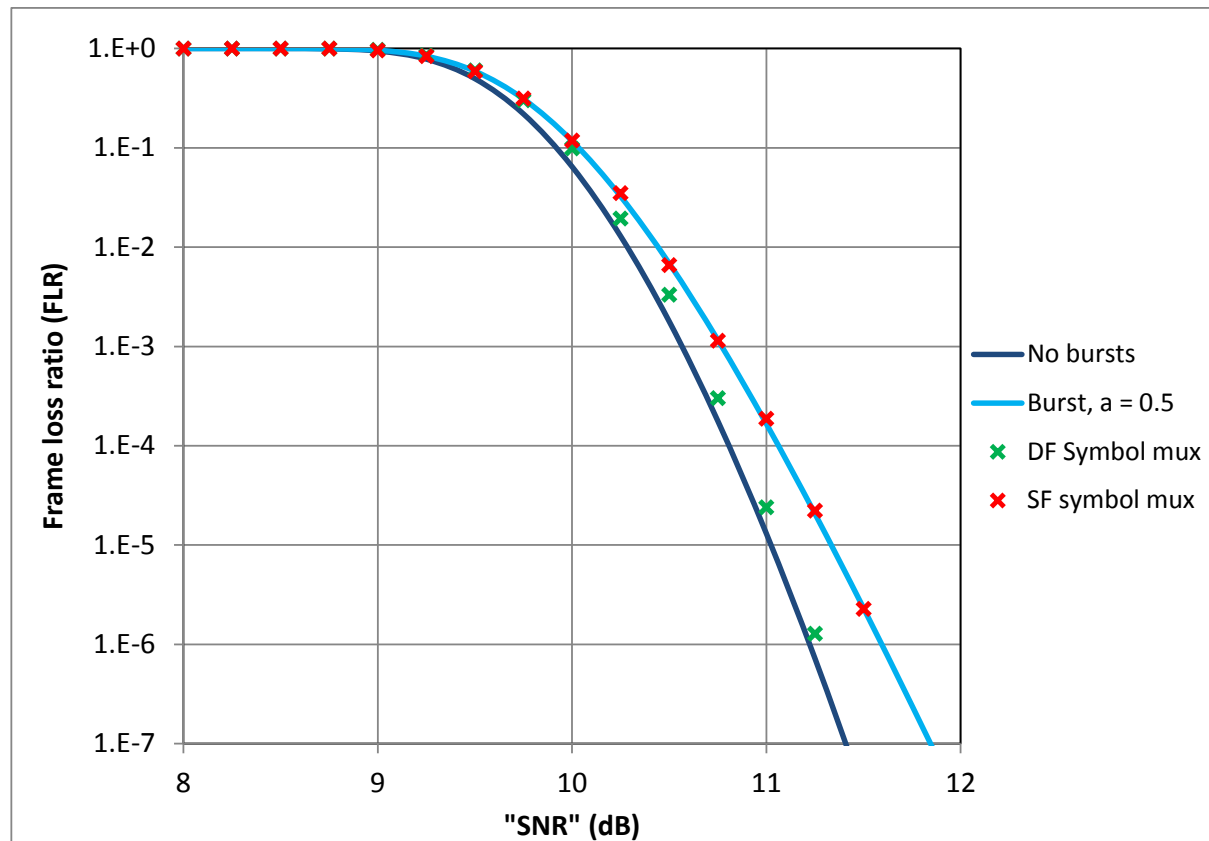
$$P(2) = \sum_{i=2}^{2m+1} \frac{\text{floor}(i/2)}{m} a^i (1-a) + \sum_{i=2m+2}^{4m-1} \frac{2m - \text{floor}(i/2)}{m} a^i (1-a) = 0.0333 \quad (9)$$

$P(1) = 0.9667$, $P(2) = 0.0333$, $P(3) = 3.2E-8$ etc.

Symbol interleaving for RS codes

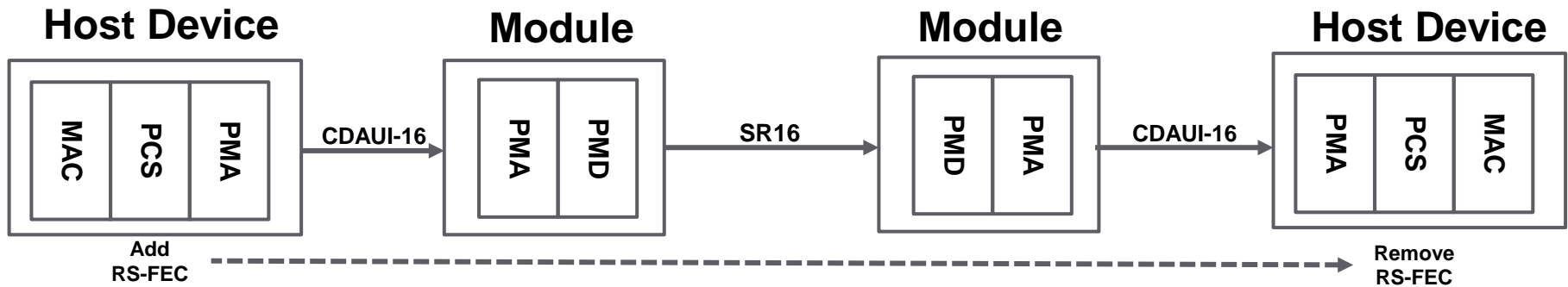
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Monte Carlo analysis of symbol interleaving shows that, as expected, 1:2 same FEC (SF) with bursts shows the same performance as for non-interleaved FEC with bursts and 1:2 different FEC (DF) with bursts has only slightly worse performance than for random errors. **RS(528,514)** results shown below:



What about multi-part links with FEC?

If the FEC bytes are added at the source PCS layer and then the correction is applied only at the destination PCS layer as in:



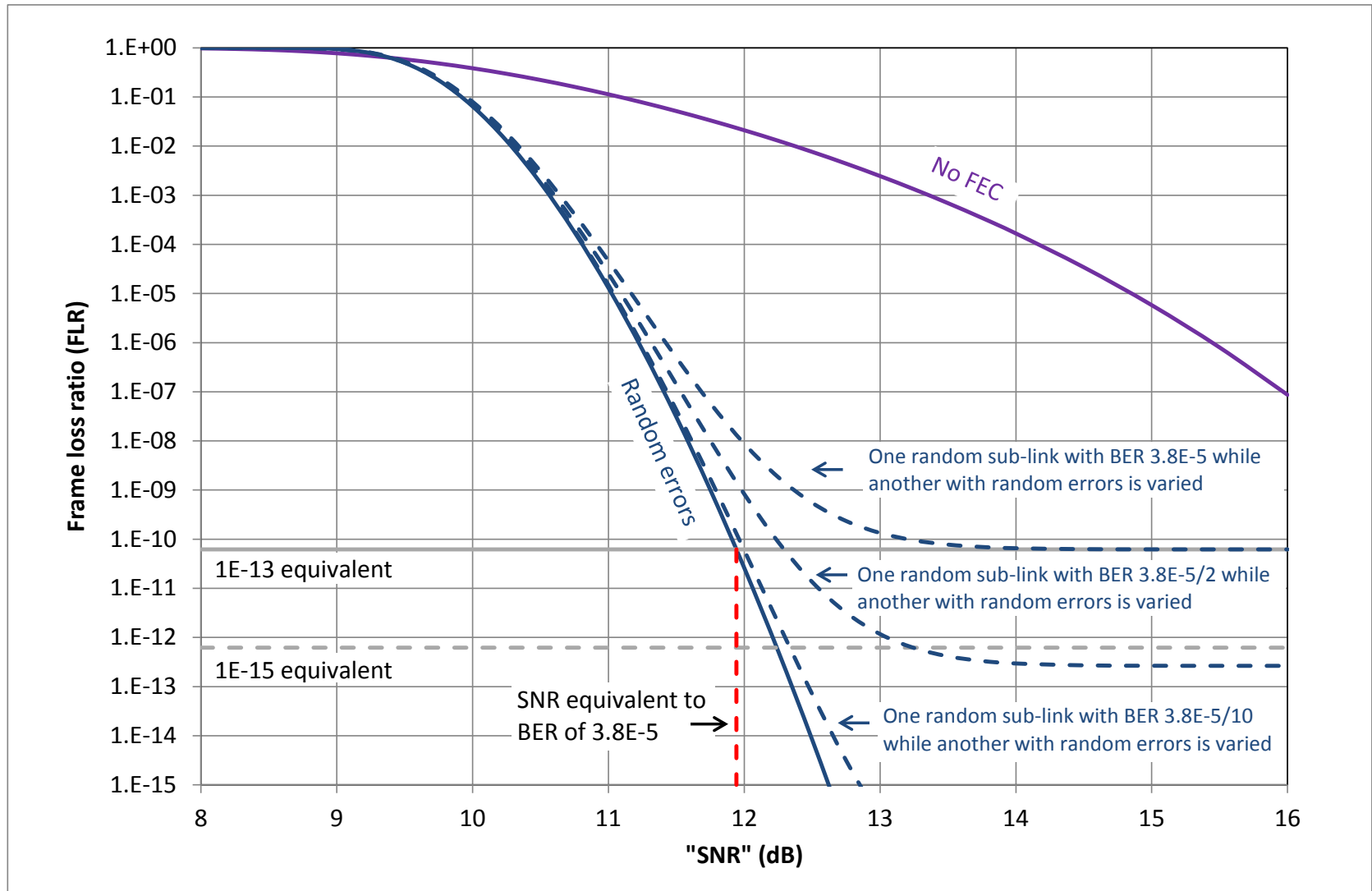
Source: [gustlin 3bs 02 0514](#)

Then the worst case input BER for the FEC decoder must be met by the concatenation of all of the sub-links.

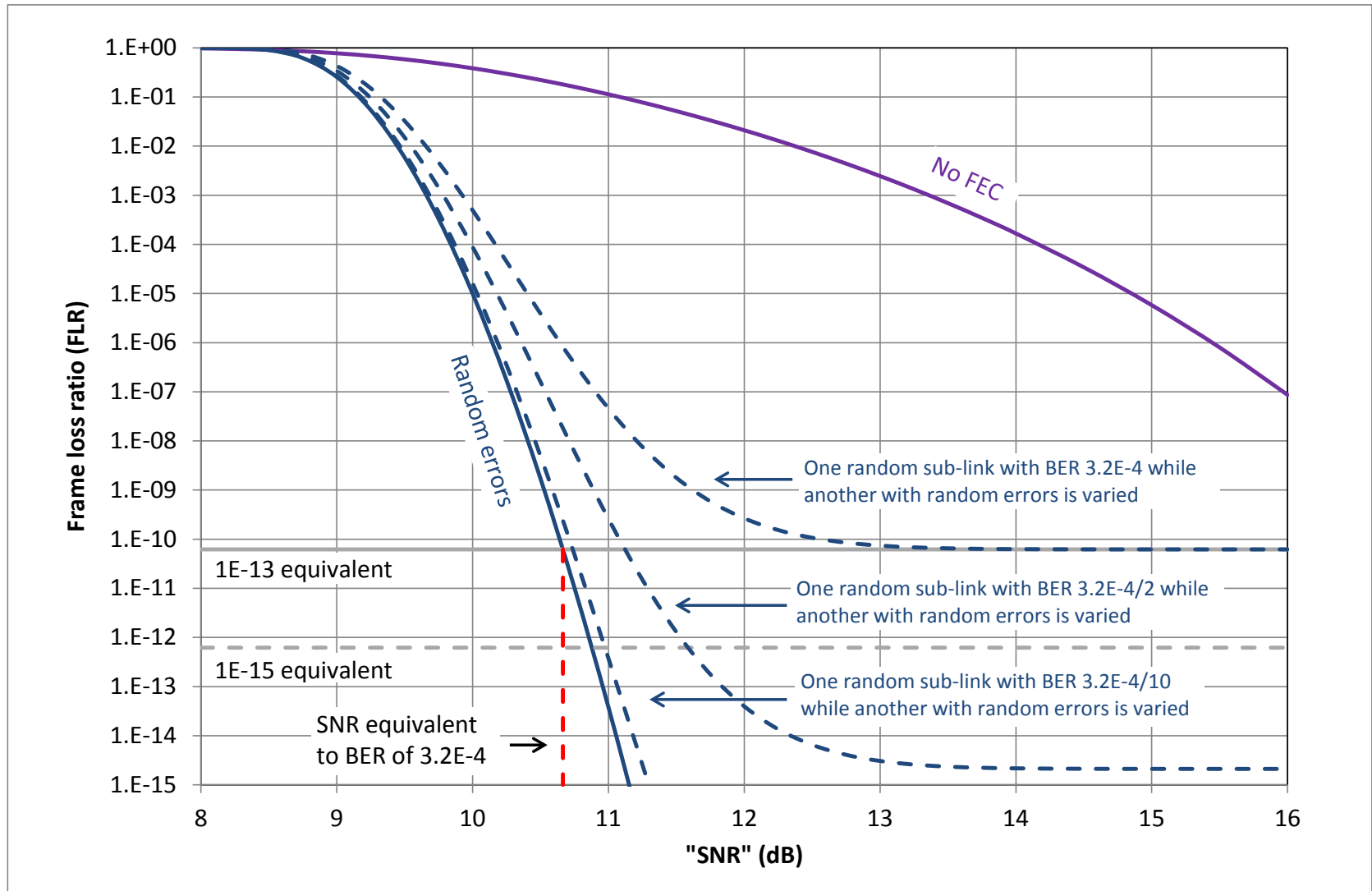
In the case of CAUI-10 -> SR10 -> CAUI-10, the worst case BER for each of the sub-links is $1E-12$ which is the same as the end-to-end requirement. This situation is tolerated on the basis that it is unlikely that all three sub-links will be at the worst case BER at the same time and if two of them were, then the end-to-end BER would still be $2E-12$.

The situation for a pair of sub-links sharing the same RS(528,514) or RS(544,514) protection is shown on the next two slides.

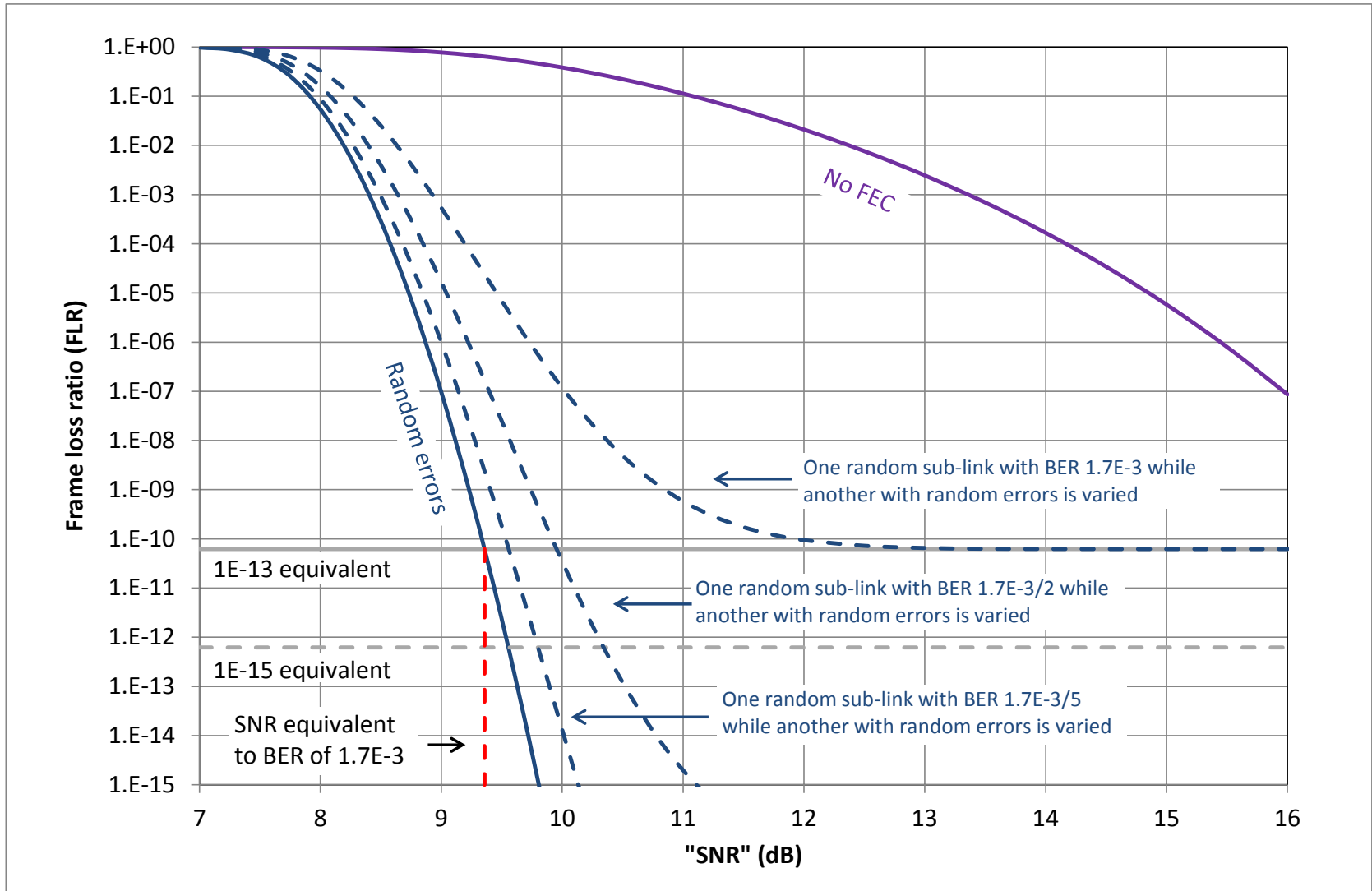
RS(528,514) multi-part (random)



RS(544,514) multi-part (random)



BCH(2858,2570) multi-part (random)



Sub-links with bursts

The curves on the previous slides are all for random errors.

What if there are burst errors in the electrical sub-links but random errors in the optical part of the link, or bursts in both?

Similar principles as previously can be used for this, except that instead of calculating the probability of 8 or more symbol errors, the calculation finds the probability of 7, 6, 5, 4, 3, 2, and 1 symbol errors due to the bursty sub-link. This is then combined with the probability of 1, 2, 3, 4, 5, 6, or 7 symbol errors due to the random sub-link or due to a bursty sub-link.

Plots covering mixtures of random and bursty sub-links as well as two bursty sub-links are included in Annex 2 and Annex 3 to this presentation.

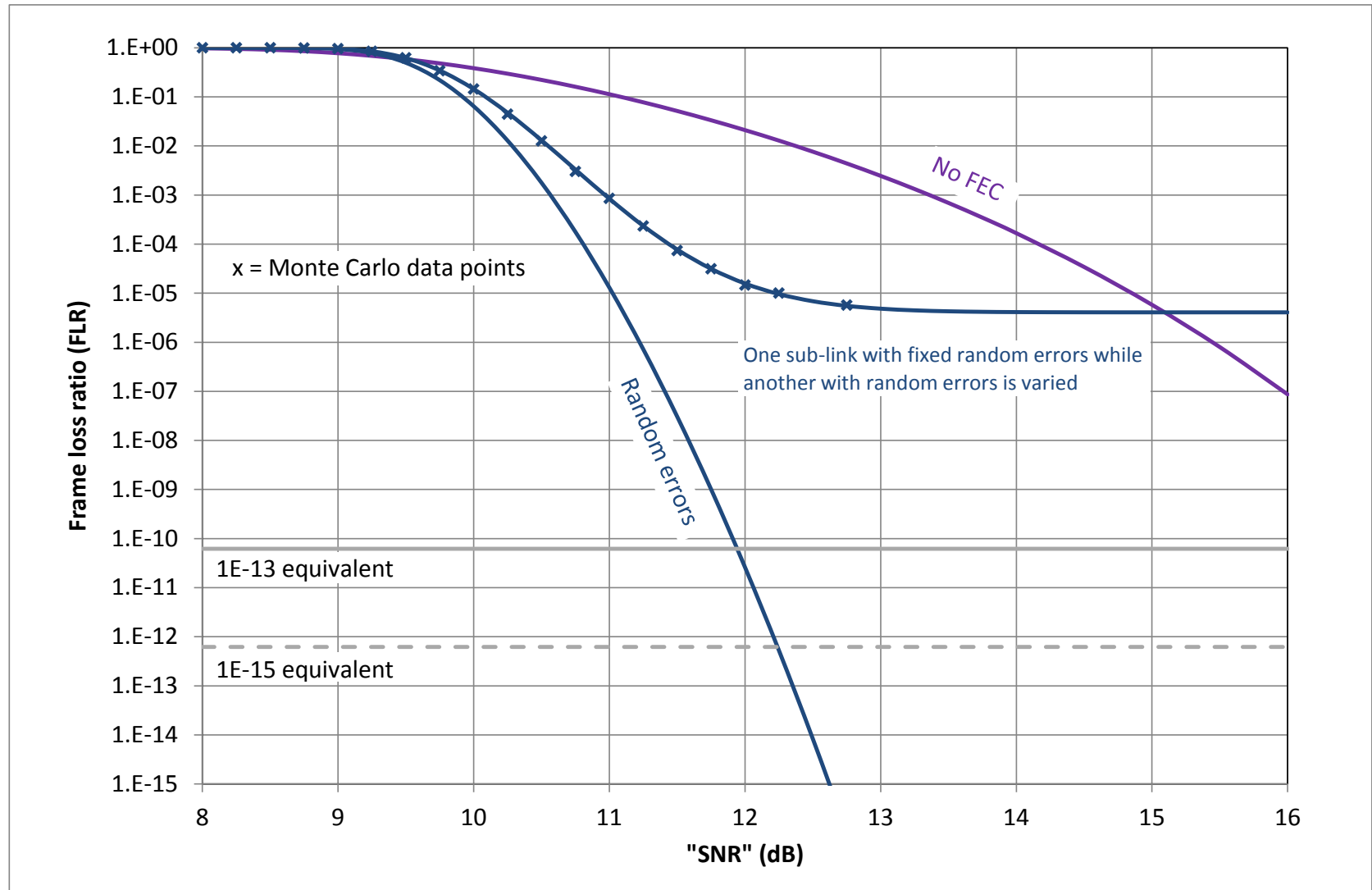
Annex 2 contains plots for a BER floor that is reachable by a Monte Carlo simulation in an attempt to validate the accuracy of each model. These show a good match between the analytical model and the Monte Carlo results.

Annex 3 then uses these models to generate plots similar to the previous two slides for each case.

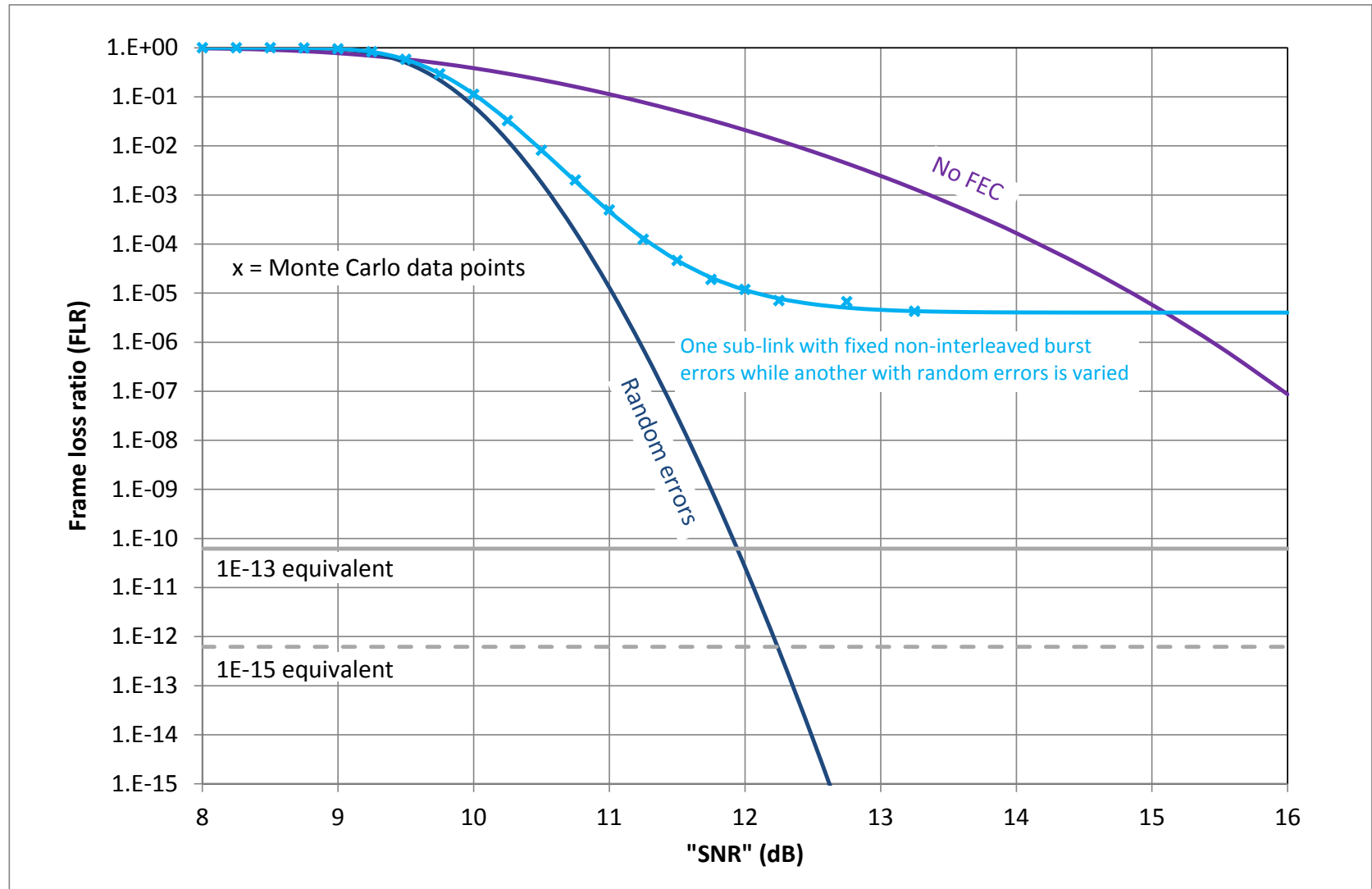
Annex 2

Plots with high FLR floors and Monte Carlo simulation results to validate models

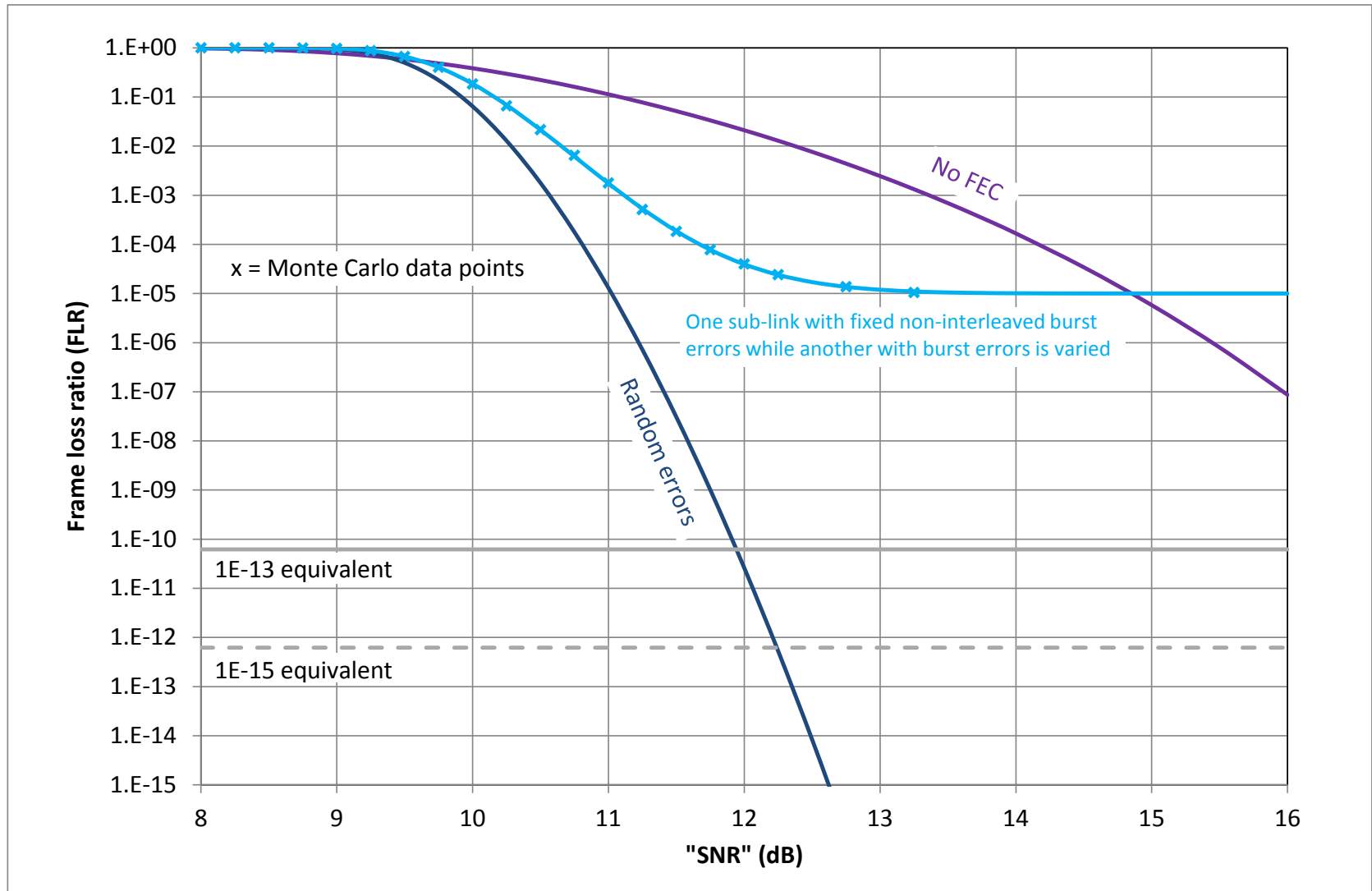
RS(528,514) multi-part (random + random)



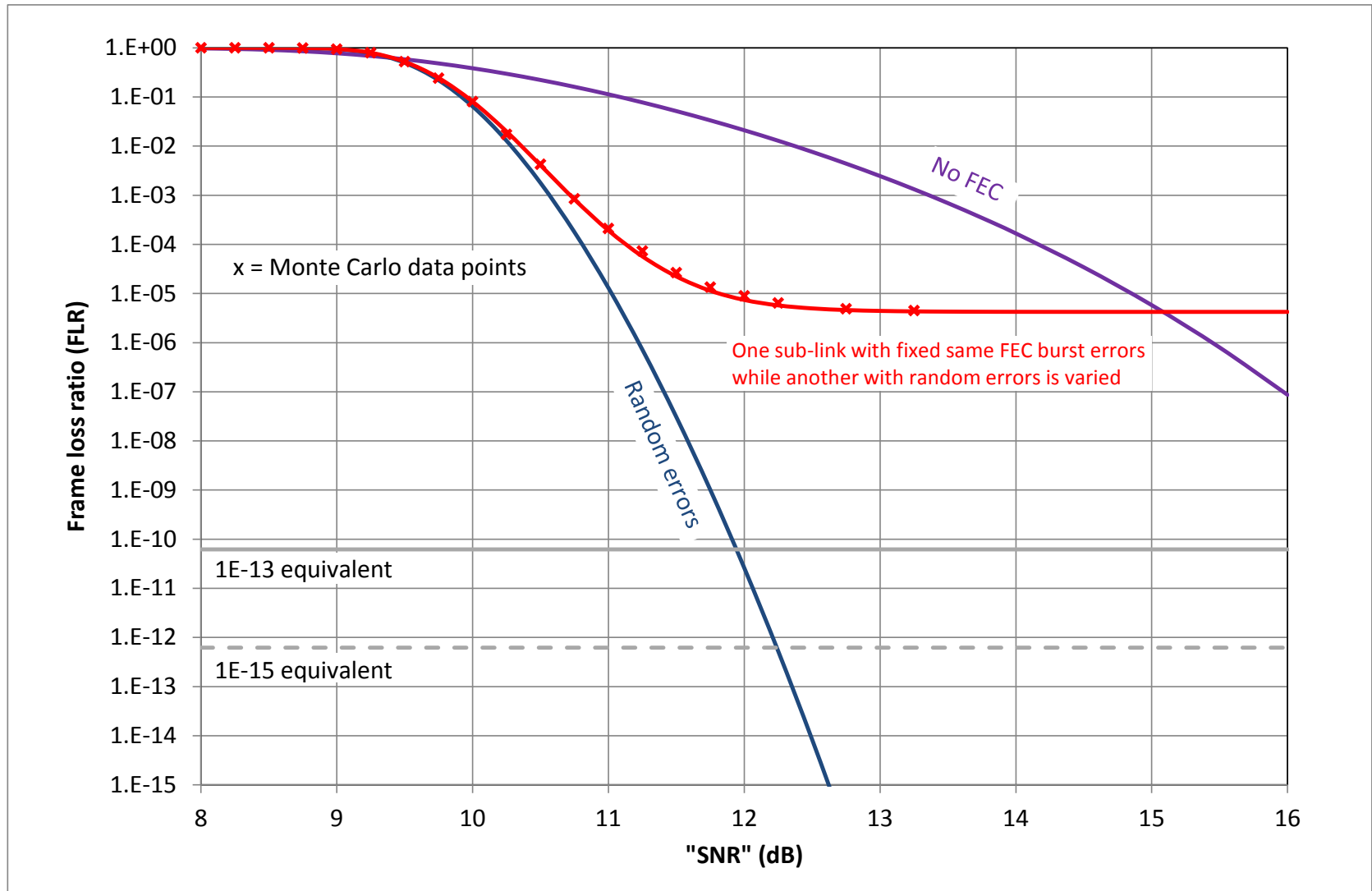
RS(528,514) multi-part non-int (burst + random)



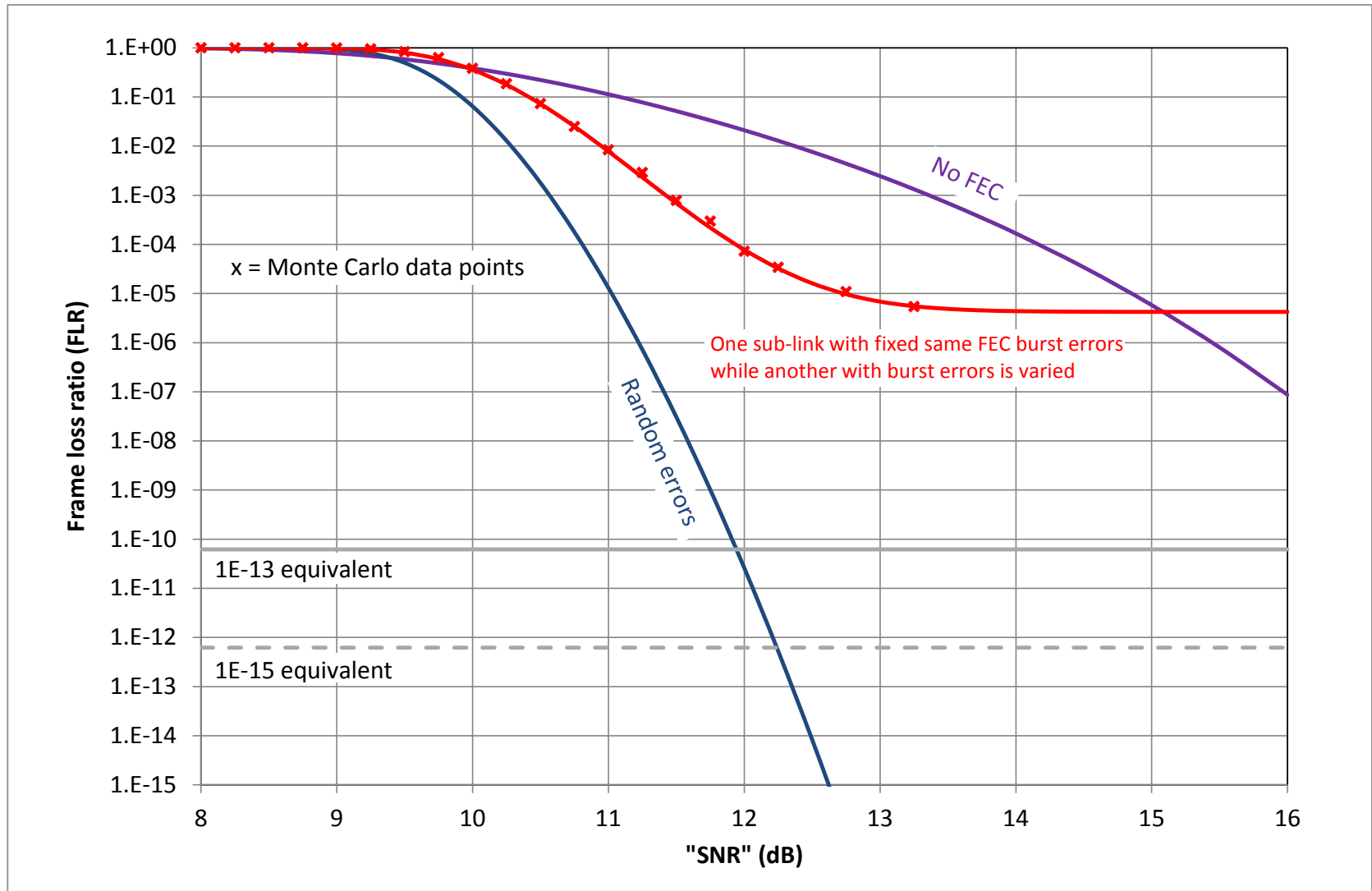
RS(528,514) multi-part non-int (burst + burst)



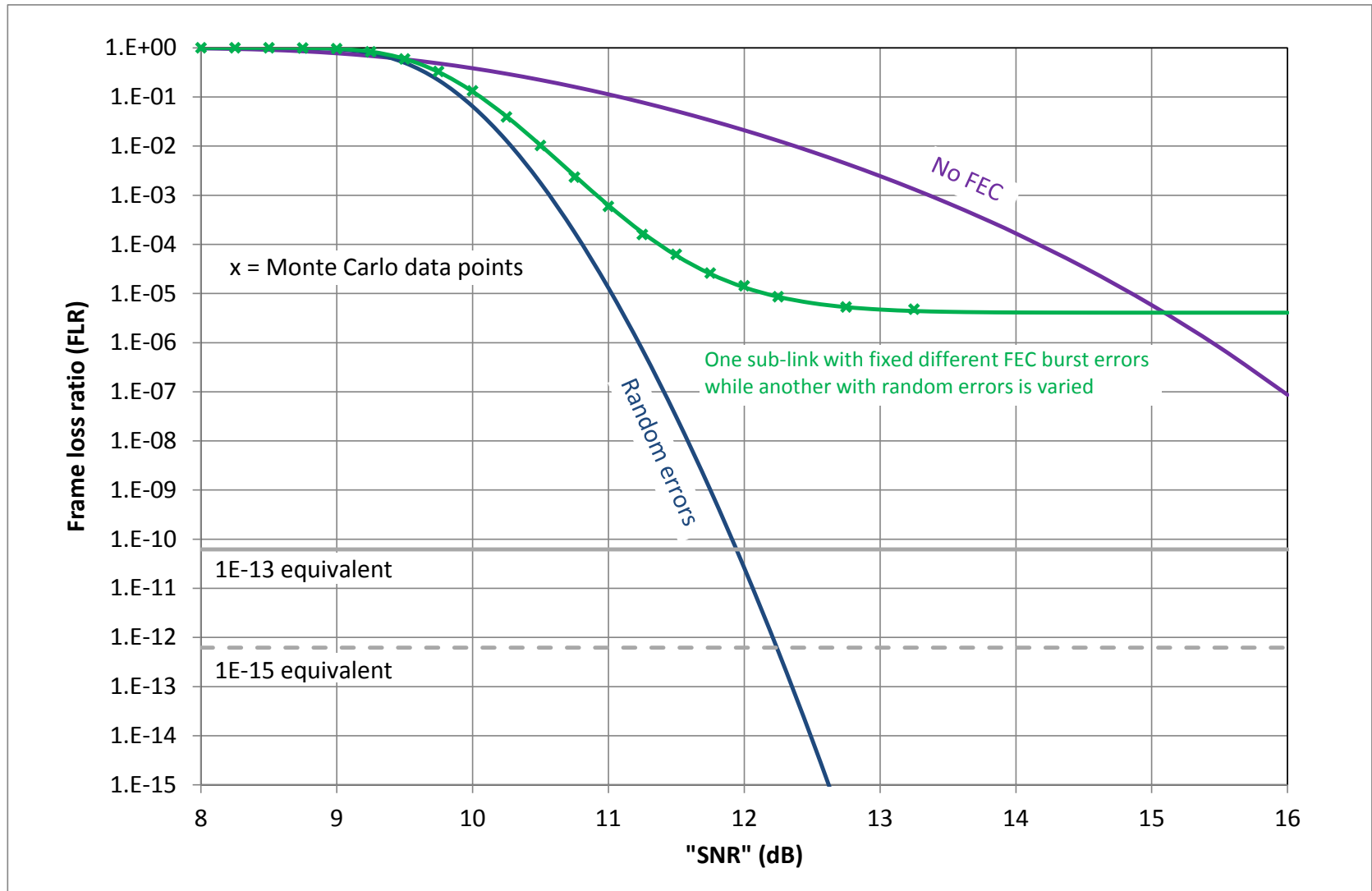
RS(528,514) multi-part (1:2 SF burst + random)



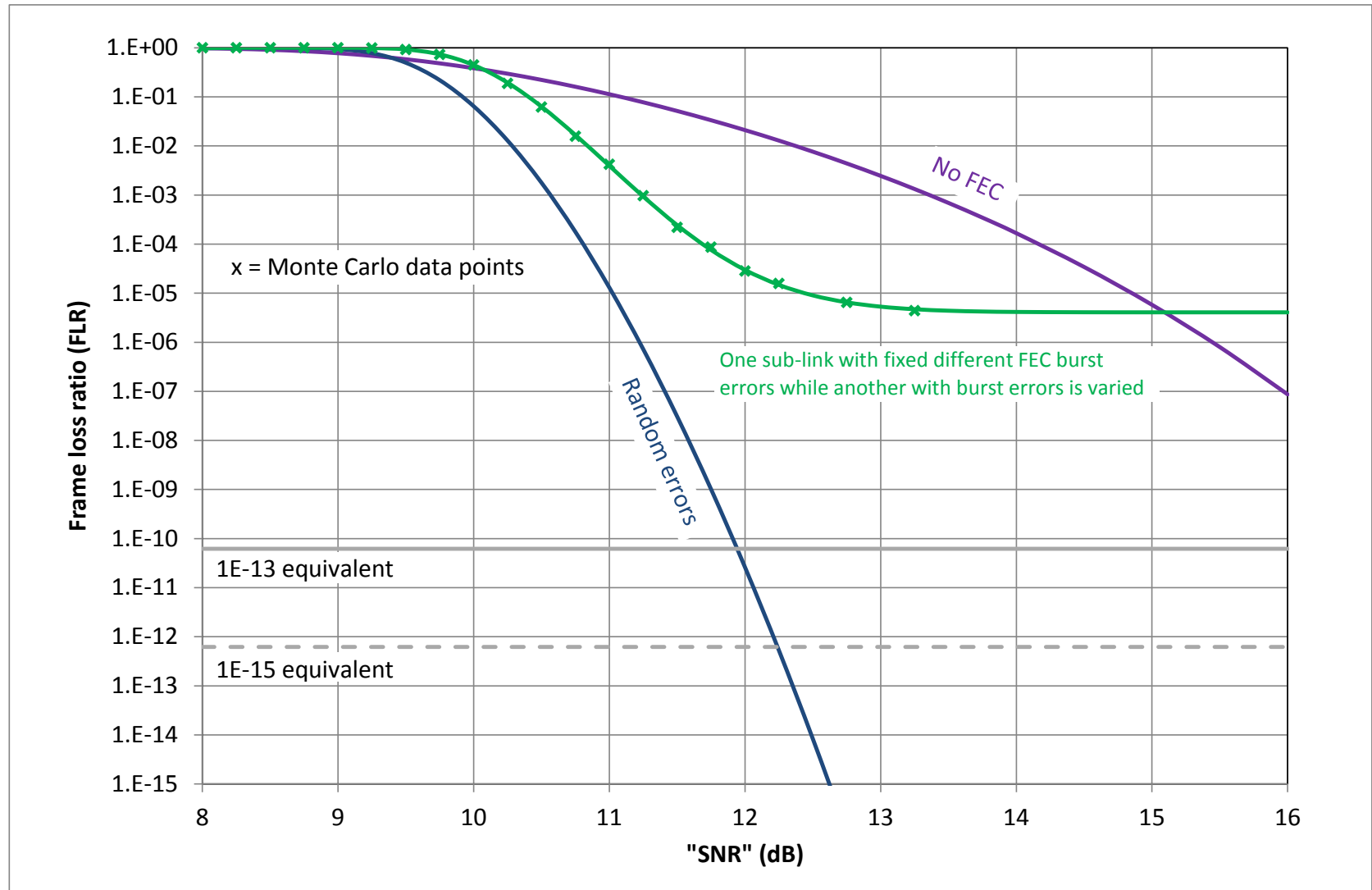
RS(528,514) multi-part (1:2 SF burst + burst)



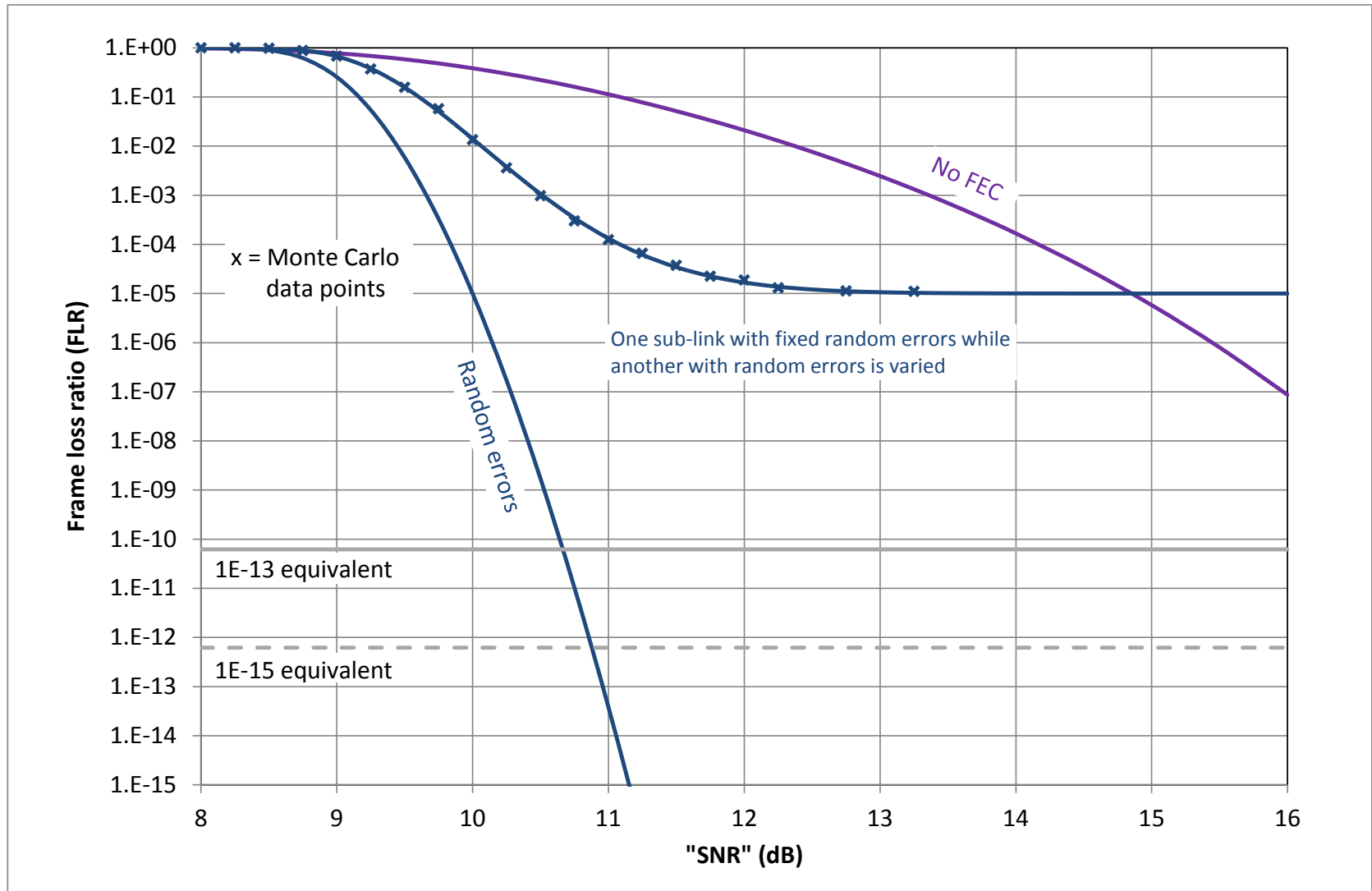
RS(528,514) multi-part (1:2 DF burst + random)



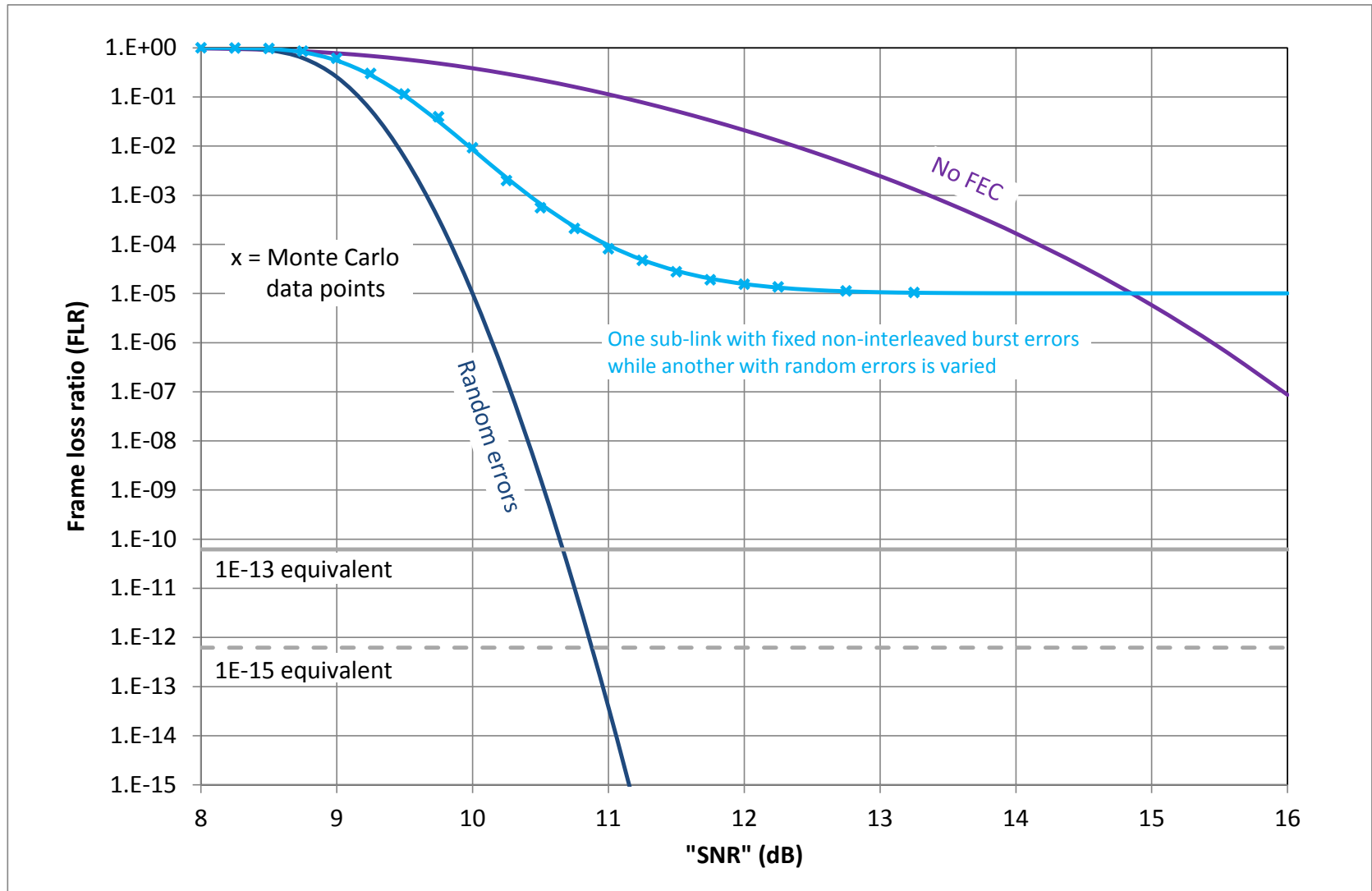
RS(528,514) multi-part (1:2 DF burst + burst)



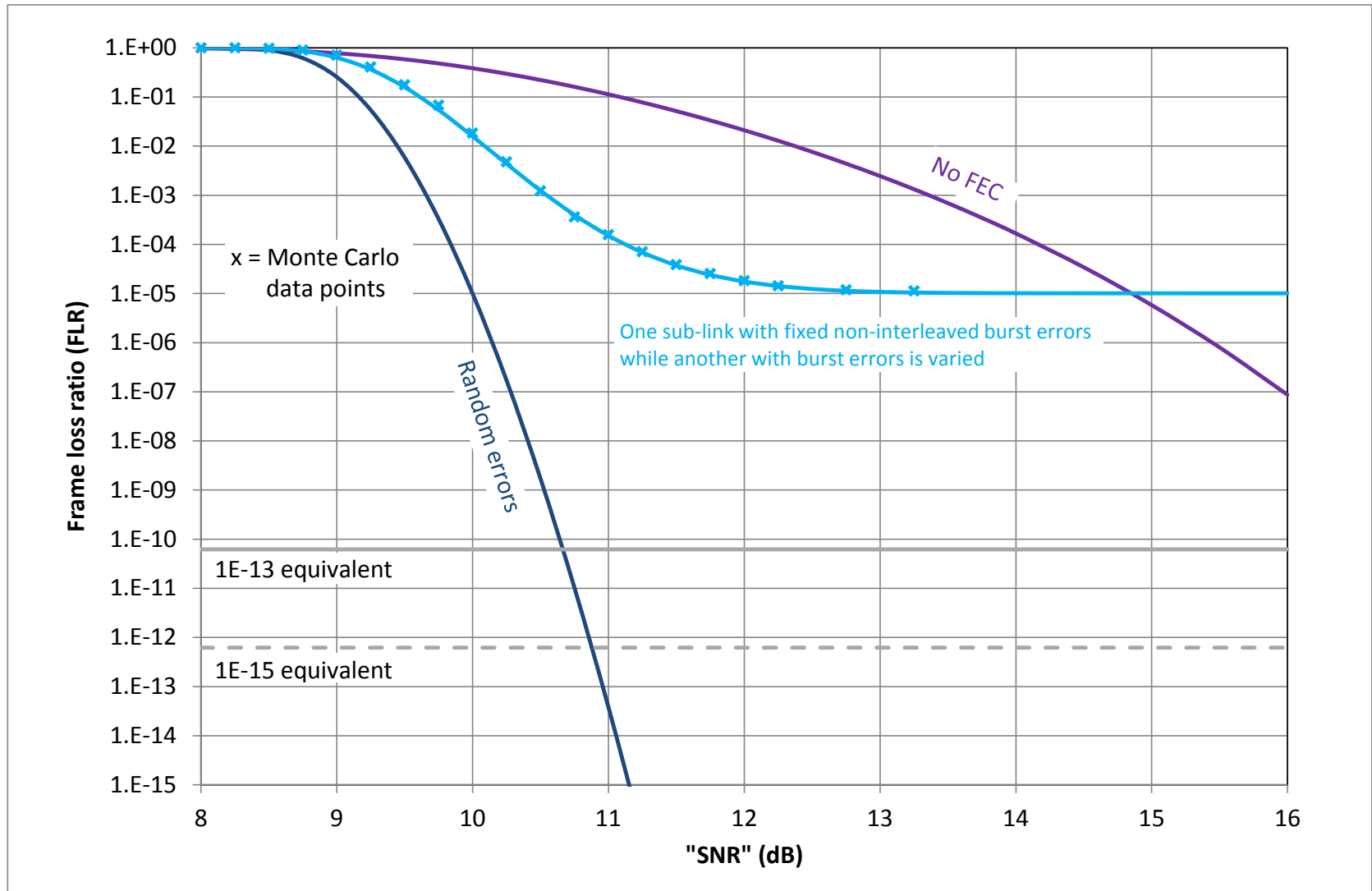
RS(544,514) multi-part (random + random)



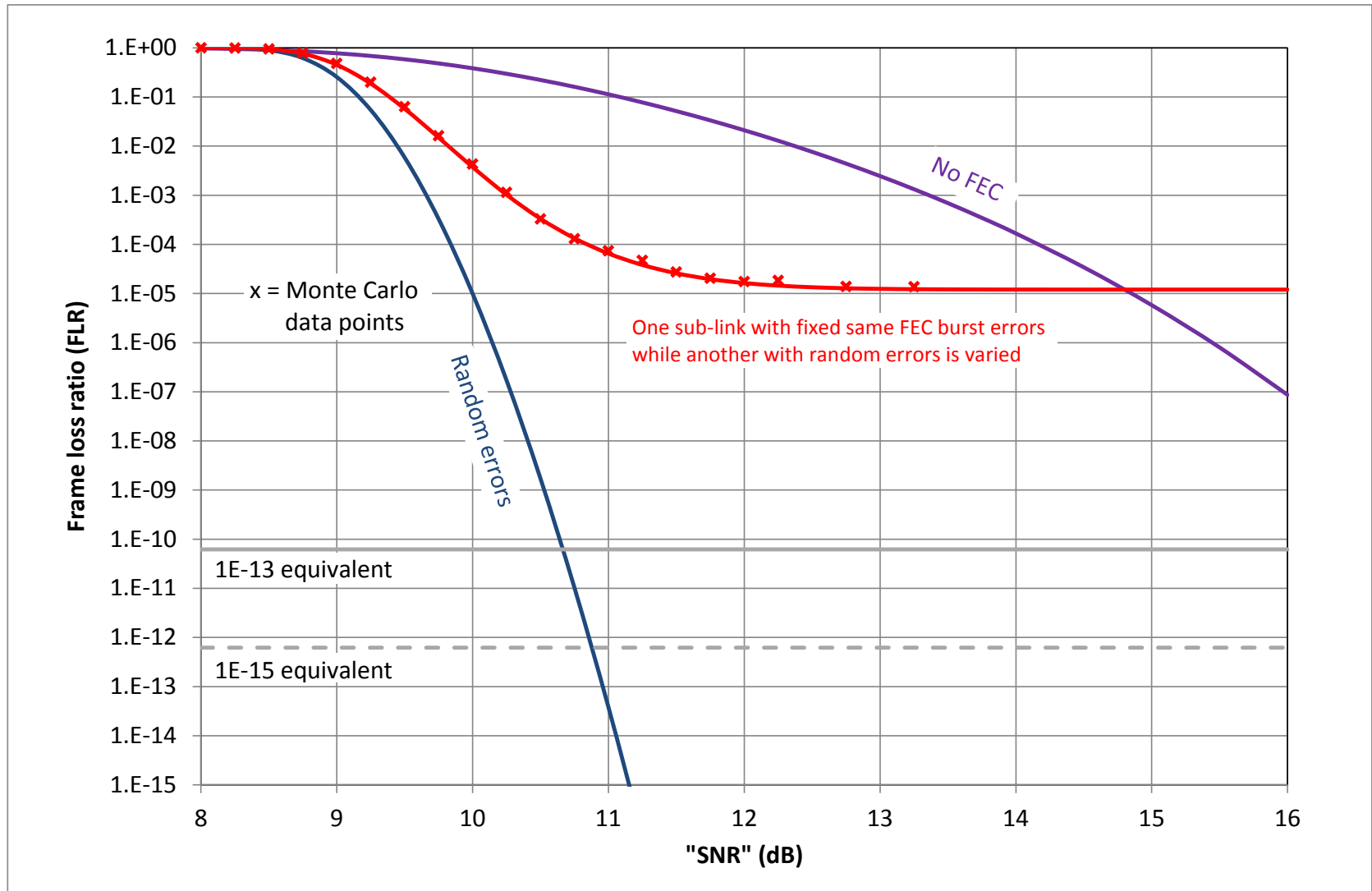
RS(544,514) multi-part non-int (burst + random)



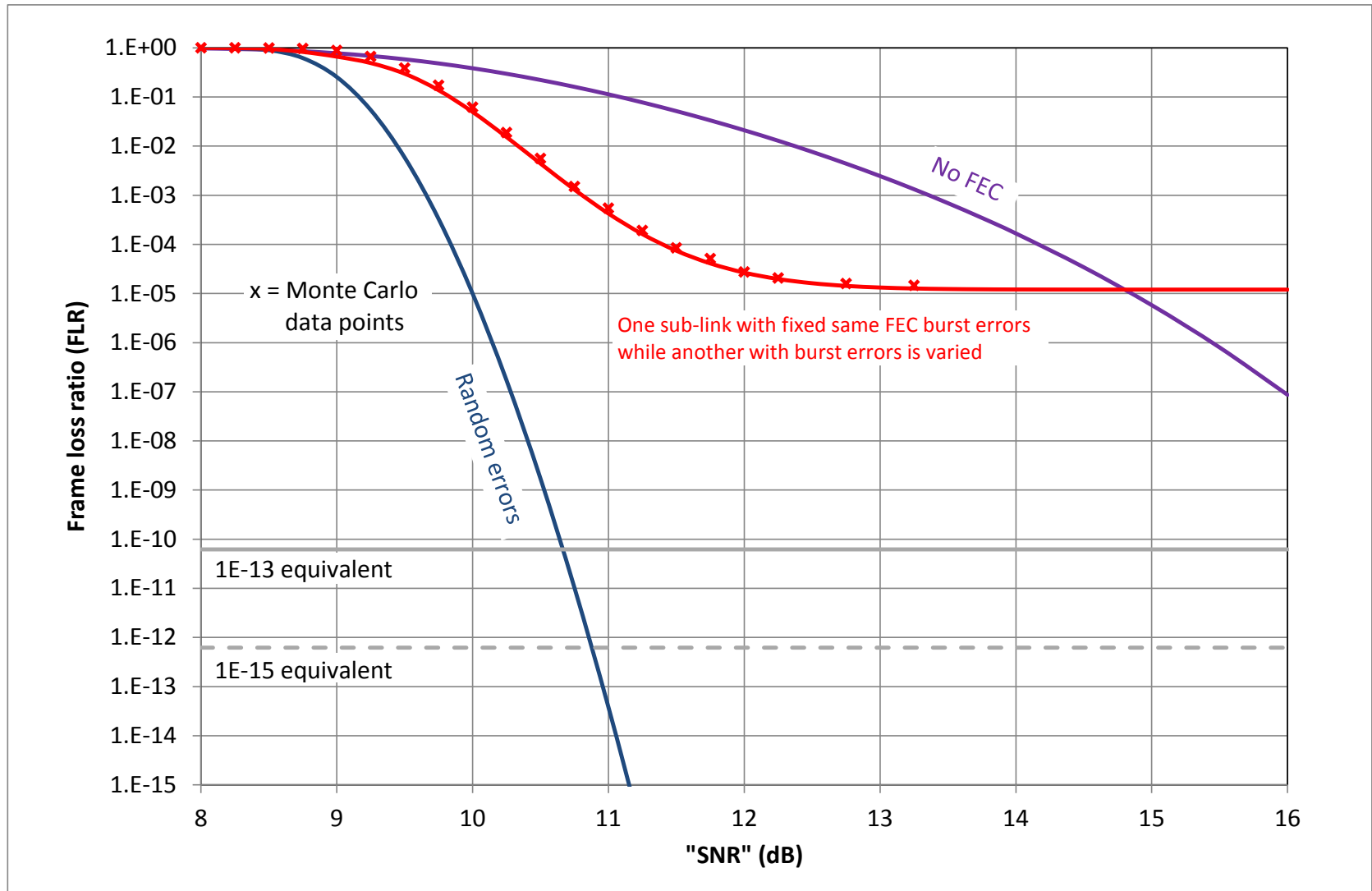
RS(544,514) multi-part non-int (burst + burst)



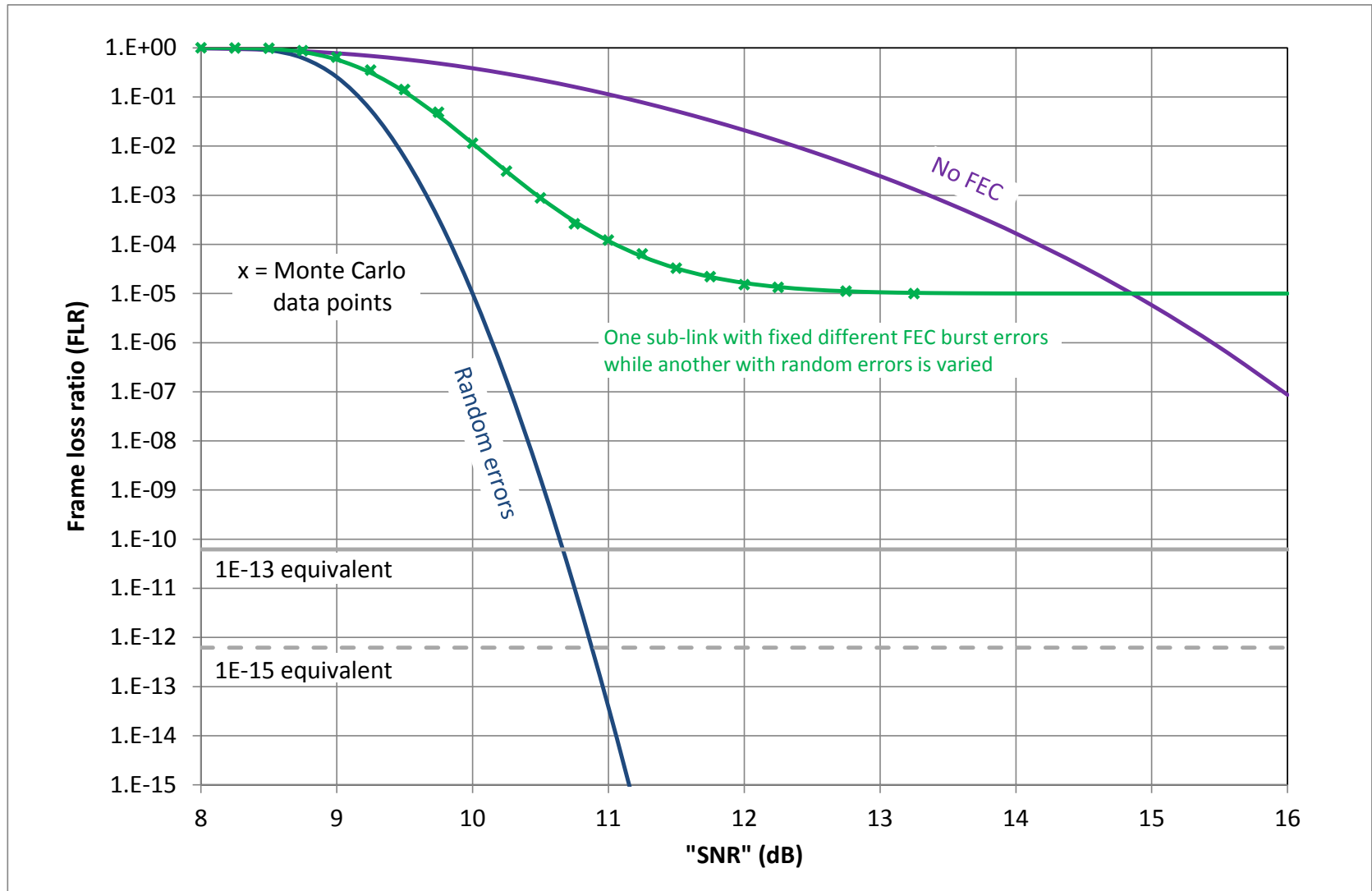
RS(544,514) multi-part (1:2 SF burst + random)



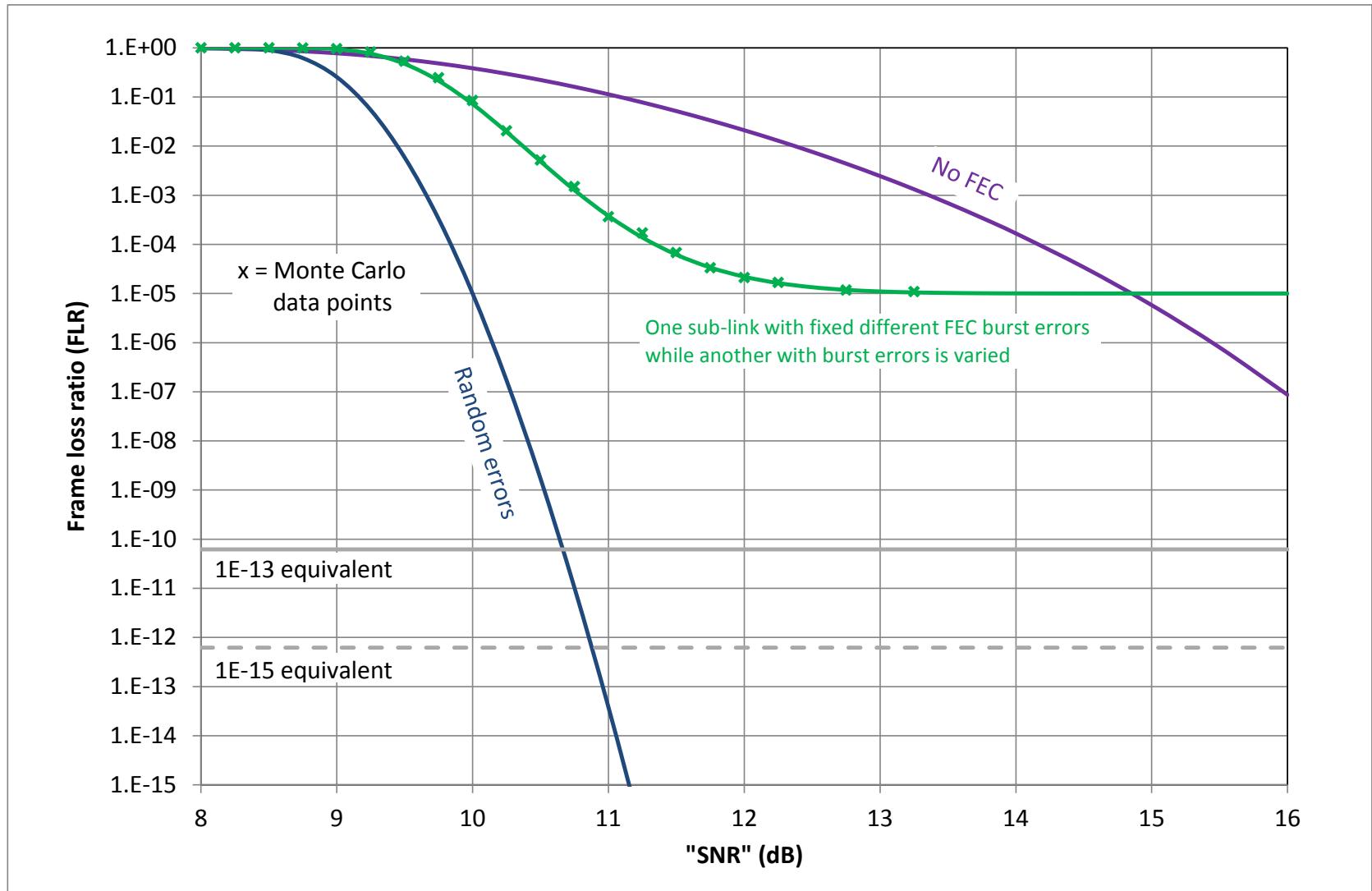
RS(544,514) multi-part (1:2 SF burst + burst)



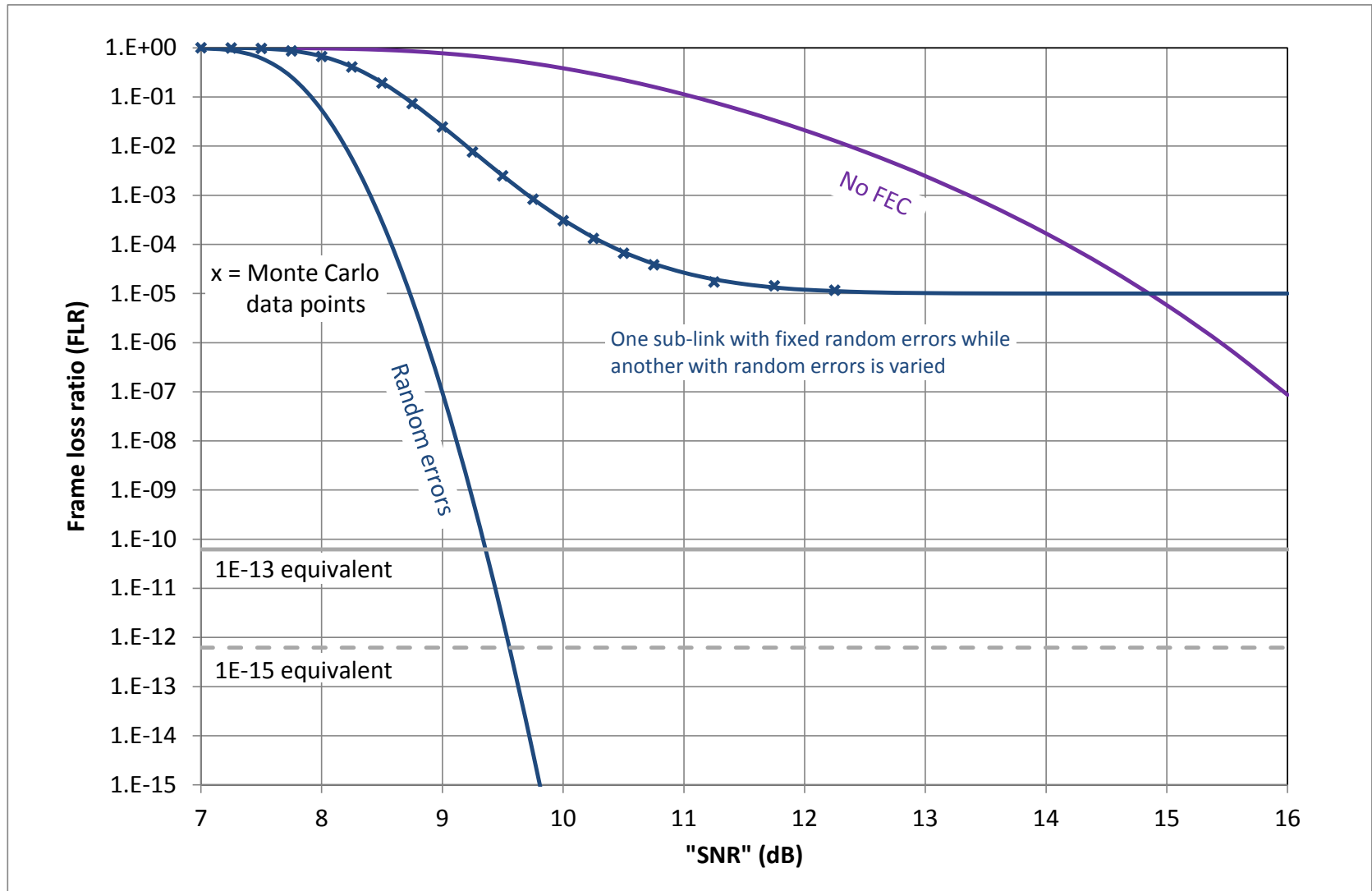
RS(544,514) multi-part (1:2 DF burst + random)



RS(544,514) multi-part (1:2 DF burst + burst)

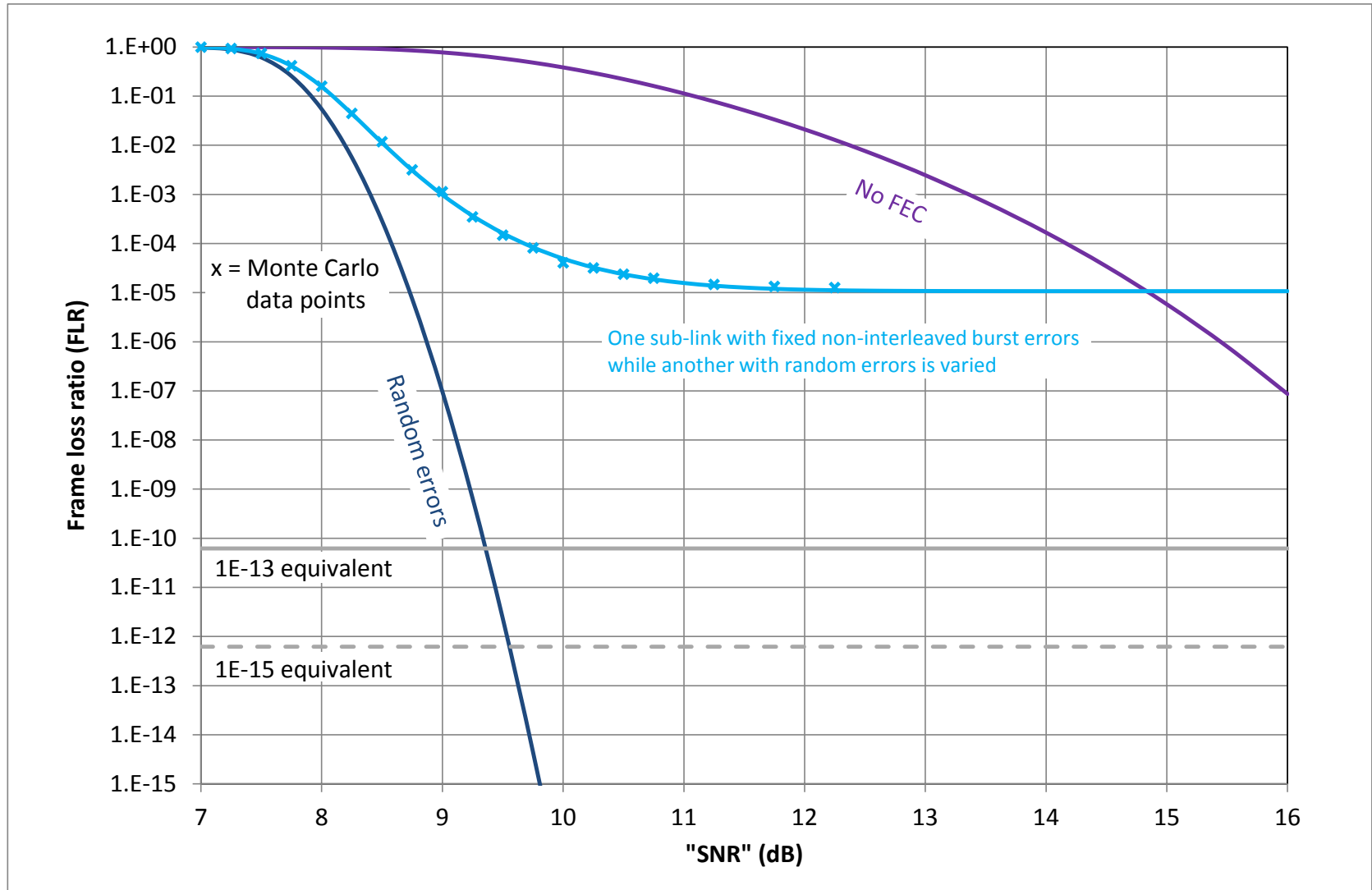


BCH(2858,2570) multi-part (random + random)

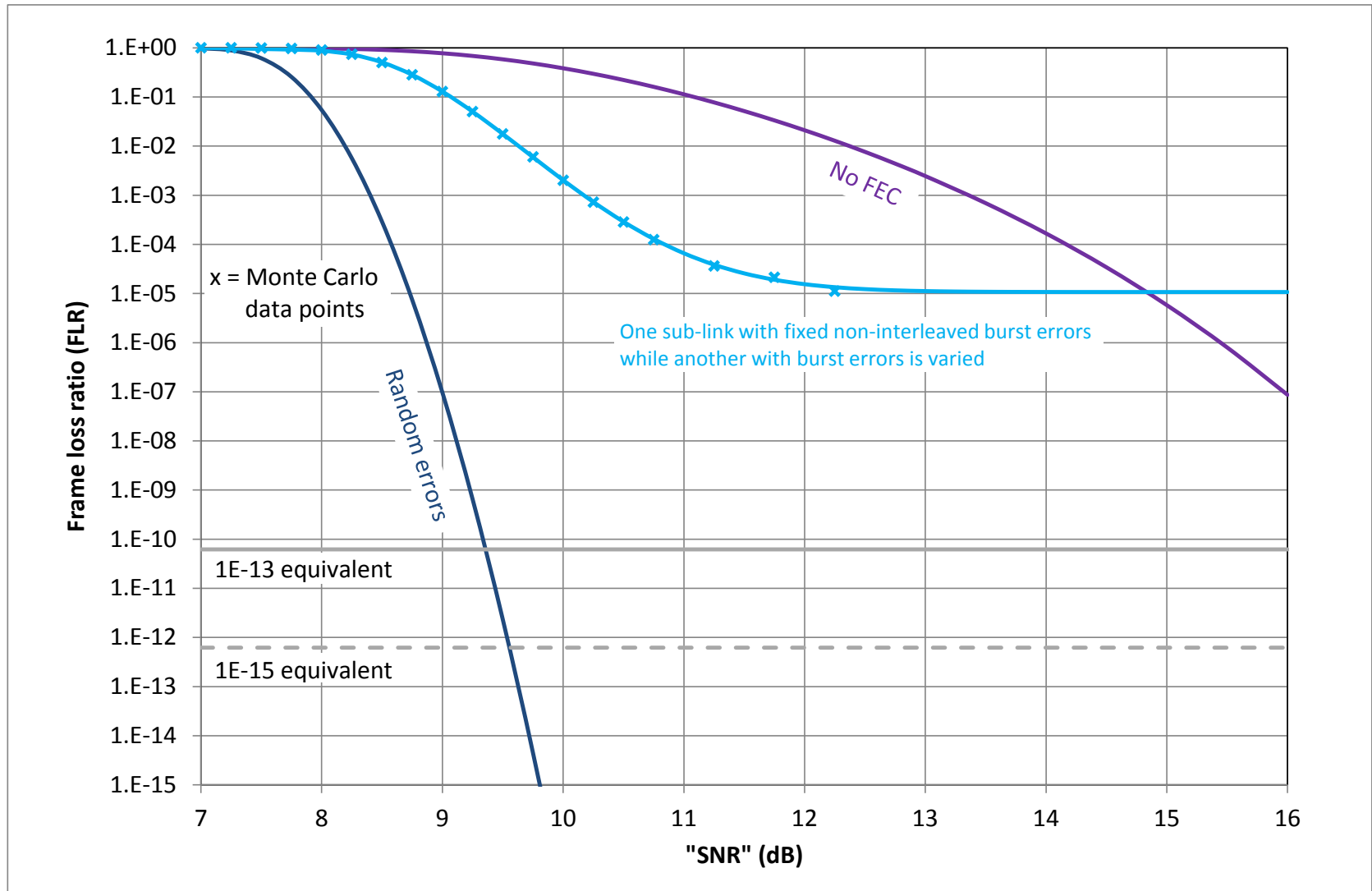


BCH(2858,2570) multi-part non-int (bst + rand)

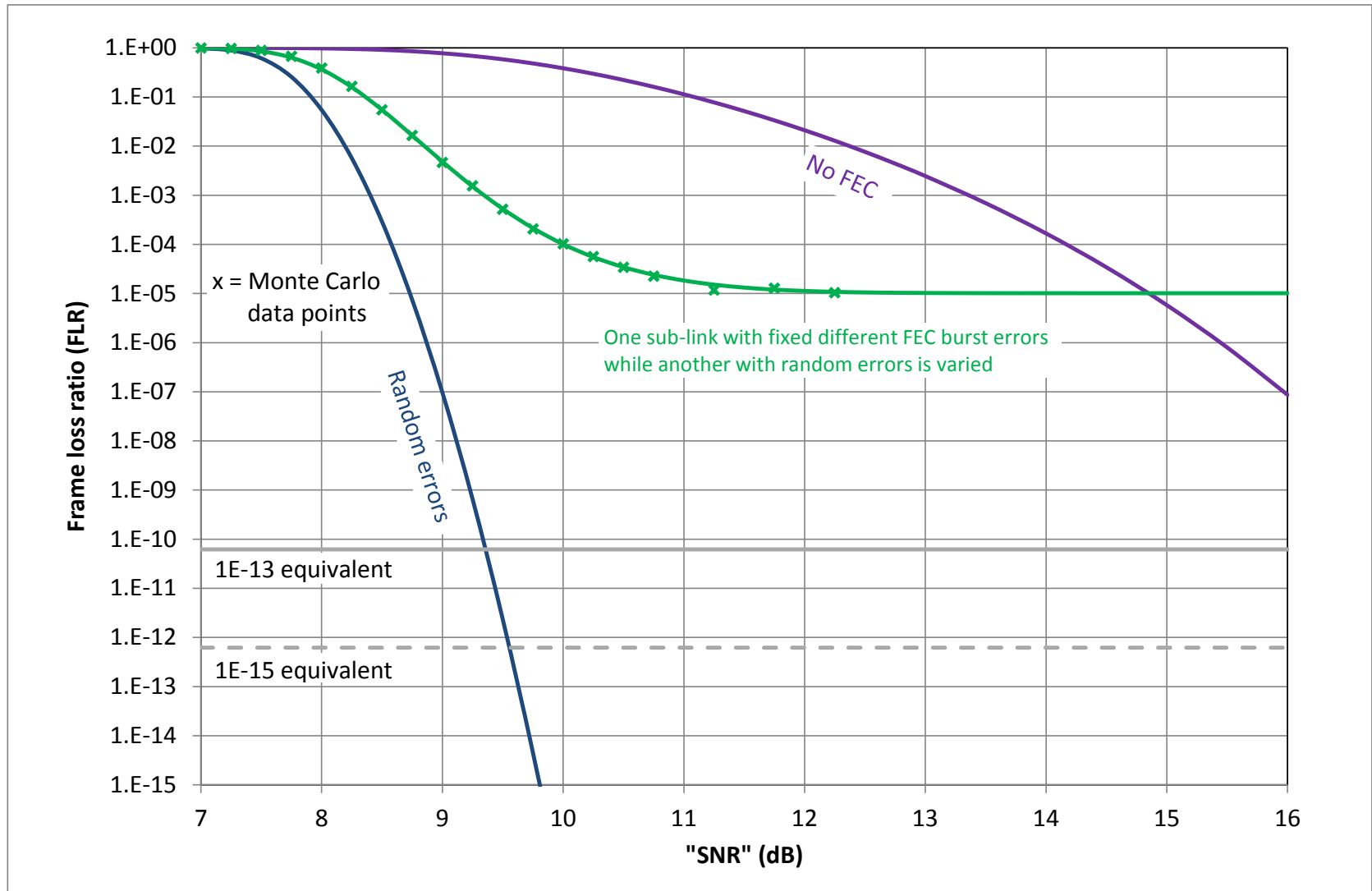
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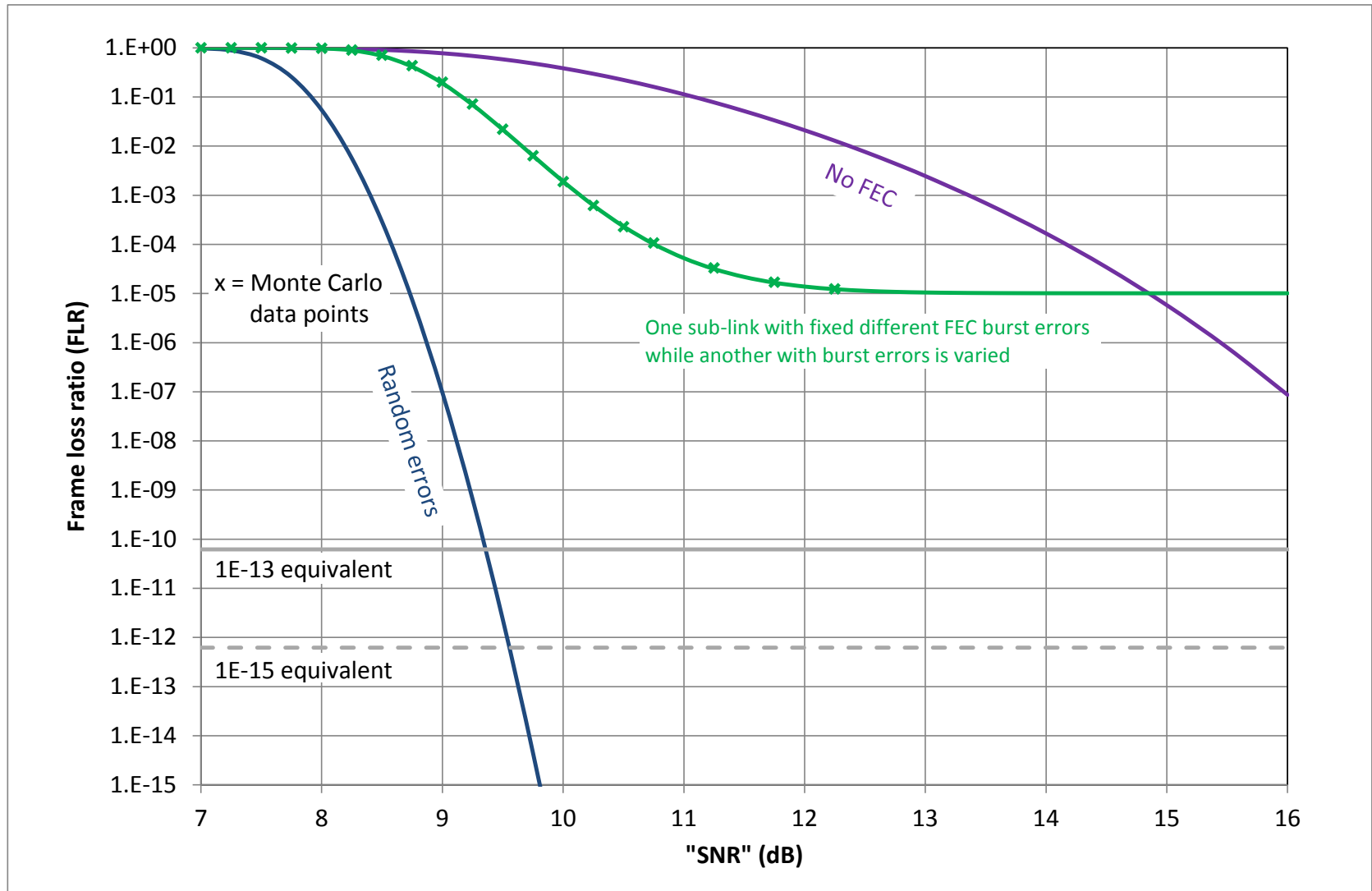
BCH(2858,2570) multi-part non-int (bst + bst)



BCH(2858,2570) multi-part (1:2 DF bst + random) *



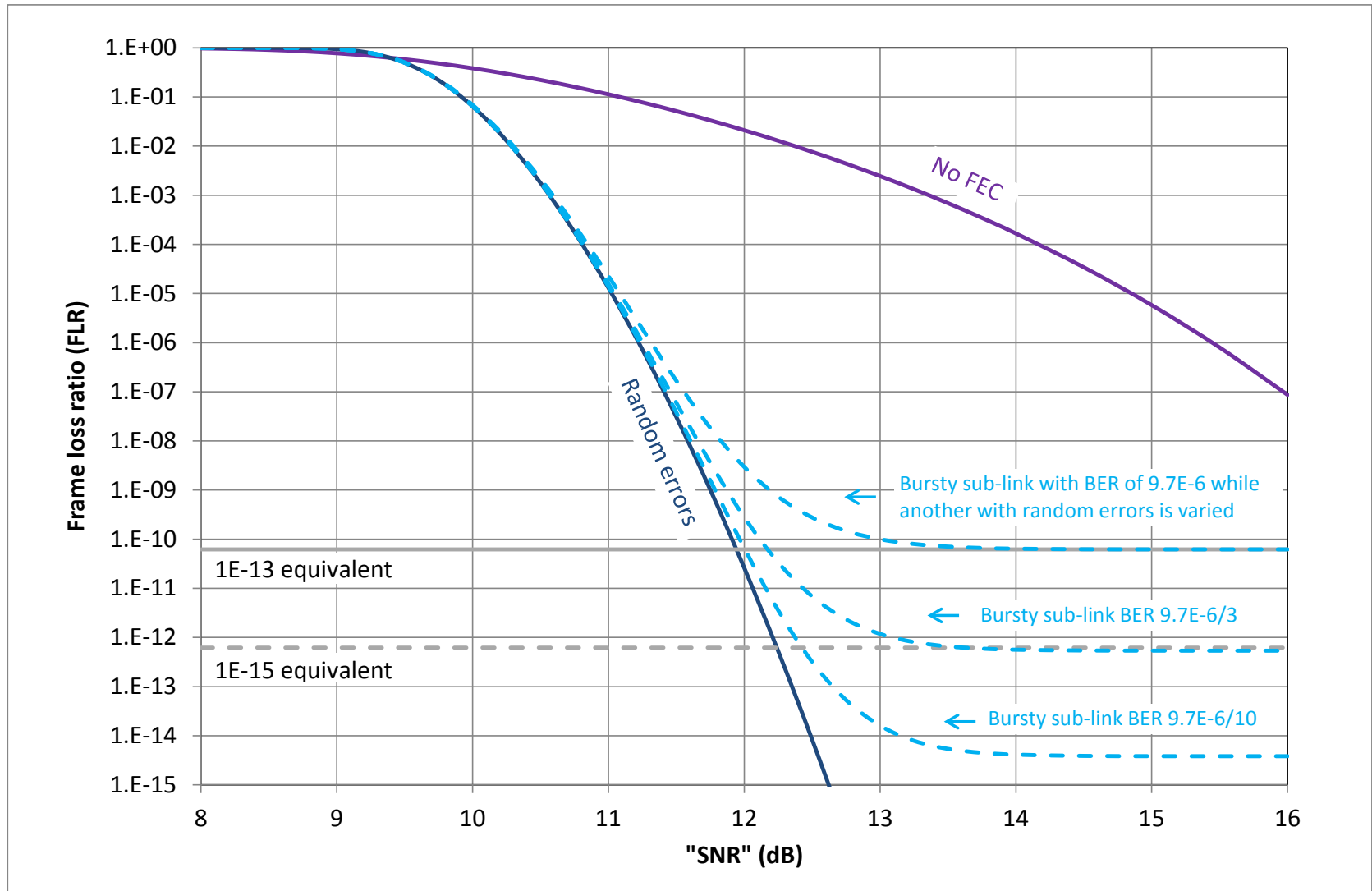
BCH(2858,2570) multi-part (1:2 DF burst + burst) *



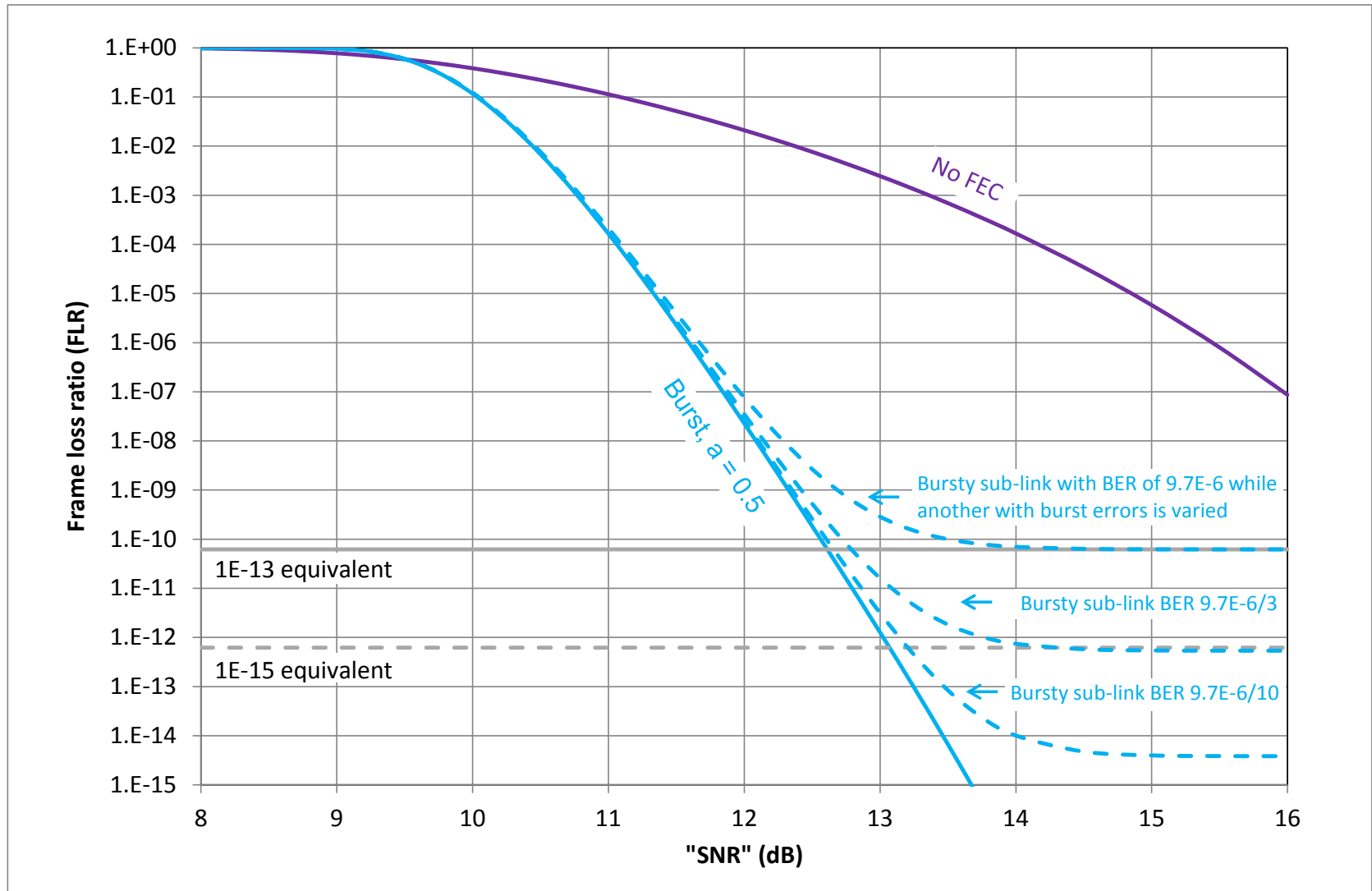
Annex 3

Plots with BER beyond the reach of Monte Carlo analysis

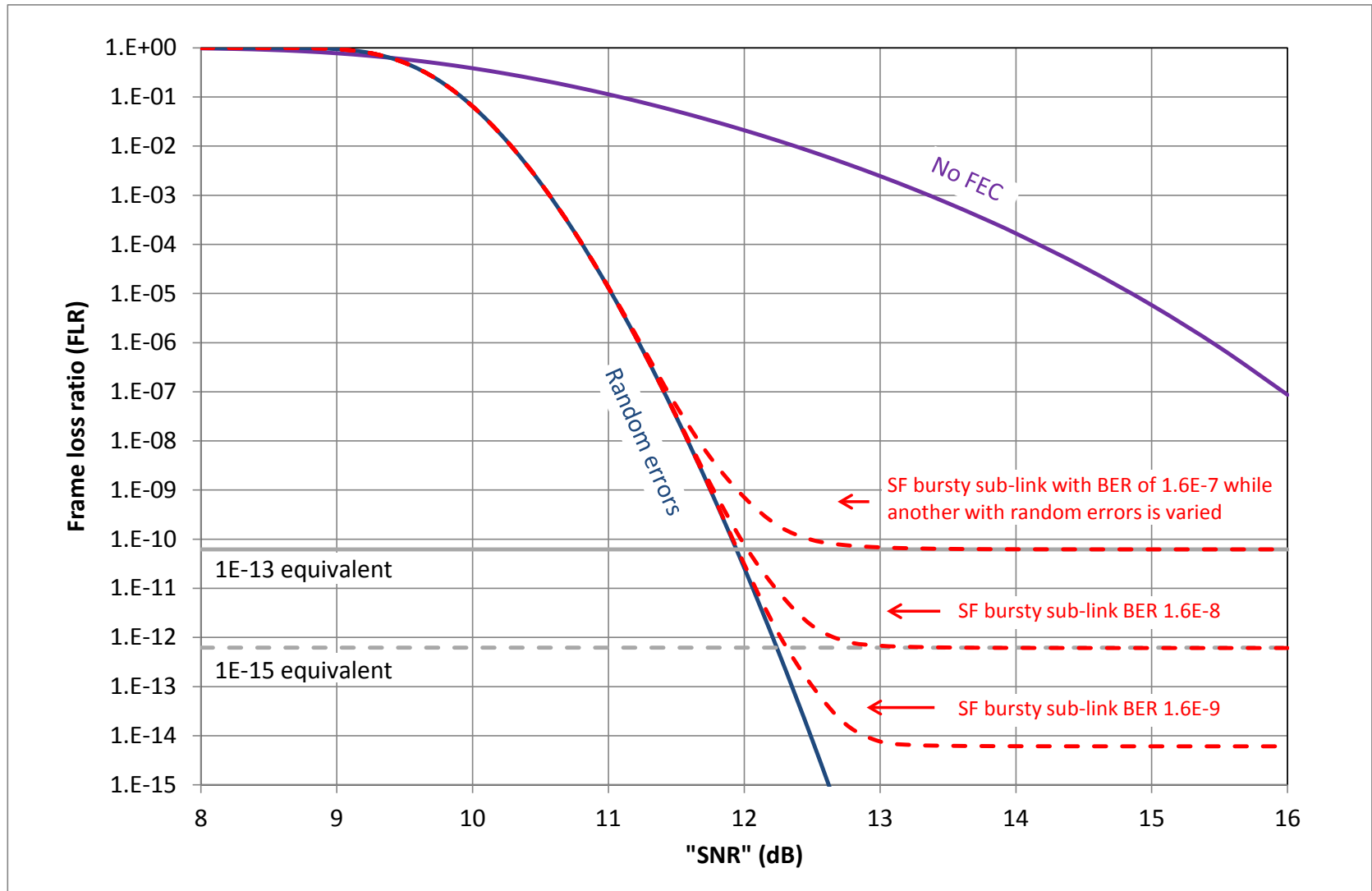
RS(528,514) multi-part non-int (burst + random)



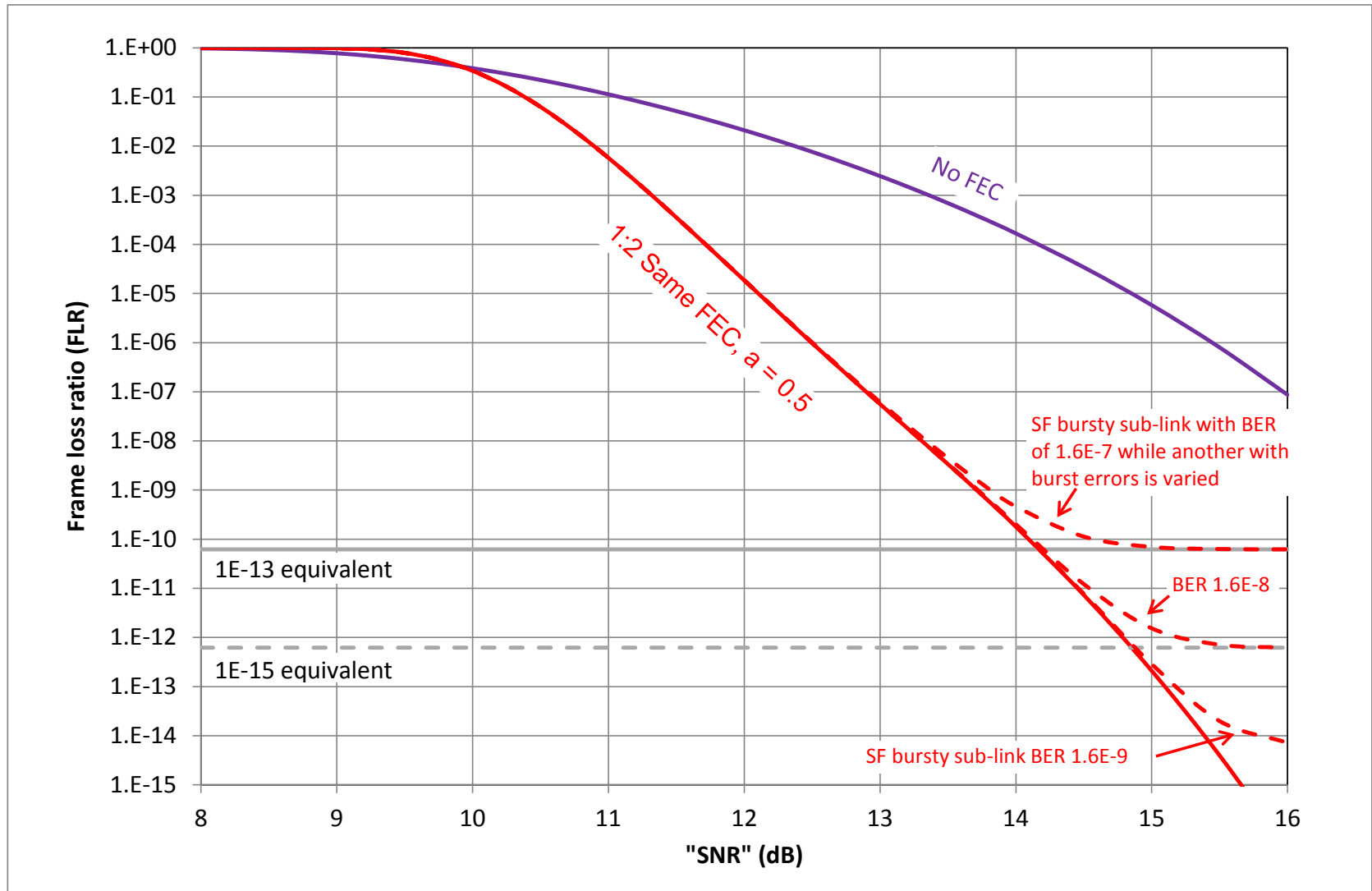
RS(528,514) multi-part non-int (burst + burst)



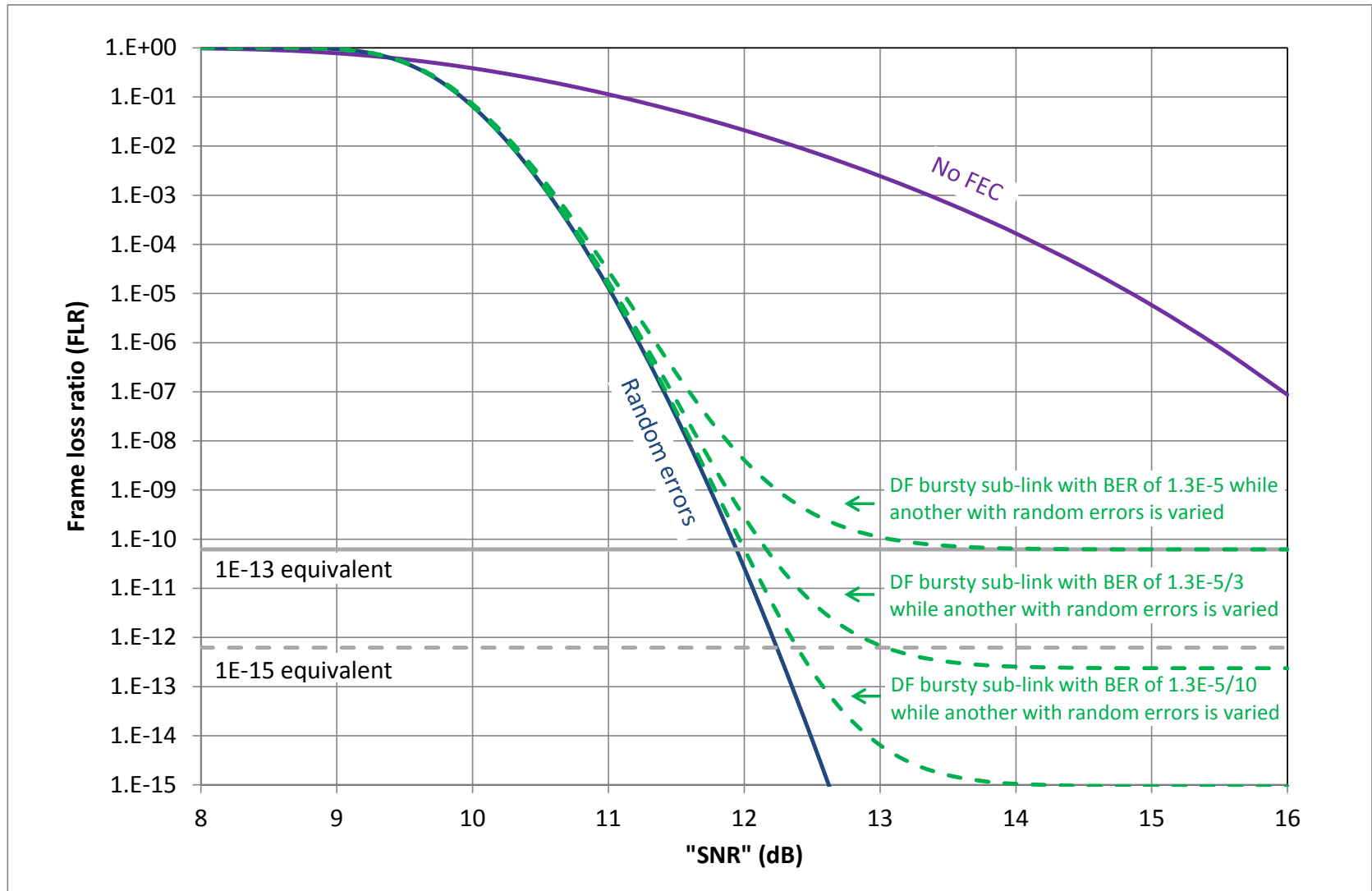
RS(528,514) multi-part (1:2 SF burst + random)



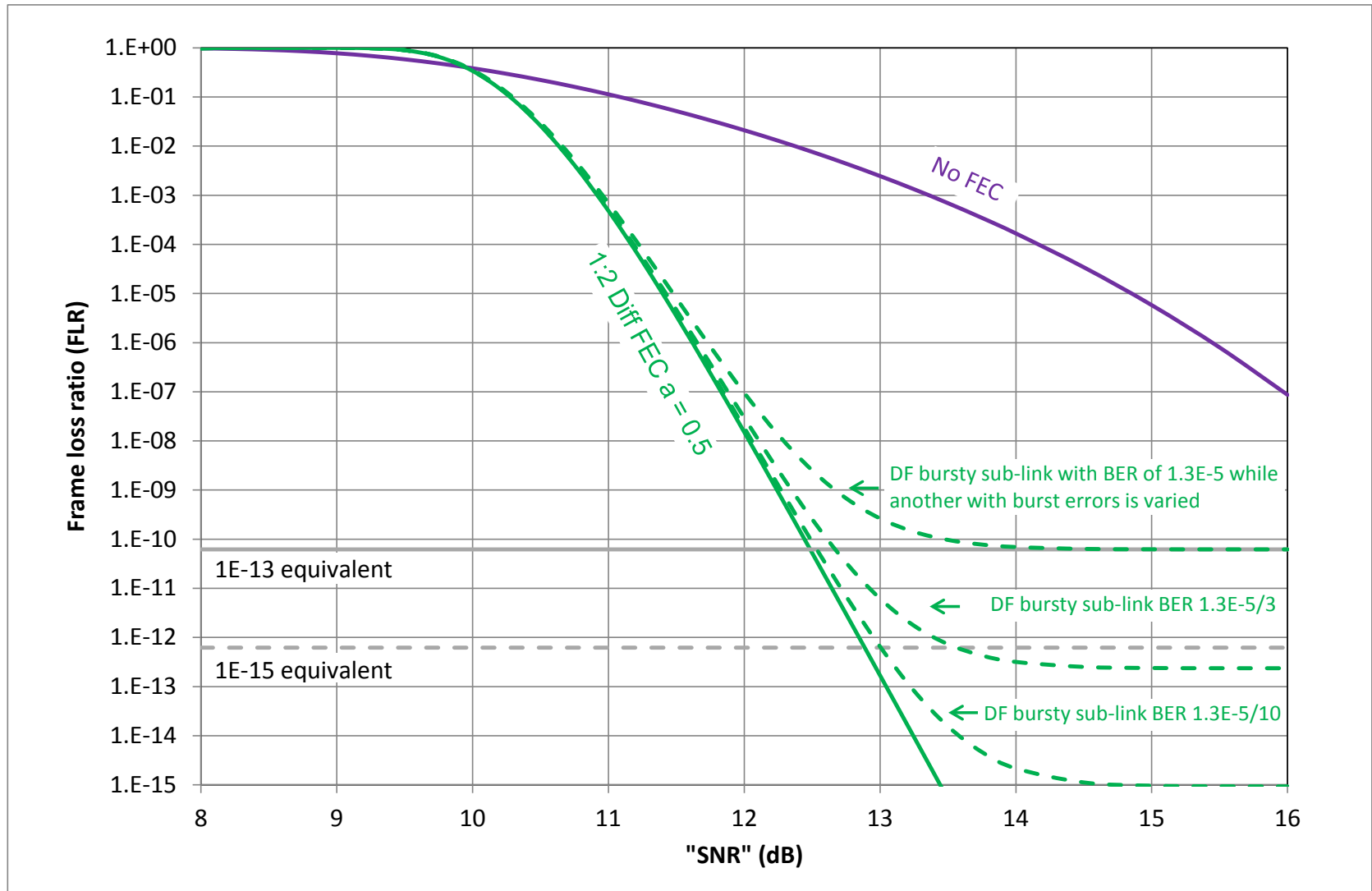
RS(528,514) multi-part (1:2 SF burst + burst)



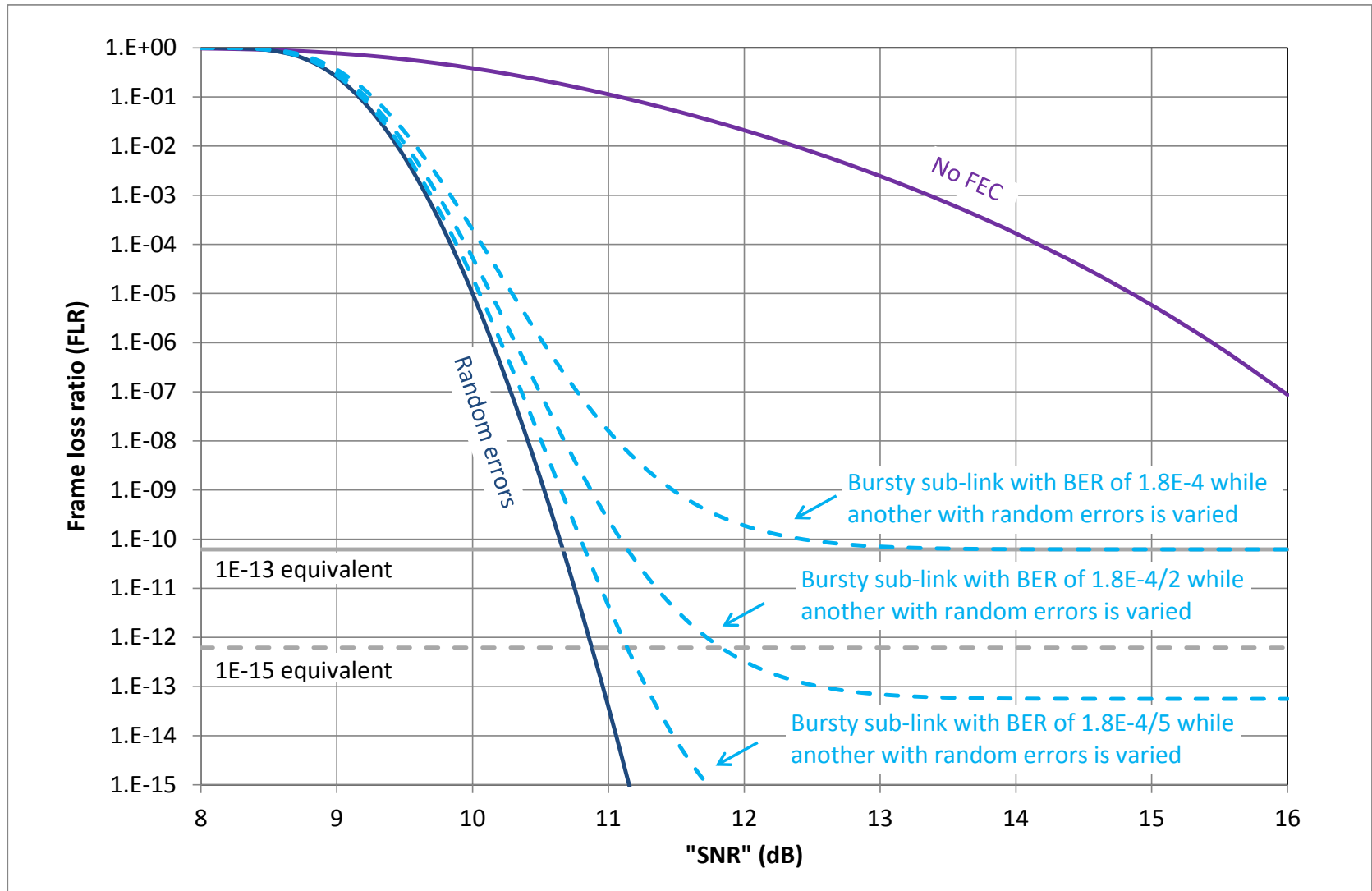
RS(528,514) multi-part (1:2 DF burst + random)



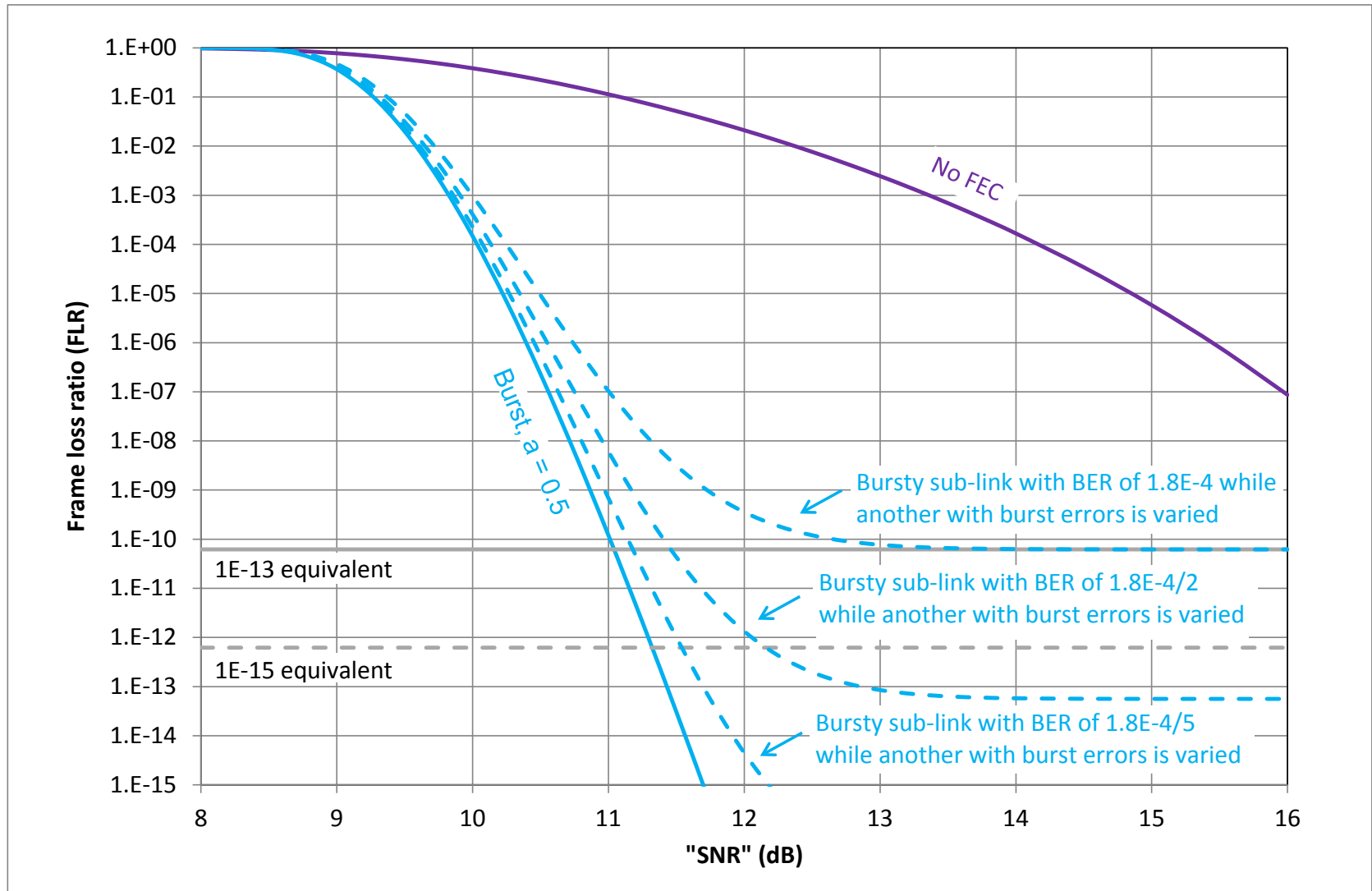
RS(528,514) multi-part (1:2 DF burst + burst)



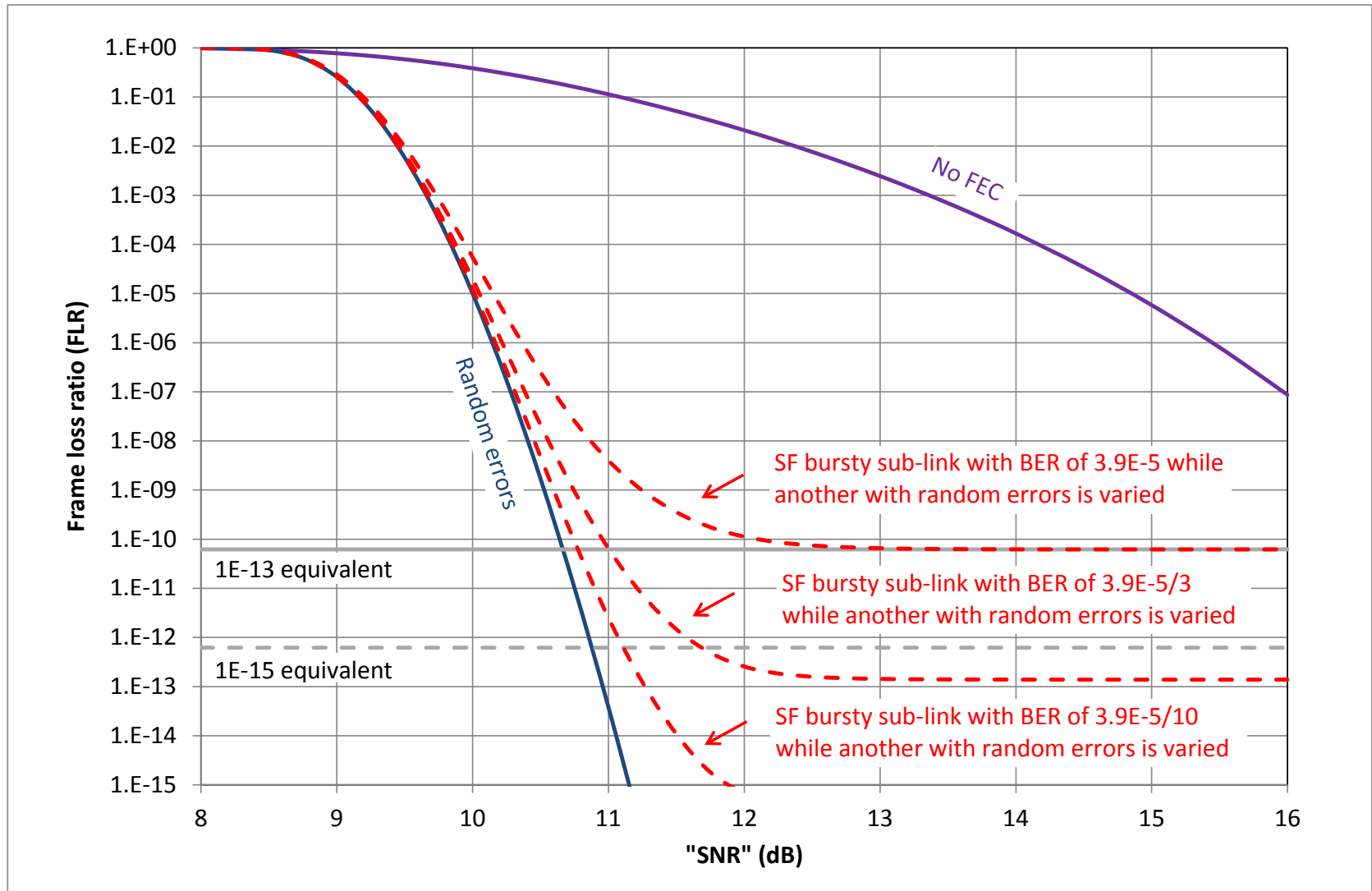
RS(544,514) multi-part non-int (burst + random)



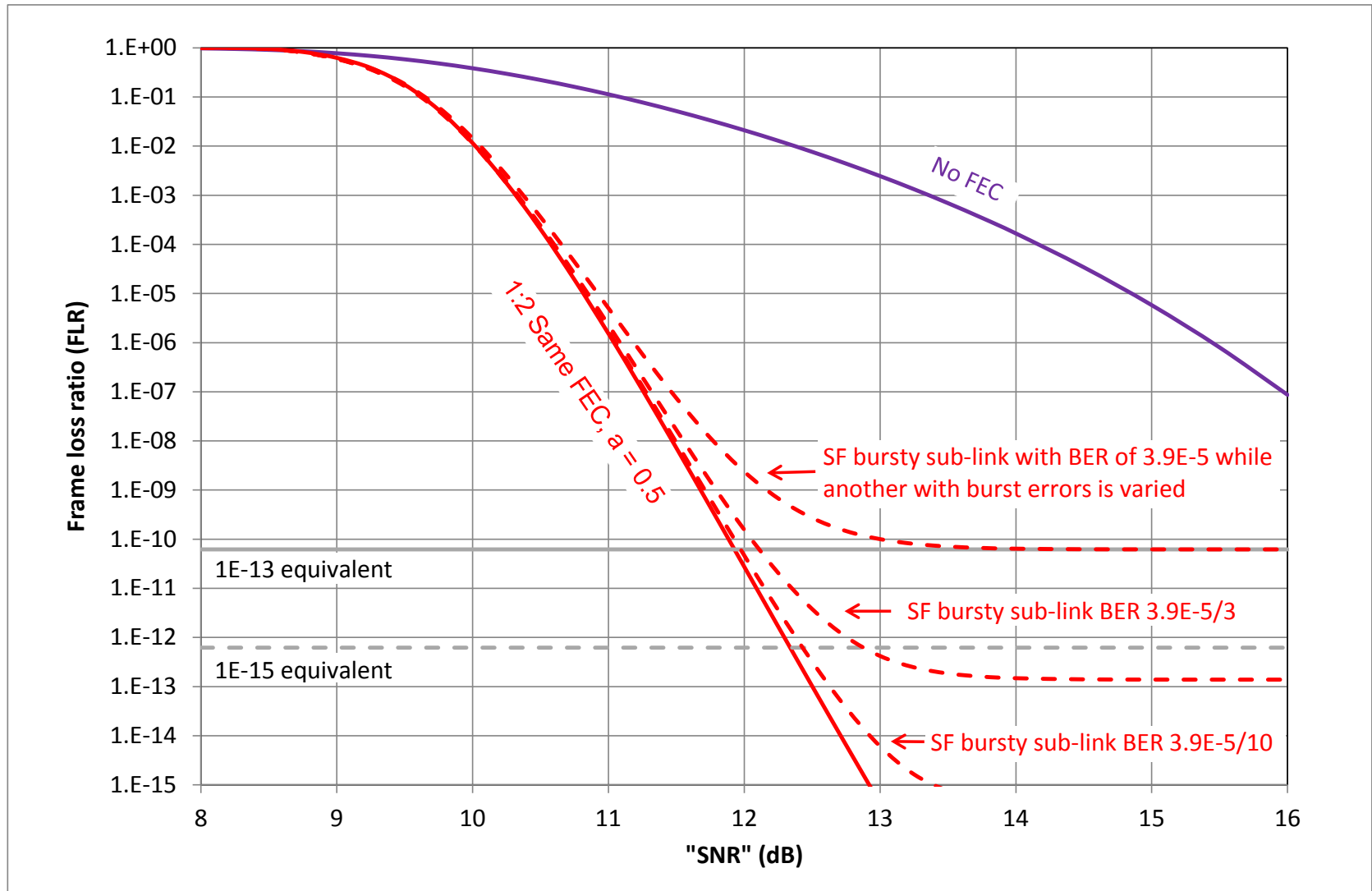
RS(544,514) multi-part non-int (burst + burst)



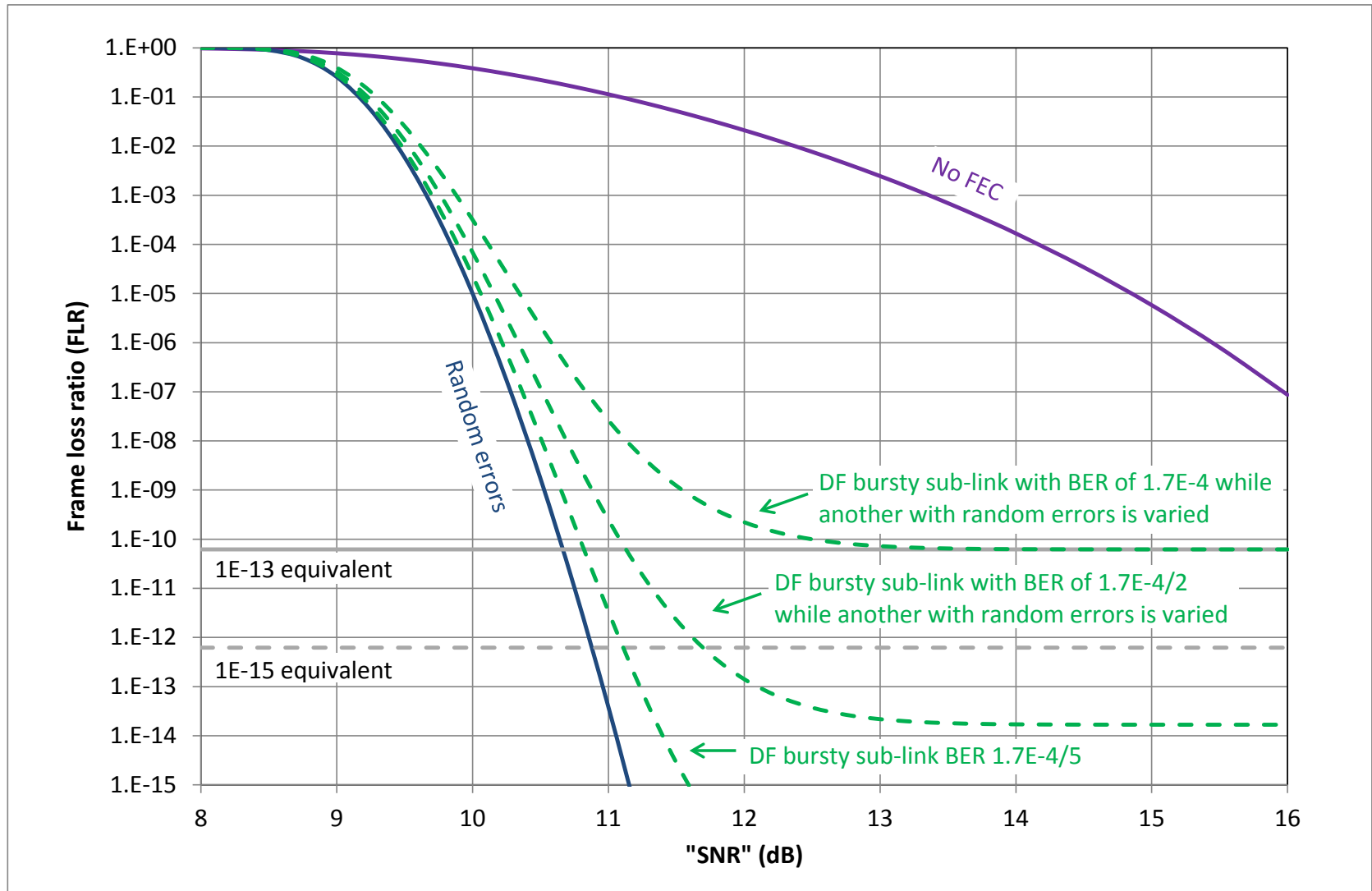
RS(544,514) multi-part (1:2 SF burst + random)



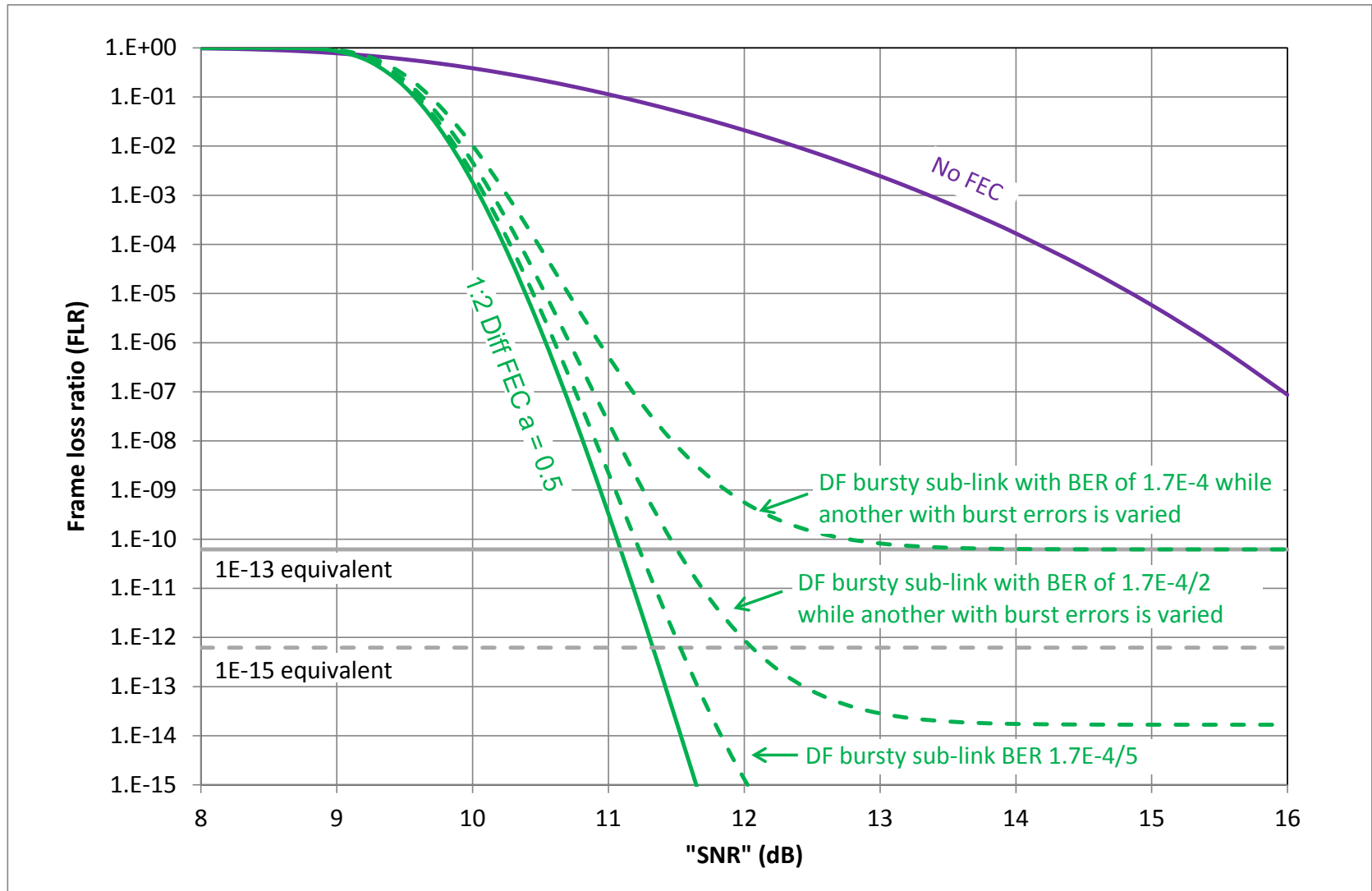
RS(544,514) multi-part (1:2 SF burst + burst)



RS(544,514) multi-part (1:2 DF burst + random)

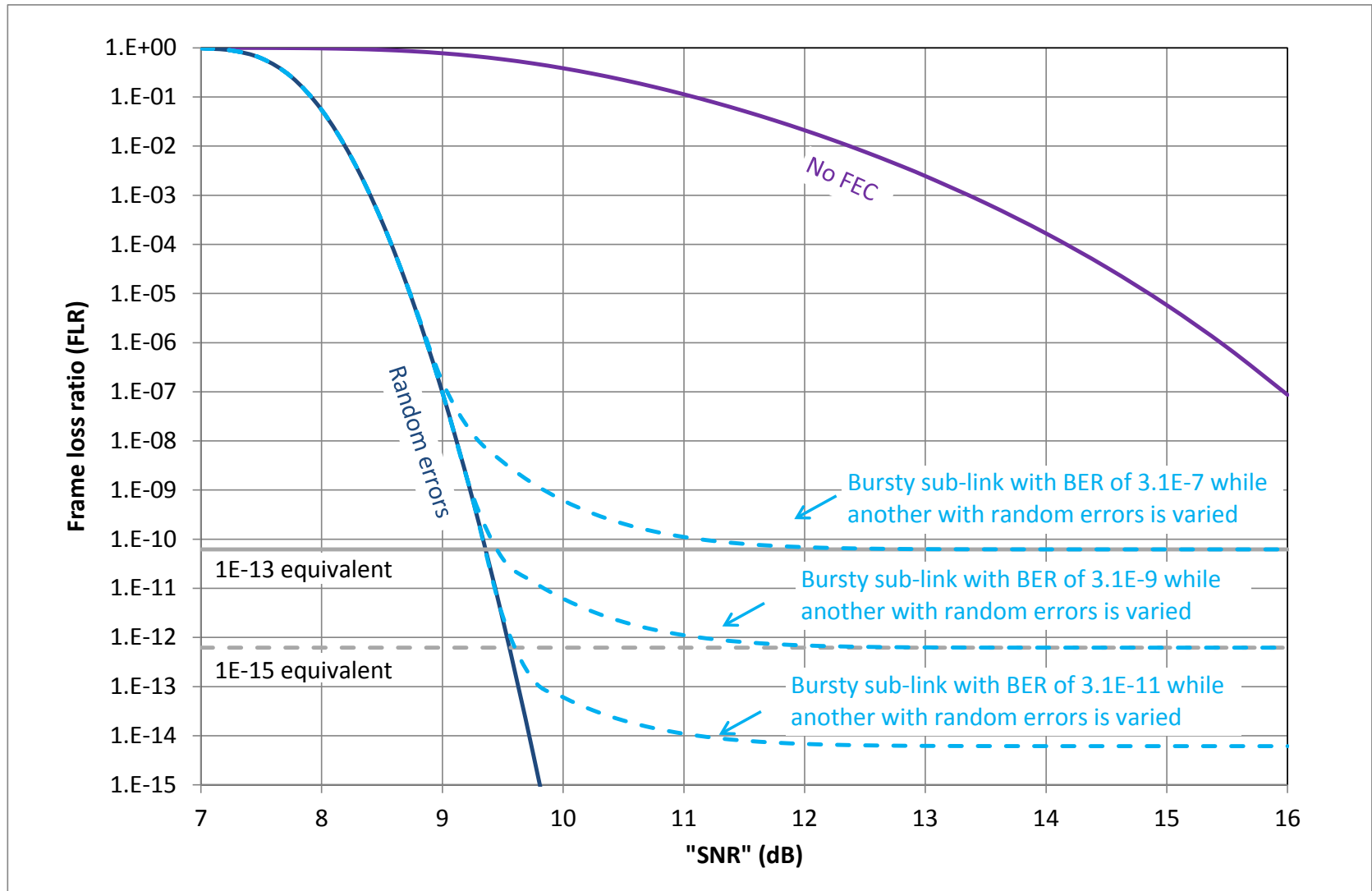


RS(544,514) multi-part (1:2 DF burst + burst)



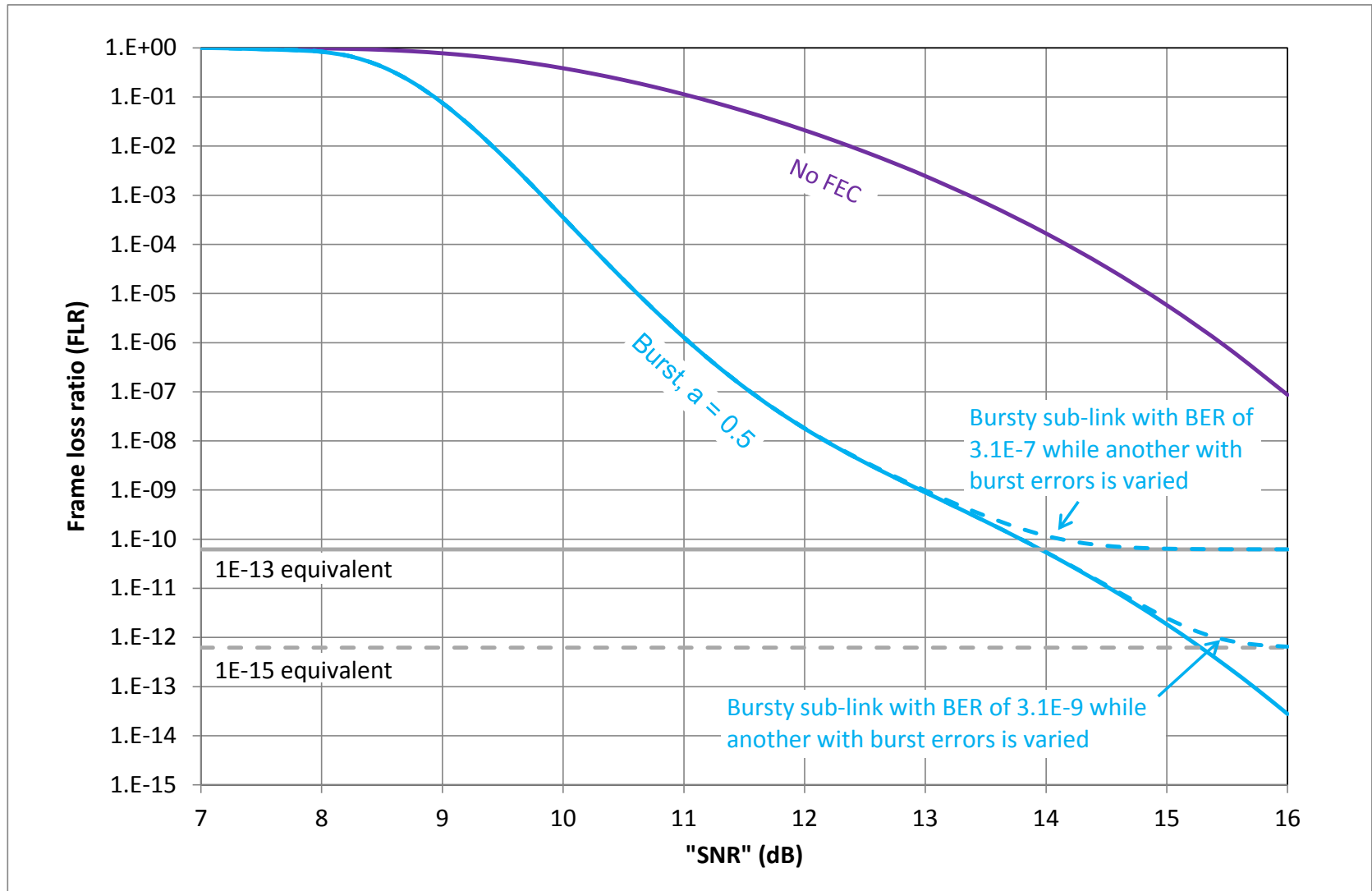
BCH(2858,2570) multi-part non-int (bst + rand)

*

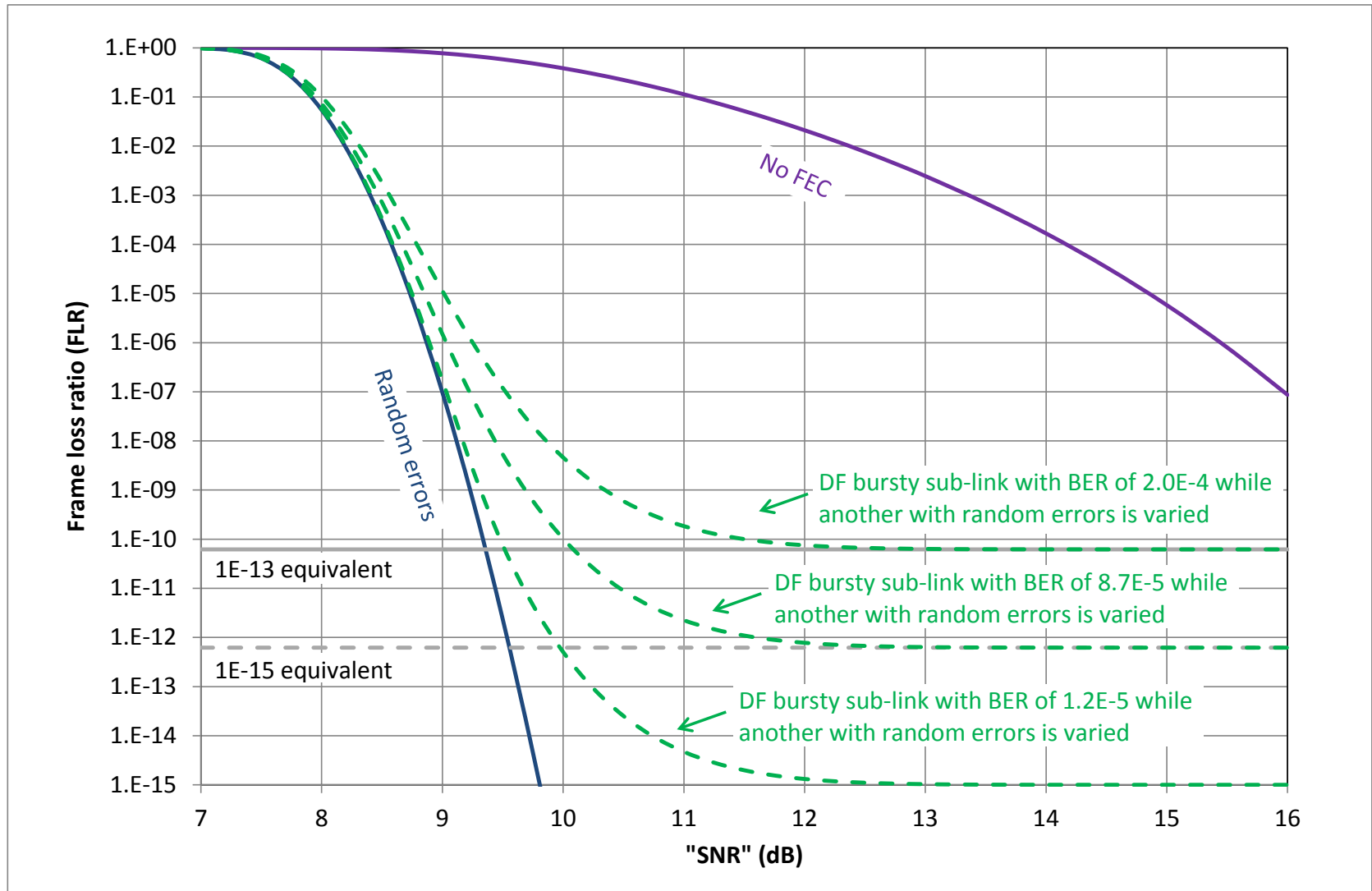


BCH(2858,2570) multi-part non-int (bst + bst)

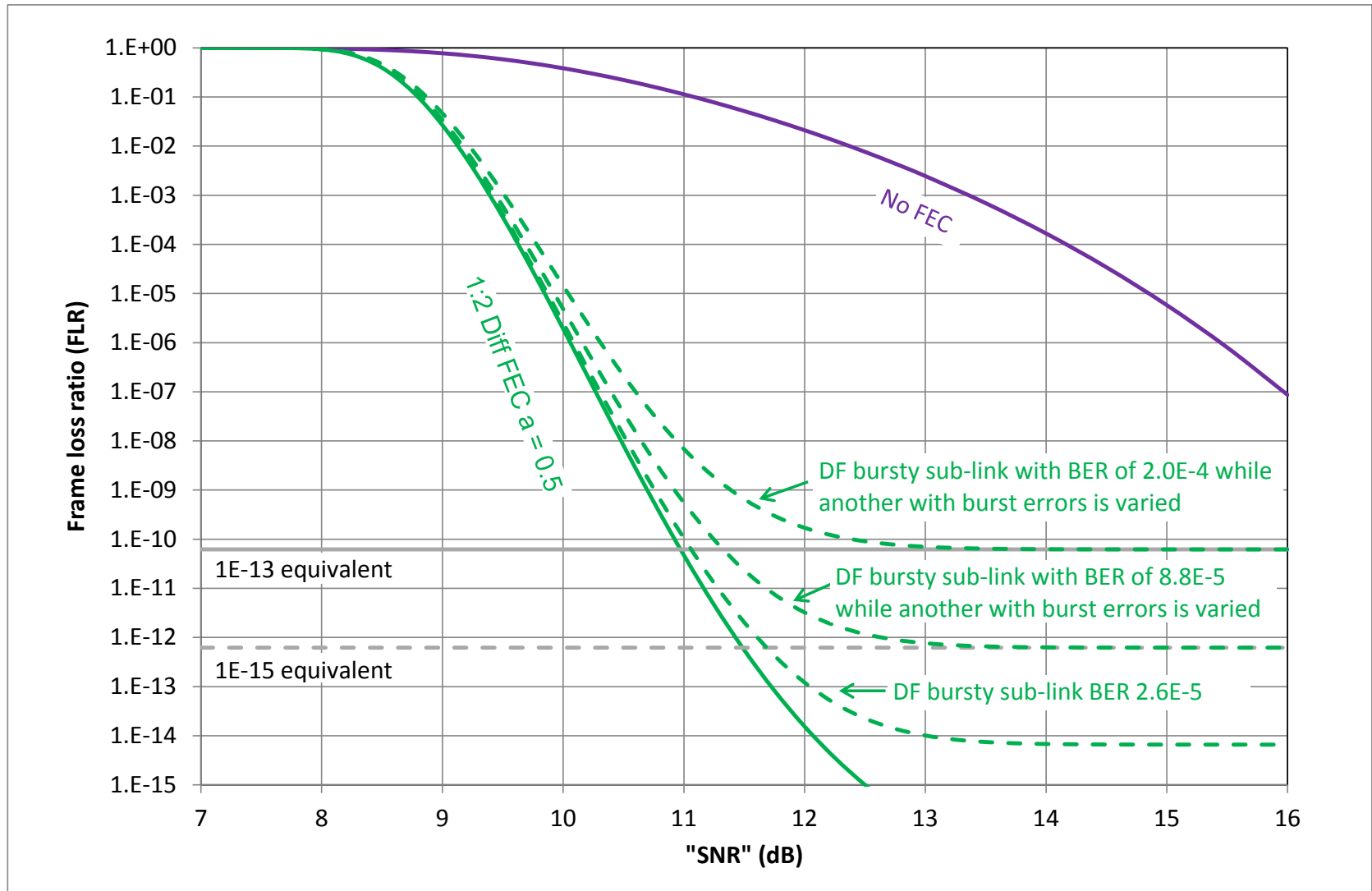
*



BCH(2858,2570) multi-part (1:2 DF bst + random) *



BCH(2858,2570) multi-part (1:2 DF burst + burst) *



Thanks!